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Book Reviews

BOOK REVIEWS

The frame for the Book Reviews section this time is formed by the books of two very well known and successful SIAM Fellows. In his featured review Akhtar Khan introduces the new book by Boris Mordukhovich, *Variational Analysis and Applications*. He praises the very extensive work as being "not only of great benefit to the research community actively engaged in variational analysis" but also encouraging "newcomers who are willing to learn the subject." The section concludes with my review of Gilbert Strang's very contemporary book on *Linear Algebra and Learning from Data*, which from my point of view fulfills all wishes for an introductory mathematical work for data scientists.

In between we find 6 more reviews of current books. Krešimir Josić reports on the book *A Course in Networks and Markets* by Rafael Pass and recommends it because of its mathematical depth. Anita Layton reviews the book *Mathematical Modelling and Biomechanics of the Brain*, starting with a report on a very drastic incidental real experiment. She recommends the book for beginners and established researchers in the field. The area of optimal control is addressed in two further books: Alain Bensoussan's book *Estimation and Control of Dynamical Systems*, reviewed by Jan Palczewski, and Paweł Olejnik, Jan Awrejcewicz, and Michal Fečkan's *Modeling, Analysis and Control of Dynamical Systems with Friction and Impacts*, reviewed by David Simpson. Finally, we have two books on systems biology in our section. Mehrshad Sadria and Anita Layton recommend the book *A First Course in Systems Biology* by Eberhard Voit as a good introduction into the field. And for advanced readers Mehdi Sadeghpour recommends the 3rd edition of the book *Stochastic Modelling for Systems Biology* by Darren Wilkinson.

Volker H. Schulz
Section Editor
sirev.br@gmail.com

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Book Reviews

Edited by Volker H. Schulz

Featured Review: Variational Analysis and Applications. *By Boris S. Mordukhovich.* Springer, Cham, 2018. $139.39. xix+622 pp., hardcover. ISBN 978-3-319-92773-2.

Variational analysis provides a unified framework to study a wide range of problems arising in optimization, calculus of variations, control, nonlinear and set-valued analysis, variational inequalities, partial differential equations, perturbation theory, inverse problems, economics, mechanics, and many other related domains. Rockafellar and Wets [10], in their celebrated monograph, coined this term to reflect the breadth of the topics they envisioned to be covered under the umbrella of variational analysis. In recent years, variational analysis has witnessed explosive growth in theory, numerics, and applications. Some of the recent monographs devoted to the techniques of variational analysis are by Attouch, Buttazzo, and Michaille [1], Aubin and Frankowska [2], Borwein and Zhu [3], Dontchev and Rockafellar [4], Ioffe [5], Khan, Tammer, and Zălinescu [6], Mordukhovich [7, 8], and Penot [9]. This new monograph is a welcome addition to the existing literature on this vibrant and expanding field.

The material of this monograph is organized into ten chapters.

Chapter 1 presents the ingredients necessary to develop the theory of generalized derivatives. It begins with an introduction of normals and tangential approximation of closed sets, their equivalent formulations, and characterizations. It also introduces the coderivatives of set-valued maps, which is one of the central themes of this monograph. The first-order subgradients are defined by taking normals to the epigraphs and by employing coderivatives. Various characterizations of the limiting subdifferential, as well as the subgradient of the distance function, are also given.

Chapter 2 conducts a thorough study of extremal principles for finitely many and countably many systems of sets and their applications. Extremal principles play a vital role in variational analysis and are akin to the well-known separation theorems, though without any convexity requirement on the data. As a direct implication of the extremal principle, the normal cone intersection rule is derived, and sum formulae for subdifferentials are given. It should be noted that the normal cone intersection rule plays a fundamental role in developing comprehensive calculus rules for generalized differentiation in variational analysis.

Chapter 3 covers the crucial properties of set-valued maps that are attributed to their well-posedness. The discussion includes the Lipschitzian stability, metric regularity, and covering/linear openness. Although these properties are intrinsic features of set-valued maps, their complete qualitative and quantitative characterizations are given by employing the coderivatives. Furthermore, utilizing the coderivative characterizations and calculus rules, it is shown that the metric regularity fails to hold for many significant classes of variational systems given as solution maps to parametric generalized equations, variational inequalities, and others.

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Chapter 4 gives calculus rules for subdifferentials of extended real-valued functions. The main emphasis is on the subdifferentiation of marginal functions, composite maps, and minima and maxima. An important contribution of this chapter is in terms of several extended versions of the classical mean value theorem in the absence of differentiability, and its exciting applications in variational analysis. The discussion includes the mean value theorem and via symmetric subgradients, the approximate mean value theorem, and subdifferential characterizations from the approximate mean value theorem.

Chapter 5 is devoted to the use of variational analysis machinery for the exploration of set-valued monotone operators, which play a significant role in variational inequalities, optimal control, partial differential equations, and other related fields. Various useful characterizations of the global monotonicity are given using coderivatives. The results include maximal monotonicity via regular coderivatives and via limiting coderivatives, coderivative criteria for local monotonicity, and strong local maximal monotonicity.

