EDUCATION

In this issue, the Education section of *SIAM Review* presents two papers. The first is "Dimensional and Scaling Analysis," written by Gjerrit Meinsma. This article is motivated by the fundamental use and the power of dimensional analysis in mathematical models of physical phenomena. In the introduction, the author writes that dimensional analysis "is easy to explain and fantastically useful in mathematical modeling, and yet it is not well appreciated in the mathematical community." In a mathematical model, any variable obtains a numerical value according to some systems of units (meters, grams, dollars, etc.). The numerical value of any physical quantity can be expressed in terms of the fundamental units: meters, kilograms, seconds, degrees Celsius, and amperes. For most quantities this is clear from the definition, while in other cases it needs to be deduced from the rule that all terms in a particular equation must have the same measurement units (dimension). A model is scale-invariant if it is independent of the system of units used to measure the quantities that appear in it.

The author formulates the famous Buckingham π-theorem, whose general idea can be summarized as follows. Suppose we consider a model containing variables $x = (x_1, \dots, x_k)$ and parameters $p = (p_1, \dots, p_m)$, in which n fundamental dimensions are involved. Then $k+m-n$ dimensionless quantities π_i can be defined, which are products and quotients of the original variables and parameters. Each scalar model equation (representing a physical law) can be reformulated in terms of π. Many interesting examples illustrate the application of the π-theorem. The author also discusses a scaling approach and presents a scaling π-theorem which makes fewer assumptions than the "physical" π-theorem.

The article is accessible to upper-level undergraduate students.

The second article is "Smooth Random Functions, Random ODEs, and Gaussian Processes," a collaborative work of Silviu Filip, Aurya Javeed, and Lloyd N. Trefethen. In this paper, the authors present in an accessible way the approach to defining paths of a Brownian motion by using smooth random functions.

In the first step, a periodic random function is described as a sum of Fourier modes with random amplitudes. The coefficients in the Fourier series are samples from a normal distribution with zero expectation. Two choices for the variance of that distribution are given. In the first case, the variance depends on the period-parameter $L > 0$ and the wavelength parameter $\lambda > 0$. In the second case, the random coefficients have variances (essentially) independent of λ, and the random functions obtained are called "big." Smooth random functions are periodic functions on an interval $[-L/2, L/2]$ which are entire and $2\pi/\lambda$ band-limited. An equivalent definition uses trigonometric polynomial interpolation through the random data, sampled from a corresponding normal distribution. Nonperiodic smooth random functions are obtained by using periodic functions on a longer interval and then truncating. When the parameter L grows to infinity the periodicity disappears. The notions are illustrated by examples referring also to the online collection at www.chebfun.org. As $\lambda \to 0$, indefinite integrals of big smooth random functions converge to the paths of the standard Brownian motion.

In a further step, smooth random functions are used as forcing functions or as coefficients in ordinary differential equations. The authors illustrate this idea on several modeling examples involving both additive and multiplicative noise. As $\lambda \to 0$, the solutions of the random ODEs containing a big smooth random function converge to solutions of Stratonovich stochastic differential equations.

The authors emphasize that the use of smooth random functions provides conceptual and computational simplicity but not necessarily computational efficiency (speed). Next to illustrative examples, the paper contains discussion on the origins of different ideas and developments in that area with many useful references.

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