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Book Reviews

Introduction, 727

Featured Review: Crowds in Equations: An Introduction to the Microscopic Modeling of Crowds (Bertrand Maury and Sylvain Faure), *Dante Kalise*, 729

Lectures on the Fourier Transform and Its Applications (Brad Osgood), *Colin Fox*, 731

Numerical Methods for Conservation Laws: From Analysis to Algorithms (Jan Hesthaven), *Christian Klingenberg*, 735

Applications of the Topological Derivative Method (Antonio André Novotny, Jan Sokolowski, and Antoni Zochowski), *Stephan Schmidt*, 737

500 Examples and Problems of Applied Differential Equations (Ravi O. Agarwal, Simon Hodis, and Donald O'Regan), *Volker H. Schulz*, 737

Algorithms for Data Mining and Machine Learning (Xin-She Yang), *Volker H. Schulz*, 739

BOOK REVIEWS

These are difficult times, but some circumstances make the work in our Book Reviews section even more difficult. Due to delivery difficulties during the coronavirus pandemic, some publishers temporarily switched from the delivery of physical books for reviews to the delivery of ebooks. This is an understandable and obvious measure at the moment. However, the Springer-Verlag publishing house has decided, even independently of and before the crisis, to send review copies only as PDF bearing the ugly watermark "Book Review Copy for Personal Use Only" and no longer provides printed copies of the respective books. Many reviewers prefer a physical book and consider it a small compensation for the considerable work they invest in the quality of their reviews. We have already noticed that under these conditions it is much more difficult to attract reviewers to Springer books. So in future we will probably only be able to report on books from this publisher to a lesser extent, even though very good books often come from this publishing house.

Nevertheless, we have been able to compile six informative reviews in this section. It starts with the featured review of Dante Kalise on the book *Crowds in Equations: An Introduction to the Microscopic Modeling of Crowds*, by Bertrand Maury and Sylvain Faure. This review was very diligently written by an expert in dynamical systems and control and praises the book in its summary as an "updated account of the mathematical modeling of crowd motion phenomenass."

Among the other reviews, the book recommendation by Christian Klingenberg, senior expert in conservation laws, stands out. It is on the book *Numerical Methods for Conservation Laws: From Analysis to Algorithms*, by Jan Hesthaven. He recommends the book as one that provides "a comprehensive and up-to-date view of the subject of numerics for conservation laws." Furthermore, the young expert on shape and topology optimization Stephan Schmidt takes a closer look at the book *Applications of the Topological Derivative Method*, by Antonio André Novotny, Jan Sokołowski, and Antoni Żochowski. Stephan recommends it for experts searching for cutting edge research results rather than for novices looking for an introduction into the field. In addition we have three more reviews which provide balanced opinions on the respective books.

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Book Reviews

Edited by Volker H. Schulz

Featured Review: Crowds in Equations: An Introduction to the Microscopic Modeling of Crowds. *By Bertrand Maury and Sylvain Faure.* World Scientific, Hackensack, NJ, 2019. $78.00. x+190 pp., hardcover. Advanced Textbooks in Mathematics. ISBN 978-1-78634-551-6.

This book presents a comprehensive overview of the mathematical methods and challenges arising in the modeling of crowd motion. The authors, two leading experts in the field, present a complete revision of the many mathematical models for crowds, with a focus on microscopic and granular matter models together with an outlook on topics such as macroscopic models and experimental data. The book is driven by the need to generate mathematical models which can effectively account for the phenomena observed in human crowds, including congestion, interaction with obstacles, and rational strategies.

The book is structured into ten chapters. Chapter 1 begins with an introduction describing relevant motivations and mathematical concepts in crowd motion such as active particles, individual decisions, and interaction rules.

