

$$(1). \max Z = 10x_1 - 5x_2 + x_3$$

$$A = \begin{pmatrix} 5 & 3 & 1 \\ -5 & 1 & -10 \end{pmatrix} \quad b = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$$

化为标准型:

$$\begin{pmatrix} 5 & 3 & 1 & 0 \\ -5 & 1 & -10 & 1 \end{pmatrix}$$

用大M法

$$\begin{pmatrix} 5 & 3 & 1 & 0 & 1 \\ -5 & 1 & -10 & 1 & 0 \end{pmatrix} \quad \text{且有 } \max Z = 10x_1 - 5x_2 + x_3 - Mx_5$$

$$\text{则有 } x_5 = -5x_1 - 3x_2 - x_3 + 10$$

$$\text{即有 } (10+5M, -5+3M, 1+M, \dots, 10M)$$

求解见下表

| $C_j$ |       |         |         |       |       |       |       |                 |
|-------|-------|---------|---------|-------|-------|-------|-------|-----------------|
| $C_B$ | $X_B$ | $x_1$   | $x_2$   | $x_3$ | $x_4$ | $x_5$ | $b$   | $\theta$        |
| $-M$  | $x_5$ | 5       | 3       | 1     | 0     | 1     | 10    | $2 \rightarrow$ |
| 0     | $x_4$ | -5      | 1       | -10   | 1     | 0     | 15    | $m$             |
| $J_j$ |       | $10+5M$ | $-5+3M$ | $1+M$ | 0     | 0     | $10M$ |                 |
|       | $x_1$ | 1       | 0.6     | 0.2   | 0     |       | 2     |                 |
|       | $x_4$ | 0       | 4       | -9    | 1     |       | 25    |                 |
| $J_j$ |       | 0       | -11     | -1    | 0     |       | -20   |                 |

此时所有非基变量 ( $x_2, x_3, x_5$ ) 检验数均为非零, 故有唯一解  $(2, 0, 0)$ ,  $\max Z = 20$

用两阶段法则有:

$$\min W = x_5$$

| $X_B$ | $x_1$     | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $b$ | $\theta$ |
|-------|-----------|-------|-------|-------|-------|-----|----------|
| $x_5$ | <u>5</u>  | 3     | 1     | 0     | 1     | 10  | 2        |
| $x_4$ | -5        | 1     | -10   | 1     | 0     | 15  | $m$      |
| $J_j$ | <u>-5</u> | -3    | -1    | 0     | 0     | -10 |          |

|             |   |     |     |   |    |
|-------------|---|-----|-----|---|----|
| $x_1$       | 1 | 0.6 | 0.2 | 0 | 2  |
| $x_4$       | 0 | 4   | -4  | 1 | 25 |
| $\lambda_j$ | 0 | 0   | 0   | 0 | 0  |

最优解为  $X = (2, 0, 0, 25, 0)$   $\min W = 0$

则有

|             |       |       |       |       |     |
|-------------|-------|-------|-------|-------|-----|
| $X_i$       | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $b$ |
| $x_1$       | 1     | 0.6   | 0.2   | 0     | 2   |
| $x_4$       | 0     | 4     | -4    | 1     | 25  |
| $\lambda_i$ | 10    | -5    | 1     | 0     | 0   |
| $\lambda_i$ | 0     | -11   | -1    | 0     | -20 |

此时非基变量检验数非0, 故解为  $(2, 0, 0)$ .  $\max Z = 20$

$$(2). \min Z = 5x_1 - 6x_2 - 7x_3$$

$$A = \begin{pmatrix} 1 & 5 & -3 \\ 5 & -6 & 10 \\ 1 & 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 15 \\ 20 \\ 5 \end{pmatrix}$$

化为标准型

$$\begin{array}{cccccc} 1 & 5 & -3 & -1 & 0 & 0 \\ 5 & -6 & 10 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