Chapter 6 offers a thorough treatment of optimality conditions for nonsmooth optimization problems, which, arguably, is one of the most significant aspects of variational analysis. The given optimality conditions cover a wide variety of scenarios such as nonsmooth problems with a single as well as multiple geometric constraints. The treatment of finitely many inequality and equality constraints is covered in detail. Examples of various optimality conditions and a comparison of the adopted tools used for deriving them make for quite an enjoyable read. The second part of the chapter deals with bilevel programming problems. The fundamental strategy is to reduce the bilevel programs to nondifferentiable ones and then use the results developed in the first part of this chapter. The topics covered include the optimistic and the pessimistic versions, the so-called value function approach, the notion of partial calmness, and the weak sharp minima. Optimality conditions for bilevel programs with smooth and Lipschitzian data are given.

Chapter 7 establishes the utility of the techniques of variational analysis for semi-infinite programs involving infinite linear and convex inequality constraint systems. It is shown that the tools of variational analysis not only offer new insight, but they also render powerful results on Lipschitzian stability and optimality conditions for semi-infinite programs with arbitrary index sets. The chapter begins with the stability aspects of infinite linear inequality systems, and the discussion involves the Lipschitz-like property, the strong Slater condition, and a coderivative computation for parametric infinite linear systems. This chapter also offers a complete coderivative characterization of Lipschitzian stability. The study of optimization problems with infinite linear constraints includes necessary optimality conditions with general nonsmooth cost functions and infinite inequality constraints and applications to water resource optimization. Another main topic of study is infinite linear systems under block perturbations where the focus is on the stability aspects using coderivatives. Infinite convex systems DC (difference of convex) optimization is explored and includes the use of metric regularity and subgradients.

Chapter 8 is a continuation of the study of semi-infinite optimization problems. The focus, however, is now on the nonconvex problems under varying assumptions on the data of the infinite systems such as differentiability, Lipschitz continuity, and lower semicontinuity. The chapter begins with constrained semi-infinite optimization problems and proceeds under the differentiability assumption, giving various constraint qualifications such as the extended Mangasarian–Fromovitz constraint qualification, the perturbed Mangasarian–Fromovitz constraint qualification, and the nonlinear Farkas–Minkowski constraint qualification. Characterizations of both the regular and the basic normal cones to the constraint set are given and subsequently used for deriving optimality conditions. The focus then shifts to nonsmooth semi-infinite programs with inequality constraints given by locally Lipschitzian functions. For this, some point-based upper estimates for the basic subdifferential of the supremum function, as well as optimality conditions, are presented. The next part deals with nonsmooth cone-constrained optimization problems where the key approach is to reduce the semi-infinite programs to problems of cone-constrained (or conic) programming in infinite-dimensional spaces. The given results include the subgradients of scalarized supremum functions, point-based optimality, and qualification conditions. The chapter concludes with a study of nonconvex semi-infinite problems with countable constraints.

Chapter 9 is devoted to set-valued optimization problems that have attracted a great deal of attention in recent years and provide a uniform framework for studying nonsmooth and multiobjective optimization problems. The chapter continues the use of a dual-space variational approach for set-valued optimization problems. It begins with a discussion of various notions of minimizers and subdifferentials induced by ordering cones. An exciting presentation of relevant topics such as variational principles for ordered maps, limiting monotonicity for set-valued maps, the variational principle of Ekeland's type, and the subdifferential variational principle is supplied. The existence results for unconstrained and constrained optimization problems of relative Pareto-type minimizers employ a variety of conditions such as subdifferential Palais–Smale conditions. A variety of optimality conditions for set-valued optimization problems are given.

Chapter 10 presents applications of variational analysis to microeconomic modeling. The welfare economics models are explored by utilizing a set-valued optimization framework, and various notions of optimality and optimality conditions more relevant to the considered economic models are given. The chapter begins with multiple models of welfare economics. Several critical generalizations of the so-called second fundamental theorem of welfare economics, which ensures the existence of marginal prices supporting local Pareto-type optimal allocations, are given. The chapter provides extended versions of the second welfare theorem for local strong, strict, and weak Pareto optimal allocations of the nonconvex economy.

To provide an accessible introduction to variational analysis, a significant portion of the results is given in a finite-dimensional setting. The monograph contains 331 exercises which are designed to help readers acquire a deep understanding of the material. These exercises provide hints to infinite-dimensional extensions and technical details of the given results, and they also contain some open problems and conjectures. Each chapter concludes with commentaries providing interesting information on the historical development of the subject, the sources of the presented material, and possible extensions. The bibliography comprises 790 entries. A significant part of the material presented in this monograph is based on the research of the author, who is not only one of the leading experts in the dynamic field of variational analysis but also a very active and passionate researcher.

In conclusion, this exceptionally well-written research monograph, *Variational Analysis and Applications*, by Boris S. Mordukhovich presents material which is of profound importance and is meticulously chosen and organized. The mathematically precise and crisp presentation style would undoubtedly help the reader in acquiring a deep understanding of the subject. I believe that this monograph would not only be of great benefit to the research community actively engaged in variational analysis but would also encourage newcomers who are willing to learn the subject.

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Akhtar A. Khan
Rochester Institute of Technology
Rochester, NY

A Course in Networks and Markets. *By Rafael Pass.* MIT Press, Cambridge, MA, 2018. \$55.00. xvi+246 pp., hardcover. ISBN 978-0-262039-78-9.

The world has always been interconnected. What has changed over the past few decades is our ability to quantify interactions between objects and entities, as well as the available computational power to analyze the resulting data. Adequate mathematical tools are essential to make such analysis possible, and for its results to be interpretable. The tools that have been developed for this purpose fall under the general heading of network science—a field that has gained considerable popularity over the past 20 years.