Chapter 2 focuses on the analysis of the simplest one-dimensional microscopic model in crowd motion, the so-called follow-the-leader model [1]. Here, a string of $N+1$ one-dimensional pedestrians characterized by their positions

$$x_1(t) < x_2(t) < \cdots < x_{N+1} \tag{1}$$

moves according to the dynamics

$$\frac{dx_i}{dt} = \varphi(x_{i+1} - x_i)\,, \quad 1 \le i \le N\,, \tag{2}$$

where φ assigns a nonnegative speed to any nonnegative distance, such as

$$\varphi(w) := \begin{cases} U(1 - \exp(-(w - w_m)/w_s)) & \text{for } w \ge w_m\,, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where U represents a desired speed, while w_m and w_s are reference distances. Such a model is the starting point for a detailed study of well-posedness, equilibria and stability analysis, wave propagation, and variants accounting for delays and inertia effects. A second take on microscopic models follows in Chapter 3, with the study of the so-called social force model proposed by Helbing and Molnár [2]. A cornerstone of multiagent systems and crow motion modeling, the dynamics governing the social force reads

$$m_i \frac{du_i}{dt} = \frac{m_i}{\tau}(U_i - u_i) + \sum_{i \ne j} f_{ij} + \sum_k f_{ik}^w\,, \tag{4}$$

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describing the evolution of agents with mass m_i, velocity u_i, desired velocity U_i, characteristic time τ, and subject to attraction-repulsion forces f_{ij} from the other individuals and to forces f_{ik}^w from obstacles. This chapter presents a thorough study of this model, including well-posedness, the existence of spurious oscillations, the modeling of a vision cone, overdamping, and gradient flows.

Chapter 4 introduces the modeling of crowd motion phenomena in the framework of granular flows. Drifting away from the previous models in which the interactions between agents are accounted for through potentials, the authors discuss the modeling of pedestrians as hard spheres with a desired velocity which can pack and collide without overlapping. Models of this type read

$$\frac{dq}{dt} = P_{C_q}(U(q)) \quad \text{in } [0,T], \tag{5}$$

where q stands for the positions (centers) of N agents, $U(q)$ the set of desired velocities for a given configuration, and P_{C_q} denotes the Euclidean projection on the closed convex cone C_q of feasible velocities. The mathematics behind these models include the theory of differential inclusions and convex analysis. The authors study several aspects of granular flows: their variational formulation as a saddle-point problem, two-dimensional models, the construction of numerical schemes, convex analysis issues, and a gradient flow framework.

Chapters 5 and 6 explore further models that can be considered within the microscopic realm. Chapter 5 focuses on cellular automata dynamics over two-dimensional lattices, and Chapter 6 presents compartment models in which the dynamics governing the number of people in a given compartment of a building are described by balance equations of the form

$$\frac{dN}{dt} = f - \Phi\,, \tag{6}$$

$$\Phi = \begin{cases} C & \text{if } N > 0\,, \\ \underset{\Phi\in[\tilde{0},C]}{\operatorname{argmin}} \left\|\tilde{\Phi} - f\right\| & \text{if } N = 0\,, \end{cases} \tag{7}$$

representing the inflow and outflow of pedestrians in a room with a given number of doors. Dynamics of this type can be constructed to account for complex layouts (many-compartment models), which the authors study throughout the chapter, along with its numerical implementation and interesting extensions.

Chapter 7 moves on to the multiscale character of crowd motion phenomena, introducing the study of macroscopic models. Instead of describing the position of a set of agents through a system of nonlinear differential equations with one equation of motion per agent, the authors focus on macroscopic models in which the evolution of the density of agents in space and time, $\rho(x,t)$, is governed by a continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0\,, \tag{8}$$

where pedestrians are advected by a velocity field $u = u(x,t)$. Much of the relevant macroscopic modeling for pedestrian flows is based on advection fields which depend on the density, for example, with $u(\rho) = U(1 - \rho/\rho_{max})$, where U and ρ_{max} are reference values for the velocity and the density, respectively. The authors present an exhaustive report of all the different modeling choices for the advection velocity field, including links with the microscopic model, two-dimensional extensions, the Hughes model, and congestion effects. This latter topic is developed in more detail as a macroscopic counterpart of the granular model from Chapter 4.

Chapter 8 presents technical aspects concerning the calculation of shortest paths and optimal velocities over complex layouts including obstacles, exits, and congestion. These calculations are implicit in many of the models presented in previous chapters, such as in the macroscopic Hughes model for pedestrian motion, and relate to dynamic programming and the Eikonal equation. Chapter 9 discusses how to effectively incorporate existing real-world data into the different models discussed throughout the book. Data on reference distances between humans, congestion, cone of vision, and the effect of obstacles, among many others, can be utilized to generate predictive models. In Chapter 10, the authors develop a detailed discussion on the mathematical modeling of characteristic phenomena in crowds such as the *faster-is-slower* effect, the role of obstacles, and *stop-and-go* waves. The book closes with two appendices on ordinary differential equations and constrained optimization, and online materials include the Python library `cromosim` for crowd motion simulation available at http://www.cromosim.fr/.

