An approximation algorithm for the minimum soft capacitated disk multi-coverage problem

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Abstract. In this paper, we study the minimum soft capacitated disk multi-coverage problem, given a set V of n points and a set S of m sensors on the plane, each sensor s has the same integer capacity U, and each point v has an integer demand d_v. Each sensor s regulates its power p_s to form a circular coverage area $disk(s, r_s)$ (a disk with s as the center and r_s as the radius), and the relationship between the power p_s and its coverage radius r_s is: $p_s = c \cdot r_s^\alpha$, where $c > 0$, $\alpha \geq 1$ are constants. The minimum soft capacitated disk multi-coverage problem is to find a set of disks supported by the power $\{p_s\}_{s \in S}$ such that each disk $disk(s, r_s)$ (chosen once) the number of points allocated at most U, each point v is allocated at least d_v times, the goal is to minimize the number of selected disks. We obtain a 4-approximation of this problem based on an LP rounding algorithm.

Keywords: Soft capacity · Multi-coverage · LP rounding · Approximation algorithm.

1 Introduction

Capacity coverage problem has been studied by a large number of scholars, and the set cover problem with capacity is a very classic capacity coverage problem. The set cover problem is that, given a set of elements E, and a set of sets $\mathcal{S}$ defined on the elements. The minimum set cover problem is to find a subset $\mathcal{S}' \subseteq \mathcal{S}$ such that each element $e \in E$ is covered by a set $S \in \mathcal{S}'$ contain it and minimize the number of the sets of selected subsets $\mathcal{S}'$. The set cover problem with capacity is a generalization of the set cover problem. Each set S is assigned an integer capacity U_S, and each set S covers elements that do not exceed its capacity.

Capacity constraints are further divided into hard capacity constraints and soft capacity constraints. The set cover problem with hard capacity constraints is that each set $S \in \mathcal{S}$ is selected at most $m(S)$ times, and allocate at most $U_S m(S)$ elements of each set S. Wolsey [17] uses a greedy algorithm to obtain an $O(logn)$-approximation for the set cover problem with hard capacity constraints. This approximation is currently the best approximation for this problem. The set cover problem of the soft capacity constraint is that each set $S \in \mathcal{S}$ can be allowed

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to open an infinite number of copies. The vertex cover problem with capacity constraints and the facility location problem with capacity constraints have also been extensively studied. More related studies on soft capacity limitation are [3, 4, 7, 8, 13, 14].

The coverage problem in geometric space is rarely studied. Bandyapadhyay et al. [1] first studied this problem, and designed an LP rounding algorithm for the metric set cover problem with hard capacity constraints, and obtained a two-standard approximation of $(21, 9)$ to this problem, which allows each selected ball to be expanded by a factor of 9. And the special case of the same capacity can get a two-standard approximation of $(21, 6.47)$, each ball only needs to be expanded by a factor of 6.47. For the Euclidean capacity coverage problem in R^d space, Bandyapadhyay et al. [1] devised a plane subdivision technique and a $O(\epsilon^{-4d}log(\frac{1}{\epsilon}), 1+\epsilon)$-approximation is proposed, which allows the ball to be expanded by an arbitrarily small constant. Later, Bandyapadhyay [2] improved the results on the metric capacity coverage problem, obtaining an $O(1)$-approximation, but only needing to expand by a factor of 5 each ball, and for the special case of the same capacity, each ball only needs to expand 4.24 times.