The aim of network science is to describe, quantify, and explain the web of interactions many of us experience daily. For instance, the flow of traffic, information in social networks, and money across financial institutions can be described using stochastic or deterministic processes on graphs. A first course on network science thus provides an excellent way to introduce advanced undergraduate and beginning graduate students to a number of topics in mathematics and statistics, and their applications to biology, sociology, economics, and other fields. Indeed, by now there a number of excellent textbooks that can be used for this purpose: Some focus on the statistical structure of networks [1, 4], while others describe applications of network science to economics [3, 2], the analysis of social networks [5], and other fields.

David Easley and Jon Kleinberg's book *Networks, Crowds, and Markets* (EK) has been particularly popular. It offers a gentle introduction to a plethora of fascinating topics at the intersection of network theory and economics and is appropriate for a class of nonmathematics majors with some background and interest in mathematics. Unfortunately, this popular approach makes EK by itself less than ideal for a course aimed at students of mathematics or mathematical economics.

The book *A Course in Networks and Markets* (RP) by Rafael Pass is designed precisely for this audience. While the topics presented in RP are a subset of those presented in EK, they are treated in more mathematical depth. Pass develops an abstract approach describing the interactions between rational agents whose loss or gain depends on their collective choice of actions. In most of the book, this is framed in the setting of game theory. Using only the assumption that the interacting agents act rationally to maximize their utility, RP proves a number of important results explained intuitively, or proved less generally, in EK. The book thus complements and extends EK by providing an abstract foundation for the analysis of networks of interacting rational agents.

The main strength of RP is one of the main strengths of mathematics in general: A proper set of definitions and a careful framing can reveal common principles between seemingly different problems. This approach allows for techniques and ideas to be applicable across different settings. For instance, RP shows that the flow of information in a network of interacting agents, the development of traffic jams, as well as trades and auctions can be analyzed using a common, abstract framework. RP also provides a good mathematical introduction to voting paradoxes. While other mathematicians have written excellent books on the topic, the approach taken here shows how voting can naturally be viewed in a game theoretic setting. Using the same general framework, RP covers web searches, the wisdom and foolishness of crowds, as well as the notion of common knowledge.

There are a few issues with the book that anyone considering adopting it as a textbook should be aware of: The book contains no problems or exercises. While EK does provide a selection of problems on related topics, these are far too simple for an advanced course. Thus anyone adopting RP as a textbook will have to develop their own set of exercises, or look for them in other books. Moreover, the notation is not always consistent. For instance, some vectors are denoted using an arrow ($\vec{a}$), while others are not (a_{-i}). There are also a number of typos that are perhaps unavoidable in a first edition.

A missed opportunity in EK and RP is the lack of discussion of computational methods, and the lack of computational examples and exercises. I found that a course on network theory and economics provides an excellent opportunity to introduce students to computational and applied statistical techniques. For instance, real social network data can be used to demonstrate that your friends, on average, have more friends than you do. Many others have used a similar approach in their network science courses, and we have shared data sets, exercises, and ideas. It may be a good time to collect these disparate resources and make them widely available.

Who, then, is this book for? The abstract framework which is the main strength of the book unfortunately also limits its audience. The book could be used to supplement EK. However, the interrelatedness of the ideas and approaches means that presenting only selected topics from RP should be done with care. Overall, despite some minor flaws, RP provides a good introduction to game theory, as well as topics in mathematical economics and network science. Motivated undergraduates, beginning graduate students, and nonspecialists with an interest in these fields will find it informative, interesting, and rewarding.

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Krešimir Josić
University of Houston

Mathematical Modelling and Biomechanics of the Brain. *By Corina Drapaca and Siv Sivaloganathan*. Springer, New York, 2019. $109.99. x+155 pp., hardcover. Fields Institute Monographs. ISBN 978-1-4939-9809-8.

Most of us probably know of Phineas P. Gage (1823–1860), if not by name, at least his story, which is arguably the most well-known case of traumatic brain injury. Gage was an American railroad construction foreman remembered for his improbable survival of a horrific accident. A large iron rod was driven completely through his head, destroying much of his brain's left frontal lobe. That injury reportedly had profound effects on his personality and behavior over the remaining 12 years of his life. Indeed, friends saw him as "no longer Gage." It is fair to say that Gage's case greatly influenced the 19th-century discussion on the brain and how injuries to specific areas of the brain could induce mental changes.

The human brain has continued to be the subject of extensive investigation. Much of the investigation aims at understanding its behavior and function. Despite plentiful evidence that mechanical factors play an important role in regulating brain activity, current research efforts focus mainly on the biochemical or electrophysiological activity of the brain. In this sense, the monograph by Drapaca and Sivaloganathan, *Mathematical Modelling and Biomechanics of the Brain*, provides an exceptional perspective by focusing on the biomechanics of the brain.

The authors discuss the use of mathematical models and the applications of fundamental concepts from continuum mechanics in the analysis of brain biomechanics. The manuscript focuses on areas of brain biomechanics in which the authors themselves (and their students) have been engaged for the past two decades. It begins with an introductory chapter reviewing relevant areas of continuum mechanics and its extensions. Subsequent chapters range from one focused on mechanical models of hydrocephalus to a subsequent chapter on models of traumatic brain injuries, aneurysms, and strokes. The final chapter discusses a number of models of tumor growth and provides concluding remarks that reinforce the purpose of the monograph.

