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A new splitting preconditioner for the iterative solution of complex symmetric indefinite linear systems

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ABSTRACT

In this paper, we consider preconditioned simplified Hermitian normal splitting (PSHNS) iteration method for solving complex symmetric indefinite linear systems, analyze the convergence of the PSHNS iteration method and discuss the spectral properties of the PSHNS preconditioned matrix. Using discrete Sine transform (DST), we apply a fast algorithm to solve the subsystem during the preconditioning process. Numerical experiments arising from the Helmholtz equation show the effectiveness and robustness of the PSHNS preconditioner. In addition, the GMRES method with the PSHNS preconditioner demonstrates meshsize-independent and wavenumber-insensitive convergence behavior.

1. Introduction

Many scientific and engineering applications often require the solution of large and sparse complex symmetric linear system

$$Ax = b, \quad A \in \mathbb{C}^{N\times N} \text{ and } x, b \in \mathbb{C}^N. \tag{1}$$

The nonsingular coefficient matrix A can be rewritten as:

$$A = W + iT, \tag{2}$$

where $W,\ T \in \mathbb{R}^{N\times N}$ are real symmetric matrices, and $i = \sqrt{-1}$ denotes the imaginary unit. More detailed backgrounds and additional references can be found in [1–4].

Recently, a variety of splitting iteration methods have been developed to solve the complex symmetric linear system $(W+iT)x = b$, where $W, T \in \mathbb{R}^{N\times N}$ are symmetric positive semi-definite matrices, and at least

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one of them is positive definite. Based on the special structure of the coefficient matrix A and the Hermitian and Skew-Hermitian splitting (HSS) [5] iteration method, Bai et al. [6] proposed a modified HSS (MHSS) iteration method. In order to accelerate the convergence rate of MHSS iteration method, they established the preconditioned MHSS (PMHSS) [7] iteration method. In [8], Hezari et al. presented the preconditioned generalized SOR (PGSOR) iterative method to solve the block two-by-two real equivalent form. Numerical results have shown that the PGSOR iteration method can lead to better computing efficiency than MHSS iteration method.

After discretizing the Helmholtz equation with the second-order finite difference scheme, we need to solve the problem $(W+iT)x=b$ with real symmetric indefinite matrix W and symmetric positive definite matrix T. As the matrices $\alpha I+W$ and $\alpha V+W$ may be indefinite and singular, thus the MHSS and PMHSS iteration methods converge slowly or stagnate. More recently, motivated by the ideas suggested in [9] and [10], Wu [11] developed the HNS iteration method and simplified HNS (SHNS) iteration method for such problem. The SHNS iteration method is written as follows:

Algorithm 1. The SHNS iteration method

Let $x^{(0)} \in \mathbb{C}^N$ be an arbitrary initial guess. For $k=0,1,2,\ldots,$ until the sequence of iterates $\{x^{(k)}\}_{k=0}^{\infty}$ converges, compute the next iterate $x^{(k+1)}$ according to the following procedure:

$$\begin{cases}(\alpha I+iW)x^{(k+\frac{1}{2})}=(\alpha T-W^2)x^{(k)}+i\alpha b,\\ (\alpha T+W^2)x^{(k+1)}=(\alpha I-iW)x^{(k+\frac{1}{2})}-i\alpha b,\end{cases} \tag{3}$$

where α is a given positive constant.

In this paper, to accelerate convergence rate, analogous to the deriving techniques discussed in [7], we establish a preconditioned SHNS (PSHNS) iteration method and construct a new preconditioner, named as the PSHNS preconditioner. Furthermore, we show that the PSHNS iteration method is unconditionally convergent and investigate the spectral properties of the preconditioned matrix. To decrease CPU times, we apply the discrete Sine transform to solve the subsystem during the implementation process of the PSHNS preconditioned GMRES method. Numerical experiments arising from the Helmholtz equation illustrate the effectiveness of this approach.

