上机作业 4

1. 计算方程 $x^{41}+x^3+1=0$ 的全部根。

根据习题 6.10，将其化为求矩阵的特征值问题，结果如下：

```
-0.9525 + 0.0000i
-0.9681 + 0.1209i
-0.9681 - 0.1209i
-0.9563 + 0.2738i
-0.9563 - 0.2738i
-0.9105 + 0.4255i
-0.9105 - 0.4255i
-0.8369 + 0.5678i
-0.8369 - 0.5678i
-0.7391 + 0.6959i
-0.7391 - 0.6959i
-0.6207 + 0.8059i
-0.6207 - 0.8059i
-0.4853 + 0.8945i   0.9337 + 0.3925i
-0.4853 - 0.8945i   0.9337 - 0.3925i
-0.3370 + 0.9592i   0.8552 + 0.5326i
-0.3370 - 0.9592i   0.8552 - 0.5326i
-0.1802 + 0.9980i   0.7537 + 0.6554i
-0.1802 - 0.9980i   0.7537 - 0.6554i
-0.0197 + 1.0094i   0.2898 + 0.9464i
-0.0197 - 1.0094i   0.2898 - 0.9464i
 0.1392 + 0.9925i   0.6323 + 0.7534i
 0.1392 - 0.9925i   0.6323 - 0.7534i
 1.0143 + 0.0809i   0.4172 + 0.8711i
 1.0143 - 0.0809i   0.4172 - 0.8711i
 0.9872 + 0.2404i   0.5076 + 0.8106i
 0.9872 - 0.2404i   0.5076 - 0.8106i
```

2. 求 A 的特征值，x 取 0.9 1.0 1.1，结果分别如下：

```
17.4397 + 0.0000i
 6.8195 + 0.0000i
 2.8704 + 0.6429i
 2.8704 - 0.6429i

17.4765 + 0.0000i
 2.8682 - 0.6887i
 2.8682 + 0.6887i
 6.7871 + 0.0000i

17.5131 + 0.0000i
 2.8655 - 0.7321i
 2.8655 + 0.7321i
 6.7559 + 0.0000i
```

