SIGEST

The SIGEST article in this issue is "Bounds on the Singular Values of Matrices with Displacement Structure," by Bernhard Beckermann and Alex Townsend. A key motivation for the work is that techniques for reducing the complexity of a computational task often rely on the existence of a low rank approximation to a high-dimensional matrix. Equivalently, that high-dimensional matrix will have relatively few dominant singular values.

The authors consider matrices with displacement structure, including Pick matrices, Cauchy matrices, Löwner matrices, and Krylov matrices, as defined in equations (4.1), (4.5), (4.9), and (5.1), respectively, which occur in a diverse range of applications. Each such matrix X satisfies a Sylvester displacement equation of the form $AX - XB = MN^*$ for certain matrices A, B, M, and N, where M and N are $m \times r$ and $n \times r$, respectively. The matrix pair (M, N) is called an (A, B)-generator of length r of X; it is particularly important to know how small r can be compared with m and n. The analysis employs an extremal problem for rational functions from complex approximation theory, and takes advantage of the properties of A and B to derive *explicit* bounds on the singular values and numerical rank of X. An important ingredient is the concept of the *Zolotarev number* associated with two disjoint sets E and F in the complex plane. The fundamental result, Theorem 2.1, is shown to provide useful bounds on the singular values of X when A, B, E, and F can be chosen appropriately. The resulting singular value bounds and corresponding bounds on the ϵ-rank (that is, the smallest integer k such that X can be approximated to precision $\epsilon\|X\|_2$ by a rank k matrix) are collected in Tables 1.1 and 1.2, respectively. In addition to providing novel technical results at the intersection between applied linear algebra and complex analysis, this work gives theoretical support for the growing number of fast algorithms based on low rank techniques.

The original article appeared in *SIAM Journal on Matrix Analysis and Applications* (SIMAX) in 2017. For this SIGEST version, the authors have added extra material in order make the presentation accessible to SIREV's wide readership. They have also discussed subsequent developments in the area and provided many new references.

The Editors