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Book Reviews

BOOK REVIEWS

This time, the Book Reviews section has a large portion of books devoted to system dynamics and applications. The section starts with the featured review by Jie Sun on the book *Synchronization: From Coupled Systems to Complex Networks* by Stefano Boccaletti, Alexander N. Pisarchik, Charo I. Del Genio, and Andreas Amann. The review informs in detail on the content of the book and its relation to the current literature on the subject. One can learn that such a thing as spontaneous symmetry breaking is a mathematical possibility in coupled identical systems, which from many perspectives arouses great interest in this book. System dynamics is also the topic of the book *Hidden Dynamics: The Mathematics of Switches, Decisions and Other Discontinuous Behaviour* by Mike R. Jeffrey, which is reviewed by Christian Kuehn. Here nonsmooth dynamics are of central interest and discussed in a very inspiring nonstandard manner, as Christian concludes.

Biological system dynamics are considered in the volume *Understanding Complex Biological Systems with Mathematics*, edited by Ami Radunskaya, Rebecca Segal, and Blerta Shtylla. This book is reviewed by Anita Layton, who applauds the editors of the book for organizing the related workshop with a special project-style format and for putting together this volume of research papers.

We have four more insightful reviews on methodological and application aspects. Here I would like to put particular emphasis on a young voice in this section. Gennadij Heidel, who finished his Ph.D. this year on a closely related topic, reviews the book *Tensor Numerical Methods in Scientific Computing* by Boris N. Khoromskij. In the era of data science, well-founded expositions like this should be kept "handy as a reference for tensor methods," as Gennadij recommends.

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Book Reviews

Edited by Volker H. Schulz

Featured Review: Synchronization: From Coupled Systems to Complex Networks. *By Stefano Boccaletti, Alexander N. Pisarchik, Charo I. Del Genio, and Andreas Amann.* Cambridge University Press, Cambridge, UK, 2018. $55.99. x+255 pp., hardcover. ISBN 978-1-107-05626-8.

From coordinated social behavior to collective neuronal dynamics and quantum coherence with a good number of practical applications, synchronization is a fascinating phenomenon that appears in many areas of science and engineering. Although the concept of synchronization is not at all a new one—arguably traced back to the age of Enlightenment when Huygens discovered the synchronized oscillation of clocks—research into synchronization was rare until about three decades ago when nonlinear dynamics and chaos started to become a hot topic and the numerical simulation of dynamical systems began to be mainstream thanks to the rapid development and wide adoption of modern computers. Over several decades of work from scientists all around the world, research into synchronization has yielded countless results, with several books and review articles written, notably the beautifully crafted introduction to the topic by Strogatz [23], the comprehensive overview by Pikovsky, Rosenblum, and Kurths [19], the focused review of synchronized oscillation in chaotic systems [6], and that for complex networks by Arenas et al. [2]. In this context, the recent book *Synchronization: From Coupled Systems to Complex Networks* by Boccaletti, Pisarchik, Genio, and Amann represents the latest in this outstanding collection.

The book starts with an introduction of the commonly encountered concepts, tools, and terminology in nonlinear dynamics (Chapter 1: "Introduction and Main Concepts", 20 pages). In particular, the first section introduces the mathematical description of several types of dynamical systems: systems whose state $\boldsymbol{x} \in \mathbb{R}^n$ evolves in time according to some underlying function. In general, a deterministic, continuous-time dynamical system can be described by a set of ODEs in the form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t, \boldsymbol{x}(t)), \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n \to \mathbb{R}^n$ is the state variable of the system, $t \in \mathbb{R}$ represents time, $\dot{\boldsymbol{x}}(t)$ refers to the time derivative $d\boldsymbol{x}(t)/dt$, and $\boldsymbol{F} = (F_1, \dots, F_n) : \mathbb{R}^n \to \mathbb{R}^n$ is a vector function. When the function is linear, that is, $\boldsymbol{F}(t, \boldsymbol{x}) = A\boldsymbol{x}$ for some matrix A, the system is called linear, while for more general cases of $\boldsymbol{F}$ the system is nonlinear. The system is called autonomous if the function $\boldsymbol{F}$ depends solely on $\boldsymbol{x}$ and not t; in other words, when the rules governing the system's dynamics is time-invariant; otherwise, the system is nonautonomous. The concepts of conservative and dissipative systems are also presented in particular from the perspective of physical systems. Furthermore, chaotic systems are introduced via an explicit example and briefly analyzed using tools from time series, phase space, and power spectrum. Among the possible states of sys-

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tem (1), those that are asymptotically stable (e.g., stable equilibriums, periodic orbits) form the so-called attractors and the set of initial conditions under which the system evolves toward an attractor is the corresponding basin of attraction. These concepts are useful in particular in control analysis of a system because the design of a controller is, in some sense, the task of modifying the effective basin of attraction and/or the attractor itself. Apart from auxiliary concepts such as phase portraits, Poincaré map, riddled basin, and a few others most of which are demonstrated through examples, the remaining part of the chapter mainly concerns the definition of stability and tools for analyzing it, including linear stability analysis and Lyapunov exponents, both of which can be used to determine local stability (or instability). Finally, given that a system's parameters can influence its behavior, in particular, stability, the topic of bifurcation is discussed and illustrated in several examples. Particular focus is placed on systems that exhibit "route to chaos" with several interesting examples in which chaos emerges from simple attractors as a consequence of changing the system's parameters.