2. The PSHNS iteration method and the PSHNS preconditioner

In order to accelerate convergence rate and enhance stability of the HNS or SHNS iteration method, it is very key and important to implement preconditioning technique. Let $V \in \mathbb{R}^{N\times N}$ be a given symmetric positive definite matrix and precondition the problem (1) as

$$\widetilde{A}\widetilde{x}=\widetilde{b}, \tag{4}$$

where $\widetilde{A}=V^{-\frac{1}{2}}AV^{-\frac{1}{2}}$, $\widetilde{x}=V^{\frac{1}{2}}x$ and $\widetilde{b}=V^{-\frac{1}{2}}b$. It is easy to obtain

$$\widetilde{A}=\widetilde{W}+i\widetilde{T}, \tag{5}$$

where $\widetilde{W}=V^{-\frac{1}{2}}WV^{-\frac{1}{2}}$ and $\widetilde{T}=V^{-\frac{1}{2}}TV^{-\frac{1}{2}}$ are real symmetric indefinite matrix and symmetric positive definite matrix, respectively.

Applying the SHNS iteration method to the preconditioned linear systems (4) and (5), we have

$$\begin{cases}(\alpha I+i\widetilde{W})\widetilde{x}^{(k+\frac{1}{2})}=(\alpha\widetilde{T}-\widetilde{W}^2)\widetilde{x}^{(k)}+i\alpha\widetilde{b},\\ (\alpha\widetilde{T}+\widetilde{W}^2)\widetilde{x}^{(k+1)}=(\alpha I-i\widetilde{W})\widetilde{x}^{(k+\frac{1}{2})}-i\alpha\widetilde{b}.\end{cases} \tag{6}$$

Using the same approach suggested in [7], we can describe the PSHNS iteration method as follows.

Algorithm 2. The PSHNS iteration method

Given an arbitrary initial guess $x^{(0)} \in \mathbb{C}^N$, for $k = 0, 1, 2, \dots$, until the sequence of iterates $\{x^{(k)}\}_{k=0}^{\infty}$ converges, compute the next iterate $x^{(k+1)}$ according to the following process:

$$\begin{cases} (\alpha V + iW)x^{(k+\frac{1}{2})} = (\alpha T - WV^{-1}W)x^{(k)} + i\alpha b, \\ (\alpha T + WV^{-1}W)x^{(k+1)} = (\alpha V - iW)x^{(k+\frac{1}{2})} - i\alpha b, \end{cases} \tag{7}$$

where α is a given positive constant and $V \in \mathbb{R}^{N\times N}$ is a symmetric positive definite matrix.

After applying the straightforward derivations, we can reformulate the iteration scheme (7) into the fixed point form

$$x^{(k+1)} = M(V;\alpha)x^{(k)} + N(V;\alpha)b, \quad k = 0, 1, 2, \dots, \tag{8}$$

where $M(V;\alpha) = (\alpha T + WV^{-1}W)^{-1}(\alpha V - iW)(\alpha V + iW)^{-1}(\alpha T - WV^{-1}W)$ denotes the iteration matrix of the PSHNS iteration method and $N(V;\alpha) = 2\alpha(\alpha T + WV^{-1}W)^{-1}W(\alpha V + iW)^{-1}$. Assuming that the matrix V be commutable with the matrix W, and then the matrix WA can be split into

$$WA = B(V;\alpha) - C(V;\alpha), \tag{9}$$

where $B(V;\alpha) = \frac{1}{2\alpha}(\alpha V + iW)(\alpha T + WV^{-1}W)$ and $C(V;\alpha) = \frac{1}{2\alpha}(\alpha V - iW)(\alpha T - WV^{-1}W)$. Therefore, the PSHNS iteration scheme is induced by the matrix splitting (9). Moreover, the splitting matrix $B(V;\alpha)$ can be used as a preconditioner for the matrix $WA \in \mathbb{C}^{N\times N}$, named as the PSHNS preconditioner.

