# MHCDMC Rouding

Qinghui Zhang[1], Weidong Li[2], Qian Su[1] Xuejie Zhang[1, *]

1. School of Information Science and Engineering, Yunnan University, Kunming 650504, China

2. School of Mathematics and Statistics, Yunnan University, Kunming 650504, China

October 31, 2022

**Abstract**

abstract

**keywords**: high performance computing; resource allocation; scheduling; approximation algorithm; bag-of-tasks

*Correspondence: xjzhang@ynu.edu.cn (X. Zhang)

# 1 Introduction

With their maneuverability and increasing affordability, unmanned aerial vehicles (UAVs) have many potential applications in wireless communication systems [1]. In particular, UAV-mounted mobile base stations (MBSs) can be deployed to provide wireless connectivity in areas without infrastructure coverage such as battlefields or disaster scenes. Unlike terrestrial base stations (BSs), even those mounted on ground vehicles, UAV-mounted MBSs can be deployed in any location and move along any trajectory constrained only by their aeronautical characteristics, in order to cover the ground terminals (GTs) in a given area based on their known locations.

After a major natural disaster, the ground-based communication facilities are usually destroyed and communication is interrupted, and important communication information is blocked, which endangers the lives of the affected people and aggravates the difficulty of post-disaster rescue. UAVs have wide application prospects in the field of emergency communication because of their advantages such as rapid deployment and the ability to provide effective air-ground line-of-sight links to cover the affected areas by equipping emergency base stations [1].

In order to protect people's life and property and speed up the post-disaster reconstruction and recovery work, we need to provide communication security for users as soon as possible. According to the actual communication demand, some potential optional UAV deployment locations are selected by relying on information such as population distribution and disaster situation. Selecting the minimum number of UAVs to restore the communication network in that area under the constraints of communication needs is a critical issue. Since larger UAVs have larger energy reserves compared to smaller UAVs, larger UAVs can transmit signals with higher power and have greater bandwidth capacity to obtain better signal coverage performance. Therefore, in this paper we assume that the UAV base station with higher signal transmitting power has a larger bandwidth capacity.

# 2 Related work

related work

# 3 System model

In a disaster area, we have pre-planned $m$ loci and possible MBSs to be deployed in. Although deploying MBSs in each location can satisfy the communication needs of all users, it is very inefficient and impractical. We need to select as few loci as possible to deploy the corresponding MBSs as soon as possible to restore communication services for $n$ users. We denote the set of users and MBS loci by $U$ and $A$, respectively. For each user $u_j \in U$, there is a bandwidth requirement $BR_j$. The signal transmit power of MBS $a_i \in A$ is $p_i$, and the bandwidth resource is limited to $BW_i$. In order to resume communication for users as soon as possible while ensuring that the communication signal-to-noise ratio of all users is not less than $SINR_{min}$, we need to select as few deployed MBSs as possible. we introduce decision variables $x_{ij}$ and $y_i$. $y_i$ denotes whether to choose $a_i$ as the final MBS deployment policy, and when $y_i = 1$ indicates that MBS $a_i$ is chosen. $x_{ij}$ is used to indicate whether to make $a_i$ allocate bandwidth resources for $u_j$ to provide services, and when $x_{ij} = 1$ indicates that $u_j$ is served by $a_i$.

The communication between the UAV-enabled MBS and the user uses an air-to-ground communication link in the sub-6 GHz band, where Line of Sight (LoS) dominates. A The path loss between $u_j$ and $a_i$ can be expressed as:

$$L_{ij}(\text{dB}) = 20\lg(d_{ij}) + 20\lg(\frac{4\pi f}{c}) + \eta_{LoS}, \tag{1}$$

where $d_{ij}$ denotes the distance between $a_i$ and $u_j$, $f$ denotes the carrier frequency, $c$ represents the speed of light, and $\eta_{LoS}$ represents the shadow fading loss of LoS, which is a constant. the signal-to-noise ratio between $a_i$ and $u_j$ is:

$$SINR_{ij} = \frac{G_{ij}p_i}{N_I + N_0}, \tag{2}$$

$G_{ij}$ denotes the channel gain between $a_i$ and $u_j$, $N_I$ denotes the interference noise power in this environment, and $N_0$ denotes the white noise power. The channel gain $G_{ij}$ is affected by the path loss and satisfies the following relationship:

$$G_{ij}p_i(\text{dB}) = p_i(\text{dB}) - L_{ij}(\text{dB}). \tag{3}$$

According to the above relationship, $u_j$'s data rate $DR_j$ can be expressed as

$$DR_j = BR_j\log_2(1 + SINR_{ij}). \tag{4}$$

Based on the above definition, we can get the integer programming form of the problem.