Taken together, the monograph provides a focused, albeit brief, introduction to a vibrant, interdisciplinary, and rapidly growing area of research, with a target audience of applied mathematicians, engineers, and theoreticians and clinical scientists and experimentalists. The chapters have the excellent feature of being essentially self-contained, adding suggestions, longer term aims, and a comprehensive set of references, which provides an ideal launching pad for those interested in delving further into these research fields. In terms of background preparation, it is suitable for students at the graduate level with an applied mathematical or engineering undergraduate training background who wish to move into interdisciplinary research in neurosurgery, neurosciences, bioengineering, biomechanics, and neuroengineering.

Below I will go into the subjects of the key chapters in more detail. Chapter 2 deals with the classical theory of continuum mechanics. The presentation is in terms of its rigorous, modern development based on Noll and Truesdell's axiomatic framework, which permits a unified study of deformable materials. As mentioned by the authors, in the mathematical description of material response to mechanical loading, there are two important basic assumptions which form the foundation of continuum mechanics: (1) The mechanical stress at a given material point at time t is determined by the past history of the deformation in a neighborhood of the considered point, i.e., the principle of determinism and local action. (2) The response of a material is the same for all observers, which is known as the principle of material objectivity. Now these principles are too general to properly characterize the nature of specific materials. Consequently, further simplifications of the relationship between mechanical stress and deformation are necessary. Such simplifications arise, for instance, from assumptions of infinitesimal deformations or for finite deformations, that a material is simple, homogeneous, and nonaging, has preferred directions of deformation, and experiences internal constraints like incompressibility, inextensibility, and rigidity. The chapter provides a

Fig. 1 *Phineas P. Gage and his "constant companion," his inscribed tamping iron. Photo originally from the collection of Jack and Beverly Wilgus, and now in the Warren Anatomical Museum, Harvard Medical School.*

review of these concepts, as well as specific constitutive laws that have been used in brain research. In addition, there is a discussion of some modern theories that generalize classical continuum mechanics and may prove very useful in future studies of brain biomechanics.

Chapter 3 tackles the mechanics of hydrocephalus. Apparently, hydrocephalus is a serious neurological disorder suffered by mankind since time immemorial. One out of every 1,000 babies is born with hydrocephalus, making it as common as Down syndrome and more common than spina bifida or brain tumors. The disorder is characterized by an abnormal accumulation of cerebrospinal fluid in the brain's ventricles and in pediatric cases often by an increased intracranial pressure. Currently, the general consensus in the medical community is that hydrocephalus is a heterogeneous group of disorders, rather than a single disease entity, and therefore the pathophysiology of hydrocephalus is much more complex and obscure than the clinical or radiological presentation of hydrocephalus, going beyond simply ventricular dilatation of the brain. Together with gross macroscopic changes, hydrocephalus results in significant changes to the brain tissue, not only of its morphology, but also its dynamics, biochemistry, metabolism, and maturation. Successful treatment does not always reverse the injuries caused by hydrocephalus; early therapeutic intervention plays a crucial role in determining the reversibility of lesions and, hence, the overall outcome. Realistic biomechanical models of hydrocephalus can advance our understanding about the pathophysiology of hydrocephalus and play an important role in predicting its evolution as well as the outcome of its treatment. This is an area where the authors and coworkers have made extensive contributions and hence are in an ideal position to provide a panoramic view of the state of the field. The authors provide some basic facts about brain anatomy and mechanisms involved in the onset and evolution of hydrocephalus and review some of the existing mathematical models of hydrocephalus.

Chapter 4 is focused on the modeling of traumatic brain injuries, cerebral aneurysms, and strokes—which the authors point out are among clinical conditions with the highest rates of fatality or long-term disability. A frightening fact: an estimated six million people in the United States have an unruptured brain aneurysm, or 1 in 50 people. To ameliorate these somber statistics, the authors note that it is of great importance to understand how mechanical trauma to the head causes brain injuries, and what the characteristic signatures of the onset of aneurysms and strokes might be. While aneurysms and strokes are caused by abnormalities in the complex chemo-mechanical interactions between cerebral blood flow and the vasculature, traumatic brain injuries can damage vasculature and brain cells not only locally, but also more globally through the functional network established among neurons. The authors review some of the mathematical models of these conditions that have appeared in the literature.

In Chapter 5, the authors discuss tumors of the central nervous system based on histology, molecular mechanisms, rate of brain invasion, and a soft tissue-type grading system. They point out that the classification of various benign and malignant brain tumors can be developed and improved by the integration of *in silico*, *in vitro*, and *in vivo* studies of brain tumors, ultimately leading to better diagnosis, treatment protocols, and, hopefully, outcomes. The authors provide a review of some of the modeling approaches proposed in the literature to predict tumor growth and therapeutic outcome.

Mathematical Modelling and Biomechanics of the Brain by Drapaca and Sivaloganatha provides a useful entry into the field of brain biomechanics. The self-contained chapters with extensive references are ideal for beginning graduate students, or for established researchers looking to move into this exciting interdisciplinary field of research. In conclusion, it is a welcome addition to the biomathematics literature.

Anita T. Layton
University of Waterloo

Estimation and Control of Dynamical Systems. *By Alain Bensoussan*. Springer, Cham, 2018. $139.99. xii+547 pp., hardcover. ISBN: 978-3-319-75455-0.

This is no ordinary book. Neither textbook nor monograph, it is a guide to the world of deterministic and stochastic control, leading the reader from classical results to the most recent methods through the eyes of one of the founders of stochastic control theory. This view and the subjective choice of material make this book different. I have become accustomed to watching my Ph.D. students struggle to recreate a broader picture of stochastic control from research papers and highly technical monographs. This book can change that.

