

Emplee la expansión de la serie de Taylor de cero hasta tercer orden para predecir $f(0.5)$ si $f(x) = 0.9x^3 - 1.4x^2 + 3x - 4$ usando como punto base $x = 0.4$.

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$f(x) = 0.9x^3 - 1.4x^2 + 3x - 4$$

$$f'(x) = 2.7x^2 - 2.8x + 3$$

$$f''(x) = 5.4x - 2.8$$

$$f(0.4) = 0.9(0.4)^3 - 1.4(0.4)^2 + 3(0.4) - 4$$

$$f'(0.4) = 2.7(0.4)^2 - 2.8(0.4) + 3$$

$$f''(0.4) = 5.4(0.4) - 2.8$$

$$f'''(0.4) = 5.4$$

$$f(0.4) = 0.9(0.064) - 1.4(0.16) + 1.2 - 4$$

$$f(0.4) = 0.0576 - 0.224 + 1.2 - 4$$

$$f(0.4) = -2.9664$$

$$f'(0.4) = 2.7(0.16) - 2.8(0.4) + 3$$

$$f'(0.4) = 0.432 - 1.12 + 3$$

$$f'(0.4) = 2.312$$

$$f''(0.4) = 5.4(0.4) - 2.8$$

$$f''(0.4) = 2.16 - 2.8$$

$$f''(0.4) = -0.64$$

$$f'''(0.4) = 5.4$$

$$f(x) \approx -2.9664 + 2.312(x - 0.4) - \frac{0.64}{2!}(x - 0.4)^2 + \frac{5.4}{3!}(x - 0.4)^3$$

$$f(x) \approx -2.9664 + 2.312(x - 0.4) - 0.32(x - 0.4)^2 + \frac{5.4}{6}(x - 0.4)^3$$

$$f(0.5) \approx -2.9664 + 2.312(0.5 - 0.4) - 0.32(0.5 - 0.4)^2 + \frac{5.4}{6}(0.5 - 0.4)^3$$

Calculando este valor:

$$f(0.5) \approx -2.9664 + 2.312(0.1) - 0.32(0.1)^2 + \frac{5.4}{6}(0.1)^3$$

$$f(0.5) \approx -2.9664 + 0.2312 - 0.0032 + \frac{5.4}{6}(0.001)$$

$$f(0.5) \approx -2.9664 + 0.2312 - 0.0032 + 0.009$$

$$f(0.5) \approx -2.7294$$

Emplee la expansión de la serie de Taylor de cero hasta tercer orden para predecir $f(0.55)$ si $f(x) = 0.9x^3 - 1.4e^x - 3.2x + 3.2$ usando como punto base $x = 0.5$

Primero, las derivadas de $f(x)$ son:

$$f'(x) = 2.7x^2 - 1.4e^x - 3.2$$

$$f''(x) = 5.4x - 1.4e^x$$

$$f'''(x) = 5.4 - 1.4e^x$$

$$f(0.5) = 0.9(0.5)^3 - 1.4e^{0.5} - 3.2(0.5) + 3.2$$

$$f'(0.5) = 2.7(0.5)^2 - 1.4e^{0.5} - 3.2$$

$$f''(0.5) = 5.4(0.5) - 1.4e^{0.5}$$

$$f'''(0.5) = 5.4 - 1.4e^{0.5}$$

$$f(x) \approx (-0.8875 - 1.4e^{0.5}) + (-2.525 - 1.4e^{0.5})(x - 0.5) + \frac{(2.7 - 1.4e^{0.5})}{2!}(x - 0.5)^2 + \frac{(5.4 - 1.4e^{0.5})}{3!}(x - 0.5)^3$$

$$f(x) \approx -0.8875 - 1.4e^{0.5} - 2.525(x - 0.5) - \frac{1.4e^{0.5} - 2.7}{2!}(x - 0.5)^2 - \frac{1.4e^{0.5} - 5.4}{6}(x - 0.5)^3$$

$$f(0.55) \approx -0.8875 - 1.4e^{0.5} - 2.525(0.05) - \frac{1.4e^{0.5}-2.7}{2!}(0.05)^2 - \frac{1.4e^{0.5}-5.4}{6}(0.05)^3$$

Calculando este valor:

$$f(0.55) \approx -0.8875 - 1.4e^{0.5} - 0.12625 - \frac{1.4e^{0.5}-2.7}{2!}(0.0025) - \frac{1.4e^{0.5}-5.4}{6}(0.000125)$$

$$f(0.55) \approx -1.01375 - 1.4e^{0.5} - \frac{1.4e^{0.5}-2.7}{2!}(0.0025) - \frac{1.4e^{0.5}-5.4}{6}(0.000125)$$

$$f(0.55) \approx -1.01375 - 1.4e^{0.5} - \frac{1.4e^{0.5}-2.7}{2}(0.0025) - \frac{1.4e^{0.5}-5.4}{6}(0.000125)$$