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On Euler-extrapolated Hermitian/skew-Hermitian splitting method for complex symmetric linear systems[☆]

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ABSTRACT

In this paper, we propose an Euler-extrapolated Hermitian/skew-Hermitian splitting (E-HS) iterative method for solving a class of complex symmetric linear systems. The convergence and optimal parameter of the E-HS method are presented. In addition, some spectral properties of the preconditioned matrix are also studied. Numerical experiments verify the effectiveness of the E-HS method either as a solver or a preconditioner.

1. Introduction

In this paper, we focus on the solution of the following nonsingular system of linear equations

$$Ax \equiv (W + iT)x = b \tag{1.1}$$

where $W, T \in \mathbb{R}^{n\times n}$ are both symmetric positive semi-definite matrices, $b \in \mathbb{C}^n$ is a given vector, $x \in \mathbb{C}^n$ is an unknown vector and $i = \sqrt{-1}$ is the imaginary unit. Here and in the sequel, we assume $W, T \neq 0$, which implies that A is not skew-Hermitian but non-Hermitian matrix. Complex linear systems of the form (1.1) come from many problems in scientific computing and engineering applications, see [1–4].

In recent years, many efficient iterative methods have been proposed to solve the complex system (1.1). For example, based on the HSS method [5], Bai et al. designed the modified HSS (MHSS) method [6] and the preconditioned MHSS (PMHSS) method [7]. Moreover, Bai discussed the preconditioned Krylov subspace

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method [8], Axelsson and Kucherov presented the C-to-R method [9] and Salkuyeh et al. established the generalized SOR (GSOR) method [10], see also [11–15].

In this paper, based on the special structure of the matrix A, we construct an Euler-extrapolated Hermitian/skew-Hermitian splitting (E-HS) method, which contains its equivalent form

$$(T - iW)x = -ib. \tag{1.2}$$

In particular, we weaken these conditions to the requirement that both of the matrices W and T are symmetric positive semi-definite satisfying $null(W) \cap null(T) = \{0\}$. Moreover, we also strengthen these conditions to the requirement that both of the matrices W and T are symmetric positive definite.

The organization of this paper is as follows. In Section 2, we establish the E-HS method. In Section 3, we study the convergence and optimal parameter of the E-HS method and the spectral properties of the corresponding preconditioned matrix. Numerical experiments verified the effectiveness of the E-HS method in Section 4. Finally, we make some conclusions in Section 5.

2. The E-HS method and its implementation

Multiplying the Euler's formula $e^{-i\theta} = \cos(\theta) - i\sin(\theta)$ by the two sides of the system (1.1) yields

$$\tilde{A}x = \big(\cos(\theta)W + \sin(\theta)T + i\big(\cos(\theta)T - \sin(\theta)W\big)\big)x = e^{-i\theta}b, \tag{2.1}$$

where $\theta \in [2k\pi, \frac{\pi}{2} + 2k\pi]$, and k is an integer. Without loss of generality, we take $\theta \in [0, \frac{\pi}{2}]$. Thus, we establish the following Euler-extrapolated Hermitian/skew-Hermitian splitting (E-HS) iterative method.

Algorithm 2.1 (*The E-HS Method*). Given arbitrary initial guesses $x^{(0)} \in \mathbb{C}^n$, for $k = 0, 1, 2, \ldots$ until the sequence of iterates $\{x^{(k)}\}_{k=0}^{\infty} \in \mathbb{C}^n$ converges, compute $x^{(k+1)}$ according to the following procedure

$$\big(\cos(\theta)W + \sin(\theta)T\big)x^{(k+1)} = i\big(\sin(\theta)W - \cos(\theta)T\big)x^{(k)} + e^{-i\theta}b, \tag{2.2}$$

where $\theta \in [0, \frac{\pi}{2}]$.

Remark 2.1. It is worth mentioning that the E-HS method reduces to the HS method of the system (1.1) when choosing $\theta = 0$, and it also retrieves the HS method of the system (1.2) when choosing $\theta = \frac{\pi}{2}$. Moreover, Eq. (2.1) can be regarded as a combination of Eqs. (1.1) and (1.2), i.e., (2.1) $=$ $\cos(\theta) \cdot (1.1) + \sin(\theta) \cdot (1.2)$ with $\cos^2(\theta) + \sin^2(\theta) = 1$, which is called Euler-extrapolated technique.