In summary, this book provides an updated account of the mathematical modeling of crowd motion phenomena. The focus on microscopic models makes it an accessible reference for a wide audience including mathematicians with an interest in agent-based phenomena and practitioners in transport engineering. The material on granular matter models and gradient flows should be of interest to a more specialized audience. The book is also suitable for delivering a short course on crowd motion phenomena.

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Lectures on the Fourier Transform and Its Applications. *By Brad Osgood.* AMS, Providence, RI, 2019. $115. xvi+693 pp., hardcover. ISBN 978-1-4704-4191-3.

This book is derived from lecture notes written for course EE261 on Fourier analysis taught within Electrical Engineering (EE) at Stanford University, covering the classical material on function representation and deterministic systems with a treatment of distribution theory; stochastics are not covered beyond some content on univariate random variables. Lectures from around 2008 can be found online.

Chapter 1 introduces Fourier series, with the transform being treated in later chapters. A footnote on page 1 addresses this sequence:

> Many books for engineers launch right into Fourier transforms. . . . It's a defensible choice, but it's not my choice.

There are good reasons why most EE courses "launch right into" the transform; by the end of a course like this it is much easier to understand series in terms of the transform, whereas moving from series to transforms without the notion of distributional convergence is fraught with technical issues if one wishes to be precise. Indeed, exactly these issues can be found in the present text that utilizes the space $L^2([0,1])$ for functions with period 1, but is vague about other periods and is silent regarding unknown periods. The convergence of series to transforms (on pages 100–103) is a standard demonstration by example that remains vague about the nature of convergence.

From a practicing electrical engineer's viewpoint, these technical issues ultimately stem from periodic functions being an idealization, principally useful for signal synthesis, whereas signals analyzed in the real world are only approximately periodic, at best. No physical system analyzes measured signals to produce the "true" Fourier series. The human ear, for example, effectively analyzes short-time weighted averages with the longest weighting kernel being of a few seconds duration, while most are much shorter, down to a few milliseconds [1]. This has the practical advantage that we humans can respond to sounds in finite time. The same practicalities produce the "line width" displayed by spectrum analyzers. The delta function as a short-time limit of a pulse, and the impulse response of a system, can be a more natural place to start for electrical engineers than periodic functions that bring technical issues related to causality and observability.

Quirky Section Titles and Content. Beyond the chapter titles, section headings become comedic. For example, in Chapter 1 you find subsection 1.3, "It All Adds Up," because the section introduces sums, then subsection 1.3.1, "Lost at c" because c is used to denote coefficients and sounds like "sea." Get the great jokes? Chapter 2 has subsection 2.2.2, "For Whom the Bell Curve Tolls"—that derives the transform of a Gaussian, which is a Gaussian—and later comes section 3.6.7, "The Central Limit Theorem: The Bell Curve Tolls for Thee."

The text is also replete with anecdotes about the author; in subsection 1.3.2, "Fourier Coefficients," we learn that the author's friend Randy Smith plays the trumpet, followed by a few paragraphs on the brand and some constructional details of Randy's trumpet. The index has an entry to "Smith, R.," should you wish to revise this content, but there is no entry for "Smith, P.H."

I watched a few of the online lectures—which seem quite good to me—and get the impression that the author has doubled-down on the anecdotal material and jokey headings for this text.

What My Students Said. After reading the first three chapters or so, I began to wonder who this book is written for. I estimate that in the first three chapters about 50% of the material is discursive or jokey and somewhat sloppy in terms of technical details. Presumably this is what one review on the back cover refers to as "Fourier analysis with a swing in its step" and another as "a light touch on the technicalities." EE261 would imply a 2nd-year undergraduate course at my university, and the book is published in the AMS series of Pure and Applied Undergraduate Texts founded by Paul Sally. So I took this text to my 2nd-year Electronics students to ask their opinions. They flicked through some pages and immediately complained about the use of i as the unit imaginary number, since i is unambiguously (small-signal) current in electronics. They also complained about some section headers; I think one was subsection 4.5.2, "A Fourier Transform Hit Parade, Part 1," saying that they did not like a section heading with "Part 1" when Part 2 did not follow, and indeed they could not find Part 2. (Subsection 4.6.2, "A Fourier Transform Hit Parade, Part 2," does exist, but one cannot find it in the contents nor in the index.)