The power cover problem is a variant of the set cover problem. Given a set V of n points and a set S of m sensors on the plane, each sensor $s \in S$ adjusts its power p_s and its sensing radius r_s to satisfy the relationship: $p_s = c \cdot r_s^\alpha$, where $c > 0$ and $\alpha \geq 0$ are constants. The minimum power cover problem is to find an power assignment such that all points $v \in V$ are covered and minimize the total power of the sensor $s \in S$. Li et al. [9] studied the minimum power partial cover problem, requiring at least k points to be covered, and designed a primal dual algorithm to obtain a 3^α-approximation. Dai et al. [6] proposed an $O(\alpha)$-approximation for this problem in the plane. The minimum power partial cover problem with penalty is a generalization of the minimum power partial cover problem, which requires that uncovered points will be given a penalty. Liu et al. [12] obtained a 3^α approximation based on the primal dual algorithm. And for the minimum power cover problem with submodular penalty, Liu et al. [11] proposed a $(3^\alpha + 1)$-approximation. A polynomial time approximation scheme (PTAS) is proposed for the case when the penalty is linear. If each point v has a cover requirement, the minimum power multiple coverage problem is posed, which requires finding a power assignment such that each point v is at least covered by its coverage requirement. When all points and sensors are on a line, Liang et al. [10] proposed an optimal algorithm based on dynamic programming when the maximum covering requirement of points is upper bounded by a constant. Ran et al. [15] designed a PTAS for the minimum power partial multiple coverage problem.

The minimum power coverage problem with capacity constraints is a difficult problem to solve. In this paper, we first study the minimum soft capacitated disk multi-coverage problem, where the disk is generated by power. The main structure of this paper is as follows, the second part introduces the basic knowledge related to minimum soft capacitated disk multi-coverage problem; the third part introduces the linear programming of this problem; the fourth part introduces an

LP rounding algorithm and related theoretical proofs for this problem. Finally, the fifth part summarizes the problem.

2 Preliminary

On the plane, given a set of n points V, a set of m sensors S. Each sensor s has the same integer capacity of U, and each point v has an integer requirement of d_v. Each sensor s can adjust its power p_s, the coverage of any sensor s with power p_s is a circular area with radius r_s, and the relationship between p_s and r_s is :

$$p_s = c \cdot r_s^{\alpha}$$

Where $c > 0$, $\alpha \geq 1$ are constants. Use $disk(s, r_s)$ to represent the circular coverage area with center s and radius r_s, any point $v \in disk(s, r_s)$ indicates that point v can be covered by sensor s.

In this paper, we consider the minimum soft capacitated disk multi-coverage (MSCDM) problem. The problem is to find a set of disks supported by the power $\{p_s\}_{s \in S}$ such that each disk (choose once) $disk(s, r_s)$ the number of points allocated at most U, each point v is assigned at least d_v times, the goal of minimizing the number of selected disks.

In the optimal solution of the MSCDM problem, for each disk in the optimal solution, there will be at least one point on its boundary, otherwise the radius of the disk can be reduced to cover the same point. Therefore, each sensor has at most n disks of different radius, and all sensors have at most mn disks to be considered. Next, we use D to denote such a disk, $\mathcal{D}$ to denote the set of all disks. For any $D \in \mathcal{D}$, we use $V(D)$ to represent the set of points covered by the disk D. For any $\mathcal{D}' \subseteq \mathcal{D}$, we use $V(\mathcal{D}')$ to represent the point set covered by the union of all disks in $\mathcal{D}'$, p_D represents the power of the disk D, r_D represents the radius of the disk D, and c_D represents the center of the disk D. In addition, since each sensor s has the same integer capacity U, all the disks D of different radius produced by each sensor s have the capacity U. Here, we assume that the power of all sensors is unlimited.

So we can also define the MSCDM problem as follows, given a set of points V and a set of disks $\mathcal{D}$ on the plane, each disk D has a power p_D and an identical integer capacity U, each point v has an integer demand d_v. The MSCDM problem is to find an allocation scheme such that the number of points where each disk D is selected once allocated does not exceed its capacity U, each point $v \in V$ is allocated at least d_v times, and the total number of selected disks is minimized.