$$\min \quad \sum_{a_i \in A} y_i \tag{5}$$

$$s.t. \quad x_{ij} \leq y_i \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A. \tag{5a}$$

$$\sum_{u_j \in U} (x_{ij} \cdot BR_j) \leq y_i \cdot BW_i, \quad \forall a_i \in A. \tag{5b}$$

$$\sum_{a_i \in S} x_{ij} = 1, \qquad\qquad\qquad \forall u_j \in U. \tag{5c}$$

$$x_{ij} = 0, \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A \text{ such that } SINR_{ij} < SINR_{\min}, \tag{5d}$$

$$x_{ij} \in \{0, 1\}, \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A \tag{5e}$$

$$y_i \in \{0, 1\}, \qquad\qquad\qquad \forall a_i \in A. \tag{5f}$$

Constraint (5a) means that $u_j$ can be served by $a_i$ only after the MBS $a_i$ is selected. Constraint (5b) is the bandwidth resource capacity resource constraint of each MBS, the sum of bandwidth demand of users it serves cannot exceed its own capacity. Constraint (5c) of indicates that every user must be served. Constraint (5d) means that if $u_j$ is served by $a_i$, then the SINR of $u_j$ has to be greater than $SINR_{min}$. Constraints (5e) and (5f) are two integer decision variable constraints. We relax the integer programming (5) to be able to obtain its linear programming (6):

$$\min \quad \sum_{a_i \in A} y_i \tag{6}$$

$$s.t. \quad x_{ij} \leq y_i, \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A. \tag{6a}$$

$$\sum_{u_j \in U} (x_{ij} \cdot BR_j) \leq y_i \cdot BW_i, \quad \forall a_i \in A. \tag{6b}$$

$$\sum_{a_i \in S} x_{ij} = 1, \qquad\qquad\qquad \forall u_j \in U. \tag{6c}$$

$$x_{ij} = 0, \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A \text{ such that } SINR_{ij} < SINR_{\min}, \tag{6d}$$

$$x_{ij} \geq 0, \qquad\qquad\qquad \forall u_j \in U, \forall a_i \in A \tag{6e}$$

$$0 \leq y_i \leq 1, \qquad\qquad\qquad \forall a_i \in A. \tag{6f}$$

We can obtain the optimal solution $\sigma = (x, y)$ of LP 6 in polynomial time, which is a fractional solution. In this solution, for any $x_{ij} > 0$ , we can ensure that $SINR_{ij} \geq SINR_{\min}$. In a real scenario, MBS can enhance its own signal coverage through spectrum multiplexing and other methods. We define $SINR'_{\min}(< SINR_{\min})$ as a new minimum SINR after using the

multiplexing technique. When $SINR'_{min} \leq SINR_{ij} \leq SINR_{\min}$, $a_i$ can also provide services with better signal quality for $u_j$ by adopting multiplexing techniques. In addition, MBSS can also enhance the signal coverage by increasing the signal power. Let $p'_i$ be the power after enhancement.

For the two variables $SINR'_{\min}$ and $p'_i$, they have a limited variation. In this paper, we do not focus on how small $SINR'_{\min}$ is or how large $p'_i$ is; we only give the definition of the magnitude of their variable as follows.

$$\varepsilon^p = \max_{a_i \in A} \frac{p_i' - p_i}{p_i}, \tag{7}$$

$$\varepsilon^{SINR} = \frac{SINR_{\min} - SINR'_{\min}}{SINR_{\min}}. \tag{8}$$

# 4 Rounding

In this section, we present an efficient rounding algorithm for the solution $\sigma = (x, y)$ of LP (6). The rounding algorithm consists of three parts, namely "determining superior and inferior servers", "clustering of servers" and "selecting the final servers". Finally, we demonstrate through theoretical analysis that the approximation ratio of the algorithm is () when the SINR and the power are varied by $\varepsilon^p$ and $\varepsilon^{SINR}$, respectively.

## 4.1 Determining Superior and Inferior Servers

The values of $x$ and $y$ are in the range of 0 to 1 in $\sigma = (x, y)$. Given a parameter $0 < \alpha \leq 1/2$, and each server with non-zero $y_i$ can be classified according to the relationship with $\alpha$. When $y_i = 1$, we call ai a superior server and denote their set by $S$. When $0 < yi \leq \alpha$, we call $a_i$ an inferior server and denote their set by $I$.

## 4.2 Clustering of Servers

Clustering of Servers

## 4.3 Selecting the Final Servers

Selecting the Final Servers

# 5    Experimental Results

# 6    Conclusion and future work

We feel that the local assignment algorithm in Section will find application in related areas.

# Acknowledgement

# References

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**Algorithm 1** Determining Superior and Inferior Servers
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**Input:** A fraction solution of LP(6) $\sigma$, a parameter $\alpha$ and the magnitudes of variable of power

and SINR $\varepsilon^p$ and $\varepsilon^{SINR}$.

**Output:** A fraction solution $\overline{\sigma} = (\overline{x}, \overline{y})$ containing only superior and inferior servers.

1:  Initialize $\overline{x} \leftarrow x$, $\overline{y} \leftarrow y$.

2:  $\mathcal{S} \leftarrow \{a_i | \forall a_i \in A \text{ and } \overline{y}_i = 1\}$

3:  $\mathcal{I} \leftarrow \{a_i | \forall a_i \in A \text{ and } 0 < \overline{y}_i \le \alpha\}$.