Chapter 2 of the book, titled "Low-Dimensional Systems," (56 pages) is focused on synchronization in systems of a small number of coupled units—generally either two or three. The chapter starts with a nice historical note of the term synchronization and discussion of its relevance in science and engineering fields. What follows is a more detailed presentation of different types of coupling between two oscillators (either unidirectionally coupled or bidirectionally coupled) and those among three oscillators. Next, the book introduces phase oscillators and presents the now-famous example of coupled Kuramoto oscillators with detailed analysis regarding the synchronization profile of two coupled Kuramoto oscillators. It then moves onto complete synchronization between two chaotic oscillators which is studied together with linear stability analysis, and the concept of conditional Lyapunov exponents is illustrated via an example. Global stability is also briefly mentioned and the usual way of analyzing it through the construction of a Lyapunov function. Continuing on to the topic of phase synchronization includes cases where the the system is not periodic with no natural phase variable, the book surveys several ways of defining phases including geometric phase, Hilbert transform, Poincaré maps, and (in a good amount of detail) the technique of isochrons. Several commonly used measures of phase synchronization are presented. Through an interesting example of three coupled chaotic Rössler oscillators, commonly observed features of phase synchronization are summarized. Most of the rest of the chapter is devoted to types of synchronization beyond complete and phase synchronization: lag and anticipating synchronization that often appear in time-delay systems and in systems of nonidentical oscillators; oscillation death and chaos suppression as well as coherence resonance; and generalized synchronization, meaning that the dependence between two oscillators is a general and possibly nonlinear function—the challenge is usually to detect the existence and identify the form of such a function through (numerical) observations [24]. The chapter ends with a short section titled "A Unifying Mathematical Framework for Synchronization," which describes some of the authors' own past work as an attempt to formalize the various types of synchronization and unify their mathematical definition. Although an in-depth mathematical analysis is not explicitly presented in the book itself, several references (in particular, [4]) are provided for those who wish to further explore this prominant topic.

In Chapter 3 ("Multistable Systems, Coupled Neurons, and Applications," 47 pages), an extended preface is provided to discuss the phenomena of multistability and collective (synchronized) behavior in coupled network systems that appear in various physical and biological systems. The main sections cover several types of systems that exhibit multistability, including the basic setup of unidirectionally coupled systems and systems modulated by common external forces as well as bidirectionally coupled systems. Tools and concepts such as linear stability analysis, bifurcation, and basins of attraction are used, when applicable, to analyze the emergence of synchronized states and their properties. The interesting phenomenon of intermittency, where the system switches between different parts of the phase space, is reported and discussed. In addition to complete synchronization, anticipation synchronization and generalized synchronization have also been observed in these systems. Having illustrated most of the above features using systems of coupled Rössler-like oscillators, the chapter continues with a fascinating section devoted to the dynamics and synchronization occurring in systems of coupled neurons. Given the important role played by synchronization in information transfer and processing in the brain and other biological systems, researchers have devoted great efforts to develop mathematical models for these systems in the attempt to understand the fundamental mechanisms that govern their dynamics and functioning [7]. Two notable models are presented: the Rulkov map [21] and the Hodgkin–Huxley model [9]. When these relatively simple neuron models are coupled, their dynamics can exhibit a rich set of patterns including features that are reminiscent of real physiological processes such as several kinds of neuron dynamics and various types of synchronization. Finally, the chapter ends with a section on chaos synchronization and secure communication, a topic that started in the 1990s and since received considerable attention over the past three decades.

Chapter 4, titled "High-Dimensional Systems" (61 pages), mainly concerns systems of coupled phase oscillators. The chapter starts with a nice review of the widely used Kuramoto oscillator model described by the coupled ODEs [11]

$$\dot{\vartheta}_k = \omega_k + \frac{K}{N}\sum_{j=1}^{N}\sin(\vartheta_j - \vartheta_k),\ k = 1, \ldots, N. \tag{2}$$

This model is derived from considering coupled Stuart–Laudau oscillators in the limit of large parameter values $\alpha \to \infty$ and $\beta \to \infty$ while keeping α/β constant. The Kuramoto model is a minimal model that captures two salient features in natural systems: (1) heterogeneity, in that individual oscillators have different natural frequencies, and (2) interaction represented by the existence of coupling. Here there are N coupled oscillators, where $\vartheta_k \in [0, 2\pi)$ is the phase variable of the kth oscillator and ω_k is its oscillating frequency in the absence of coupling. Parameter K is the coupling strength. By tuning the parameter K, this model exhibits a transition from incoherence to synchronization. The case of $N = 3$ is examined in some detail, with the Kuramoto order parameter explicitly defined; simulation results in the large N limit are provided and discussed together with an analytical approximation of the critical coupling strength near which the transition from incoherence to coherence occurs. The case of time-varying coupling and its origin in modeling synaptic neuron connections are also discussed. Next, the chapter considers a spatially embedded system where the coupling between oscillators depends on the embedded distance. Extensive research on these systems has revealed many interesting phenomena that are summarized and discussed in the book. For instance, for coupled identical systems it was long believed that the system settles into either a coherent phase-locking state or an incoherent state, yet research in the early 2000s showed that *spontaneous symmetry breaking* is indeed possible [12, 13], in which a system of identical phase oscillators (symmetric setup) can actually result in an asymmetric outcome, where a portion of the oscillators synchronize among themselves while the rest drift around in some seemingly random fashion; such states were later named by Abrams and Strogatz as "chimera states" [1] and have since attracted much research interest. Despite a substantial amount of recent research on chimera states including several experimental projects (see the condensed review [17]), several open problems remain, including the very issues of defining a chimera state rigorously and characterizing its stability, both of which are mentioned in the book in view of recent developments. Continuing with the phase oscillator model, the chapter discusses a few other interesting patterns of synchronization with explicit examples such as Bellerophon states, amplitude death, oscillation death, and chimera death. The chapter ends with a section on systems with time delay, with particular attention given to autosynchronization (synchronization between a system's current and time-delayed states) and delay-based chaos control (e.g., the classic OGY method [16] and the Pyragas method [20]).