3 Linear Programming for the MSCDM Problem

First, we define that for any $v \in V$ and $D \in \mathcal{D}$, if the point $v \in V(D)$ is assigned to the disk D, $x_{vD} = 1$; if the disk $D \in \mathcal{D}$ is selected, $y_D = 1$. The integer linear programming of the MSCDM is given as follows:

$$\begin{aligned} \min & \sum_{D:D\in\mathcal{D}} y_D \\ s.t.\ & x_{vD} \le y_D,\ \forall v \in V(D),\ \forall D \in \mathcal{D}, \\ & \sum_{v:v\in D} x_{vD} \le y_D \cdot U,\ \forall D \in \mathcal{D}, \\ & \sum_{D:D\in\mathcal{D}} x_{vD} \ge d_v,\ \forall v \in V(D), \\ & x_{vD} \in N_0,\ \forall v \in V,\ \forall D \in \mathcal{D}, \\ & y_D \in N_0,\ \forall D \in \mathcal{D}, \end{aligned} \tag{1}$$

where the first constraint indicates that the point $v \in V(D)$ is assigned to the disk $D \in \mathcal{D}$, and the disk D must be selected; The second constraint indicates that the number of points allocated to the disk D does not exceed its capacity; The third constraint indicates that the point $v \in V(D)$ is assigned at least d_v times; Relax constraints $x_{vD} \in N_0$, $y_D \in N_0$ to $x_{vD} \ge 0$ and $y_D \ge 0$, the corresponding relaxed linear programming is as follows:

$$\begin{aligned} \min & \sum_{D:D\in\mathcal{D}} y_D \\ s.t.\ & x_{vD} \le y_D,\ \forall v \in V(D),\ \forall D \in \mathcal{D}, \\ & \sum_{v:v\in D} x_{vD} \le y_D \cdot U,\ \forall D \in \mathcal{D}, \\ & \sum_{D:D\in\mathcal{D}} x_{vD} \ge d_v,\ \forall v \in V(D), \\ & x_{vD} \ge 0,\ \forall v \in V,\ \forall D \in \mathcal{D}, \\ & y_D \ge 0,\ \forall D \in \mathcal{D}, \end{aligned} \tag{2}$$

Lemma 1. *Let (x, y) be a feasible solution of program (2), where y is an integer, x may be a fraction, We can find a feasible solution (z, y) with the same feasible multi-coverage in polynomial time, where z, y are both integers.*

Proof. Let $\mathcal{C}$ be a feasible multi-coverage for the program (2) and $f(\mathcal{C})$ be the maximum value of the number of points covered by $\mathcal{C}$. For each disk $D \in \mathcal{C}$, let $m(D)$ denote the number of times the disk D appears in $\mathcal{C}$. We build a directed associative graph $G = (L, V, E)$, in which a copy of each disk in $\mathcal{C}$ is used as a vertex of the vertex set L of the graph G, that is,

$$L = \{v_i(D) | D \in \mathcal{C}, 1 \le i \le m(D)\},$$

For each vertex $v_i(D) \in L$, $v \in V(D)$, there is an edge $(v_i(D), v) \in E$ with a capacity of 1. Add a source point s, and for each vertex $v_i(D) \in L$, $1 \le i \le m(D)$, add an edge $(s, v_i(D))$ with an integer capacity of U. Add another sink point t, and for each $v \in V$, add an edge (v, t) of an integer capacity d_v.

Consider the maximum flow of this network, since the maximum flow of this network is an integer, we can get another solution that contains the same disk. For the MSCDM problem, $\mathcal{C}$ is a feasible solution iff

$$f(\mathcal{C}) = \sum_{v \in V} d_v,$$

Since all capacities are integers, integer feasible solutions (z, y) can be found, that is, a capacity assignment scheme can be obtained.

