STAT 621 Chapter 1

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1 Counting

Rule 1: multiplication rule: Consider a task that is finished in n successive steps. Suppose there are k_1 ways to do step 1, k_2 ways to do step 2,..., and k_n ways to do step n. Then there are $k_1 k_2 \cdots k_n$ possible ways to finish this task.

Rule 2: full permutation: $n!$.

Example Derive Rule 2 from Rule 1.

Based on these basic rules, other useful patterns can be derived, including

- Permutation: $P_k^n = n(n-1)\cdots(n-k+1)$
- Combination: $C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

  Interpretation 1: Choose k out of n distinguishable objects, how many choices are there? $(P_k^n / k!)$

  Interpretation 2: There are n balls consisting of k red ones and $n-k$ blue ones, where same-colored balls are indistinguishable. How many ways are there to arrange all n balls in a row?

  Interpretation 3: How many ways are there to divide n people into two groups of sizes k and $n-k$?
- Multiple combination: $\begin{bmatrix} n \\ n_i \end{bmatrix}$.

  Interpretation 1: analogous to 'interpretation 2' above.

  Interpretation 2: analogous to 'interpretation 3' above.

Binomial theorem and binomial coefficient

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Multinomial coefficient:

$$(x_1 + \cdots + x_k)^n = \sum_{n_1,\ldots,n_k} \begin{bmatrix} n \\ n_i \end{bmatrix} x_1^{n_1} \cdots x_k^{n_k}$$

where the summation is over all valid combinations of the nonnegative integers $n_1, \ldots, n_k$ that satisfy $\sum_{i=1}^{k} n_i = n$.

2 Probability concepts

trial
experiment

sample space
a point in the sample space

event

probability of an event (the "limit relative frequency" interpretation)

conditional probability

independent events

3 Random variables

Probability function: pmf and pdf

Distribution function: cdf

Quantiles: definition; visualization; how to find

Example Ex 1, p. 34.

Expectation (mean, location)

Variance (spread, scale)

Definition: $\text{var}(X) = E\big[\big(X - E(x)\big)^2\big]$

Property: $\text{var}(X) = E(X^2) - \big(E(X)\big)^2$

Covariance

Definition: $\text{cov}(X, Y) = E\big[\big(X - E(x)\big)\big(Y - E(Y)\big)\big]$

Property: $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$

Correlation coefficient

independence $\Rightarrow$ cov is 0 $\Rightarrow$ corr is 0

Mean and variance of the sum of random variables

4 Useful distributions

4.1 Bernoulli

p

Example Find mean and variance.

4.2 Binomial

n, p

- Assumptions on the experiment.
- Relation with Bernoulli.

Example Find mean and variance. [Hint: use relation with Bernoulli.]

4.3 Normal

μ, σ

Master the following:

- Remember its pdf formula; recognize μ and σ in a normal pdf.
- Conversion between standard and non-standard normals.
- Find a specified quantile of standard normal, i.e. given p, find z_p.
- Find a specified quantile of a nonstandard normal, i.e. given p, find x_p.
- Given z, find tail area, i.e. $F(z)$ or $1 - F(z)$.
- Given x, find tail area, i.e. $F(x)$ or $1 - F(x)$.
- Table look-up.

Example Ex 3–5, p. 55–57.

4.4 Chi-squared

k

- Remember it's non-neg.
- Know its relation with standard normal.
- Given p, find x_p.
- Given x, find tail area $F(x)$ or $1 - F(x)$.

4.5 Central Limit Theorem

Approximating binomial and χ^2 by normal. Understand the reason via CLT.

Example Ex 6, p. 58.

Example Ex 8–9, p. 60–61.

5 Other useful results

$$1 + 2 + \cdots + N =$$

$$1^2 + 2^2 + \cdots + N^2 =$$