引入  $x_6, x_7$

$$\max Z = 5x_1 - 6x_2 - 7x_3 - Mx_6 - Mx_7$$

$$\begin{pmatrix} 1 & 5 & -3 & -1 & 0 & 1 & 0 \\ 5 & -6 & 10 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ 20 \\ 5 \end{pmatrix}$$

$$x_6 = -x_1 - 5x_2 + 3x_3 + x_4 + 15$$

$$x_7 = -x_1 - x_2 - x_3 + 5$$

即  $\lambda$  为  $(5+2M, -6+6M, -7+2M, -M, 0, 0, 0.20M)$

| $X_i$    | $x_1$             | $x_2$        | $x_3$             | $x_4$                  | $x_5$ | $x_6$ | $x_7$ | $b$             | $\theta$ |
|----------|-------------------|--------------|-------------------|------------------------|-------|-------|-------|-----------------|----------|
| $x_6$    | 1                 | 5↑           | -3                | -1                     | 0     | 1     | 0     | 15              | 3        |
| $x_5$    | 5                 | -6           | 10                | 0                      | 1     | 0     | 0     | 20              |          |
| $x_7$    | 1                 | 1            | 1                 | 0                      | 0     | 0     | 1     | 5               | 5        |
| $\Delta$ | 5+2M              | <u>-6+6M</u> | -7+2M             | -M                     | 0     | 0     | 0     | 20M             |          |
| $x_2$    | 0.2               | 1            | -0.6              | -0.2                   | 0     |       | 0     | 3               |          |
| $x_5$    | 6.2               | 0            | 6.4               | -1.2                   | 1     |       | 0     | 38              |          |
| $x_7$    | 0.8               | 0            | 1.6               | 0.2                    | 0     |       | 1     | 2               |          |
| $\Delta$ | 6.2+0.8M          | 0            | <u>-10.6+5.6M</u> | 1.2M+2                 | 0     |       | 0     | 38M-18          |          |
| $x_2$    | $\frac{25}{32}$   | 1            | 0                 | $-\frac{1}{8}$         |       |       | 0     | $\frac{5}{2}$   |          |
| $x_3$    | $\frac{31}{32}$   | 0            | 1                 | $-\frac{3}{16}$        |       |       | 0     | $\frac{95}{16}$ |          |
| $x_7$    | $-\frac{3}{4}$    | 0            | 0                 | $\frac{1}{2}$          |       |       | 1     | -7.5            |          |
| $\Delta$ | 5M- $\frac{7}{4}$ | 0            | 0                 | 2.25M- $\frac{51}{16}$ |       |       | 0     |                 |          |

出错

不动

用两阶段法则有

$$\min w = x_6 + x_7$$

|          | $x_1$ | $x_2$     | $x_3$       | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $b$ |
|----------|-------|-----------|-------------|-------|-------|-------|-------|-----|
| $x_6$    | 1     | 5↑        | -3          | -1    | 0     | 1     | 0     | 15  |
| $x_5$    | 5     | -6        | 10          | 0     | 1     | 0     | 0     | 20  |
| $x_7$    | 1     | 1         | 1           | 0     | 0     | 0     | 1     | 5   |
| $\Delta$ | -2    | <u>-6</u> | -2          | 1     | 0     | 0     | 0     | -20 |
| $x_2$    | 0.2   | 1         | -0.6        | -0.2  | 0     |       | 0     | 3   |
| $x_5$    | 6.2   | 0         | 6.4         | -1.2  | 1     |       | 0     | 38  |
| $x_7$    | 0.8   | 0         | 1.6         | 0.2   | 0     |       | 1     | 2   |
| $\Delta$ | -0.8  | 0         | <u>-1.6</u> | -0.2  | 0     |       | 0     | -2  |

|       |     |   |   |                |   |                |
|-------|-----|---|---|----------------|---|----------------|
| $x_2$ | 0.5 | 1 | 0 | $-\frac{1}{8}$ | 0 | $\frac{15}{4}$ |
| $x_5$ | -3  | 0 | 0 | -2             | 1 | 30             |
| $x_3$ | 0.5 | 0 | 1 | $\frac{1}{8}$  | 0 | $\frac{5}{4}$  |
| $b$   | 0   | 0 | 0 | 0              | 0 | 0              |

则有

|     |   |    |    |   |   |   |
|-----|---|----|----|---|---|---|
| $b$ | 5 | -6 | -7 | 0 | 0 | 0 |
|-----|---|----|----|---|---|---|

|       |     |   |   |                |   |                |
|-------|-----|---|---|----------------|---|----------------|
| $x_2$ | 0.5 | 1 | 0 | $-\frac{1}{8}$ | 0 | $\frac{15}{4}$ |
| $x_5$ | -3  | 0 | 0 | -2             | 1 | 30             |
| $x_3$ | 0.5 | 0 | 1 | $\frac{1}{8}$  | 0 | $\frac{5}{4}$  |

$\frac{45}{2} + \frac{35}{4} = \frac{125}{4}$   
 最优解为  $(0, \frac{15}{4}, \frac{5}{4})$      $\max z = -\frac{125}{4}$