Note that the iterative scheme (2.2) can be described using the following standard form

$$x^{(k+1)} = \mathcal{T}_\theta x^{(k)} + M_\theta^{-1}b, \; k = 0, 1, 2, \ldots,$$

where $\mathcal{T}_\theta = i\big(\cos(\theta)W + \sin(\theta)T\big)^{-1}\big(\sin(\theta)W - \cos(\theta)T\big)$ is the iteration matrix of the E-HS method and $M_\theta = e^{i\theta}\big(\cos(\theta)W + \sin(\theta)T\big)$. In fact, the iterative scheme (2.2) can also be obtained from the splitting of the original matrix $A = M_\theta - N_\theta$, where $N_\theta = ie^{i\theta}\big(\sin(\theta)W - \cos(\theta)T\big)$. Then, the splitting matrix M_θ can be seen as a preconditioner to the coefficient matrix A. More precisely, in every step of the iterative scheme (2.2) or applying the preconditioner M_θ to accelerate the convergence rate of Krylov subspace method, it is required to solve a linear system $z = M_\theta^{-1}r = \big(\cos(\theta)W + \sin(\theta)T\big)^{-1}e^{-i\theta}r$, where $r, z \in \mathbb{C}^n$ are the current and generalized residual vectors, respectively. Thus, if the matrix $\cos(\theta)W + \sin(\theta)T$ is symmetric positive definite, it can be solved exactly by the Cholesky factorization or inexactly by the CG algorithm.

3. Convergence and preconditioning properties

To discuss the convergence of the E-HS method, we first give several lemmas.

Lemma 3.1 ([7]). *Assume that $A = W + iT \in \mathbb{C}^{n\times n}$, with $W, T \in \mathbb{R}^{n\times n}$ being symmetric positive semi-definite matrices. Then the matrix A is nonsingular if and only if $null(W) \cap null(T) = \{0\}$.*

Lemma 3.2 ([16]). *Assume that $W, T \in \mathbb{R}^{n\times n}$ are symmetric positive semi-definite satisfying $null(W) \cap null(T) = \{0\}$, and let $\theta \in (0, \frac{\pi}{2})$. Then the matrix $\cos(\theta)W + \sin(\theta)T$ is symmetric positive definite.*

First, we consider that the matrices W and T are symmetric positive semi-definite satisfying $null(W) \cap null(T) = \{0\}$, and we obtain the following results.

Theorem 3.1. *Assume that $W, T \in \mathbb{R}^{n\times n}$ are symmetric positive semi-definite satisfying $null(W) \cap null(T) = \{0\}$ and let $\theta \in (0, \frac{\pi}{2})$. Then the following statements hold true:*

(1) *the eigenvalues of the iteration matrix $\mathcal{T}_\theta$ are $\lambda_j = i\frac{\sin(\theta)-\mu_j\cos(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}$, $j = 1, 2, \ldots, n$, and the spectral radius of the iteration matrix $\mathcal{T}_\theta$ is*

$$\rho(\mathcal{T}_\theta) = \max\left\{\left|\frac{\sin(\theta) - \mu_{\min}\cos(\theta)}{\cos(\theta) + \mu_{\min}\sin(\theta)}\right|, \left|\frac{\sin(\theta) - \mu_{\max}\cos(\theta)}{\cos(\theta) + \mu_{\max}\sin(\theta)}\right|\right\}, \tag{3.1}$$

where μ_j is a generalized eigenvalue of the matrix pair (W, T), $\mu_{\min}$ and $\mu_{\max}$ are the smallest and largest generalized eigenvalues of the matrix pair (W, T), respectively;

(2) *the E-HS method is convergent if the parameter θ satisfies*

$$\begin{cases} \{\theta \in (0, \frac{\pi}{2}) \mid \frac{1-\mu_{\min}}{1+\mu_{\min}} < \cot(\theta) < \frac{\mu_{\max}+1}{\mu_{\max}-1}\}, & \mu_{\max} > 1; \\ \{\theta \in (0, \frac{\pi}{2}) \mid \frac{1-\mu_{\min}}{1+\mu_{\min}} < \cot(\theta)\}, & \mu_{\max} \leqslant 1; \end{cases}$$

