EDUCATION

This issue of *SIAM Review* contains two papers in the Education section.

The first paper is "Case for First Courses on Finite Markov Chain Modeling to Include Sojourn Time Cycle Chart," authored by Samuel Awoniyi and Ira Wheaton.

Markov chains are among the most fundamental and popular forms of stochastic dynamical systems. In this article, the authors assume that the reader is familiar with the basic notions in the theory of Markov chains and focus on sojourn time cycle charts (briefly, STC charts) by discussing the role, applications, and methods of computation which could be used in an introductory class on Markov chains. An STC is associated with a specified subset of states and represents the sum of two expectations (averages): the time the Markov chain will stay, or sojourn, inside that set of states and the time the chain will be outside of it. An STC chart is a chart displaying a finite number of such STCs.

The presentation is limited to Markov chains with finite state space, which allows for a simplified explanation of the computational method. Multiple examples lead the reader through the derivation of the balance equation, computation of the relevant sojourn times, and construction and interpretation of the charts. The examples include both discrete-time and continuous-time processes. The authors comment on the role and consequences of the ergodicity assumptions versus the uniqueness of the solution to the balance equation for computing sojourn times.

In conclusion the authors express their belief that the topic should be covered when Markov chains are first introduced to students in order to encourage further exploration of Markov chain modeling and computation.

The second paper, "Multivariate Polynomial Interpolation in Newton Forms," is written by Richard D. Neidinger.

Approximation is a powerful tool facilitating both theoretical analysis and numerical methods. Frequently, we replace a nonlinear function, whose evaluation is expensive, by an approximation. A classical and natural approach is to use polynomials. We interpolate the function by constructing a polynomial matching its values at some distinct nodes in its domain. Constructive algorithms build a suitable polynomial basis, corresponding to the nodes, and produce coefficients for the interpolating polynomial, given specific function values. While univariate interpolation procedures are well understood and designed to ensure nice approximation properties, the multivariate case is much more involved and poses many theoretical and numerical challenges.

This article presents extensions of univariate techniques for Newton interpolating polynomials to the case of multivariate data points. While the ideas of those techniques have been around for many years, they have received their modern form and implementation with the recent advances in scientific computing and with the increase of computing power.

The article starts with a short section on univariate Newton interpolation establishing notation and terminology. It proceeds to discuss multivariate interpolation on a grid. Introducing the notion of multi-index, corresponding to a node in the n-dimensional space, a classical Newton polynomial is defined. The interpolating polynomial is a linear combination of classical Newton polynomials, each corresponding to a multi-index node and the function value associated with it. A generalization of the divided-difference formula and the associated algorithm are presented. Additionally, the author provides a short proof of the existence and uniqueness of the interpolating polynomial for any subset of multi-indices with corresponding points from a grid. A two-dimensional example illustrates the interpolation process. In the second half of the paper, arbitrary nodes are considered. The author discusses how a Newton-like basis, up through degree d, can be determined for an arbitrary set of $\binom{n+d}{n}$ nodes in the n-dimensional space. Some preliminary knowledge in multivariate interpolation is expected from the reader. However, the algorithms are accessible after an introductory course in numerical analysis.

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