Runge-Kutta Methods Comparison

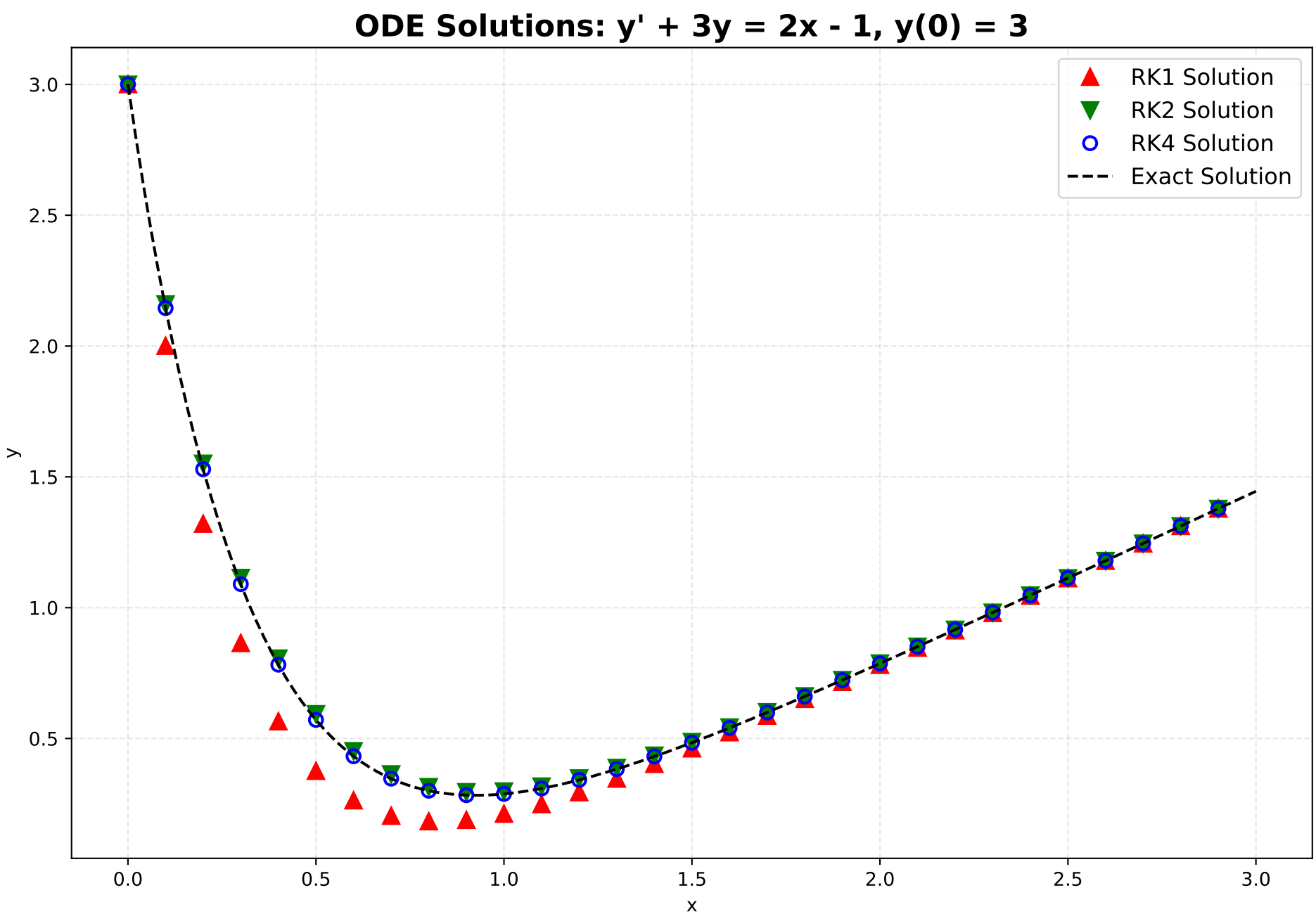
ODE Problem:
y' + 3y = 2x - 1
Also written as: dy/dx = 2x - 1 - 3y
Initial condition: y(0) = 3
Domain: x=∈ [0, 3]
Number of steps: 30
Step size h = 0.1000