(3) *the optimal parameter θ_* of the E-HS method which minimizes corresponding spectral radius is*

$$\begin{aligned} \theta_* &= \left\{arc\tan\left(\frac{\mu_{\min}\mu_{\max} - 1 + \sqrt{(1+\mu_{\min}^2)(1+\mu_{\max}^2)}}{\mu_{\min}+\mu_{\max}}\right) \in (0, \frac{\pi}{2})\right\} \\ &\left(= \left\{arc\cot\left(\frac{1 - \mu_{\min}\mu_{\max} + \sqrt{(1+\mu_{\min}^2)(1+\mu_{\max}^2)}}{\mu_{\min}+\mu_{\max}}\right) \in (0, \frac{\pi}{2})\right\}\right), \end{aligned} \tag{3.2}$$

and the corresponding optimal convergence factor of the E-HS method is given by

$$\rho(\mathcal{T}_{\theta_*}) = \frac{\sin(\theta_*) - \mu_{\min}\cos(\theta_*)}{\cos(\theta_*) + \mu_{\min}\sin(\theta_*)} \quad \left(= \frac{\mu_{\max}\cos(\theta_*) - \sin(\theta_*)}{\cos(\theta_*) + \mu_{\max}\sin(\theta_*)}\right). \tag{3.3}$$

Proof. (1) Let μ_j be an arbitrary generalized eigenvalue of the matrix pair (W, T) and $0 \neq x \in \mathbb{C}^n$ be its corresponding eigenvector. Then, we have $Tx = \mu_j Wx$, which means that

$$\big(\cos(\theta)W + \sin(\theta)T\big)x = \big(\cos(\theta) + \mu_j\sin(\theta)\big)Wx.$$

Note that $\mu_j \geqslant 0$ and $\theta \in (0, \frac{\pi}{2})$, it follows from Lemma 3.2 that $\cos(\theta) + \mu_j\sin(\theta) > 0$ and

$$\big(\cos(\theta)W + \sin(\theta)T\big)^{-1}Wx = \frac{1}{\cos(\theta) + \mu_j\sin(\theta)}x.$$

Therefore

$$\begin{aligned}\mathcal{T}_\theta x &= i\big(\cos(\theta)W+\sin(\theta)T\big)^{-1}\big(\sin(\theta)W-\cos(\theta)T\big)x\\ &= i\big(\sin(\theta)-\mu_j\cos(\theta)\big)\big(\cos(\theta)W+\sin(\theta)T\big)^{-1}Wx\\ &= i\frac{\sin(\theta)-\mu_j\cos(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}x,\end{aligned}$$

and

$$\rho(\mathcal{T}_\theta)=\max\left\{\left|\frac{\mu_j\cos(\theta)-\sin(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}\right|\right\}:\equiv\max\left\{\left|\frac{\sin(\theta)-\mu_{\min}\cos(\theta)}{\cos(\theta)+\mu_{\min}\sin(\theta)}\right|,\ \left|\frac{\sin(\theta)-\mu_{\max}\cos(\theta)}{\cos(\theta)+\mu_{\max}\sin(\theta)}\right|\right\},$$

here we use the function $f(\mu_j)=\frac{\mu_j\cos(\theta)-\sin(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}$ is increasing with respect to $\mu_j\geqslant 0$.

(2) Obviously, the E-HS method is convergent if and only if $\rho(\mathcal{T}_\theta)<1$, which is equivalent to

$$-1<\frac{\mu_j\cos(\theta)-\sin(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}<1,$$

or equivalently

$$\begin{cases}(1-\mu_j)\sin(\theta)<(\mu_j+1)\cos(\theta)\\ (\mu_j-1)\cos(\theta)<(\mu_j+1)\sin(\theta)\end{cases},\ \textit{namely},\ \begin{cases}-\dfrac{\mu_j-1}{\mu_j+1}<\cot(\theta)\\ \dfrac{\mu_j-1}{\mu_j+1}<\tan(\theta)\end{cases}. \tag{3.4}$$

Note that the function $f(\mu_j)=\frac{\mu_j-1}{\mu_j+1}$ is increasing with respective to $\mu_j\geqslant 0$, then we have

(a) if $\mu_{\max}>1$, then $\frac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)<\frac{\mu_{\max}+1}{\mu_{\max}-1}$ with $\theta\in(0,\frac{\pi}{2})$;
(b) if $\mu_{\max}\leqslant 1$, then $\frac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)$ with $\theta\in(0,\frac{\pi}{2})$.