The very last chapter (Chapter 5, 52 pages), "Complex Networks," mainly concerns the effects of network structure on its synchronizability. The initial section introduces some basic concepts of networks (graphs), in particular the adjacency matrix A and Laplacian matrix L associated with a network. What follows is the presentation of the master stability function (MSF) originally developed by Pecora and Carroll [18] for analyzing the linear stability of the complete synchronization states $x_1 = x_2 = \cdots = x_N$ associated with the coupled nonlinear dynamics of the general form

$$\dot{x}_i = f(x_i) + \sigma \sum_{j=1}^{N} W_{ij} \left(g(x_j) - g(x_i) \right). \tag{3}$$

For the case of undirected networks (symmetric coupling), the MSF, denoted by $\Lambda_{\max} : \mathbb{R} \to \mathbb{R}$, can be used to determine the stability of synchronization for an arbitrary network and such analysis can be further exploited to quantify synchronizability of a given network, revealing the important role played by the Laplacian eigenvalues of a network. The next few sections present common types of network models and their Laplacian spectra, including small-world networks [25] and preferential attachment models [3]; several bounds of the extreme eigenvalues of network Laplacian are presented with a preliminary level of derivation. Following these spectra analyses is a section on enhancing and optimizing network synchronization, which very briefly reviews the results on the topic [14, 8] that led to several interesting recent studies [22, 15]. The rest of the chapter focuses on topics of network synchronization that are relatively new and trendy, including so-called explosive synchronization [5] and synchronization in temporal networks (networks whose structure varies in time) and in multilayer networks (see [10] for a remarkable review on this emerging subject), with the last section devoted to a short exploration of a selection of experimental results and some numerical examples.

Overall, the book, which is written in the style of a research monograph, covers a broad range of interesting topics and concepts in chaos and synchronization in coupled dynamical systems and networks, and it does so via a combination of examples that often include ODEs, deduction, and figures showing results from computer simulations. The writing style is closer to a scientific report and not bound by mathematical rigor or formality, which makes the book suitable for a wide range of audience including undergraduates, as long as they have had a solid training in college-level mathematics. Several timely topics are discussed including optimal and explosive synchronization and synchronization in temporal and multilayer networks. How should one read the book? The different chapters and even many of the sections are more or less self-contained, and as a consequence, many parts of the book can be studied and understood without necessarily digesting all the preceding materials. My personal recommendation is to glance through the book, pick your favorite part(s), and start right there—flip back to the introduction if necessary.

Is the book worth having on your bookshelf? On the one hand, for whose who are wishing for a more systematic and organized presentation or a rigorous mathematical treatment of the subject, the book might not be the ideal choice: a careful and thorough reader might notice and possibly be bothered by the repetition of concepts that appear in multiple places yet are presented differently; in several parts of the book citations to key mathematical results are not provided (e.g., existence and uniqueness of solutions to an ODE could have been mentioned by citing a standard textbook on ODEs; related work on defining chaos and chaotic attractors is not provided; a good portion of the last chapter on the eigenvalues of the graph Laplacian comes from spectral graph theory but important references are missing, etc.). On the other hand, having gone through the book myself, I found many interesting and insightful comments and discussions scattered around throughout as if they were gems to be picked out among stones. The book is definitely worth reading for those not only interested in becoming familiar with the topic of synchronization and its applications but also wishing to learn more about the many emerging problems and areas of research in the field and to develop new ideas to tackle some of these open problems. For those mathematicians who are searching for new domains of research beyond their own field of expertise, once the potential obstacles of terminology and writing style are bypassed, there are many open challenges and problems in this exciting cross-disciplineary field of synchronization that are crying for a mathematical fomunation and solution.

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Tensor Numerical Methods in Scientific Computing. *By Boris N. Khoromskij.* De Gruyter, Berlin, 2018. $149.99. x+369 pp., hardcover. ISBN 978-3-11-037013-3.

This monograph presents recent results from the field of low-rank tensor methods and their application to areas of scientific computing, mainly partial integrodifferential equations arising from the natural sciences, such as the Hartree–Fock or the Fokker–Planck equations. The book reviewed here may be seen as a companion publication to the more computationally minded book [2], coauthored by Khoromskaia and reviewed in the next issue of *SIAM Review*.

In the first two chapters, the book combines two important fields of numerical analysis: approximation theory and multilinear algebra. Since the main focus of the book is on tensor-structured approximation of multidimensional differential equations, the two basic tensor formats, the canonical and the Tucker formats, are immediately introduced in the general form over Hilbert spaces, which allows the reader to trace back both continuous and discretized objects, introduced in the example sections, to the same basic concepts: the separation of spatial variables. Many of the results from approximation theory cited by the author in this chapter date back to his own work from the previous decade, most importantly the results on sinc approximation methods.

The first chapter concludes with a collection of examples of the application of sinc interpolation and sinc quadrature methods, including cases such as the Newton kernel $1/\rho$ and the Slater function $G(\rho) = \mathrm{e}^{-2\sqrt{\alpha\rho}}$ with $\rho = \sqrt{x_1^2 + \cdots + x_d^2}$. The author also briefly discusses open questions such as sinc approximation for the Helmholtz kernel. Even though the exposition is focused mainly on theory, the author goes to great lengths to provide convincing numerical results for all examples, including error plots and tables.

What follows is a detailed survey of recent developments in multilinear algebra with a special focus on the approximation of functions. Special consideration is given to high performance methods, which allow the treatment of tensors of extremely high dimension as typically arise from partial differential equations. As in the previous chapter, the author makes a point of drawing the connection between the analytical properties of the underlying continuous structures and the rank structure of the resulting tensors. In this spirit, one of the highlights of this chapter is the section on multigrid Tucker approximation of function related tensors, which shows how a cascadic multigrid based method can be used to compute tensor approximations efficiently. Thus, it provides a link between a classical method from scientific computing and multilinear algebra.

