RESEARCH SPOTLIGHTS

Numerical solution of the Navier–Stokes equations in both two and three dimensions is the subject of the first of two Research Spotlights articles in the current issue. Authors Leslie Greengard and Shidong Jiang, in their article "A New Mixed Potential Representation for Unsteady, Incompressible Flow," present a new integral representation for the unsteady, incompressible Stokes or Navier–Stokes equations. Given the Helmholtz decomposition of the forcing term, the authors show that the unsteady Stokes equations in a bounded domain have an explicit solution involving heat potentials. Owing to this result, they observe that the solution to the full unsteady Stokes equation is linearly decomposable as the sum of a solution to the homogeneous, linearized system plus the particular solution, defined in Lemma 3.2, to the unsteady incompressible Navier–Stokes problem. The authors express the homogeneous solution in terms of harmonic and heat layer potentials, so that the problem boils down to determining the boundary densities from a system of integral equations. A second-order accurate implicit one-step time marching scheme is proposed first, but their subsequent analysis of the conditioning of the problem leads the authors to investigate a method for solving the unsteady Stokes equations using a rule of predictor-corrector type. Readers may appreciate the potential for extensions and improved accuracy that are outlined in the final section.

The aim of the second paper, coauthored by Jon Cockayne, Chris J. Oates, T. J. Sullivan, and Mark Girolami, is to establish rigorous foundations for what the authors define as a "Bayesian probabilistic numerical method," or Bayesian PNM. In their paper "Bayesian Probabilistic Numerical Methods," the authors begin by introducing the reader to the definition of PNMs as "algorithms whose output is a distribution over possible values of a deterministic quantity of interest, such as the value of an integral." A literature review provides the reader with pointers to work on PNMs in numerical integration, numerical linear algebra, and numerical solution of ODEs and PDEs. Expanding on this previous work, the authors propose a formal definition of a Bayesian PNM that they can use for methodological comparisons for many standard numerical tasks. The example they give as a need for such improvement is in comparing the uncertainty modeling one might run into with uncertainty quantification of PDEs: a probability distribution might be placed either on the solution of a PDE or on the discretization error of a numerical method, and a new theoretical foundation is needed to make methodological comparison possible. They explain how BPNMs can be "meaningfully composed," thereby enabling nontrivial error propagation and accumulation to be quantified. As an illustration of their theory on an application in industrial process monitoring, they use BPNMs to account for the effect of discretization on inferences made on the conductivity field when electrical impedance tomography is employed to assess material content. In the final discussion section, readers are given an assessment of the aspects of this new approach requiring further consideration and research.

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