The author states in his "Declaration of Principles" on page 665:

> Without apology I will write $i = \sqrt{-1}$. In many areas of science and engineering it's common to use j for $\sqrt{-1}$. If you want to use j in your own work, I won't try to talk you out of it. But I'll use i.

If this were simply a matter of notation for $\sqrt{-1}$ I would find no problem with choosing i or j. But to electrical engineers, j is an *operator*—rotation by $\pi/2$. Armed with its name, students can successfully search for "j operator," whereas my search for "i operator" returned nothing useful.

I agree with my students that the informal section titles are annoying. The majority of subheadings do nothing to enhance the book, and also don't do what they are supposed to do, which is to structure the book and provide a clear outline of what the text contains.

Distribution Theory but Not Operator Calculus. Chapter 4, titled "Distributions and Their Fourier Transforms," is the second-longest chapter and in some sense is the most straitlaced. The text opens with

> We've been playing a little fast and loose with the Fourier transform ... but it's also true that we haven't done anything wrong.

I agree with the first statement, but not the second. This chapter walks through what looks like a clean development of tempered distributions, the Fourier transform of generalized functions, and convolution. The material and notation look so familiar that I think I have read essentially identical material in other texts.

In a few places the author extemporizes. In section 4.1.2, "The Path, the Way," one reads:

> "Distribution" is Schwartz's term, and it has nothing to do with probability distributions, etc.

I read this line to a statistician colleague, who immediately replied that Kolmogorov's great step in the 1930s was to identify probability distributions with normalized measures and, of course, measures induce Schwartz distributions. The phrase "has nothing to do with" does not convey "includes" to me.

While the text presents distribution theory, the text does not develop or use the operator calculus that is common in EE. (Search for "Heaviside operator" to see examples.) I think of these methods as akin to metalinguistic abstraction in computer science, where complex problems are solved by creating a new language that is more natural to the problem space.

The absence of operator methods is somewhat baffling as the author makes the connection between Oliver Heaviside (who introduced operator calculus to EE), Paul Dirac, and Laurent Schwartz in a paragraph titled *Let us now praise famous men* on page 235. Schwartz made it clear that he was justifying Heaviside's symbolic calculus for use in EE, and subsequent extensions such as those made by Dirac, in the very first lines of his *Théorie des Distributions* [2]:

> *Il y a plus de* 50 *ans que l'ingénieur Heaviside introduisit ses règles de calcul symbolique, dans un mémoire audacieux où des calculs mathématiques fort peu justifiés sont utilisés pour la solution de problèmes de physique. Ce calcul symbolique, ou opérationnel, n'a cessé de se développer depuis, et sert de base aux études théoriques des électriciens.*

The penalty from not presenting the operator calculus is that all examples, derivations of Fourier transforms, and the following chapters on sampling and systems are not as concise and elegant as they could be. Perhaps this is why the current text runs to 600 pages and weighs in at 1.2 kg (hardcover), while there exist slimmer volumes that cover this material more clearly.

A "Fast and Loose" CLT. Let me present one example of sloppy mathematical detail, among many. In section 3.6.7, on the Central Limit Theorem (CLT)—the one with the subtitle "The Bell Curve Tolls for Thee"—the text "proves," using Fourier transforms, a formal statement of the CLT on page 198 that assumes the random variables are i.i.d. with mean 0 and standard deviation 1 (and nothing else).

The "proof" presents the same steps that follow the first statement of a CLT on page 194 of [3] for i.i.d. random variables, though the proof in [3] also requires that all moments are bounded. The next CLT in Theorem 5.10.5 of [3] loosens up the i.i.d. requirement and states the usual requirement that third moments are bounded—along with a condition on moments required for the non-i.i.d. case.

Armed with this information, or by just reading the "proof" in the current text, it takes a few moments to produce a counterexample to the CLT stated there. The CLT in this text is semitrue and the "proof" has incorrect steps.