In the last section of the chapter, the author introduces the tensor train (TT) format, a "multiplicative" format in the sense that the dimension splitting is introduced not in an additive form as in the canonical format, but in a multiplicative form. Basic results on the properties and the computation of a TT approximation to a full tensor from recent literature are provided. This section can be seen as a prelude to the following chapter, the quantized tensor train (QTT) format, introduced by the author in [3]. The main idea is based on the fact that many function-generated vectors of size $N = q^L$, reshaped into a $q\times\cdots\times q$ hypercube, admit a low-rank representation, thus allowing the use of the TT format even for problems which a priori do not have a high dimension. This chapter provides a summary of recent developments in QTT methods; researchers in PDE-related fields will find particularly useful the section on QTT representation of multivariate matrices, which describes the applications of the QTT format to discretized linear operators.

Finally, after these rather general surveys, the book concludes with its longest chapter, which is a collection of examples for the application of tensor methods to integrodifferential equations. The treatment is both mathematically rigorous and supported by illustrative numerical results.

As a Ph.D. researcher with a background in both multilinear algebra and PDE-constrained optimization, I have learned a tremendous amount from this monograph. To my knowledge, it is unique in explicitly combining multilinear algebra with scientific computing. After the previous monograph by Hackbusch [1], which explores the connection between multilinear algebra and approximation theory in Hilbert spaces, the present book takes the next step and collects applications of these ideas explicitly, while also giving a survey of most recent theoretical results.

While it is not necessary to be an expert in approximation theory to grasp the main concepts of the book, the style of the exposition is decidedly rigorous. I recommend it wholeheartedly to readers with a strong background in linear algebra multivariate analysis, preferably with some experience with multilinear algebra, approximation theory, functional analysis, or PDEs. For my part I certainly intend to keep it handy as a reference for tensor methods in the context of scientific computing.

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Markov Chains and Mixing Times. Second Edition. *By David A. Levin and Yuval Peres.* American Mathematical Society, Providence, RI, 2017. $84.00. xvi+447 pp., hardcover. ISBN 978-1-4704-2962-1.

This is the second edition of a book originally published in 2008.

Markov processes, and Brownian motion in particular, have been a great success of modern probability theory as they formed a path toward answering certain questions in mathematical analysis and partial differential equations such as the heat equation. Basically, probability theory gave a way to transform a problem originally posed in three (two spatial and one temporal) dimensions into infinite dimensions (the space of continuous functions) by constructing a suitable probability measure intimately linked to the heat equation (and variants thereof).

It took a few more decades for people to realize that a number of important problems, arising mostly in statistical physics and in computer science, can be formulated within the context of finite state Markov chains. Owing to this realization, new questions were asked and new techniques were devised. One such question is the rapidity of convergence of the law of the state X_t at time t to its stationary distribution π as measured by, say, the total variation distance $d(t) = \sup_B |\mathbb{P}(X_t \in B) - \pi(B)|$. The mixing time t_{mix} is the smallest t for which $d(t)$ is less than a small ε. To deal with this and many other problems, various types of techniques and points of view were applied. This book is an authoritative exposition of all these techniques, in which the reader will encounter elements of potential theory, geometry, algebra, probability, and even physics. Even though the book is written with the reader who only possesses "the basics" in mind, it is undoubtedly the case that the reader who has been previously exposed to some of the areas mentioned above will be better able to appreciate the techniques, ideas, and methods of proofs.

It may appear strange that while the "general theory of Markov processes" was developed early on, it took a while for the theory for finite chains to appear, but this is not surprising. Here is another example: while certainly people knew how to maximize smooth functions of many variables with smooth constraints, it took a while for the theory (and, in particular, algorithms) for the maximization of linear functions with linear constraints to appear. This is due to at least two reasons, the first being the need for it in applied situations and the second being the availability of computers that either facilitate the solution or pose new questions.

A Markov chain in finite state space is, from one point of view, the best and simplest of all worlds: there always is a stationary distribution and, given the obvious condition of irreducibility (from each state, any other state can be visited), this distribution is unique. Moreover, getting rid of often annoying periodicity, owing to the fact that the parameter ranges over integers, the law of the state X_t at time t converges to the stationary distribution, in fact exponentially fast as seen by elementary linear algebra. However, the dependence of $d(t)$ on the size n of the state space is often such that, as $n \to \infty$, $d(ct_{\mathrm{mix}})$ converges to 1 if $c < 1$ or to 0 if $c > 1$. When this is the case, the chain exhibits cutoff; physically, this means that the chain approaches stationarity in a rather abrupt manner. This is not always the case and deciding whether or not a chain exhibits cutoff is a matter of ongoing research. The question of cutoff first appears in the 1980s in a celebrated result by Aldous and Diaconis [1] showing that the mixing time of a Markov chain modeling the usual riffle-shuffle in a deck of $n = 52$ playing cards is of the order of magnitude of $\log_2 n$, whence the popular fact that seven shuffles suffice for randomizing the order of the cards.