Although the title does not reveal it, this book is mostly concerned with deterministic and stochastic control. Deterministic control theory is often used to introduce methods which are then applied to study stochastic problems. The presentation starts from explicitly solvable linear-quadratic problems (with full and partial observation) and then moves to general methods for nonlinear systems. Principal agent problems and differential games are covered in detail in final chapters. “Estimation” in the title refers mostly to classical statistical techniques (both static and dynamic) but presented with a control twist (this part of the book can be read without any prior knowledge and interest in control methods). Stochastic control with partial observation is limited to linear-quadratic systems.

1. Estimation. The title could suggest that the presentation of estimation techniques is geared towards control problems. This is only partly true. Stochastic control problems with partial information are only studied in the linear-quadratic framework; i.e., the state is modeled as a linear system with Gaussian noise, and the functional is quadratic in the state and control. Broader treatment of partially observed controlled systems would be very welcome, but the reader can always consult another book by the same author [1], where they will find the theory presented with complete proofs as well as the general theory of control with partial observation.

The innovations approach in the framework of discrete-time Kalman filters is beautifully presented in section 4.7, explaining some of the magic that many feel when faced with the classical introduction of the approach in continuous time (section 7.1). Most of Chapter 4 is a crash course in frequentist and Bayesian statistics which, in my opinion, is too concise to learn the material: a reader unfamiliar with the basics of mathematical statistics may need to consult a statistics textbook. Chapter 5 covers mostly generalized linear models (GLMs) and can be skipped by readers interested in optimal control. Sections 5.1–5.4 present classical material commonly found in statistical textbooks. However, in section 5.5 one finds a dynamic version of GLMs where the parameter of interest evolves over time. Here, one needs to use approximations (or rather postulate a certain form of conditional distribution)—a very pragmatic approach. I was excited to see such practical thoughts (one can find more of them in Chapter 4) which are rarely seen in the world of stochastic control theory but often crucial for real applications of this theory.

2. Control of Linear Systems. Chapters 2 and 3 present classical problems in control of deterministic linear systems, i.e., systems of the form $x'(t) = F(t)x(t) + G(t)v(t)$, where F, G are known matrix-valued functions and v is the control variable. It is interesting to read about controllability and observability problems before turning to optimal control problems, which are more familiar to the stochastic control audience. In those problems, the functional to be minimized is of linear-quadratic form where the running cost is a quadratic function of the state and control, and the terminal cost is quadratic in the terminal state. This theory is notationally cumbersome, but a lot can be derived explicitly without resorting to general results, thus providing useful insights. An infinite horizon problem akin to controllability is discussed in depth.

In the stochastic version the state is perturbed by a Brownian motion, i.e., in the simplest case, it satisfies the stochastic differential equation $dx(t) = \big(F(t)x(t) + G(t)v(t)\big)dt + dW(t)$, where $(v(t))$ is a stochastic process adapted to the filtration generated by the Brownian motion $(W(t))$. The objective is the expectation of the above-mentioned quadratic functional. In Chapter 8, the author builds on intuitions developed for the deterministic problem to show tricks that apply in several types of stochastic problems, including a risk-sensitive functional in section 8.3 (called "exponential-of-integral payoff" in the book).

Chapter 9 in the only place where the reader will see the interplay between control and estimation. The state $x(t)$ is not directly observable by the controller. Instead, all the control actions must be based on observations of another linear process whose dynamics depends on $x(t)$. The book introduces a fundamental concept of separation principle which states that the solution of the problem can be split into two phases: filtering and then control of a fully observable system. Here, again, the linear form of the state dynamics allows for explicit methods of solution, and relevant "tricks" are nicely presented.

It might seem that the framework of linear-quadratic control is all but dead from the research perspective. Just the opposite. This is a model of choice for new control problems such as mean-field games, control with non-Gaussian noise (e.g., fractional Brownian motion or Rosenblatt process), and principal-agent problems.

3. Control of Nonlinear Systems. There are two broad methods for solution of optimal control problems: Pontryagin's maximum principle and dynamic programming. The first formulates necessary conditions for optimality which, only under additional assumptions (such as convexity), become sufficient. Dynamic programming, on the other hand, is aimed at providing sufficient conditions. The book follows a well-trodden path of explaining those two concepts for deterministic systems in Chapter 10 before moving to stochastic systems in Chapter 11. Apart from discussing the maximum principle and dynamic programming equation (Hamilton–Jacobi–Bellman equation, or HJB equation), it also directs the reader's attention to more delicate issues. The link between the two concepts is intuitively explained (more formal presentation can be found, e.g., in Chapter 5 of [4]). I like very much that the maximum principle is related to the Gâteaux differential of the functional at the optimum, providing a first-order condition interpretation of the procedure.

The treatment of stochastic systems involves a lot of technicalities. Instead of delving into details, the author replaced many proofs with sketches or intuitive derivations. These highlight main ideas which are often less clear in formal treatments and allow the reader to absorb a large number of sophisticated techniques in a relatively short time. Topics covered include a weak approach to stochastic control (via Girsanov transformation), the relationship to backward stochastic differential equations (BSDEs), and viscosity solutions to deal with cases when the value function is not smooth enough to apply the (generalized) Itô formula. The latter theory is vast and covered in an excellent monograph [3], but the present book provides sufficient details to allow the reader to appreciate this methodology and understand its usefulness. It should be noted that the presentation of the HJB approach is limited to problems in which control affects the drift only. The interested reader can find more general results, e.g., in [4].