So, the above conditions (a) and (b) both guarantee the validity of the inequalities (3.4).

(3) Using the same strategy of the HSS method, we need minimize the spectral radius $\rho(\mathcal{T}_\theta)$. If θ_* is such a minimize point, by making use of the function $f(\theta)=\frac{\mu_j\cos(\theta)-\sin(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}$ is decreasing with respect to $\theta\in(0,\frac{\pi}{2})$, it must satisfy $\mu_{\max}\cos(\theta_*)-\sin(\theta_*)\geqslant 0$, $\mu_{\min}\cos(\theta_*)-\sin(\theta_*)\leqslant 0$ and

$$\frac{\sin(\theta_*)-\mu_{\min}\cos(\theta_*)}{\cos(\theta_*)+\mu_{\min}\sin(\theta_*)}=\frac{\mu_{\max}\cos(\theta_*)-\sin(\theta_*)}{\cos(\theta_*)+\mu_{\max}\sin(\theta_*)},\quad \theta_*\in(0,\frac{\pi}{2}).$$

After some computations, we obtain the results of (3.2). □

Following, we consider that the matrices W and T are symmetric positive definite.

Theorem 3.2. *Assume that $W,T\in\mathbb{R}^{n\times n}$ are symmetric positive definite and $\theta\in[0,\frac{\pi}{2}]$. Then the matrix $\cos(\theta)W+\sin(\theta)T$ is symmetric positive definite, and the following statements hold true:*

(1) *the eigenvalues of the iteration matrix $\mathcal{T}_\theta$ are $\lambda_j=i\frac{\sin(\theta)-\mu_j\cos(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}$, $j=1,2,\dots,n$, and the spectral radius of the iteration matrix $\mathcal{T}_\theta$ is given by (3.1), where μ_j is an eigenvalue of the matrix $W^{-1}T$, $\mu_{\min}$ and $\mu_{\max}$ are the smallest and largest eigenvalues of the matrix $W^{-1}T$, respectively;*
(2) *the E-HS method is convergent if the parameter θ satisfies*

$$\begin{cases}\big\{\theta\in(0,\frac{\pi}{2})\mid \dfrac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)<\dfrac{\mu_{\max}+1}{\mu_{\max}-1}\big\}, & \mu_{\min}\leqslant 1,\quad \mu_{\max}>1;\\ \big\{\theta\in(0,\frac{\pi}{2})\mid \dfrac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)<\dfrac{\mu_{\max}+1}{\mu_{\max}-1}\big\}\bigcup\big\{\frac{\pi}{2}\big\}, & \mu_{\min}>1,\quad \mu_{\max}>1;\\ \big\{\theta\in(0,\frac{\pi}{2})\mid \dfrac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)\big\}, & \mu_{\max}=1;\\ \big\{\theta\in(0,\frac{\pi}{2})\mid \dfrac{1-\mu_{\min}}{1+\mu_{\min}}<\cot(\theta)\big\}\bigcup\big\{0,\ \frac{\pi}{2}\big\}, & \mu_{\max}<1;\end{cases}$$

(3) *the optimal parameter θ_* of the E-HS method which minimizes corresponding spectral radius is*

$$\theta_* = \left\{ arc\tan\left(\frac{\mu_{\min}\mu_{\max} - 1 + \sqrt{(1+\mu_{\min}^2)(1+\mu_{\max}^2)}}{\mu_{\min}+\mu_{\max}} \right) \in [0, \frac{\pi}{2}) \right\} \bigcup \left\{ arc\cot\left(\frac{1 - \mu_{\min}\mu_{\max} + \sqrt{(1+\mu_{\min}^2)(1+\mu_{\max}^2)}}{\mu_{\min}+\mu_{\max}} \right) \in (0, \frac{\pi}{2}] \right\}, \tag{3.5}$$

and the corresponding optimal convergence factor of the E-HS method is given by (3.3).