The book is partitioned into two parts. In the first part there is an introduction to (finite) Markov chains with examples and methods. The classical reference on the subject is the 1976 book of Kemeny and Snell *Finite Markov Chains* [4] that follows a matrix approach. The approach of the current book is, whenever possible, more probabilistic. The reason for this is that, even in cases where the state space is finite, the structure of the space (e.g., a subset of the set $\{0,1\}^{\mathbb{Z}^d}$) makes it unnatural to be thinking in terms of matrices. Several examples are presented, some of which are not just examples but classes of important chains. Sometimes, given a possibly unnormalized probability measure π on a large combinatorial set, one is interested in exhibiting a random element with law π. This has numerous practical and theoretical applications (in simulation, in statistical physics, in maximizing a real-valued function on the set). One then designs dynamics by means of a Markov chain that is as nice as possible and possesses π as an invariant measure. This is done in Chapter 3 by the Metropolis method and the Glauber method (specific for state spaces that are included in sets of functions). The concept of mixing time (in terms of the total variation and other kinds of distances) is the subject of Chapter 4. Chapter 5 shows the intimate link between total variation (and hence mixing) and coupling. The quite unsuccessful motto that probability theory is measure theory plus independence can be strengthened by adding coupling as well. The reader will get a taste of this in this book. An irreducible and aperiodic Markov chain converges in law to its stationary distribution because at some finite random time the state has precisely the stationary distribution (a coupling construction originally conceived by Doeblin in [3]). Couplings provide upper bounds on $d(t)$. A strong stationary time τ (Chapter 6) is a finite random time such that X_τ has law π and τ is independent of X_τ. Given enough symmetry, such times may exist. Chapter 7 switches the point of view and makes use of a geometric quantity called bottleneck ratio which is a manifestation of the isoperimetric or Cheeger constant in Riemannian geometry: in a compact Riemannian n-dimensional manifold M, if such a constant is small, then it means that one can split M in two pieces by a very "small" $(n-1)$-dimensional manifold; thus a small Cheeger constant means very bad bottlenecks. Clearly, chains possessing bottlenecks converge slowly to stationarity and thus the mixing time can be lower bounded by a constant that is inversely proportional to the Cheeger constant. In Chapter 8, a random walk on a group is a generalization of a random walk in a (subset of the d-dimensional) lattice: replace the up, down, left, and right by a set of generators of the group. The symmetric group $\mathcal{S}_n$ of bijections of $\{1,\dots,n\}$ into itself is a canonical example. Since transpositions generate $\mathcal{S}_n$, the corresponding random walk models the silly mixing of a deck of n cards by randomly interchanging two cards are a time. The usual mixing (riffle-shuffle) is also analyzed in this chapter.

Often, chains possess (or are designed to possess) various types of symmetry, the most important of which is time-reversibility: when in stationarity, the law of the process $(X_t, t \in \mathbb{Z})$ is invariant by the time-reversal operation. This amounts to the algebraic conditions $\pi(x)P(x,y) = \pi(y)P(y,x)$ for all x and y, often known as "detailed balance equations." In this case, the chain can be thought of as a purely resistive electric circuit (or network): to each pair of vertices x and y attach a resistor of value $1/\pi(x)P(x,y)$. In such a case, one can talk about voltages (corresponding to harmonic functions and, in particular, expected hitting times) but also about currents (which is the new aspect introduced by the circuit analogy). It is then seen that the effective resistance between two distinct vertices a and b is inversely proportional to the probability that the chain, starting from a, will visit b before returning to a. All that is explained in Chapter 9. Chapters 9 and 10 study various kinds of random times: hitting times, random target times, commute times, and cover times and relationships between them and mixing times.

Chapter 12, the last of the first part, switches to algebra: it is unavoidable not to mention the use of eigenvalues and eigenvectors in analyzing powers (and hence convergence) of the transition matrix. Every Markov chain possesses an eigenvalue at 1; this is the trivial eigenvalue. Assuming irreducibility and aperiodicity, every nontrivial eigenvalue of the transition probability matrix P is in the interior of the complex unit circle. Writing $\lambda_1 = 1, \lambda_2, \dots, \lambda_k$ for the distinct eigenvalues of P we have that $P^t = \Pi_1 + \lambda_2^t \Pi_2 + \cdots + \lambda_k^t \Pi_k$, where Π_j denotes projection onto the generalized eigenspace of λ_j. Hence, the rate of convergence of P^t to Π_1 is governed by $\lambda_*^t := \max_{2\le j\le k} |\lambda_j|^t$. For a reversible chain, the quantity $\gamma_* = 1 - \lambda_*$ is called spectral gap. The larger the spectral gap, the faster the convergence. Reversible chains yield a self-adjoint P under the inner product $\langle f,g\rangle_\pi := \sum_x f(x)g(x)\pi(x)$ and hence all eigenvalues are real. In this case, one can apply an ℓ^2 point of view and derive information about the chain using Dirichlet forms (Chapter 13). The relaxation time is the inverse of the spectral gap and it is comparable to the mixing time in the sense explained in Chapter 12.

Part 2 of the book ("The Plot Thickens") starts with Chapter 13 deriving bounds on the spectral gap via Mu-Fa Chen's method of contractions, explains David Wilson's method for lower bounds on the mixing time, shows how the spectral gap is related to the bottleneck ratio, explains the path method, and concludes with a short section on the important class of expander graphs. Chapter 14 discusses the relationship between path coupling and the transportation metric and presents an application of counting the size of a large combinatorial set (the set of all proper q-colorings of a certain graph) using Markov chains. The Ising model, a simple mathematical model of ferromagnetism, is the subject of Chapter 15. Chapter 16 shows an application of the concept of shuffling cards to DNA sequences in biology. Chapter 17 discusses a number of topics revolving around martingales that are introduced for the first time at this point. One particular topic is the notion of evolving sets, a set-valued Markov chain obtained from the original one. Chapter 18 is devoted to the cutoff phenomenon discussed earlier. Chapter 19 is devoted to the so-called lamplighter walks. Chapters 20 and 21 go beyond the main chains of the book and discuss continuous-time chains and chains with a countably infinite state space. Chapters 23 and 24 are new in this edition. Chapter 23 studies the interacting particle system exclusion process and, in particular, derives estimates for its mixing time. Chapter 24 relates mixing times and hitting time parameters to stationary stopping times. Chapter 25 discusses the so-called coupling from the past, illustrating an idea ubiquitous in mathematics: when studying the asymptotic behavior of a dynamical system it is often possible to bring down $+\infty$ to "now" by starting the system at time $-\infty$. This is an idea that works even for non-Markovian systems (see [2]) and gives, *inter alia*, methods for constructing random elements whose law is exactly (and not only approximately) the stationary distribution of the chain. This chapter has been written by J. Propp and D. Wilson. Several old and new open problems are discussed in Chapter 26. The book concludes with appendices discussing background or specialized material such as simulation.