BSDEs appear in two areas described above: giving dynamics of the adjoint process in the stochastic maximum principle and in weak formulation of the dynamic programming approach. There is a further link with nonlinear parabolic PDEs (with nonlinearity in the first derivative). The existence and uniqueness of solutions to such PDEs is proved in the first part of Chapter 12. It is worth mentioning that the PDEs appearing in stochastic analysis are usually defined on the whole space, while the classical PDE theory is concerned with boundary value problems. Solutions to BSDEs with a driver of quadratic growth, which are linked to the above PDEs, are explored in section 12.3. A reader unfamiliar with this theory may need further reading to appreciate the results as some technical details are omitted, e.g., in which sense to interpret the solution of the PDE, or under what conditions the link between BSDEs and PDEs is valid.

In Chapter 13 the reader will find a presentation of techniques for explicit solution of optimal control problems motivated by financial applications. These problems involve control of the diffusion term (not covered in the theoretical part of the book) or problems of control and stopping. An interesting study of non-Markovian stochastic control problems is presented in Chapter 14. There, the reader will appreciate in more details the difference between strong and weak formulation of the problem as well as the use of backward stochastic partial differential equations.

4. Principal Agent Problems, Differential Games, and Target Problems. The last part of the book, Chapters 15–18, is concerned with problems which are related to control or which use control techniques. Some of the theory is relatively new (developed in the last 10 years). The coverage is very wide. The presentation is centered on ideas and general mathematical techniques, with assumptions largely left out. The reader should not expect to find theorems ready to cite in their work but will benefit from a coherent and uncluttered view of various difficult problems in applications of control theory.

In the framework of differential games, there are multiple concepts of equilibria depending on the information available to each player. Open-loop equilibria are related to Pontryagin's maximum principle, while closed-loop equilibria are related to Bellman principle and HJB equations. Examples of non-zero-sum and zero-sum games are presented, including the linear-quadratic case. The reader will also find a deep discussion of the case when Issac's condition for a zero-sum game fails: this gives rise to a duality gap between an upper and a lower value of the game. Interpretation of those values as values of the game with different information structure available to players is provided. Solution techniques for stochastic games are presented as generalizations of those for deterministic games. Chapter 17 discusses Stackelberg differential games in which players are no longer equal. In Stackelberg games there are two players: a leader and a follower. The follower chooses his strategy in response to the leader. There are multiple variations of this response mechanism which are cleanly presented in the book. I found it very helpful to see examples which showed how each of these different mechanisms affects players' strategies.

Principal-agent problems also feature two players, but the principal does not control the underlying dynamics directly. Instead, he offers a contract (remuneration) to the agent to steer him toward applying controls favored by the principal. The contract itself does not have to be of feedback form, so a direct application of stochastic control methods is not possible. The approach of [2] is deeply rooted in BSDE methods, while the presentation in this book tries to offer a more direct analytical approach. To appreciate it fully the reader should be familiar with the aforementioned monograph.

The last chapter of the book shows a small selection of hedging problems on the Black–Scholes market: superreplication with convex constraints and quantile hedging. To fully understand the material, the reader must already be familiar with continuous-time contingent claim pricing beyond what is discussed in Chapter 13. The solution of a quantile hedging problem is particularly worth mentioning because the author uses elegant probabilistic arguments.

5. Conclusions. This book is a great resource for graduate students and those who want to learn and understand stochastic control theory. It is also a great read for experts who want to gain a broader overview of the subject and wish to see connections between different techniques. The book is not an ultimate reference with the strongest statements of all results and complete proofs. I concur with the author that it is better to present ideas clearly rather than have proofs mired in technicalities. What could be further expanded is an introduction to each chapter on its content and how it relates to the rest of the material in the book to help readers see the broad picture and the connections between different topics and techniques. Nevertheless, this is an excellent book and a great complement to the current offering in stochastic control.

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Jan Palczewski
University of Leeds

Stochastic Modelling for Systems Biology. *By Darren J. Wilkinson.* Chapman & Hall/CRC Press, 2018. $119.95. 384 pp., hardcover. ISBN 978-1-138549-28-9.

This book describes the mathematical methods for the modeling, simulation, and statistical inference of biological systems encountered primarily in the fields of systems and synthetic biology, biophysics, physical cell biology, and similar disciplines. The primary focus of the book is the stochastic modeling of these systems, as randomness and noise are essential to the description of biological processes at the microscopic (molecular) and mesoscopic levels.

The book is divided into four parts. Part I is an introduction to the modeling of biomolecular systems, or biochemical networks, defined by chemical species and reactions among them. Different representations of biochemical networks that can be found in different contexts, such as graphical representations (Petri nets) and representations using Systems Biology Markup Language, are given here. Part II of the book provides a background in probability theory and Markov processes needed for the methods of stochastic simulation and inference discussed in the rest of the book. Emphasis is on the properties of standard distributions (discrete and continuous) and various techniques used for random number generation from these distributions. Part III, stochastic chemical kinetics, describes the theory behind the stochastic simulation methods that are most commonly used to simulate biochemical networks. The Gillespie algorithm and the next reaction method along with their extensions are highlighted. A basic introduction to the extension of the stochastic simulation methods to the spatially extended systems is provided, which proves to be a good starting point for the modeling of systems where spatial effects are of interest. Part IV of the book is concerned with Bayesian inference methods for stochastic models of biochemical networks. In this part, different inference approaches are briefly introduced and some examples are given.

