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CORRECTION

Series B

Correction to: On the pervasiveness of difference-convexity in optimization and statistics

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After the publication of the DOI version of [4], the authors were alerted to two examples in the literature that show the value function of, respectively, a constraint-only [2], or an objective-only [3] perturbed (nonconvex) quadratic program (QP) may not be piecewise linear-quadratic, which forms a subclass of the class of piecewise quadratic functions. The latter class of piecewise functions is the principal conclusion of Part (c) of Proposition 9 in the paper [4]. These examples do not contradict the proof of the proposition but invalidate the statement of this part of the proposition. Below we present the correct statement of this part without repeating its proof. We use the same notations as in [4] throughout this erratum; in particular, "dc" stands for difference-of-convex.

Corrected Part (c) of Proposition 9 in [4]. Suppose that Q is copositive on D_∞. It holds that

(c) there exist a finite family $\mathcal{F} \triangleq \{S_F\}$ of polyhedra in $\mathbb{R}^{m+k}$ and finitely many quadratic functions $\{\text{qp}_F\}$ such that $\text{qp}_{\text{opt}}(q,b) = \min_{F:(q,b)\in S_F} \text{qp}_F(q,b)$; hence $\text{qp}_{\text{opt}}(q,b)$ is a piecewise quadratic function on $\text{dom}(Q,D)$. □

We should mention that the reference [1, pages 234–238] contains a fair amount of analysis of the value function $\text{qp}_{\text{opt}}(q,b)$, albeit for a fixed pair (q,b) only. In

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particular, [1, Theorem 3.129] is related to Parts (c) and (d) of [4, Proposition 9] for a fixed (q, b).

Another oversight in [4] is in the statement of Theorem 12. To apply Proposition 11, we need the value function $\mathrm{qp}_{\mathrm{opt}}(q, b)$ to be quadratic on each polyhedral piece S_F in the family $\mathcal{F}$ in Part (c) of Proposition 9. While this is true when Q is positive semidefinite, the statement of the theorem is incomplete without the required quadratic (as opposed to the min of quadratic) property. Below we present the correct statement of this theorem.

Corrected Theorem 12 in [4]. Suppose that Q is copositive on D_∞ and that $\mathrm{dom}(Q, D)$ is convex. Then the value function $\mathrm{qp}_{\mathrm{opt}}(q, b)$ is dc on $\mathrm{dom}(Q, D)$, provided that $\mathrm{qp}_{\mathrm{opt}}$ is a quadratic function on each polyhedral member in the family $\mathcal{F}$ in the above corrected Part (c) of Proposition 9 in [4]. In particular, $\mathrm{qp}_{\mathrm{opt}}(q, b)$ is dc on $\mathrm{dom}(Q, D)$, if Q is positive semidefinite. □

We should point out that while not satisfying the quadratic condition in the above theorem, the two numerical examples in [2,3] can be verified to be dc, by the mixing property (Corollary 10 in [4]). For the one in [3] the domain of the value function is $\mathbb{R}^2$; for the other one in [2], the domain of the function, which is restricted to the (closed) fourth quadrant in $\mathbb{R}^2$ in the reference, can be easily extended to an open convex set containing this quadrant.

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