EDUCATION

This issue of *SIAM Review* presents two papers in the Education section.

The first is "Coin-Flipping, Ball-Dropping, and Grass-Hopping for Generating Random Graphs from Matrices of Edge Probabilities," by Arjun S. Ramani, Nicole Eikmeier, and David F. Gleich.

The theory of random graphs gained a lot of popularity with the rise of wireless technology, and especially with the increasing attention paid to social networks. Studies of existing networks are frequently concerned with the presence of motifs. These structures are defined as subgraphs or connectivity patterns that are found in the real network more frequently than expected, where the expectation is determined by the occurrence of the subgraph in random graphs. The authors compare extracting motifs from networks to the statistical test aimed at identifying an information pattern in a noisy signal. While statistical testing for goodness of fit to theoretical distributions is well established, a test on graphs lacks the theoretical probability distribution of the given motif. An empirical approximation of this distribution might be used when generating many random graphs to simulate the real network of interest. To this end, we need a way of generating random graphs very efficiently. In addition to social networks, random graphs are used also in performance evaluation and benchmarking of computer networks, as well as evaluation of algorithms.

In this paper, the authors explain and illustrate how to generate random graphs efficiently using the adjacency matrix. In this setting, the random graph model is described by a matrix of probabilities for each edge. They discuss the most widely used random graph distributions according to the Erdős–Rényi model, the Kronecker model, the Chung–Lu model, and a stochastic block model. Given a matrix of probabilities P, one may generate a random adjacency matrix A where the a_{ij} is 1 with probability p_{ij} by simulating biased coin flips. However, coin-flipping is inefficient and does not lead to sparse networks. Ball-dropping and grass-hoping are the names of other methods presented in this paper. The authors give pseudocodes for the methods, provide intuitive understanding, and discuss why the methods provide the correct distribution for the Erdős–Rényi model. The paper concludes with a number of problems, gradated from easy to research level, whose solutions are available in the supplementary material.

The paper is accessible for undergraduate students who have some knowledge of probability and discrete mathematics. It contains many relevant references, which can be used to study the topic or motivate the use of random graphs in various areas of applied mathematics and computer science. The computer codes are available through the github repository https://github.com/dgleich/grass-hopping-graphs.

The second paper, titled "Obligate Mutualism in a Resource-Based Framework," is written by Roger Cropp and John Norbury. It presents mathematical models for describing mutualist interactions in nature.

Mutualistic interactions between two species involves the exchange of goods or services. While each species involved in a mutualism does receive a benefit from the interaction, the two partners do not necessarily benefit equally. When each organism cannot survive without the other, scientists talk about an obligate mutualism: both organisms are obligated to rely on one another. Ecologists claim that mutualism is ubiquitous in nature, and naturally this phenomenon is a subject of intensive research.

The authors discuss mathematical models representing the dynamical systems of obligate mutualistic interactions in various cases. One example is that of "nurse plants" in local environments which facilitate the establishment of other plant species. Another example is the obligate pollination, which occurs between certain plants and their pollinators, usually insects or birds. Experts in the field recognize that obligate mutualism is not adequately represented by using the framework of linear dynamical systems. The authors discuss an approach which allows us to represent population interactions that vary from competition through mixotrophy and facultative mutualism to predation interactions, while maintaining stable population coexistence equilibrium points. This smooth continuum of population interactions can be extended to include the case of obligate mutualism. Such models require specific terms that describe the dependence of one population on another population and are more complex in nature than the simple Lotka–Volterra conservative normal models. Another key feature of the models, used in this paper, is the introduction of an explicit constraint on the growth of every population. This is accomplished by including a finite resource, which may be either a recycling limiting nutrient or another population. In the presented examples, the authors illustrate the dynamics of the species, and comment on the impact of parameters, equilibrium points, and insight gained from the model.

The paper is of interest to mathematics students who would be intrigued by applications of dynamical systems theory, as well as for ecology students who would like to learn advanced models for population dynamics.

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