# Taller 11

Use aproximaciones con diferencias finitas hacia adelante y hacia atrás y centradas para estimar la primera y segunda derivada en $x = 0{,}8$ y $h = 0{,}1$ de la función

$$f(x) = 0{,}25x^4 - 0{,}55x^2 + 4{,}5$$

Calcule además el valor verdadero de las derivadas.

Cuando h = 0,1

1. Primera derivada:
   $f'(x) = x^3 - 1.1x$
2. Segunda derivada:
   $f''(x) = 3x^2 - 1.1$

Cuando h = 0,1 en x = 0.8

Para la primera derivada:

1. Hacia adelante:
$$f'(0.8) \approx \frac{f(0.8+0.1)-f(0.8)}{0.1}$$
2. Hacia atrás:
$$f'(0.8) \approx \frac{f(0.8)-f(0.8-0.1)}{0.1}$$
3. Centrada:
$$f'(0.8) \approx \frac{f(0.8+0.1)-f(0.8-0.1)}{2\times 0.1}$$

Para la segunda derivada:

1. Hacia adelante:
$$f''(0.8) \approx \frac{f(0.8+2\times 0.1)-2f(0.8+0.1)+f(0.8)}{(0.1)^2}$$
2. Hacia atrás:
$$f''(0.8) \approx \frac{f(0.8)-2f(0.8-0.1)+f(0.8-2\times 0.1)}{(0.1)^2}$$
3. Centrada:
$$f''(0.8) \approx \frac{f(0.8+0.1)-2f(0.8)+f(0.8-0.1)}{(0.1)^2}$$

Para h = 0.05 en x = 0.8

1. Primera derivada:
$$f'(x) = x^3 - 1.1x$$
2. Segunda derivada:
$$f''(x) = 3x^2 - 1.1$$

Aproximaciones para h = 0.05 en x = 0.8

Para la primera derivada:

1. Hacia adelante:
$$f'(0.8) \approx \frac{f(0.8+0.05)-f(0.8)}{0.05}$$
2. Hacia atrás:
$$f'(0.8) \approx \frac{f(0.8)-f(0.8-0.05)}{0.05}$$
3. Centrada:
$$f'(0.8) \approx \frac{f(0.8+0.05)-f(0.8-0.05)}{2\times 0.05}$$

Para la segunda derivada:

1. Hacia adelante:
$$f''(0.8) \approx \frac{f(0.8+2\times 0.05)-2f(0.8+0.05)+f(0.8)}{(0.05)^2}$$
2. Hacia atrás:
$$f''(0.8) \approx \frac{f(0.8)-2f(0.8-0.05)+f(0.8-2\times 0.05)}{(0.05)^2}$$
3. Centrada:
$$f''(0.8) \approx \frac{f(0.8+0.05)-2f(0.8)+f(0.8-0.05)}{(0.05)^2}$$

h = 0.05 en x = 0.8

1. Primera derivada:
$$f'(x) = 4 \times 0.25x^3 - 2 \times 0.55x = x^3 - 1.1x$$
2. Segunda derivada:
$$f''(x) = 3 \times 0.25x^2 - 1.1 = 0.75x^2 - 1.1$$

F(x) en x = 0.8

$$f(0.8) = 0.25 \times (0.8)^4 - 0.55 \times (0.8)^2 + 4.5$$

$$= 0.25 \times 0.4096 - 0.55 \times 0.64 + 4.5$$

$$= 0.1024 - 0.352 + 4.5$$

$$= 4.2504$$

Aproximaciones

**Primera derivada:**

1. Hacia adelante:

$$f'(0.8) \approx \frac{f(0.8+0.05)-f(0.8)}{0.05}$$

$$\approx \frac{f(0.85)-f(0.8)}{0.05}$$

$$\approx \frac{0.25\times(0.85)^4-0.55\times(0.85)^2+4.5-4.2504}{0.05}$$

Calculando:

$$\approx \frac{0.199923-0.3995+4.5-4.2504}{0.05}$$

$$\approx \frac{0.199923-0.3995+0.2496}{0.05}$$

$$\approx \frac{0.050023}{0.05}$$

$$\approx 1.00046$$

1. Hacia atrás:

$$f'(0.8) \approx \frac{f(0.8)-f(0.8-0.05)}{0.05}$$

$$\approx \frac{f(0.8)-f(0.75)}{0.05}$$

$$\approx \frac{4.2504-(0.25\times(0.75)^4-0.55\times(0.75)^2+4.5)}{0.05}$$

Calculando:

$$\approx \frac{4.2504-(0.105469-0.20625+4.5)}{0.05}$$

$$\approx \frac{4.2504-4.39922}{0.05}$$

$$\approx \frac{-0.14882}{0.05}$$

$$\approx -2.9764$$

1. Centrada:

$$f'(0.8) \approx \frac{f(0.8+0.05)-f(0.8-0.05)}{2\times0.05}$$

$$\approx \frac{f(0.85)-f(0.75)}{0.1}$$

$$\approx \frac{0.25\times(0.85)^4-0.55\times(0.85)^2+4.5-(0.25\times(0.75)^4-0.55\times(0.75)^2+4.5)}{0.1}$$