Presentation of EE. A nice aspect of this book, for EE students, is that the author uses the Fourier kernel $e^{-2\pi ist}$, which is the standard kernel in EE, notwithstanding the use of i rather than j and the use of s as the transform variable, which is typically used as the transform variable in the Laplace transform in EE and hence is the variable in "complex impedances." There are many other choices of kernel out there, such as $e^{-i\omega t}$, but the kernel $e^{-2\pi ist}$ has the desirable property that if t is in seconds, then s is in Hz (Hertz)—the same units found on spectrum analyzers and frequency generators. The author denotes frequency in Hz by ν (page 3), but does not connect s with ν, other than saying: "If t has dimension time... s must have dimension 1/time." Given that radial frequency ω also has units of 1/time, there is more that could be usefully said.

The author initially experiments with the notation $\hat{f}$ for the Fourier transform of function f, and $\check{g}$ for the inverse transform of g, leading to $\hat{f}^{\check{}}$, which should be $\check{\hat{f}}$, for the inverse transform of $\hat{f}$. I found that the caron could be indistinguishable from a blemish on the paper. Fortunately, the author later mainly utilizes the standard operator notation $\mathcal{F}$ and $\mathcal{F}^{-1}$ for forward and inverse transforms.

The author goes on to say

> The square magnitude $|\hat{f}(s)|^2$ is called the power spectrum... or the spectral power density... or the energy spectrum.

I guess that the author is trying to be helpful with these potential names, but really they are virtually useless as they mix up units of energy and power. Nonobservance of units permeates this text.

I list here just a few other examples that jarred: On page 170 we are told that the phase of complex V is $\tan^{-1}(\frac{\operatorname{Re} V}{\operatorname{Im} V})$, as if the phase of V equals the phase of $-V$. In problem 8.16 concerning a half-wave power supply, the text says

> Ideally the low-pass filter would only pass the DC component.

This implies the less-than-ideal property that the power supply could only produce output at $t = +\infty$.

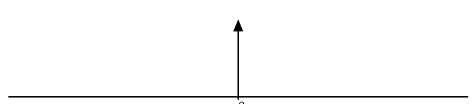
Section 8.8 describes causality, but does not say why the mathematical requirement of causality is tacked on at the end, and not included with linear and shift-invariant as basic properties of systems. The Kramers–Kronig relations are not to be found.

Only Real Functions Are Plotted! During the process of reading this book I realized that there are no plots of complex-valued functions. I presume that I don't need to remind you that the Fourier transform is typically complex valued. In a book with many, many plots of Fourier transforms, somehow the author has contrived to only show real-valued transforms. Actually, I exaggerate; there is a *single* plot showing the imaginary part of the transform in an exercise in Chapter 5, and two other exercises show the phase of the Fourier transform. This book does less than nothing to teach students how to visualize and work with complex-valued functions.

The Height of (Area under) Delta Functions Is Typically Not Shown on Plots. A useful standard is for delta functions to be plotted as an arrow, with the height of the arrow on the vertical axis denoting the *area* under the delta function. After all, 1δ is a different (generalized) function to 2δ, and it is useful to indicate this on plots. The author writes:

> You probably also learned to represent δ graphically as a spike sticking up from 0. The spike is often tagged to have height 1, or strength 1, whatever "height" or "strength" mean for something that's supposed to be ∞ at 0:

and presents the graph



Note the absence of indication of the (unit) height of this spike, indeed the complete absence of any vertical axis. I immediately hear my collective past teachers reminding me that a graph without units, let alone without an axis, is meaningless.

Saying that a delta function is "supposed to be ∞ at 0" is mathematical nonsense. A suitable defining feature is that the δ returns 1 when integrated over any open interval containing 0. To not mark this unit integral as the height of the delta function is a substantial disservice to readers. For example, on page 373 the Fourier transform of $\cos(9\pi t/2)$ is shown as two spikes with unmarked heights. I don't know how one is supposed to differentiate this diagram from the transforms of $2\cos(9\pi t/2)$ or $20\cos(9\pi t/2)$.

Many fabulous texts, and courses, derive formulas for the DFT, FFT, and Nyquist's sampling theorem, as well as the implications of finite-time measurement, exactly, succinctly, and intuitively by using accurately scaled and annotated graphical methods. This text does not support any of these desirable methods or teaching outcomes.

In Summary. After reading this book I have learned that the author worked in the dean's office, he plays the trombone, his friend Randy plays the trumpet, the author does not know why orchestras tune to an oboe playing an A, and many more anecdotes. Honestly, I don't want to know any of these things.

There is solid and well-written material in this text that can also be found in other texts. However, this text mixes that material in equal measure with material that you would need to avoid or correct.