Proof. The proof is analogous to Theorem 3.1, here is omitted. □

Remark 3.1. Based on Theorems 3.1–3.2, we can get $\rho(\mathcal{T}_0) = |\mu_j|$ and $\rho(\mathcal{T}_{\frac{\pi}{2}}) = \frac{1}{|\mu_j|}$ when $\mu_j \neq 1$, then $\rho(\mathcal{T}_{\frac{\pi}{4}}) = 0(< 1)$ when $\mu_j = 1(\neq 1)$. So, an optimal parameter θ_* in (3.2) or (3.5) must exist such that $\rho(\mathcal{T}_{\theta_*}) < 1$ and the E-HS method is always convergent by taking optimal parameter θ_*.

On the other hand, the clustered spectrum of the preconditioned matrix often leads to rapid convergence of the GMRES method [17], thus, we have the following results.

Theorem 3.3. *Under the assumption of Theorems 3.1–3.2, the eigenvalues of the matrix $M_\theta^{-1}A$ are*

$$\xi_j = 1 + i\frac{\mu_j\cos(\theta) - \sin(\theta)}{\cos(\theta) + \mu_j\sin(\theta)},\ j = 1, 2, \ldots, n,$$

namely, $\Re(\xi_j) = 1$ and $\Im(\xi_j) = \frac{\mu_j\cos(\theta)-\sin(\theta)}{\cos(\theta)+\mu_j\sin(\theta)}$ is an eigenvalue of the matrix $(\cos(\theta)W + \sin(\theta)T)^{-1}(\cos(\theta)T - \sin(\theta)W)$ with $\cos(\theta) + \mu_j\sin(\theta) > 0$, where μ_j is a generalized eigenvalue of the matrix pair (W, T) or an eigenvalue of the matrix $W^{-1}T$.

Proof. Let ξ_j be an eigenvalue of the matrix $M_\theta^{-1}A$, it follows from Theorems 3.1–3.2 that

$$\xi_j = 1 - \lambda_j = 1 + i\frac{\mu_j\cos(\theta) - \sin(\theta)}{\cos(\theta) + \mu_j\sin(\theta)}.$$

Therefore, the proof is completed. □

Remark 3.2. Based on the above discussion, by taking the optimal parameter θ_* for the E-HS preconditioner, the eigenvalues of the preconditioned matrix $M_\theta^{-1}A$ are

$$1 + i\frac{\mu_j\cos(\theta_*) - \sin(\theta_*)}{\cos(\theta_*) + \mu_j\sin(\theta_*)},\ j = 1, 2, \ldots, n.$$

So, they are contained within the complex disk centered at 1 with radius $r = \frac{\mu_j\cos(\theta_*)-\sin(\theta_*)}{\cos(\theta_*)+\mu_j\sin(\theta_*)}$ strictly less 1, which is desirable property for Krylov subspace acceleration.

4. Numerical experiments

In this section, one example to illustrate the effectiveness of the E-HS method is compared with MHSS [6] and GSOR [10] methods in terms of the iteration steps (IT) and the elapsed CPU time in seconds (CPU) for solving complex symmetric linear system (1.1). In practical implementations, we choose the null vectors as an initial guess and the stopping criterion $RES = \|b - Ax^{(k)}\|_2/\|b\|_2 < 10^{-6}$. All the computations results are run in MATLAB 2017a, and the linear sub-systems can be solved exactly by a Cholesky factorization.

Table 1
Numerical results for different iterative methods and preconditioners.

