SIGEST

In this section we present "Multilevel Markov Chain Monte Carlo," by T. J. Dodwell, C. Ketelsen, R. Scheichl, and A. L. Teckentrup. This is the highlighted SIGEST version of an article that first appeared in *SIAM/ASA Journal of Uncertainty Quantification* in 2015. Multilevel Monte Carlo (MLMC) is a means to improve the efficiency of Monte Carlo computations in the case where the samples come with discretization errors. It cleverly combines samples at different discretization levels (and hence with different accuracies and computational costs) using what statisticians might call a recursive control variates technique. As the name suggests, MLMC shares some similarities with the classical multigrid method in numerical PDEs:

- it exploits a hierarchy of discretization levels;
- it can offer dramatic improvements in efficiency over standard approaches;
- it is more of a general philosophy than a specific tool;
- making it work effectively on a new problem class typically requires new insights.

However, there are also key differences; most notably, there are no V or W cycles with MLMC—we only pass through the hierarchy in one direction. MLMC was first designed by Giles [30] in the context of numerical simulation of stochastic differential equations, with a view to option valuation in mathematical finance. Related work can also be traced back to Heinrich [41]. Since those seminal papers, the design, implementation, and analysis of MLMC methods for stochastic problems have become a very active field, with an order-of-magnitude improvement of computational cost achieved in a range of settings.

This SIGEST paper develops MLMC in the direction of inverse problems that are solved via Metropolis–Hastings Markov chain Monte Carlo methods. The authors present a very general methodology for multilevel Metropolis–Hastings in large-scale Bayesian inversion, along with a computational complexity analysis. As with many MLMC developments, a key challenge is to find a tight coupling for computations at neighboring levels in order to control the overall variance. Here the coupling involves parallel Markov chains. This general theory is then applied to a single-phase Darcy flow problem in groundwater modeling, with computational experiments confirming the relevance of the complexity analysis.

In producing this SIGEST article, the authors have extended section 1 in order to motivate their work and introduce relevant background material for a nonspecialist audience. They also discuss recent developments that have taken place in the area.

The Editors