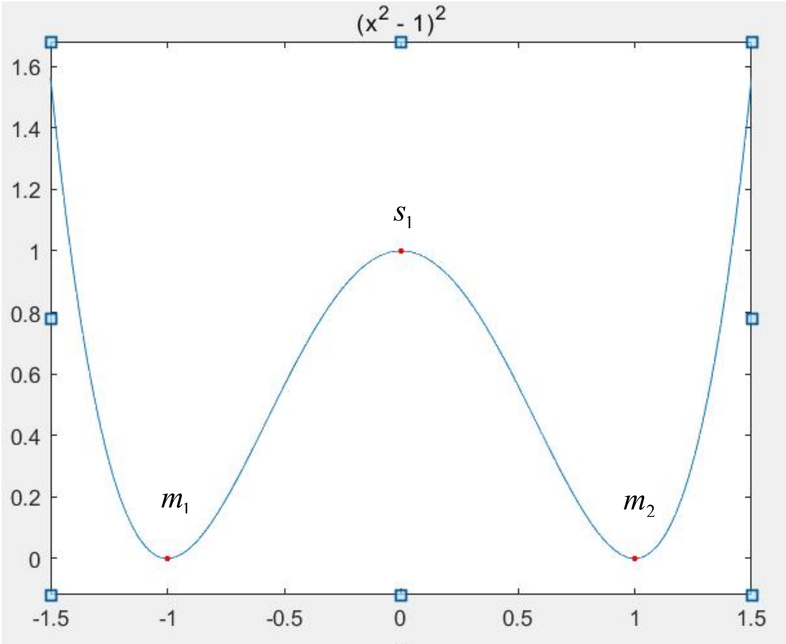
Empirical studies

Here we choose the objective function $f(x)=(x^2-1)^2$



For an initial point $w_0 \in B_1$, we consider the **Thm 1 :**

First, $P(w^{\varepsilon}(T^1(\varepsilon)) \in B_2) \to q_{12}q_1^{-1}$, and we can calculate that

$$q_{12}q_1^{-1} = \frac{\left|1-\dfrac{1}{|s_2-m_1|^{\alpha}}\right|}{1+\dfrac{1}{|s_0-m_1|^{\alpha}}} = 1$$

when we let $s_0 \to -\infty$ and $s_2 \to +\infty$.

We can also observe that there are only 2 basins, so the transition must be $B_1 \to B_2$.

Second, $P(\varepsilon^{\alpha}T^1(\varepsilon) \geq u) \leq e^{-q_1 u}$ for any u ≥ 0, it means $\varepsilon^{\alpha}T^1_{w_0}(\varepsilon) \xrightarrow{d} \exp(q_1)$

so we only need to verify

$$\varepsilon^{\alpha}E_{w_0}(T^i(\varepsilon)) \to q_1^{-1}$$

here $q_1 = q_{12} = 1$.

SDE : $$dw_t^{\varepsilon} = -\nabla f(w_t^{\varepsilon})dt + \varepsilon dL_t^{\alpha}$$

Discrete : $$w_{t+\Delta t} = w_t - 4w_t(w_t^2 - 1)\Delta t + \varepsilon S_{\Delta t}^{\alpha}$$

here $S_{\Delta t}^{\alpha} \sim S\alpha S(\Delta t^{1/\alpha})$, $w_0 \in B_1 := \{x : |x+1| < \delta\}$.