4 An LP Rounding Algorithm for the MSCDM Problem

In this section, we give the optimal solution of the linear programming (2) (x^*, y^*), and use an LP rounding algorithm to get a feasible solution $(\bar{x}, \bar{y})$, where $\bar{y}$ is an integer and $\bar{x}$ may be a fraction. Define two basic concepts, let the heavy disk set be

$$\mathcal{D}_H = \{D \in \mathcal{D} | y_D > \frac{1}{2}\},$$

The light disk set is

$$\mathcal{D}_L = \{D \in \mathcal{D} | 0 < y_D \leq \frac{1}{2}\}.$$

The detailed steps of our algorithm are as follows. First, we initialize $(\bar{x}, \bar{y})$ to (x^*, y^*), for each point $v \in V$, while $\sum_{D \in \mathcal{D}_L, \bar{x}_{vD} > 0} \bar{y}_D$ $> \frac{1}{2} d_v$, find a subset of light disks $\mathcal{D}_{L_v} \subseteq \mathcal{D}_L$ that covers the point v satisfying

$$\sum_{D \in \mathcal{D}_{L_v}, \bar{x}_{vD} > 0} \bar{y}_D > \frac{1}{2} d_v.$$

We select a disk with the largest radius D_{max} in $\mathcal{D}_{L_v}$ (if there is more than one disk with the largest radius, choose one of them), set $\bar{y}_{D_{max}} := \sum_{D \in \mathcal{D}_{L_v}} \bar{y}_D$. Let $\mathcal{D}_{LH}$ be the set of disks that become heavier, add D_{max} to the set of disks $\mathcal{D}_{LH}$, and set $\bar{y}_{D_{max}} := \lceil \bar{y}_{D_{max}} \rceil$ (disk D_{max} becomes a heavy disk). Perform the above operation for each point $v \in V$.

Then, expand the radius of each disk $D \in \mathcal{D}_{LH}$ by three times, let $\mathcal{D}^3_{LH}$ represent such a set of disks. For any point $v \in V(\mathcal{D}_{L_v})$, let D^3_{max} be the disk with the radius of D_{max} expanded by three times, and set $\bar{x}_{vD^3_{max}} := \bar{x}_{vD_{max}} + \sum_{D \in \mathcal{D}_{L_v} \setminus \{D_{max}\}} \bar{x}_{vD}$. For each disk $D \in \mathcal{D}_{L_v} \setminus \{D_{max}\}$, set $\bar{x}_{vD} := 0$ and $\bar{y}_D := 0$. For the disk D^3_{max}, according to the triangle inequality, it must cover all points in $\mathcal{D}_{L_v}$. And it has enough capacity to satisfy all points in $\mathcal{D}_{L_v}$, because

$$\sum_{v \in V(\mathcal{D}_{L_v})} \bar{x}_{vD^3_{max}} = \sum_{D \in \mathcal{D}_{L_v}} \sum_{v \in D} \bar{x}_{vD} \leq \sum_{D \in \mathcal{D}_{L_v}} \bar{y}_D U = \bar{y}_{D_{max}} U.$$

Finally, we get a set of heavier disks $\mathcal{D}^3_{LH}$ with a three times larger radius and a set of initially heavy disks $\mathcal{D}_H$. Let the set of all heavy disks after the LP rounding be $\bar{\mathcal{D}}_H$. Obviously there is,

$$\bar{\mathcal{D}}_H = \mathcal{D}^3_{LH} \cup \mathcal{D}_H.$$

For any point $v \in V$, it is obvious that

$$\sum_{D \in \mathcal{D}_{L_v}} \bar{x}_{vD} \leq \sum_{D \in \mathcal{D}_{L_v}, \bar{x}_{vD} > 0} \bar{y}_D \leq \frac{1}{2} d_v,$$

where $\mathcal{D}_{L_v}$ is the set of all light disks covering the point v after the LP rounding.

In order to ensure that each point v is assigned at least d_v times, So each point v is assigned to the disk $D \in \bar{\mathcal{D}}_H$ at least $\frac{1}{2} d_v$ times. That is, for any point $v \in V$, satisfy

$$\sum_{D: D \in \bar{\mathcal{D}}_H} \bar{x}_{vD} \geq \frac{1}{2} d_v.$$

that is, to satisfy

$$d_v \leq 2 \sum_{D: D \in \bar{\mathcal{D}}_H} \bar{x}_{vD} \leq 2 \sum_{D: D \in \bar{\mathcal{D}}_H} \lceil \bar{x}_{vD} \rceil \leq 2 \sum_{D: D \in \bar{\mathcal{D}}_H} \lceil \bar{y}_D \rceil$$

So for each disk $D \in \bar{\mathcal{D}}_H$, each heavy disk needs to be selected $2\lceil \bar{y}_D \rceil$ times. That is, according to the LP rounding algorithm we find a feasible solution $(\bar{x}, \bar{y})$, where $\bar{y}$ is integral. Our algorithm pseudocode is shown in Algorithm 1.