A colleague from Stanford confirmed that EE261 is a graduate-level course. I cannot imagine that this text (and course) is intended for graduate students in EE who might progress; at least, it would be a great disservice to impose this text on such students.

I recommend that you look elsewhere for a text on Fourier transforms.

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Numerical Methods for Conservation Laws: From Analysis to Algorithms. *By Jan Hesthaven.* SIAM, Philadelphia, 2018. $89.00. xvi+555 pp., softcover. ISBN 978-1-611975-09-3. https://doi.org/10.1137/1.9781611975109.

The book *Numerical Methods for Conservation Laws* by Jan Hesthaven deals with hyperbolic conservation laws,

$$U_t + \nabla \cdot F(U) = G(U),$$

where F and G are given functions of the unknown function $U = U(t, x)$ with $t \in R^+$ and $x \in R^d$, where $d = 1, 2,$ or 3.

Classical examples of these equations are

- the Euler equations of compressible, inviscid gas dynamics (possibly with a gravitational source term G);
- the equations of ideal magnetohydrodynamics.

The subject of numerical methods for these partial differential equations was propelled into life by von Neumann's endeavor during the Manhattan project in the early 1940s, and ten years later later by Godunov, when he wanted to compute the flow around a reentry space vehicle in order to transport humans into space and back.

In contrast to computing elliptic partial differential equations, where numerics (finite elements) could mimic the theoretical development (Hilbert spaces) that came before, the same was not possible for hyperbolic conservation laws. Glimm (in 1965) had been able to prove convergence of a variant of Godunov's numerical method, providing a proof of existence in one space dimension, but nothing of that sort has yet been possible in two or three space dimensions. A multidimensional existence theory even for the compressible Euler equations still eludes us today. This means that the subject of the numerics of conservation laws had to take off on its own, and it did so from the late 1970s onward.

Thus, it makes sense that in those heady times after 1980, when new numerical methods for conservation laws were being developed, textbooks on the subject of numerics were published. In the 1990s there were books by Godlewski and Raviart [3], [4], Randy LeVeque [7], [8], and Dietmar Kröner [6] on finite volume methods, and in the first decade of the 21st century there came a book by Hesthaven and Warburton [5] on discontinuous Galerkin methods.

It should be noted that numerical methods of conservation laws were and still are quite sought after by practitioners, who are constantly on the prowl for ever better methods for their particular applications. Some of their legacy codes are based on the philosophy of these books, and Randy LeVeque's two books in particular have been quite influential. If they wanted something more recent than the publication of these textbooks, they had to contend with review articles (like [9]) or simply read the newest developments in original articles.

Thus, it is high time for a new book. The book at hand by Jan Hesthaven is highly welcome, especially since it attempts to give a comprehensive overview of the subject up to the present day and thus fills a gap. It differs from previous textbooks on numerical conservation laws in covering a broader range of numerical methods. In particular, it discusses

- finite difference methods,
- finite volume methods,
- high order finite volume methods,
- discontinuous Galerkin methods,
- spectral methods.

Possibly the author had the excellent treatises of LeVeque in mind, because there is emphasis in Hesthaven's book on aspects that are not covered in LeVeque's books. An example is the thorough treatment given to higher order methods, an area that has been developed extensively in the 21st century, and thus was not so well developed at the time Randy LeVeque wrote his books. Hesthaven's book spends one third of its almost 600 pages on a thorough treatment of high order finite volume methods.

The emphasis of this book is on numerical methods rather than numerical analysis, so should be quite useful for practitioners. The book also has classroom use in mind, albeit at an advanced graduate level. The many MATLAB routines it contains will come in quite handy there.

That this text is deeply rooted in the achievements of the past literature can be gauged from its emphasis on problems in one space dimension. The extension to two or three dimensions can in some cases be achieved in a straightforward way, however, there are also new things yet to be discovered in computing multidimensional compressible flow. Notions like vorticity do not exist in one space dimension, and thus warrant multidimensional schemes. Examples of multidimensional schemes are Phil Roe's active flux scheme [2] or residual distribution schemes [1], but these are outside the scope of this book.

In summary, this book is a welcome addition to the literature, providing a comprehensive and up-to-date view of the subject of numerics for conservation laws.