In particular, if we choose $V = W^2$, then the iteration scheme (7) reduces to

$$\begin{cases} (\alpha W + iI)\widetilde{x}^{(k+\frac{1}{2})} = (\alpha T - I)x^{(k)} + i\alpha b, \\ (\alpha T + I)x^{(k+1)} = (\alpha W - iI)\widetilde{x}^{(k+\frac{1}{2})} - i\alpha b. \end{cases} \tag{10}$$

Similar to the above deriving process, it holds that

$$A = B(\alpha) - C(\alpha),$$

where $B(\alpha) = \frac{1}{2\alpha}(\alpha W + iI)(\alpha T + I)$ and $C(\alpha) = \frac{1}{2\alpha}(\alpha W - iI)(\alpha T - I)$. Then, the splitting matrix $B(\alpha) = \frac{1}{2\alpha}(\alpha W + iI)(\alpha T + I)$ can be considered as a preconditioner for the complex symmetric indefinite matrix $A \in \mathbb{C}^{N\times N}$.

3. Convergence analysis

In this section, we will prove the convergence properties of the PSHNS iteration method and discuss the spectral distribution of the preconditioned matrix.

Theorem 1. *Assume that $A = W + iT \in \mathbb{C}^{N\times N}$, with $W \in \mathbb{R}^{N\times N}$ being symmetric indefinite and $T \in \mathbb{R}^{N\times N}$ being symmetric positive definite. Let α be a positive constant and $V \in \mathbb{R}^{N\times N}$ be a symmetric positive definite matrix. Then the spectral radius $\rho(M(V;\alpha))$ of the PSHNS iteration matrix satisfies $\rho(M(V;\alpha)) \le \sigma(\alpha)$, where $\sigma(\alpha) \equiv \max_{\mu_j \in sp(V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}})} |\frac{\alpha\mu_j - 1}{\alpha\mu_j + 1}|$, where $sp(V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}})$ denotes the spectrum of the matrix $V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}}$. Therefore, it holds that $\rho(M(V;\alpha)) \le \sigma(\alpha) < 1$, $\forall \alpha > 0$, i.e., the iteration converges unconditionally.*

Proof. It is easy to have

$$\begin{aligned}
\rho(M(V;\alpha)) &= \rho(\widehat{M}(V;\alpha)) = \rho((\alpha V - iW)(\alpha V + iW)^{-1}(\alpha T - WV^{-1}W)(\alpha T + WV^{-1}W)^{-1}) \\
&= \rho((\alpha V - iW)(\alpha V + iW)^{-1}WV^{-\frac{1}{2}}(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} - I)(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} + I)^{-1}V^{\frac{1}{2}}W^{-1}) \\
&= \rho(V^{\frac{1}{2}}W^{-1}(\alpha V - iW)(\alpha V + iW)^{-1}WV^{-\frac{1}{2}}(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} - I)(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} + I)^{-1}) \\
&= \rho((\alpha V^{\frac{1}{2}}W^{-1}V^{\frac{1}{2}} - iI)(\alpha V^{\frac{1}{2}}W^{-1}V^{\frac{1}{2}} + iI)^{-1}(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} - I)(\alpha V^{\frac{1}{2}}W^{-1}TW^{-1}V^{\frac{1}{2}} + I)^{-1}).
\end{aligned}$$

If we define $U_\alpha = (\alpha V^{\frac{1}{2}} W^{-1} V^{\frac{1}{2}} - iI)(\alpha V^{\frac{1}{2}} W^{-1} V^{\frac{1}{2}} + iI)^{-1}$, then it holds that U_α is a unitary matrix and $\|U_\alpha\|_2 = 1$. It is not difficult to prove that the matrix $V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}}$ is a symmetric positive definite matrix. Thus we have

$$\rho(M(V;\alpha)) \le \|(\alpha V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}} - I)(\alpha V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}} + I)^{-1}\|_2 = \max_{\mu_j \in sp(V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}})} \left| \frac{\alpha\mu_j - 1}{\alpha\mu_j + 1} \right|.$$

Obviously, $\mu_j > 0$ holds for all $\mu_j \in sp(V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}})(1 \le j \le n)$. So we can verify $\rho(M(V;\alpha)) \le \sigma(\alpha) < 1$. $\square$

Theorem 1 shows that the convergence speed of the PSHNS iteration method is bounded by $\sigma(\alpha)$, which depends on the eigenvalues of the matrix $V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}}$. Now we attempt to find α such that the upper bound $\sigma(\alpha)$ be minimum. This fact is precisely described as the following corollary.