Just as with the first edition of the book, this is a highly trusted account of the field, starting from elementary topics and rapidly moving to research problems. Large, but not all, portions of the book are accessible to undergraduate students. The book can be used as a graduate textbook but also as a rapid introduction for a researcher wishing to enter this area.

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Hidden Dynamics: The Mathematics of Switches, Decisions and Other Discontinuous Behaviour. *By Mike R. Jeffrey.* Springer, Cham, 2018. $112.99. xviii+521 pp., hardcover. ISBN 978-3-030-02106-1

The first key question I always ask when reading a book review is how the work under review fits into the existing monograph and textbook landscape in the area. Are there other comparable alternatives? Is the new book particularly timely? Do we urgently need a book on the subject? So let me start with these questions in mind.

Nonsmooth differential equations have a long history and even the modern dynamics theory of the subject reaches back to at least the middle of the 20th century. Yet, there are still relatively few books available organizing progress in the subject. The first landmark monograph covering substantial efforts of the Russian mathematical and engineering school is the book by Filippov [3] from 1988 (from 1985 in its original Russian version). Later came Brogliato's work with a focus on mechanics and control [1] as well as elegant shorter accounts by Kunze [7] and Leine and Nijmeijer [9]. The final two books with a strong nonlinear dynamical systems focus on nonsmooth systems are the monographs by di Bernardo et al. [2] from 2008 and the book by Simpson from 2010 [10]. Based on this bird's eye view, it is evident that we not only need another book summarizing the rapid recent progress, but actually such a book is long overdue! In fact, this was a key initial reason as to why I was particularly interested in Jeffrey's contribution to nonsmooth systems [5].

The book has fourteen chapters, the first two of which are introductory and provide a general perspective as well as as a concrete setting for planar systems. In the planar setup the geometric view is particularly appealing. Planar nonsmooth ordinary differential equations (ODEs) serve as zeroth-order benchmark problems for the entire subject. Instead of a differentiable, or even continuous, vector field, one studies systems with discontinuities, which are assumed to occur on submanifolds of codimension one or higher. For planar piecewise smooth ODEs, one typically focuses on a single curve separating two regions in which the vector field is smooth. On the curve itself there is no a-priori given definition of the flow. Without additional knowledge we have to define the dynamics on the curve, where one switches between vector fields. The term "hidden dynamics" can be interpreted as capturing the need to use the given vector fields, and potentially even additional hidden terms, to uncover dynamics near a switching curve. Of course, a similar philosophy is then applied to switching for higher-dimensional surfaces/manifolds/varieties. These geometric objects are also called switching layers. The theme of focusing on the effects induced by a switching layer gets exploited throughout the last twelve chapters of the book. The stage is set in Chapters 3–5 with definitions of nonsmooth flows, trajectories, switching multipliers, sliding, and related concepts. Chapters 6–8 cover somewhat classical material, or at least relate to classical constructions ranging from singularity theory to (linear) stability and bifurcations. In this context, two key differences for nonsmooth ODEs are that the vector field can be tangent to a switching layer and that equilibria may lie on the switching layer. These issues induce already considerable difficulty for piecewise linear ODEs. Chapters 9–13 attack these issues for nonlinear problems and form the theoretical cornerstone contribution of the book. The presentation focuses on bifurcation phenomena, breaking of determinancy via multiple possible trajectory extensions near switching, and desingularization, finally culminating in a detailed analysis of the two-fold singularity. Via desingularization of the switching layer, one actually observes that desingularized smooth systems display phenomena very close to, or even exactly studied already, in the context of multiple time scale dynamics [6]. The dynamics near the switching layer becomes slow, while the external vector fields are governed by fast motion. Finally, in Chapter 14, Jeffrey discusses several modern applications of the theory.

Instead of delving deeper into the material, let me continue to a general view on the book. The first thing one notices when starting to read is that the style is quite different from a "classical" mathematics monograph. It is neither written in a pure mathematics theorem-proof format, nor in a purely example-application driven format. Instead it seems to me that Jeffrey's intention is to talk applied mathematics directly to the reader and to communicate his views on and excitement about nonsmooth geometric dynamics. In my personal opinion, we need more bravery like this in book writing. Attempting a different style in such a major undertaking as a book takes courage and should be applauded. Of course, whether or not the creative approach has worked for this book can probably only be decided over the next few decades by current and future readers.

So, what is the target audience? I think the book is very readable, even for those with a relatively low-level background. The minimum requirement probably would be a first course in nonlinear dynamical systems [11], although a second course certainly would be helpful as well [4], especially with a focus on bifurcation theory [8] or multiscale systems [6] since these subjects are linked particularly tightly with the viewpoint of Jeffrey. Therefore, the potential audience is actually very broad, including all research-oriented applied mathematicians with an interest in nonsmooth systems. In terms of using it as a textbook, one is charged to extract the essential messages out of Jeffrey's book as well as from the the other sources mentioned above [2, 7, 10, 9, 3, 1] to really provide a comprehensive short mathematical course on the topic. Yet Jeffrey's book seems ideally suited for a masters-level seminar course in which students present the book chapter-by-chapter to each other.