Theorem 1. *The algorithm 1 computes a feasible solution $(\bar{x}, \bar{y})$ to the program (2), where $\bar{y}$ is integral.*

According to the algorithm 1, we can find that for each disk $D \in \bar{\mathcal{D}}_H$, the value of rounding y_D is an integer, so we can apply lemma 1 to get an integral capacitated cover where $(\bar{x}, \bar{y})$ are both integers. For each disk $D \in \mathcal{D}_{LH}$, its cost is expanded by a constant factor. We can derive the following lemma 2 and theorem 2.

Lemma 2. $\sum_{D \in \mathcal{D}_{LH}} \bar{y}_D \leq 2 \sum_{D: D \in \mathcal{D}_L} y_D^*$.

Proof. Let $(\hat{x}, \hat{y})$ be a feasible solution of (2), since for each point $v \in V$ and each $D \in \mathcal{D}_{LH}$, $\hat{y}_D > \frac{1}{2} d_v$, $\bar{y}_D := \lceil \hat{y}_D \rceil$. If $d_v = 1$, there is $\bar{y}_D := 1$, then

$$\sum_{D: D \in \mathcal{D}_{LH}} \bar{y}_D = \sum_{D \in \mathcal{D}_{LH}} 1 \leq \sum_{D \in \mathcal{D}_{LH}} 2\hat{y}_D \leq 2 \sum_{D \in \mathcal{D}_L} y_D^*$$

Algorithm 1: ROUNDING

Input: A point set V; a disk set $\mathcal{D}$; an integer capacity U; a demand function $d : V \to N_0$;

Output: $\bar{\mathcal{D}}_H$.

Initial, set $(\bar{x}, \bar{y}) := (x^*, y^*)$, $\mathcal{D}_{LH} := \phi$, $\mathcal{D}^3_{LH} := \phi$, $\bar{\mathcal{D}}_H := \phi$.
for $v \in V$ **do**
  while $\sum_{D \in \mathcal{D}_L, \bar{x}_{vD} > 0} \bar{y}_D > \frac{1}{2} d_v$ **do**
    Find a subset $\mathcal{D}_{L_v} \subseteq \mathcal{D}_L$ of light disks covering point v such that $\sum_{D \in \mathcal{D}_{L_v}, \bar{x}_{vD} > 0} \bar{y}_D > \frac{1}{2} d_v$.
    for $D \in \mathcal{D}_{L_v}$ **do**
      $D_{max} := \arg\max_{D \in \mathcal{D}_{L_v}} r_D$ and set $\bar{y}_{D_{max}} := \sum_{D : D \in \mathcal{D}_{L_v}} \bar{y}_D$.
      $\mathcal{D}_{LH} := \mathcal{D}_{LH} \cup \{D_{max}\}$.
for $D \in \mathcal{D}_{LH}$ **do**
  Set $\bar{y}_D := \lceil \bar{y}_D \rceil$. ($D$ becomes a heavy disk).
Set $\mathcal{D}^3_{LH} := \{disk(c_D, 3r_D) | D \in \mathcal{D}_{LH}\}$.
for $v \in V(\mathcal{D}_{L_v})$ **do**
  Let $D^3_{max} \in \mathcal{D}^3_{LH}$ be a disk whose radius is 3 times larger than $D_{max} \in \mathcal{D}_{LH}$.
  Set $\bar{x}_{vD^3_{max}} := \bar{x}_{vD_{max}} + \sum_{D \in \mathcal{D}_{L_v} \setminus \{D_{max}\}} \bar{x}_{vD}$.
  for $D \in \mathcal{D}_{L_v} \setminus \{D_{max}\}$ **do**
    for $v \in V(D)$ **do**
      Set $\bar{x}_{vD} := 0$ and $\bar{y}_D := 0$.
Set $\bar{\mathcal{D}}_H = \mathcal{D}^3_{LH} \cup \mathcal{D}_H$.
for $D \in \bar{\mathcal{D}}_H$ **do**
  Select each disk $2\lceil \bar{y}_D \rceil$ times.