Corollary 1. *Assume that the conditions of Theorem 1 be satisfied. Let the μ_{min} and μ_{max} denote the lower and the upper bounds of the eigenvalues of the symmetric positive definite matrix $V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}}$, respectively. Then the optimal parameter α is*

$$\alpha_* \equiv arg \min_{\alpha} \left\{ \max_{\mu_{min} \le \mu \le \mu_{max}} \left| \frac{\alpha\mu - 1}{\alpha\mu + 1} \right| \right\} = \frac{1}{\sqrt{\mu_{min}\mu_{max}}}$$

and the bound for $\rho(M(V;\alpha))$ is

$$\sigma(\alpha_*) = \frac{\sqrt{\mu_{max}} - \sqrt{\mu_{min}}}{\sqrt{\mu_{max}} + \sqrt{\mu_{min}}} = \frac{\sqrt{\kappa(V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}})} - 1}{\sqrt{\kappa(V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}})} + 1},$$

where $\kappa(V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}}) = \frac{\mu_{max}}{\mu_{min}}$ denotes the spectral condition number of the matrix $V^{\frac{1}{2}} W^{-1} T W^{-1} V^{\frac{1}{2}}$.

Proof. The proof is similar to that of Theorem 2.2 in [11], hence it is omitted.

Next, we discuss the spectral distribution of the preconditioned matrix $B(V;\alpha)^{-1} WA$. From Theorem 1, it is not difficult to show that all the eigenvalues λ_i of $B(V;\alpha)^{-1} WA$ satisfy $|1-|\lambda_i|| < 1(1 < i < N)$. Therefore, the smaller the spectral radius becomes, the more cluster all the eigenvalues of the preconditioned matrix round $(1, 0)$. This desirable property can lead to fast convergence rate of the preconditioned GMRES [12] method. We note that the PSHNS iteration can provide an effective preconditioner for Krylov subspace methods like the GMRES method.

4. Numerical examples

In this section, we test some numerical experiments to illustrate the effectiveness of the HNS and PSHNS preconditioners. The complex symmetric indefinite linear systems arise from discretizing the two-dimensional and three-dimensional Helmholtz equations. All computations are carried out in MATLAB (version R2010b). In our implementations, we choose the initial guess $x_0 \in \mathbb{C}^N$ and set the right-hand side $b = A * (1 + i) * \mathbf{ones}(N, 1)$. Note that Its and CPU denote iterations and CPU-time for computing approximation, respectively. Let the stopping criterion be $\frac{\|r_j\|}{\|r_0\|} \le 1.e - 6$ and choose $V = W^2$. GMRES and GMRES($\sharp$) [12] denote the unrestarted GMRES method and its restarted method, respectively. The maximum number of iterations allowed is set to 5000 and the mark "–" shown in Tables 1 and 2 indicates that the iterative method cannot converge until the maximum iterations. In all tables, α is numerical optimal.

Example 1. We consider the 2-D Helmholtz equation [13].

$$\begin{cases} -\Delta u - k^2 u + i\sigma_2 u = f(x, y), & (x, y) \in \Omega \equiv [0, 1] \times [0, 1], \\ u|_\Gamma = g(x, y), & (x, y) \in \Gamma, \end{cases} \tag{11}$$

Table 1
Its and CPU for preconditioned GMRES and GMRES(50) for Example 1.