3. Matlab 代码：

6 个函数：

```
function [v,b] = house(x)
n=length(x);
y=max(x);
x=x/y;
s=x(2:n)'*x(2:n);
v=ones(n,1);
v(2:n)=x(2:n);
if s==0
    b=0;
else
    a=sqrt(x(1)^2+s);
    if x(1)<=0
        v(1)=x(1)-a;
    else
        v(1)=-s/(x(1)+a);
    end
    b=2*v(1)^2/(s+v(1)^2);
    v=v/v(1);
end
% H=eye(n)-b*(v*v');
end

function [Q,H] = Hessenberg(A)
%Hessenberg 分解
H=A;
[~,n]=size(H);
```

```
for k=1:n-2
    [v,b]=house(H(k+1:n,k));
    H1=eye(n-k)-b*v*v';
    H2=eye(n);
    for  i=k+1:n
        for  j=k+1:n
            H2(i,j)=H1(i-k,j-k);
        end
    end
    if k==1
        Q=H2;
    else
        Q=Q*H2;
    end
    H(k+1:n,k:n)=H1*H(k+1:n,k:n);
    H(1:n,k+1:n)= H(1:n,k+1:n)*H1;
end
end

function [Q,H]=QR_Iteration(A)
%QR 迭代
H=A;
[~,n]=size(H);
a=n-2;
m=n-1;
s=H(m,m)+H(n,n);
t=H(m,m)*H(n,n)-H(m,n)*H(n,m);
x=H(1,1)*H(1,1)+H(1,2)*H(2,1)-s* H(1,1)+t;
y=H(2,1)*(H(1,1)+H(2,2)-s);
z=H(2,1)*H(3,2);
Q=eye(n);
for k=0:n-3
    Pk=eye(n);
    [v,b]=house([x,y,z]');
    q=max(1,k);
    H(k+1:k+3,q:n)=(eye(3)-b*v*v')* H(k+1:k+3,q:n);
    pk=eye(3)-b*v*v';
    Pk(k+1:k+3,k+1:k+3)=pk(1:3,1:3);
    if k<a+1
        Q=Q*Pk;
    end
    r=min(k+4,n);
    H(1:r,k+1:k+3)= H(1:r,k+1:k+3)*(eye(3)-b*v*v');
```

```
    x=H(k+2,k+1);
    y=H(k+3,k+1);
    if  k<n-3
        z=H(k+4,k+1);
    end
end
[v,b]=house([x,y]');
Pk=eye(n);
pk=eye(2)-b*v*v';
Pk(n-1:n,n-1:n)=pk(1:2,1:2);
if n-2<a+1
    Q=Q*Pk;
end
H(n-1:n,n-2:n)=(eye(2)-b*v*v')*H(n-1:n,n-2:n);
H(1:n,n-1:n)=H(1:n,n-1:n)*(eye(2)-b*v*v');

function [x]=Ifschur(A)
[~,n]=size(A);
y=zeros(1,n-1);
x=1;
for i=1:n-1
    y(i)=A(i+1,i);
end
m=0;
for i=1:n-1
    if abs(y(i)-0)<1e-5
        m=m+1;
    end
end
if m==0
    x=1;
else
    z=zeros(1,m);
    j=1;
    i=1;
    while(i<n)
        if abs(y(i)-0)<1e-5
            z(j)=i;
            j=j+1;
        end
        i=i+1;
    end
    i=1;
    while(i<m)
```

```
        if z(i+1)-z(i)>2
            x=1;
            break;
        end
        i=i+1;
    end
    if i>=m
        x=0;
    end
end
end

function [r]=EigValue(A)
[~,n]=size(A);
r=zeros(1,n);
y=zeros(1,n-1);
for i=1:n-1
    y(i)=A(i+1,i);
end
m=0;
for i=1:n-1
    if abs(y(i)-0)<1e-5
        m=m+1;
    end
end
if m==0
    x=1;
else
    z=zeros(1,m);
    j=1; i=1;
    while(i<n)
    if abs(y(i)-0)<1e-5
        z(j)=i;
        j=j+1;
    end
    i=i+1;
    end
end
if z(1)==2
    p=[1,A(1,1)+A(2,2),A(1,1)*A(2,2)-A(1,2)*A(2,1)];
    r(1:2)=roots(p);
    j=1;
    while j<m
        if z(j+1)-z(j)==1
```

```
            r(z(j+1))=A(z(j+1),z(j+1));
        end
        if(z(j+1)-z(j)==2)

p=[1,-(A(z(j+1)-1,z(j+1)-1)+A(z(j+1),z(j+1))),A(z(j+1)
-1,z(j+1)-1)*A(z(j+1),z(j+1))-A(z(j+1)-1,z(j+1))*A(z(
j+1),z(j+1)-1)];
            r((z(j+1)-1):z(j+1))=roots(p);
        end
        j=j+1;
    end
    if n-z(m)==1
        r(n)=A(n,n);
    else

p=[1,-(A(n-1,n-1)+A(n,n)),A(n-1,n-1)*A(n,n)-A(n-1,n)*
A(n,n-1)];
        r(n-1:n)=roots(p);
    end
else
    r(1)=A(1,1);
    j=1;
    while j<m
        if z(j+1)-z(j)==1
            r(z(j+1))=A(z(j+1),z(j+1));
        end
        if(z(j+1)-z(j)==2)

p=[1,-(A(z(j+1)-1,z(j+1)-1)+A(z(j+1),z(j+1))),A(z(j+1)
-1,z(j+1)-1)*A(z(j+1),z(j+1))-A(z(j+1)-1,z(j+1))*A(z(
j+1),z(j+1)-1)];
            r((z(j+1)-1):z(j+1))=roots(p);
        end
        j=j+1;
    end
    if n-z(m)==1
        r(n)=A(n,n);
    else

p=[1,-(A(n-1,n-1)+A(n,n)),A(n-1,n-1)*A(n,n)-A(n-1,n)*
A(n,n-1)];
        r(n-1:n)=roots(p);
    end
end
```

```
function [r]=Implicit_QR(A)
[~,n]=size(A);
[~,H] = Hessenberg(A);
u=10e-5;
for  i=2:n
    if  abs( H(i,i-1))<=(abs(H(i,i))+
abs(H(i-1,i-1)))*u
        H(i,i-1)=0;
    end
end
H22=H;
x=Ifschur(H22);
while x==1
    [~,H22]=QR_Iteration(H22);
    for  i=2:n
        if  abs( H22(i,i-1))<=(abs(H22(i,i))+
abs(H22(i-1,i-1)))*u
            H22(i,i-1)=0;
        end
    end
    x=Ifschur(H22);
end
[r]=EigValue(H22);
r=r';
end
```

主程序：

```
A=zeros(41,41);
A(1,38)=-1;A(1,41)=-1;
for i=2:41
    A(i,i-1)=1;
end
% [r1]=Implicit_QR(A);
disp(eig(A));

x=0.9;
B=[9.1 3.0 2.6 4.0;4.2 5.3 4.7 1.6;3.2 1.7 9.4 x;6.1 4.9
3.5 6.2];
[r2]=Implicit_QR(B);
% disp(r2);
disp(eig(B))
x=1.0;
B=[9.1 3.0 2.6 4.0;4.2 5.3 4.7 1.6;3.2 1.7 9.4 x;6.1 4.9
```

```
3.5 6.2];
[r3]=Implicit_QR(B);
disp(r3);
% disp(eig(B));
x=1.1;
B=[9.1 3.0 2.6 4.0;4.2 5.3 4.7 1.6;3.2 1.7 9.4 x;6.1 4.9
3.5 6.2];
[r4]=Implicit_QR(B);
disp(r4);
% disp(eig(B));
```