RESEARCH SPOTLIGHTS

The issue of accurate function approximation is an important topic in numerical analysis. The first Research Spotlights article, "Frames and Numerical Approximation," by Ben Adcock and Daan Huybrechs, promotes the use of regularized truncated frame-based approximation for this purpose. Given a Hilbert space $\mathcal{H}$, a frame is an indexed family of elements whose span is dense in $\mathcal{H}$, which are often not orthogonal or linearly independent, but which satisfy a frame condition. The frame condition is an expression bounding, both below and above, the ℓ_2-norm of the coefficients of a function with respect to the frame elements by constants (the lower and upper frame bounds, respectively) times the Hilbert space norm of the function. The authors address complications arising when using the first N frame elements to derive a numerical approximation. As the frame elements are not typically orthogonal or independent, a system involving an ill-conditioned Gram matrix must be solved. A key result is that the authors are able to link the ill-conditioning to the behavior of the frame bounds. They therefore propose using a truncated SVD of the Gram matrix to compute the desired approximate projections, and tie the convergence of the resulting function approximation to how well the function can be approximated by vectors of coefficients with small norms. Three examples are studied in detail, providing evidence that regularized truncated frame-based function approximation could be a valuable addition to the suite of tools used in function approximation. The article concludes with opportunities for further comparison and investigation.

As defined in the article "A New Class of Efficient and Robust Energy Stable Schemes for Gradient Flows," by Jie Shen, Jie Xu, and Jiang Yang, a gradient flow refers to a dynamic that is driven by a free energy and given dissipation mechanism. Examples outlined include physical problems modeled by PDEs in gradient flow form, such as crystallization and thin films. The purpose of this second article in Research Spotlights is to study and expand the use of the "scalar auxiliary variable" (SAV) approach and to couple it with specific time-stepping schemes for the solution of a large class of gradient flow problems. In comparison with the other methods to which they compare, the SAV approach is applicable to a larger class of gradient flows. It is easier to implement than the closely related IEQ approach since it requires solving decoupled linear equations with constant coefficients, yet it retains desirable features of the IEQ approach, providing schemes that are unconditionally stable about a modified energy and are linear and second-order accurate. Following an introduction of the SAV approach and discussion of how to combine it with either the Crank–Nicolson or BDF second-order schemes, the authors demonstrate the efficiency and second-order accuracy on several gradient flows. They expand on these ideas by combining the SAV approach with higher-order BDF schemes and adaptive time stepping for further gains in accuracy and efficiency. The authors caution that the key to getting physically consistent results using the SAV approach is in the way the energy terms are split, and that this will change from case to case. Nevertheless, they are able to highlight the versatility of the SAV approach by concluding the paper with a demonstration of the approach on three distinct applications whose gradient flows have different properties.

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