Method	Preconditioner	k	10	20	30	40	50
		m^2	32^2	64^2	96^2	128^2	160^2
GMRES	No-prec	Its	65	142	270	479	694
		CPU	0.216	6.066	59.471	147.089	528.242
	ILU(A, 0)	Its	39	91	215	413	561
		CPU	0.184	2.786	34.994	252.296	666.298
	HNS	α	31.8	12.8	21.3	25.3	15.4
		Its	10	7	7	8	6
		CPU	0.061	0.457	1.729	3.892	10.051
	PSHNS	α	1780.4	1830.7	3112.5	5728.3	2812.7
		Its	3	3	3	3	3
		CPU	0.029	0.072	0.133	0.202	0.294
GMRES(50)	No-prec	Its	108	610	–	–	–
		CPU	0.232	5.834	–	–	–
	ILU(A, 0)	Its	39	172	1078	–	4263
		CPU	0.118	2.668	46.415	–	341.759
	HNS	α	27.2	11.9	26.7	31.2	16.7
		Its	10	7	7	8	6
		CPU	0.061	0.456	1.746	3.936	10.042
	PSHNS	α	1806.1	1476.5	2951.2	5682.3	2792.3
		Its	3	3	3	3	3
		CPU	0.030	0.073	0.134	0.205	0.298

Table 2
Its and CPU for preconditioned GMRES and GMRES(20) for Example 2.

Method	Preconditioner	k	10	15	18	21	25
		m^3	20^3	25^3	30^3	35^3	40^3
GMRES	No-prec	Its	55	81	107	143	200
		CPU	1.713	8.939	23.159	66.595	171.295
	ILU(A, 0)	Its	34	65	123	150	242
		CPU	0.590	4.469	31.796	77.823	272.556
	PSHNS	α	787.6	857.2	904.9	884.5	937.1
		Its	3	3	3	3	3
		CPU	0.551	0.786	1.186	1.612	2.398
GMRES(20)	No-prec	Its	263	901	3003	–	–
		CPU	3.881	24.307	137.168	–	–
	ILU(A, 0)	Its	71	203	2006	1523	2376
		CPU	0.560	3.979	80.785	86.523	223.305
	PSHNS	α	813.2	876.3	973.8	1026.2	963.5
		Its	3	3	3	3	3
		CPU	0.497	0.789	1.187	1.602	2.315

where k and Γ denote the wavenumber and the boundary of $\Omega \equiv [0,1] \times [0,1]$, respectively. Let σ_2 be 0.1. We apply the center-difference scheme to the above 2-D Helmholtz equation, then it follows that

$$A = T_m \otimes I_m + I_m \otimes T_m - k^2h^2(I_m \otimes I_m) + i\sigma_2(I_m \otimes I_m), \tag{12}$$

where the equidistant step-size $h = \frac{1}{m+1}$ is used in the discretization on the direction x, y and the natural lexicographic ordering is employed to the unknowns. Moreover, $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix, $\otimes$ denotes the Kronecker product, and T_m is tridiagonal matrix defined by $T_m = tridiag(-1, 2, -1)$. From (12), we set the matrices W and T be $W = T_m \otimes I_m + I_m \otimes T_m - k^2h^2(I_m \otimes I_m)$ and $T = \sigma_2(I_m \otimes I_m)$, respectively. Let V_m denote the m-by-m discrete Sine transform (DST) matrix [14] which is given by $[V_m]_{i,j} = \sin(\frac{ij\pi}{m+1})$ and the inverse of the DST matrix is also given by $V_m^{-1} = \frac{2}{m+1}V_m$. Using Theorem 4.5.2 in [15], we have

$$V_m^{-1}T_mV_m = D, \tag{13}$$

where D is a diagonal matrix with diagonal entries $\lambda_k = 4\sin^2(\frac{k\pi}{2(m+1)}), k = 1, \ldots, m$.

Using DST, the subsystem $(\alpha W + iI)z = r$ of the PSHNS preconditioned GMRES and GMRES(50) methods can be reformulated as

$$[\alpha(D \otimes I_m + I_m \otimes T_m - k^2h^2(I_m \otimes I_m)) + i(I_m \otimes I_m)]\overline{z} = \overline{r}, \tag{14}$$

where $\overline{z} = (V_m^{-1} \otimes I_m)z$ and $\overline{r} = (V_m^{-1} \otimes I_m)r$. From the special structure of (14), $\overline{z}$ can be obtained by solving the m tridiagonal linear systems, and then we can get the solver z by using fast DST. In this example, we test ILU(A, 0) [16], HNS and PSHNS preconditioners to enhance convergence behavior.