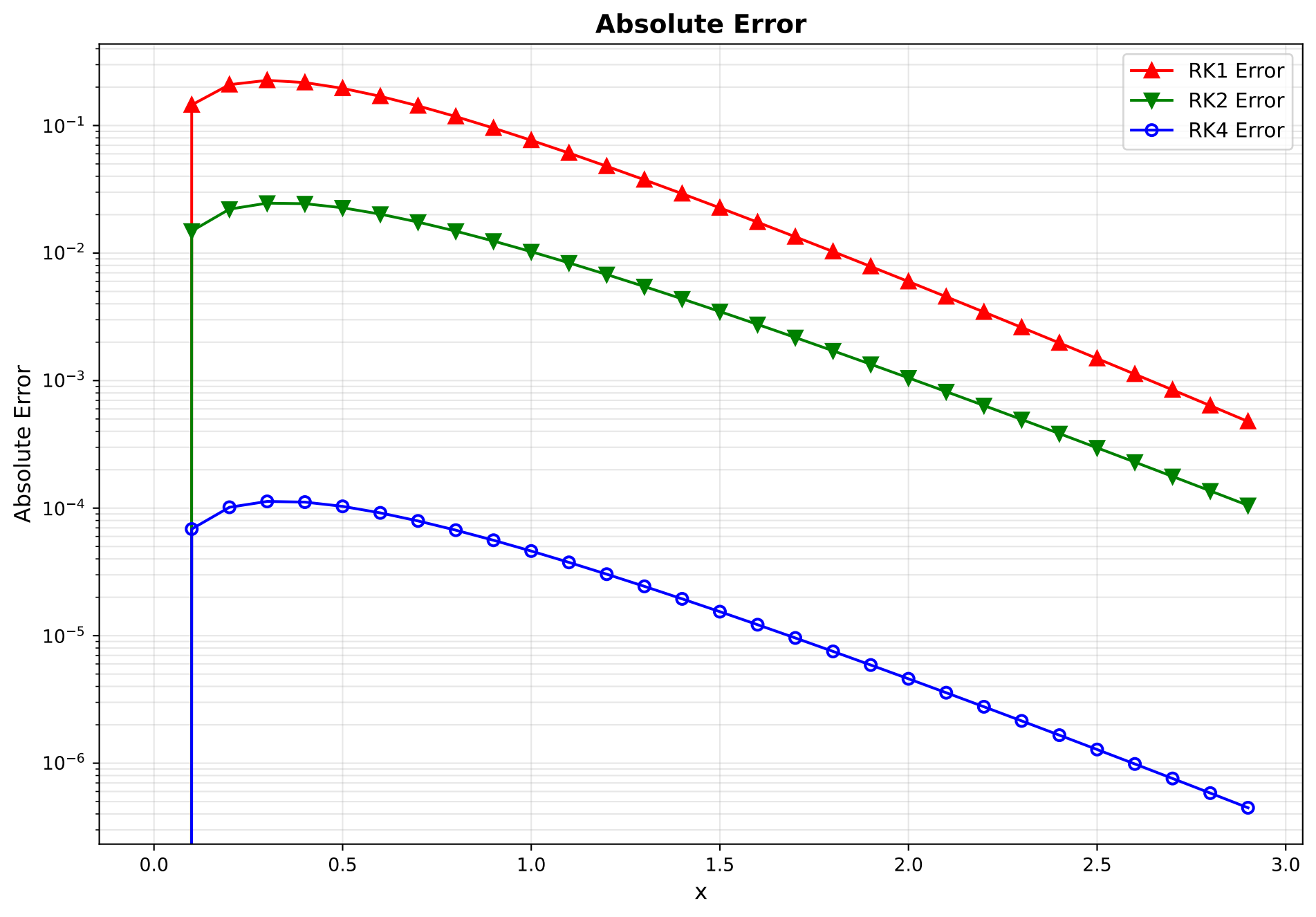
Exact Solution:
y(x) = (2x)/3 - 5/9 + (32/9)·exp(-3x)

Methods:
• RK1: First-order method
• RK2: Second-order method
• RK4: Fourth-order method

Error:
RK4 vs RK1: 1068.5x more accurate
RK4 vs RK2: 234.9x more accurate

Conclusion:
RK4 demonstrates far better accuracy compared to both RK1 and RK2 methods


ODE Solutions: y' + 3y = 2x - 1, y(0) = 3
RK1 Solution
RK2 Solution
RK4 Solution
Exact Solution
y
x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Absolute Error
RK1 Error
RK2 Error
RK4 Error
Absolute Error
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x