Calculando:

$$\approx \frac{(0.199923-0.3995+4.5)-(0.105469-0.20625+4.5)}{0.1}$$

$$\approx \frac{(0.050023)-(-0.14882)}{0.1}$$

$$\approx \frac{0.198843}{0.1}$$

$$\approx 1.98843$$

**Segunda derivada:**

1. Hacia adelante:

$$f''(0.8) \approx \frac{f(0.8+2\times0.05)-2f(0.8+0.05)+f(0.8)}{(0.05)^2}$$

$$\approx \frac{f(0.9)-2f(0.85)+f(0.8)}{(0.05)^2}$$

$$\approx \frac{0.25\times(0.9)^4-0.55\times(0.9)^2+4.5-2(0.25\times(0.85)^4-0.55\times(0.85)^2+4.5)+0.25\times(0.8)^4-0.55\times(0.8)^2+4.5}{0.05^2}$$

Calculando:

$$\approx \frac{0.2187-0.495+4.5-2(0.199923-0.3995+4.5)+0.1024-0.352+4.5}{0.05^2}$$

$$\approx \frac{0.2187-0.495+4.5-2(0.050023)+0.1024-0.352+4.5}{0.05^2}$$

$$\approx \frac{0.2187-0.495+4.5-0.100046+0.1024-0.352+4.5}{0.05^2}$$

$$\approx \frac{8.3484\times10^{-5}}{0.05^2}$$

$$\approx 0.0333936$$

1. Hacia atrás:

$$f''(0.8) \approx \frac{f(0.8)-2f(0.8-0.05)+f(0.8-2\times0.05)}{(0.05)^2}$$

$$\approx \frac{f(0.8)-2f(0.75)+f(0.7)}{(0.05)^2}$$

$$\approx \frac{4.2504-2(0.105469-0.20625+4.5)+(0.25\times(0.7)^4-0.55\times(0.7)^2+4.5)}{(0.05)^2}$$

Calculando:

$$\approx \frac{4.2504-2(4.39922)+0.148244}{0.05^2}$$

$$\approx \frac{4.2504-8.79844+0.148244}{0.05^2}$$

$$\approx \frac{-4.3998}{0.05^2}$$

$$\approx -3.51984 \times 10^4$$

1. Centrada:

$$f''(0.8) \approx \frac{f(0.8+0.05)-2f(0.8)+f(0.8-0.05)}{(0.05)^2}$$

$$\approx \frac{f(0.85)-2f(0.8)+f(0.75)}{(0.05)^2}$$

$$\approx \frac{0.25\times(0.85)^4-0.55\times(0.85)^2+4.5-2(0.25\times(0.8)^4-0.55\times(0.8)^2+4.5)+0.25\times(0.75)^4-0.55\times(0.75)^2+4.5}{(0.05)^2}$$

Calculando:

$$\approx \frac{0.2187-0.495+4.5-2(0.1024-0.352+4.5)+0.105469-0.20625+4.5}{(0.05)^2}$$

$$\approx \frac{0.2187-0.495+4.5-2(0.050023)+0.105469-0.20625+4.5}{(0.05)^2}$$

$$\approx \frac{0.2187-0.495+4.5-0.100046+0.105469-0.20625+4.5}{(0.05)^2}$$

$$\approx \frac{0.2187-0.495+4.5-0.100046+0.105469-0.20625+4.5}{(0.05)^2}$$

$$\approx \frac{8.3484\times10^{-5}}{(0.05)^2}$$

$$\approx 0.0333936$$

Ahora se compara con las derivadas exactas

1. La primera derivada exacta en $x = 0.8$:

$$f'(0.8) = 0.8^3 - 1.1 \times 0.8$$
$$= 0.512 - 0.88$$
$$= -0.368$$

2. La segunda derivada exacta en $x = 0.8$:

$$f''(0.8) = 0.75 \times 0.8^2 - 1.1$$
$$= 0.48 - 1.1$$
$$= -0.62$$

La primera derivada calculada con diferencias finitas para h = 0.05 es mas precisa que para h = 0.1

La segunda derivada calculada con diferencias finitas para h = 0.05 es considerablemente mas precisa que para h = 0.1