EDUCATION

The Education section of *SIAM Review* presents three papers in this issue. In the first paper Robert M. Corless and Leili Rafiee Sevyeri discuss "The Runge Example for Interpolation and Wilkinson's Examples for Rootfinding." The authors use several classical examples in numerical analysis to discuss propagation of error and the role of sensitivity and the conditioning of the functions subject to numerical procedures. The first example was brought to the fore by Carl Runge in 1901 when exploring the approximation error occurring in polynomial interpolation. The effect illustrated by this example is also referred to as Runge's phenomenon. It consists of oscillation at the edges of the domain of the function, which is observed when the function is approximated by interpolation using a high-degree polynomial over a uniform grid. This example plays an important role and is used in many numerical analysis textbooks because it illustrates pitfalls and limitations of high-degree polynomial interpolation. It reveals that using higher degree polynomials does not always improve the accuracy of the approximation. The second example is a classical polynomial, introduced by J. H. Wilkinson in his paper "The Evaluation of the Zeros of Ill-Conditioned Polynomials. Part I" [*Numer. Math.*, 1 (1959), pp. 150–166]. It illustrates a difficulty when finding the root of a polynomial, which stems from the sensitivity of the polynomial at the location of the roots, with respect to perturbations in its coefficients. Wilkinson's polynomial is also frequently used in the context of eigenvalue calculation. In that case, computing eigenvalues of a matrix by first calculating the coefficients of the matrix's characteristic polynomial and then calculating its roots may result in a very ill-conditioned problem even if the original problem is well conditioned. The characteristic polynomial can be very sensitive to perturbations in its coefficients near its roots. The authors bring the theory of backward error analysis and conditioning to the reader's attention. This approach explains the phenomena of the presented examples and is in harmony with the numerical experience thus far.

The second paper is titled "Mathematics + Cancer: An Undergraduate 'Bridge' Course in Applied Mathematics." It is presented by Tracy L. Stepien, Eric J. Kostelich, and Yang Kuang. The authors share their insightful experience with a one-semester modeling course based on mathematical models for cancer occurrence and growth, as well as models describing the effect of certain cancer treatments. The targeted audience is undergraduate students who have completed standard courses on calculus, ordinary differential equations, linear algebra, and introductory probability and statistics. The offer of such a course supports various goals. It helps develop modeling skills for students in applied mathematics while at the same time exposing students to many mathematical ideas and enabling them to make informed choices about future specialization in the various branches of applied mathematics and statistics. Additionally, the students come in contact with compelling scientific as well as social problems and see the impact of applied mathematics on real life. Finally, a research component in the course provides invaluable experience to the enrolled students.

The paper discusses the structure and organization of the course. The main portion of it provides an overview of various modules whose suggested length is one or two weeks. The topics encompass statistical models regarding the occurrence of cancer depending on age, the growth of the population of cancer cells, the evolution of resistance to treatment, tumor dynamics at the cellular and tissue levels, the measurement and assessment of treatment efficacy, short-term forecasts of tumor progression, the use of statistics in designing clinical trials, and many others. Finally, the public health perspective is brought into the picture: how much cancer might be preventable by a healthy life style, how much is due to environmental exposure, and how much might be due to hereditary reasons or random mutations.

The reader will find a sample syllabus, homework problem sets, and computer lab descriptions in the supplementary materials. The authors themselves aim to give students experience in reading research papers and do not use textbooks. However, they mention suitable textbooks for those teachers who would prefer to follow one. Extensive references to relevant literature are included. Finally, the authors point out that their course might be used as a template for other courses of similar type based on topics drawn from other areas of the mathematical sciences.

The third contribution is the paper by Antônio Neto on "Matrix Analysis and Omega Calculus."

The first ideas giving rise to Omega calculus were presented by MacMahon in his book *Combinatory Analysis* published by Cambridge University Press in 1915–1916. The main subject there is the solution of linear Diophantine systems composed of equalities and inequalities; in this context, the author involves results on partitions of natural numbers. In that framework, the so-called Omega operator is applied systematically to each equation and inequality. The final outcome of the procedure is a generating function describing all the solutions of the Diophantine system. The methodology is referred to as MacMahon's partition analysis (MPA) or Omega calculus. Frequently, the latter is identified with the Omega package, which contains numerical methods based on MPA.

In the current paper, Antônio Neto presents an extension of Omega operator calculus to matrix-valued functions. The focus is placed on the matrix exponential functions, which have many applications. The extended Omega operator is defined based on the matrix exponential series and the Neumann series with the use of the Frobenius norm for square matrices. Application of the Omega operator to the theory of differential equations, to graph theory, and to other areas is discussed. The paper contains a compact and easy-to-apply method to compute multiple integrals involving matrix exponentials based on MPA. The author stipulates that the presented definition and method generalize and unify previous work.

The material is accessible to advanced undergraduate students in applied mathematics who have knowledge in calculus including Taylor series of scalar functions, various norms, matrix operations and inversion, and matrix exponentials.

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