4:  **for all** $u_j \in U$ **do**

5:      **if** $\sum_{a_i \in \mathcal{I} : \overline{x}_{ij} > 0} \overline{y}_i > \alpha$ **then**

6:          Construct an set $\mathcal{I}_j$ of inferior servers serving $u_j$ such that $\alpha < \sum_{a_i \in \mathcal{I}_j} \overline{y}_i \le 2\alpha$.

7:          $r \leftarrow \max_{a_i \in \mathcal{I}_j} r_i$, $\mathcal{T} \leftarrow \{a_i | r_i \le \varepsilon r/2, \forall a_i \in \mathcal{I}_j\}$.

8:          Divide the set $\mathcal{I}_j \backslash \mathcal{T}$ of servers into $\lceil \log(1/\varepsilon) \rceil$ levels such that each $l$th level server $a_i$

           has $2^{l-1} r\varepsilon < r_i \le 2^l r\varepsilon$, $0 \le l \le \lceil \log(1/\varepsilon) \rceil$.

9:          **for** $l = 0$ to $\lceil \log(1/\varepsilon) \rceil$ **do**

10:             Cut the square centered at $u_j$ with side length $2^{l+2} r\varepsilon$ to grid cells, and each grid cell

               has side length $2^{l-2} r\varepsilon^2$.

11:             For each grid cell that contains at least one server, creating a group $G_l$ consisting of

               all servers contained in the grid cell.

12:             $\mathcal{G}_l \leftarrow$ all groups in the $l$th level.

13:             **for all** $G_l \in \mathcal{G}_l$ **do**

14:                 $a_m \leftarrow \arg\max_{a_i \in G_l} r_i$.

15:                 Rerout the flow from each $a_i \in G_l \backslash \{a_m\}$ to $a_m$.

16:                 $\overline{y}_m \leftarrow 1$; $\overline{y}_i \leftarrow 0, \forall a_i \in G_l \backslash \{a_m\}$.

17:             **end for**

18:         **end for**

19:         Rerout the flow from $\mathcal{T}$ to server has maximum radii in $\mathcal{I}_j$.

20:     **end if**

21: **end for**

22: $\overline{y}_i \leftarrow 1, \forall a_i \in A, \alpha < y_i < 1$.

23: **return** $\overline{\sigma}$.
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**Algorithm 2** Clusting of Servers

**Input:** A fraction solution from Alg.2 $\overline{\sigma}$

**Output:** A fraction solution $\widehat{\sigma} = (\widehat{x}, \widehat{y})$ containing only superior and inferior servers.

1: Initialize $\widehat{x} \leftarrow \overline{x}$, $\widehat{y} \leftarrow \overline{y}$.

2: $\mathcal{S} \leftarrow \{a_i | \forall a_i \in A \text{ and } \widehat{y}_i = 1\}$

3: $\mathcal{I} \leftarrow \{a_i | \forall a_i \in A \text{ and } 0 < \widehat{y}_i \leq \alpha\}$.

4: $\mathcal{O} \leftarrow \emptyset, \mathcal{I}' \leftarrow \mathcal{I}; C_i \leftarrow \emptyset, \forall a_i \in \mathcal{S}$

5: **while** $\mathcal{I}' \neq \emptyset$ **do**

6:     **for all** $a_i \in \mathcal{S}$ **do**

7:         **for all** $a_t \in \mathcal{I}'$ **do**

8:             **if** $a_t$ intersects $a_i$ and $AC_i \geq \sum_{u_j \in U} \widehat{x}_{tj}$ **then**

9:                 Rerout the flow from $a_t$ to $a_i$, and $C_i \leftarrow C_i \cup \{a_t\}$.

10:             **end if**

11:         **end for**

12:     **end for**

13:     **if** $\mathcal{I}' \neq \emptyset$ **then**

14:         $A_i \leftarrow \{u_j | \widehat{x}_{ij} > 0, \forall u_j \in U\}, \forall a_i \in \mathcal{I}'$.

15:         $k_i \leftarrow \min \left\{ BW_i, \sum_{u_j \in A_i} BR_j \right\}$.

16:         $a_t \leftarrow \arg\max_{a_i \in \mathcal{I}'} k_i, \mathcal{O} \leftarrow \mathcal{O} \cup \{a_t\}$.

17:         **if** $k_t \equiv \sum_{u_j \in A_t} BR_t \leq BW_t$ **then**

18:

19:         **end if**

20:         **if** $k_t \equiv BW_t < \sum_{u_j \in A_t} BR_t$ and $k_t \geq 2BR_t^{\min}$ **then**

21:

22:         **end if**

23:         **if** $k_t \equiv BW_t < \sum_{u_j \in A_t} BR_t$ and $BR_t^{\min} \leq k_t < 2BR_t^{\min}$ **then**

24:

25:         **end if**

26:     **end if**

27: **end while**

28: $\overline{y}_i \leftarrow 1, \forall a_i \in A, \alpha < y_i < 1$.

29: **return** $\overline{\sigma}$.