Method	Grid			32 × 32			
	(σ_1, σ_2)	$(10^2, 10^0)$	$(10^2, 10^1)$	$(10^2, 10^2)$	$(10^2, 10^3)$	$(10^2, 10^4)$	$(10^2, 10^5)$
	$\Vert W\Vert_2/\Vert T\Vert_2$	8792	879.2	87.92	8.792	0.8792	0.0879
	α_*	0.0009	0.0091	0.0912	0.9122	9.1223	91.2235
	$\rho(\mathcal{T}_{\alpha_*})$	0.9055	0.8980	0.8258	0.8124	0.9945	0.9997
MHSS	IT(CPU)	40(1.181)	40(1.160)	36(1.081)	30(0.873)	39(1.134)	40(1.169)
MHSS–GMRES	IT(CPU)	6 (0.117)	8 (0.201)	14(0.313)	16(0.406)	14(0.361)	8 (0.257)
	α_*	1.0000	0.9983	0.8685	0.2125	0.0237	0.0024
	$\rho(\mathcal{T}_{\alpha_*})$	0.0000	0.0017	0.1315	0.7875	0.9763	0.9976
GSOR	IT(CPU)	2(0.012)	3(0.019)	9(0.037)	81(0.529)	–(–)	–(–)
GSOR–GMRES	IT(CPU)	2(0.009)	3(0.016)	7(0.026)	22(0.125)	75(1.563)	94(13.512)
	θ_*	0.0042	0.0422	0.3536	0.7824	1.2042	1.5263
	$\rho(\mathcal{T}_{\theta_*})$	0.0042	0.0412	0.3563	0.7910	0.3703	0.0433
E-HS	IT(CPU)	3(0.010)	5(0.020)	13(0.030)	58(0.162)	14(0.036)	5(0.016)
E-HS–GMRES	IT(CPU)	3(0.006)	5(0.015)	11(0.021)	16(0.063)	10(0.029)	5(0.014)

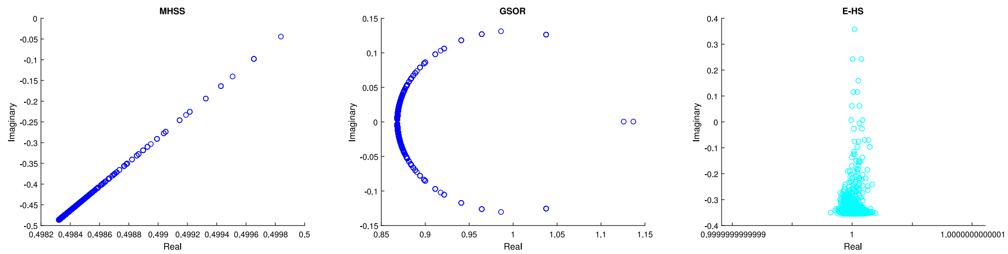

Fig. 1. Eigenvalues distribution of the three preconditioners with $Grid = 32$ and $\sigma_1 = \sigma_2 = 10^2$.

Example 1 (*[10]*). Consider the complex Helmholtz equation

$$-\Delta u + \sigma_1 u + i\sigma_2 u = f,$$

where σ_1 and σ_2 are real coefficient functions, u satisfies Dirichlet boundary conditions in $D = [0,1] \times [0,1]$ and $i = \sqrt{-1}$. We discretize the problem with finite differences on a $m \times m$ grid with mesh size $h = 1/(m+1)$. This leads to the complex symmetric linear system of the form $\big((K + \sigma_1 I) + i\sigma_2 I\big)x = b$, where $K = I \otimes V_m + V_m \otimes I$ is the discretization of $-\Delta$ by means of centered differences, wherein $V_m = h^{-2} tridiag(-1, 2, -1) \in \mathbb{R}^{m\times m}$. In our tests, the right-hand side vector b is taken as $b = (1+i)A\mathbf{1}$, with $\mathbf{1}$ being the vector of all entries equal to 1. Moreover, we also normalize the coefficient matrix on right-hand side of the above equation by multiplying both by h^2.

Table 1 lists the numerical results of the MHSS, GSOR and E-HS methods with respect to iterative methods and preconditioned GMRES methods by choosing $Grid = 32$, and Fig. 1 shows the eigenvalues distribution of the corresponding preconditioned matrices. As we observe, we can conclude that the E-HS method is more efficient than the MHSS and GSOR methods in terms of the iteration steps and CPU times.

5. Conclusions

In this paper, we established the E-HS method for solving (1.1). The convergence and optimal parameter of the E-HS method and the spectral properties of the preconditioned matrix are carefully analyzed. Numerical results show that the effectiveness of the E-HS method either as a solver or as a preconditioner.

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