For Parts I through III, an introductory background in probability theory, such as an undergraduate-level course, will be helpful. However, a reader with a good background in probability theory and Markov processes at the level of an advanced undergraduate or a graduate course will find some other parts of the book more approachable. In particular, the section about inhomogeneous Poisson processes and diffusion processes in Chapter 5 is theoretically more advanced. Similarly, the material in Part IV will be easier to follow for a reader with some familiarity with Bayesian inference methods. A background in biology and biological modeling is not required. Some introductory background in programming will be very helpful in order for the reader to readily understand the provided codes.

For the numerical implementation of the different techniques discussed, the book provides code that is written in the free statistical programming language R. This high-level language has the advantage of easy readability at the expense of slower run speed for larger and more complex systems. However, the transparency offered by R will allow one to implement the same ideas in other programming languages. The R programs that are provided in the text and the accompanying packages that can be downloaded from the book's website are very helpful for the reader in order to understand the advantages and limitations of different algorithms and techniques. Either in R or adapted to a different programming language, one can use the provided code interactively to test the algorithms and compare them against each other. One can even use the provided code, with small adaptations as necessary, to investigate one's own mathematical models. However, for large and complex systems which arise in practical problems, one may need to use a high-performance language as noted in the book. The motivated reader should be able to learn the basics of the R programming language by implementing the algorithms in this book. However, the book does not provide an introduction to R, and the uninitiated reader will have to consult other online documentation as necessary.

One of the good aspects of the book is that it covers a range of techniques for modeling, simulation, and inference of stochastic systems used for systems biology models. Some of these techniques are still being actively developed. The book introduces the the main ideas behind each technique, but implementing and exploring them using the provided R code may be necessary to truly learn some of these methods. For most of the methods discussed, the book also gives appropriate references for further reading.

Some algorithms and techniques discussed in the book, especially the material related to Bayesian inference in Chapters 10 and 11, are described only briefly. This makes it difficult for the reader to understand these methods without referring to the original literature. In some cases, the mathematical derivations given for a specific technique are insufficiently explained. For instance, a novice would appreciate more details in Chapter 11, which describes Bayesian inference for stochastic chemical kinetics.

Overall, I would recommend this book to readers interested in mathematical modeling of biological systems, in particular stochastic methods, who want to learn about different existing approaches, their basic ideas, their effective use, and their numerical implementation. To learn the theory of stochastic processes and methods of their analysis, I would recommend one of the many classic textbooks on the subject. This book may be a good choice for a graduate-level course where the emphasis is on modeling and (stochastic) simulation of biological systems. The students would also benefit from other textbooks that could be used in conjunction with this book.

MEHDI SADEGHPOUR
University of Houston

A First Course in Systems Biology. Second Edition. *By Eberhard O. Voit.* Garland Science, New York, 2017. $99.95. xii+468 pp., softcover. ISBN 978-0-8153-4568-8.

In the past two decades, there has been a growing interest in systems biology, with an ever-increasing number of publications on the topic. A search for "systems biology" on www.amazon.com yielded over 1,000 titles. The philosophy underlying systems biology is that the whole of a living system cannot be completely understood by the study of its individual parts. That seems intuitive enough. Yet the answer to the question "What is systems biology?" proves to be quite elusive. If you ask five biomedical researchers this question, you may well get 10 different answers! This diversity of opinions may be due largely to the wide range of disciplines involved and to the varying emphasis placed on the computational modeling and experimental aspects of systems biology. So, you can imagine the challenges in writing a textbook in systems biology that appeals to a wide audience. It brings to mind the blind men and an elephant.

A First Course in Systems Biology has done an excellent job illustrating the elephant. The book presents the origins, concepts, tools, state of the art, and future directions in systems biology research. Indeed, it is a powerful introductory text written by one of its internationally recognized leaders, Eberhard O. Voit, a professor and David D. Flanagan Chair in Biological Systems at Georgia Tech. Several areas addressed in the book have been advanced by the author's own research in statistical biology and metabolic engineering, which obviously adds to the book's overall value.

The fifteen chapters cover a variety of biological topics, including genes, proteins, metabolism, population dynamics, physiology, and medicine. Presentation of the materials is well balanced and accessible, often highlighting the connection between modeling and experimental data. As such, the book is suitable for readers with either a mathematical or biological background. Different chapters weigh differently on the mathematics versus biology scale, and some are more advanced than others. The reader can choose between more introductory or more advanced levels of exposition. For example, Chapter 5, "Parameter Estimation," can serve as a helpful reference for an experienced systems biologist. In contrast, Chapter 6 "Gene Systems," which begins with a description of the basic building blocks of life and then proceeds to discuss current experimental methods of gene expression analysis, is accessible even for beginners. Besides parameter estimation in Chapter 5, modeling techniques covered include graph theory, machine learning, and differential equations.

Many systems biology texts focus heavily on the mathematics. While most SIAM readers will no doubt find the math interesting, many biologists may feel overwhelmed by the technical details. This text distinguishes itself by its attempt to address the whole expanse of systems biology, with a minimalist mathematical approach, and a more practical perspective on real-world application.

If you are a biologist looking for a first look at systems biology that does not assume you dream in bifurcation and Lyapunov exponents, look no further. The book is nicely illustrated with many well-designed figures—and in color too!—with just enough math to do the job. If you prefer diving into the mathematical abyss, however, there are plenty of other systems biology texts for you.