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Applications of the Topological Derivative Method. *By Antonio André Novotny, Jan Sokołowski, and Antoni Żochowski.* Springer, Cham, 2019. $159.99. xiv+212 pp., hardcover. ISBN 978-3-030-05431-1.

The book combines a theoretical overview of topology optimization with solution procedures for a variety of applications. The theoretical part focuses on singularly perturbed domains and introduces asymptotic expansion and the Steklov–Poincaré operator as important tools. It is worth mentioning that, although the adjoint approach is described as beneficial to topology optimization several times with more convenient expressions arising, the main goal of the book seems to be the derivation of a closed form expression for the topological derivative via asymptotic analysis, in particular, of the governing PDE.

To this end, the first half of the book develops the foundations for topology optimization based on asymptotic expansion and singularly perturbed domains. The second half, beginning with Chapter 6, is based around several different application problems and their respective topological derivatives. A welcome exception is the Navier–Stokes problem discussed in section 6.3.2 and the accompanying paper [1], where the Lebesgue differentiation theorem coupled to a Darcy permeability law gives rise to a topological asymptotic expansion without the usual asymptotic analysis of the PDE solution, while still being based on singularly perturbed domains and not a material density approach. Thus, problems with gray scale densities are prevented. The book concludes with Newton-type methods for topology optimization.

To make the most out of the book, I would recommend previous experience with asymptotic analysis. Indeed, the character of the book is closer to a presentation of cutting edge research and each chapter is more like a research paper with less textbook character, giving each chapter a high degree of independence. Indeed, to utilize each chapter to the fullest, I can only recommend also following the respective accompanying papers. By doing so, the book will provide a well-rounded impression of the current state of the art of topology optimization for a variety of applications.

REFERENCE

[1] L. F. N. Sá, R. C. R. Amigo, A. A. Novotny, and E. C. N. Silva, *Topological derivatives applied to fluid flow channel design optimization problems*, Struct. Multidisc. Optim., 54 (2016), pp. 249–264.

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500 Examples and Problems of Applied Differential Equations. *By Ravi O. Agarwal, Simon Hodis, and Donald O'Regan.* Springer, Cham, 2019. $51.99. xiv+432 pp., hardcover. ISBN 978-3-030-26383-6.

As a mathematician active in the vast field of differential equations, one always strives to enrich one's lectures with practical examples to keep students engaged. Of course, these examples should illustrate certain effects discussed in the lecture to increase motivation. Whether or not the book under consideration here actually proves to enrich the teaching depends strongly on the direct style of the lecture: analytic, numerical, with modeling emphasis, etc.

Ideally this book should complement lectures on the theory and analytical solution of ordinary and partial differential equations. According to the preface it is also intended as a "supplement to some of" the authors' "previous books published by Springer-Verlag," which probably means the books [1, 2]. The book contains a lot of examples—500 is the right order of magnitude, though I have not counted them exactly. From the point of view of the analytical treatment of differential equations, there is probably everything you could wish for: Chapters 1 to 3 bring examples of first-, second-, and higher-order ODEs; Chapter 4 deals with examples treated with power series approaches; Chapter 5 deals with examples of first-order ODE systems; Chapter 6 mentions the classical Runge–Kutta method and uses it to give analytically nontreatable ODE numerical solutions; Chapter 7 gives an introduction to classical stability theory for ODEs and discusses stability properties in several examples; Chapter 8 deals with linear boundary value problems (BVPs) and examples, where the transition from BVPs for ODEs to BVPs for PDEs is fluent; Chapter 9 on nonlinear BVPs deals mainly with ODE BVPs or those that can be easily reformulated into ODE BVPs.

Each of the nine chapters is divided into three parts: in the first part selected theoretical principles are explained, some of which are taken from the books [1, 2]. In the second part, elaborated examples are presented, some of which are briefly set into a historical context. In the third part, tasks (problems) are formulated that encourage independent reflection and application of the concepts presented in the first part. In order to give an impression of the style, I will pick out a few representative examples.