Otherwise, we have

$$\begin{aligned}
\sum_{D : D \in \mathcal{D}_{LH}} \bar{y}_D &= \sum_{D \in \mathcal{D}_{LH}} \lceil \hat{y}_D \rceil \\
&\leq \sum_{D \in \mathcal{D}_{LH}} (\hat{y}_D + 1) \\
&= \sum_{D \in \mathcal{D}_{LH}} \hat{y}_D + \sum_{D \in \mathcal{D}_{LH}} 1 \\
&\leq (1 + \frac{2}{d_v}) \sum_{D : D \in \mathcal{D}_{LH}} \hat{y}_D \\
&\leq (1 + \frac{2}{d_{v_{min}}}) \sum_{D : D \in \mathcal{D}_{LH}} \hat{y}_D \\
&\leq (1 + \frac{2}{d_{v_{min}}}) \sum_{D : D \in \mathcal{D}_{L}} y^*_D \\
&\leq 2 \sum_{D : D \in \mathcal{D}_{L}} y^*_D
\end{aligned}$$

where the second inequality is based on $1 < \frac{2\hat{y}_D}{d_v}$, and the last inequality is based on $d_{v_{min}} \geq 2$ and $d_{v_{min}}$ is the minimum coverage requirement for point $v \in V$.

Theorem 2. *Algorithm 1 can get a* 4*-approximation for the MSCDM problem.*

Proof. Because for each $D \in \bar{\mathcal{D}}_H$, we choose every heavy disk $2\lceil \bar{y}_D \rceil$ times. And because for each disk $D \in \mathcal{D}_{LH}$, $\bar{y}_D := \lceil \bar{y}_D \rceil$ and have

$$\bar{\mathcal{D}}_H = \mathcal{D}^3_{LH} \cup \mathcal{D}_H,$$

for all points $v \in V$, then

$$\begin{aligned}
\sum_{D:D\in\bar{\mathcal{D}}_H} 2\lceil \bar{y}_D \rceil &= \sum_{D:D\in\mathcal{D}^3_{LH}} 2\bar{y}_D + \sum_{D:D\in\mathcal{D}_H} 2\lceil \bar{y}_D \rceil \\
&= \sum_{D:D\in\mathcal{D}_{LH}} 2\bar{y}_D + \sum_{D:D\in\mathcal{D}_H} 2\lceil y^*_D \rceil \\
&\leq 4 \sum_{D:D\in\mathcal{D}_L} y^*_D + 4 \sum_{D:D\in\mathcal{D}_H} y^*_D \\
&\leq 4 \sum_{D\in\mathcal{D}} y^*_D \\
&\leq 4OPT
\end{aligned}$$

where the first inequality is based on the lemma 2, the last inequality is based on $\sum_{D:D\in\mathcal{D}} y^*_D \leq OPT$, where OPT is the optimal value of integer programming (1).

5 Conclusion

In this paper, we first study the minimum soft capacitated disk multi-coverage problem, where the disk is generated by power, and a 4-approximation algorithm is designed by using an LP rounding algorithm and the geometric properties of the disk set.

The minimum soft capacitated power cover problem requires us to take power as a cost and be able to combine the geometric properties of the disk. the minimum soft capacitated power multi-coverage problem is a generalization of the minimum soft capacitated power cover problem, this problem has not been studied yet. It is a challenge to obtain a constant approximation like the MSCDM problem.

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