A First Course in Systems Biology is the perfect book for an introductory upper division undergraduate or preliminary graduate course in systems biology, or a biologist new to the application of dynamical systems theory to biology.

MEHRSHAD SADRIA
ANITA T. LAYTON
University of Waterloo

Modeling, Analysis and Control of Dynamical Systems with Friction and Impacts. *By Paweł Olejnik, Jan Awrejcewicz, and Michal Fečkan.* World Scientific, Singapore, 2018. $98.00. xiv+262 pp., hardcover. ISBN 978-981-3225-28-2.

A double torsion pendulum, billiards on a sloped table, and the control of a unicycle: a sample of the curious problems brought under consideration in this book.

The aim of the book is to show how the dynamics of systems with friction and/or impacts can be understood. These features are ubiquitous in mechanical components such as robotic joints, gear boxes, braking and clutch systems, and drilling and cutting tools. Friction and impacts are highly nonlinear, hence often the primary driver of complex dynamics including chaos. While friction is usually unavoidable, impacts are typically not part of the desired operating behavior as they tend to cause damage and wear. Yet the mathematicians among us will be pleased to know that a detailed bifurcation analysis of impacting solutions is still highly valuable, particularly if it reveals the existence of a coexisting stable impacting solution that the system may transition to via a sudden disturbance or random event.

This book uses the classical dynamical systems methodology of first modeling, primarily with ODEs, then analysis, via bifurcation identification and continuation. For friction and impacts, both aspects are fraught with unique challenges and complications. The modeling of friction in particular is an incredibly complex and subtle topic. In some cases a simple variation on the Coulomb friction law suffices; more sophisticated modeling is usually needed to accommodate lubrication films, surfaces with cracks, or significant deformations, for example. Similarly, an appropriate model of impacts depends on the materials involved.

On the analysis side, the models typically involve discontinuities, switches, or jumps corresponding to impacts and stick-slip transitions. Hence the models are piecewise-smooth or hybrid systems, and their analysis requires knowledge of grazing bifurcations and other bifurcations unique to nonsmooth systems.

The book begins with a review of friction modeling. At over almost 50 pages, this review is extremely comprehensive, taking us through history from Leonardo da Vinci to a wide of variety of recent studies. The remaining 13 chapters of the book consider particular problems in detail. Mostly these are not full mechanical systems, but rather "toy" systems that display the full effects and consequences of friction and/or impacts yet are simple enough that they admit a reasonably transparent and accessible mathematical analysis.

Overall the book is best suited to those with some prior experience dealing with models of mechanical systems. The book skips the simplest one-degree-of-freedom mass-spring and pendulum-type systems, described in related books such as [1, 2, 3, 4, 5], and dives straight into more complex phenomena. The mathematical analysis is achieved via many methods, such as sliding motion of Filippov systems, Melnikov functions, and the construction of maps to describe evolution between switching events. Again the book skips a review of the basic mathematical concepts of such discontinuous systems; these can be found in books with a more mathematical emphasis [6, 7, 8, 9].

While an overarching theory for friction and impacts is not provided, the collection of selected problems offers a nice survey of the field. I particularly appreciated the continued connections between experimental observations and mathematical theory, which I think serves as a good example for other areas of applied mathematics.

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David J. W. Simpson
Massey University, New Zealand

Linear Algebra and Learning from Data.
By Gilbert Strang. Wellesley-Cambridge Press, 2019. $95.00. xiv+432 pp., hardcover. ISBN 978-0-692196-38-0.

Imagine the following scenario: Like many other universities worldwide, your university has created a new Master's program in data science and you are now considering how to design a one-semester introductory course in mathematical fundamentals for a quantitatively oriented but otherwise heterogeneous group of students. As a mathematician, you ask yourself what data science exactly is, what kind of mathematics plays a role, what is most important when there is not enough time, and how you should set up such a course.

It was exactly this problem that I faced a year ago and developed my own ideas, which focused on singular value decomposition and other topics of numerical linear algebra with small excursions to calculus for neuronal networks.

Now, however, this problem is completely and convincingly solved by Gilbert Strang in his most recent book on linear algebra and learning from data. Most readers will be familiar with Gilbert Strang through his many successful efforts in teaching mathematical foundations in several books and other media, in particular linear algebra. In this book, the grandmaster goes beyond linear algebra (Chapter I) and imparts further basics, such as dealing with large and special matrices (Chapters II and IV), low-rank approximation (Chapter III), basics of statistics and optimization (Chapters V and VI), and an introduction to neural networks (Chapter VII). The writing style is dense but catchy. The reader is addressed directly, almost like in the video lectures by the author, which can be found on the internet. Lecturers will enjoy the book because it presents exactly the right topics in an excellent and coherent way, and because there is a lot of background information and numerous important references to the literature and to links on the internet. I envylessly acknowledge that this book is considerably better and more comprehensive than my own script that I wrote for my course a year ago. Although the book contains rigorous proofs for numerous statements within the text, these are not highlighted in the layout. Instead of the usual keyword "theorem," boxes serve as a structuring element containing the important statements and remarks. For each chapter there is a set of exercises and smaller software projects. The book is ideally suited as a basis for a course on mathematical basics for data science and machine learning. It is advisable to move selected chapters to a subsequent course because the wealth of material conveyed in this book is large.

I recommend this excellent book without hesitation as a course foundation for all teachers and will use it in the next semester as an accompanying material for my own course. The only complaint I can make about the book is that it hasn't been published earlier.

Volker H. Schulz
Trier University