In Example 1.9 from the first chapter, Newton's linear cooling law is first formulated as an equation and then applied to analytically determine the time of death of a corpse. In the related Problem 1.7 the question of the drying time of a wet t-shirt is formulated as a task. In view of the current societal situation when writing this review (March 2020), it is of course particularly interesting that in Examples 5.11 and 5.12 the SIR model for the spread of infectious diseases is examined analytically. This is one of the few examples in the book where the conceptual ideas leading to the mathematical model are discussed. Toward the end of the book, however, fewer and fewer modeling ideas are communicated. Example 9.3 (Circular Membrane Theory) begins as follows: "The equation for a circular membrane subjected to a normal uniform pressure can be reduced to (see [three references])

$$y'' + \frac{k}{y^2} + \frac{3}{x}y' = 0\,,\ 0 < x < 1."$$

A discussion of the solution theory of the equation follows. This structure, an equation (sometimes linked to a historical name) falls from the sky, is very often found in this book. For a book that aims to look beyond the mathematical horizon toward applications, this seems overly academic.

Chapter 6 on Runge–Kutta methods is somewhat unfortunate. Only the classical Runge–Kutta method with an arbitrary constant step size is introduced and used. An insight into the well-known limitations of this explicit 4th-order Runge–Kutta method and the necessity of step size control is not provided. Not a single relevant work on numerics of ODEs is cited that could be helpful to the interested reader, although there is an abundance of books like [4, 5, 3].

If one overlooks the fact that in very many examples one has to work out the modeling context oneself and that numerics plays almost no role at all in this book, then one can find here an abundance of ideas for possible applications of the analytical treatment of differential equations.

As usage scenarios I can imagine the following examples:

(a) I want to deal with ODEs of higher order in my lecture and need some motivating examples, so I flip through Chapter 3 of the book.

(b) I think of the keyword "cycloids" and look for a context. I find two pages in the subject index that match it.

In summary, the book provides an excellent collection of ideas to spice up a lecture on differential equations with an analytical approach and thus to increase the motivation of students.

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Algorithms for Data Mining and Machine Learning. *By Xin-She Yang.* Academic Press, London, 2019. $74.95. xii+173 pp., softcover. ISBN 978-0-128172-16-2.

A scientific book always represents the solution of a multicriterion optimization problem. On the one hand, a scientific area should be presented as widely as possible, but on the other hand, the greatest possible depth and accuracy should be achieved. In addition, the publisher also imposes a page limit, e.g., 170 pages. The book *Algorithms for Data Mining and Machine Learning* by Xin-She Yang has put the main emphasis on breath while satisfying the page limit.

This book actually covers all algorithms I have ever read and heard about in the context of data science, making it an almost encyclopedic work of 170 pages. This encyclopedic breadth in very limited space comes of course at the expense of depth, accuracy, and, unfortunately, also comprehensibility.

In Chapter 1 the book provides an outline of gradient-based optimization from ground zero to the KKT conditions in 18 pages. Mathematical basics like convexity, computational complexity, norms, probability distributions, Monte Carlo sampling, entropy, etc., are discussed in Chapter 2 in a further 23 pages. In the following 20 pages, Chapter 3 provides various optimization algorithms, with biology inspired algorithms taking up a large part. Chapter 4 covers in 23 pages basic algorithms for linear and nonlinear data fitting. Logistic regression, PCA, and the like are discussed in the next 17 pages in Chapter 5. Chapter 6 discusses basic algorithms for data mining in almost 20 pages. Support Vector Machines fill the next 10 pages of Chapter 7. Twenty pages on artificial neural networks in Chapter 8 conclude the book. Specific properties of the algorithms are barely compared with each other, rather the algorithms are put next to each other in a laundry list fashion, more or less like a list of available options to be chosen in the many software packages mentioned.

The style of the book is most comparable to a lecture. However, it does not contain any exercises. As a key feature of the book, the informal, theorem-free approach is advertised on the back cover, which in itself probably lowers the hurdle to access for many reader groups. However, "informal" here also means that many terms remain blurred if one is not already familiar with them. I had hoped to learn concepts from the book that I had only known as buzzwords before: Metropolis–Hastings algorithm and random forest algorithm. I actually learned them while reading the book, but only with strong additional support from Google.

Furthermore, as a mathematician, it bothers me when matrices are in the denominator of a fraction as in several places in the book. Because of its ambiguity we generally prohibit this sloppy substitute of a well-ordered product with the inverse matrix from the first semester on.

How can you use the book? First of all, it is an excellent tool to get a rough but encyclopedic overview of numerous algorithms in data science. In addition, it can be used to find an algorithm labeled with a keyword in association with other related algorithms via the index. For the beginner in the field of algorithms for data science, however, this book is less suitable.

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