The last, potentially most debatable, question I would like to raise here concerns the general paradigm of nonsmooth systems. Due to their high practical use for mechanical systems in the context of impact and friction in particular, their practical usefulness is established. However, finding the "best" mathematical modeling viewpoint is still somewhat under discussion. Jeffrey points out these issues (rightly so!) at the beginning of his book and the discussion recurs at various points. For example, one might argue that upon mathematically zooming in or using a finer model, we may smooth a system, so why bother with the nonsmooth view? Here I fully agree with Jeffrey's approach that the nonsmooth system is a very useful singular limit, and that smoothing should be brought in as a technical tool. The second key point often made in the context of nonsmooth models is observatiom of the evident nonuniqueness induced by a switching layer, and thus the need to analyze the resulting nondeterministic dynamics. So far, so good. Yet, this approach really involves in most cases only classical techniques from deterministic analysis, almost pathologically avoiding probabilistic methods, with very few recent exceptions. Jeffrey's book is not one of those exceptions. He focuses on the "deterministic analysis of nondeterminism." Personally, I think the stochastic approach holds a lot of promise to aid better understanding of apparent nonsmooth loss of determinism. Not only does "stochastic analysis of nondeterminism" sound natural to me, but it may bring fundamental new insights in the next few decades (and potentially lead to another useful book!).

In summary, I can definitely recommend Jeffrey's book for anyone looking to learn more about nonsmooth dynamical systems. Jeffrey lays out his view on hidden dynamics and thereby also sets out a potential future agenda, and so this book is one little puzzle piece that might hopefully motivate mathematicians of various interests to refine and extend methods in the vibrant area of applied dynamical systems.

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Mathematical Modeling of Mitochondrial Swelling. *By Messoud Efendiev.* Springer, Cham, 2018. $199.99. xiv+223 pp., hardcover. ISBN 978-3-319-99099-6.

Mitochondria is the powerhouse of the cell. That statement has remained meaningful to me, given the mitochondrial's prominent roles in the energy production of the cell, despite the unfortunate mockery that the statement has received on Tumblr as an example of impractical information taught in public schools. (If you are unaware of the statement's notoriety, just Google it.) The words "mitochondrial swelling" in the title of the book invoke for me an image of a nuclear plant on the verge of an explosion. While a bit dramatic, that imagery isn't that far from what actually happens in mitochondrial swelling: water starts rushing into an inner compartment of the mitochondria, making it swell; the compartment keeps swelling and swelling until it is bigger than an outer membrane and ruptures it, spilling all the mitochondrial content. It doesn't end well for the cell.

Potential audiences. This book caught my attention because of my interest in mitochondrial function, or rather its dysfunction, which is implicated in many diseases including diabetes. I know little about mitochondrial swelling, except that it's bad news for the cell. Therefore, I was expecting to learn more about the underlying mechanisms, signaling processes, and various contributing factors. "The mathematical models considered in this book can help to understand the swelling of mitochondria," states the back cover of the book. So, I was hopeful.

It is fair to say that the focus of the book is not on the biology, or even the biological implications of the models. Rather, the vast majority of the text is spent formulating a specific set of mathematical models of mitochondrial swelling. Those models are based on models that the author and his collaborators have published in recent years (e.g., [1, 2]). If your goal is to dig deep into the author's models and have all the relevant mathematical tools at your fingertips, this book might serve your purpose. However, if you want to learn more about, say, how mitochondrial swelling fits in with other cell biology processes, or how various diseases or pharmaceutical maneuvers might induce or suppress mitochondrial swelling, you may be disappointed.

It is helpful in a book review to identify the potential audiences. The focus of this book is very narrow. In fact, if you don't already have a solid background in cell biology and a good grasp of the mitochondrial swelling process, you will be quite lost. Clearly, the book isn't suitable for a class (undergraduate or graduate). I am not sure how helpful it would be for someone with general interest in cell biology. However, as I said before, if you want a comprehensive coverage of the mathematical models developed by the Efendiev group, then this book should provide a good guide.

Structure and content. For a book focusing on a specific biological process, I had expected it to start with a description of what mitochondrial swelling is, why it is important, and what insights one might glean from mathematical models. But no, the author gets right down to (mathematical) business. Even the preface of the book starts by talking about the types of equations the models are based on (ODEs, reaction-diffusion equations, etc.). The second paragraph gives the explicit form of the equations. There is no discussion of when mitochondrial swelling happens, or even what that process is. It is, indeed, a very honest preface of what is to come in the book.

The book consists of seven chapters. Chapter 1 contains the mathematical preliminaries, covering Sobolev spaces, Poincaré inequalities, Nemytski operators, etc. These are concepts that the reader needs to master in order to follow the derivation of the models in later chapters. Finally, in Chapter 2, the biological background of mitochondrial swelling and its role in cellular apoptosis are covered. However, Chapter 2 has all of eight pages, so it is rather terse. I imagine that much of the material will be difficult to understand for someone without a decent background in cell biology.

The models start emerging in the remaining chapters. In Chapters 3 through 7, we are introduced to a few of the author's published models, under different assumptions. For example, swelling in a cell in a test tube, or in a living organism without a controlled environment. Each chapter includes the derivation of the models, some mathematical analysis and discussion of the behavior of the solutions, and numerical simulations. I would have loved to have seen a more in-depth discussion of the functional implications of the simulation results, but alas that is not to be found.

Conclusion. If you go through this book cover to cover, you will learned *a way* to build mathematical models of mitochondrial swelling. If you then want to know what scientific insights these models can bring, such as how volume dynamics might impact mitochondrial bioenergetics, you will need additional reading material.

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Understanding Complex Biological Systems with Mathematics. *Edited by Ami Radunskaya, Rebecca Segal, Blerta Shtylla.* Springer, Cham, 2018. \$99.99. xiv+198 pp., hardcover. Association for Women in Mathematics Series. Volume 14. ISBN 978-3-319-98082-9.

The title of the book may suggest that it is an introductory text to mathematical biology, or a survey of mathematical approaches for modeling complex biology systems. But in fact it is a conference proceedings volume, published as a result of the workshop "Women Advancing Mathematical Biology: Understanding Complex Biological Systems with Mathematics," held at the Mathematical Biosciences Institute at the Ohio State University in April of 2017. The conference was organized by the editors of the volume (Radunskaya, Segal, and Shtylla).