From Table 1, we observe that the iteration counts of the GMRES and GMRES(50) methods increase rapidly with problem size, and for $k \geq 30$, the GMRES(50) method do not converge. For low wavenumber, the ILU(A, 0) [16] preconditioner can improve the convergence behaviors of the GMRES and GMRES(50) method, but it is not very effective to accelerate the convergence rate for $k \geq 30$. We see that the PSHNS preconditioner drastically reduces iteration steps and CPU times of the GMRES and GMRES(50) methods compared with the ILU(A, 0) and HNS preconditioners. In addition, the iteration steps of the PSHNS preconditioner is constant, thus the PSHNS preconditioned GMRES and GMRES(50) methods demonstrate h-independent convergence behavior. The PSHNS preconditioned GMRES and GMRES(50) methods also show wavenumber-insensitive convergence behavior.

Example 2. In this example, we consider the 3-D Helmholtz equation [13].

$$\begin{cases} -\Delta u - k^2u + i\sigma_2 u = f(x, y, z), & (x, y, z) \in \Omega \equiv [0, 1] \times [0, 1] \times [0, 1], \\ u|_\Gamma = g(x, y, z), & (x, y, z) \in \Gamma, \end{cases} \tag{15}$$

where k and Γ denote the wavenumber and the boundary of $\Omega \equiv [0, 1] \times [0, 1] \times [0, 1]$, respectively. Let σ_2 be 0.1. Using the same center-difference scheme suggested in Example 1, we have

$$A = W + iT,$$

where $W = T_m \otimes I_m \otimes I_m + I_m \otimes T_m \otimes I_m + I_m \otimes I_m \otimes T_m - k^2h^2(I_m \otimes I_m \otimes I_m)$ and $T = \sigma_2(I_m \otimes I_m \otimes I_m)$. Using DST, solving the subsystems $(\alpha W + iI)z = r$ is equivalent to solve

$$[\alpha(D \otimes I_m \otimes I_m + I_m \otimes D \otimes I_m + I_m \otimes I_m \otimes T_m - k^2h^2(I_m \otimes I_m \otimes I_m)) + i(I_m \otimes I_m \otimes I_m)]\overline{z} = \overline{r}, \tag{16}$$

where $\overline{z} = (V_m^{-1} \otimes V_m^{-1} \otimes I_m)z$ and $\overline{r} = (V_m^{-1} \otimes V_m^{-1} \otimes I_m)r$. From the special structure of (16), $\overline{z}$ can be obtained by solving the m^2 tridiagonal linear systems, and then we can get the solver z by using fast DST. In this example, we only test ILU(A, 0) and PSHNS preconditioners to enhance convergence behavior.

As seen from Table 2, we observe that the iteration counts of the GMRES and GMRES(20) methods also increase rapidly with problem size. Moreover, the GMRES(20) method do not converge for $k > 18$. When $k \geq 18$, the ILU(A, 0) preconditioner is not effective for the GMRES method. However, the PSHNS preconditioned GMRES and GMRES(20) methods require less iterations and CPU times than that of GMRES and GMRES(20) methods and their ILU(A, 0) preconditioned methods. We see that the PSHNS preconditioned GMRES and GMRES(20) methods are also independent of the problem size and show wavenumber-insensitive convergence behavior.

5. Conclusion

In this paper, we have established and analyzed the PSHNS preconditioner for solving complex symmetric indefinite linear systems. The unconditionally convergence property of the PSHNS iteration method has been shown and the spectral distribution of preconditioned matrix has been described. Fast DST is applied to the preconditioned systems to improve the computing efficiency. To reduce the computational cost, the inexact variants of PSHNS to deal with practical problems are under investigation and will be given in the future.

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