

## RESEARCH SPOTLIGHTS

The paper “Network Modularity in the Presence of Covariates,” by Beate Ehrhardt and Patrick J. Wolfe, is the first of two articles appearing in Research Spotlights. Given a network, the goal is to understand how the nodes connect in communities. When additional information about the network (e.g., a network of friends) is available in the form of covariates (e.g., year in school, gender), the authors propose using modularity to evaluate whether community assignments implied by the covariates themselves lead to a valid summary of the network structure. Modularity is a quantity defined in terms of sums of differences between an observed edge and the (weighted) product of the degrees of the nodes for that edge for nodes that are in the same community. The authors frame modularity as a measure of statistical significance and prove a central limit theorem for modularity under a nonparametric null model. With this in place, it becomes possible for the authors to associate a  $p$ -value with observed community structure based on a covariate. The  $p$ -value quantifies the “plausibility of the observed data under the null.” As their final illustration of the utility of their results, the authors consider several covariates relative to the Enron data set and follow a four-step process that leads to the calculation of the associated  $p$ -values. The authors argue that the small  $p$ -values which result match the expectation that certain covariates (e.g., department) are likely to impact email interactions. On the other hand, covariates in the example related to properties one would not expect to influence email interactions do indeed result in relatively high  $p$ -values.

The second paper, authored by Dan Wilson and Bard Ermentrout, is titled “Augmented Phase Reduction of (Not So) Weakly Perturbed Coupled Oscillators.” Phase reduction is a well-known method for complexity and dimensionality reduction for systems of ODEs with perturbation. In its standard form, it is generally less applicable to a large population of oscillators when the perturbations are large, though the authors do a thorough job of reviewing extensions that have been presented in the literature, as well as the limitations of those extensions. The focus in this paper is on an augmented phase reduction approach that can be applied under certain conditions (namely, mean field coupling of the oscillators together with restrictions on the perturbations). In order to help the reader develop the necessary intuition, the authors begin with a small example to illustrate the advantages of using expansions in phase and isostable coordinates. They move on to showing the general approach for generating the reduced form, which consists of coupled ODEs for the phase and isostable coordinate variables. To utilize the prescribed reduction approach, there are functions and parameters that must first be identified. The authors devote a section to a methodology for generating these necessary components. Computational efficiency is also discussed. The entire reduction process is demonstrated on a model with applications to circadian oscillations.

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