Structure and content. To understand the structure of the book, one might first consider the format of the "Women Advancing Mathematical Biology" workshop, which is somewhat nonstandard. Instead of attending a number of lectures, the participants of this workshop spent a week working on several research projects. Each project was chosen by a "team leader," whereas other team members were junior researchers who might be new to that particular research area. By matching senior research mentors with junior mathematicians, the organizers sought to "expand and support the community of scholars." The results of those new collaborations are summarized in this volume.

The volume contains mathematical research papers written by the individual project teams. These papers cover a wide a range of areas: There are a few papers on infectious diseases, including one that analyzes the impact of a superspreader and another on the life cycle of a tick (which many of us hate—the tick not the paper); there is a paper on the tumor-immune dynamics in multiple myeloma and a couple on fluid dynamics, in blood flow and around an organelle. In part due to the diversity of the topics and the research teams, there is little cohesion among the papers, other than that they all emerged from the same workshop. So in this sense, the book is substantially less focused than a typical monograph or conference proceedings volume.

Potential audiences. Had I been new to the format of this volume, I would have struggled to figure out what its potential audiences might be. (I organized a similar research collaboration workshop for women in math biology at NIMBioS, Tennessee, in 2016, and published a similarly styled proceedings volume afterwards.) These are not reviews or survey papers, so they are not intended to introduce the reader to the subjects. Some teams did a better job than others in motivating the research topic and describing the background materials. The papers are regular *research* papers, on a wide range of topics. Now, if one wants to be overly critical, one might say that none of the papers represents a gigantic advance in the field. Most of the papers involve an application or extension of an existing model, nothing truly ground-breaking. But that statement would, indeed, be overly critical. It is important to remember that, by design, most of the (junior) team members were new to that particular research problem, which was done to ensure a level playing field and positive team dynamics, so in one week, the teams digested the literature, learned the models and approaches, obtained results, and, in the few months following the workshop, wrote and published a paper. That is no small feat.

What are the appropriate audiences for this book then? While you probably wouldn't want to use it as a text for a class, graduate or undergraduate, it does include a decent sample of some of the interesting research going on in the field. I also think it's an excellent motivating tool for graduate students, who can see the level of productivity that can be achieved in essentially one week if one is 100% focused and works intensively.

I will end this review by applauding the editors (Radunskaya, Segal, and Shtylla) for organizing the workshops and for putting together this volume, and the project leaders for leading and mentoring the teams.

> "Achieving gender equality requires the engagement of women and men, girls and boys. It is everyone's responsibility."
>
> —Ban Ki-moon

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A Taste of Inverse Problems: Basic Theory and Examples. *By Martin Hanke*. SIAM, Philadelphia, 2017. $59.00. viii+162 pp., softcover. ISBN 978-1-611974-93-5.

Inverse problems can be found widely in a technological context where mathematical modeling plays a major role, which is simply everywhere and also where SIAM is active. Thus, this book really hits a nerve, especially with its title and compact appearance. The first impression of this book is that of a short but gentle introduction to a very important field of applicable mathematical technology, and all that within a nice and short paperback volume. What could be better than that?

Potential audiences. In the preface, one reads that the book is intended to be accessible to math students and engineers faced with solving inverse problems. However, when reading the book, one notices fairly soon that the major audience is math students and probably math colleagues. Each chapter is very tight and often there is not much motivational text between the many theorems and proofs. Illustrative pictures are limited to the barest minimum.

Structure and content. The book starts with a metaphorical amuse-gueule (French for appetizer) in the form of a discussion on numerical differentiation. This discussion leads nicely to the well-known square-root rule for the finite difference increment and could easily be done at that point. However, the latter part of this small appetizer chapter takes off quickly into the Sobolev space of weakly differentiable functions and presents some results that I'm still puzzled about what they are good for—no interpretation, no conclusion. The following parts are not named entrée, plat principal, dessert, but rather mundanely Parts I–III. Afterwards, there are appendices. The three parts of the book cover a wide range in inverse problems. Many interesting topics are dealt with in very compact chapters—roughly six chapters per part. Thus, it makes pretty good sense to not stick too closely to the gourmet terminology. In this metaphorical language, the separate chapters would be best compared with those intense and compact energy balls that you use to overcome urgent hunger.

Personally, I have some experience in the related discipline of PDE constrained optimization and thus obtained interesting insights into inverse problems in, for example, Chapter 6 on electrical impedance tomography, where obviously the standard inverse problems approach differs from the optimization approach that I am used to; Chapter 8, where I learned about the L-curve method as good discrepancy heuristics, and Chapter 14, where again I learned an iterative technique for the solution of the Cauchy problem which is quite different from the optimization approach that I am used to.

How should you read the book? My first approach for reading the book was linear from the first page to the last. This may not be the best way, since the chapters seem to be quite independent from one another. A much better and personally more fulfilling approach is to look at the content overview and jump right into the one chapter whose title grabs your attention most. Then, you will be immediately drawn into a very deep discussion on theoretical aspects in all the relevant function spaces. Each chapter takes a lot of time to work with the content in order to digest it properly. Have pencil and paper with you and do not overdo it at one time. Be prepared not to be charmed too much by motivational chatter: It's always a sincere and rigid discussion.

Conclusion: Should you taste the book? All in all, the book is quite an interesting read, if your expectation is not biased by its title. It is not a gentle introduction, but rather a compact and rigid discussion of selected topics in inverse problems. If one shares a view of the field with the author, this book can also be used as supplementary material for a course in inverse problems. The extensive usage of curly letters does not improve its readability, but once you get used to the typesetting, you can learn quite a lot from the book.

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