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T1 ________	**32133**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**B**	F4 ________

2015 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Summary

In this paper, we present a feasible model to make an efficient search plan for a downed plane that has crashed in open water. In order to approach the three sub-problems, three distinct models are developed.

In model I, we estimate the crash site in open water. We take a set of parameters such as flight height. We consider that different planes have different values of this parameters. We formulate a differential equation to analyze the falling process. For different planes, we set parameters as model inputs to study their affection on the crash site. We use the data of AirBus A320 to test our equations. The calculation result is reliable.

In model II, we consider the floating wreckage and build a drift model to simulate the behavior of the wreckage. In order to predict the future wreckage location, we study the leeway and slippage of floating object on the sea surface. We apply the Monte Carlo technique to estimate the prior search area.

In model III, we use a refinement of a model developed by Stone. We adapt it to determine the optimal search path based on detective capability of the search planes. Take a step further, we consider the efficiency of both parallel search path and the spiral search path. Results show that we can make an optimal search plan that maximize the detection probability using our model, which are both scientific and operational.

Since we consider general variables in maritime search, our model is broad enough to accommodate various situations.

Where is the lost plane

February 10, 2015

Contents

1 Introduction

Searching for the lost plane is a global concern. Using aircraft to detect the downed plane is a common method. We build a generic mathematical model to plan a useful search for a plane that crashed in open water.

1.1 Restatement of Problem

The problem contains following steps. Firstly, the plane that flies from point A to Point B falls and crashes in the open water like Atlantic. Then it breaks into several parts. The wreckage drift with wind and current. We send aircraft to search downed plane. We break down the problem into three sub-problems:

- Estimating the crash site in the open water
- Building a model that can simulate the drifting process
- Giving an optimal search plan

We interpret the falling of the plane as a pseudo-parabolic process. We can get a crash site by setting different parameters. The flight height and flight speed are essential elements. According to the force analysis, the calculation formulas for the trajectory site be established. Then we use the crashed time and site as an initial condition to analyze the movement of the floating wreckage.

We build a drift model to simulate the behavior of the wreckage. Because the wreckage in irregular shapes and the natural conditions are complex, we seek to obtain a good estimation. The appearance of the wreckage can be interpreted as a random event. We need to consider the characteristic of the wreckage and other factors like current. We need to determine the optimized search area.

When we make a search plan, we first consider the capacity of our search equipment. We give a criterion to assess the capacity. Then we set up different search route. Referring to instances in practice, we give several feasible plans at last.

1.2 Problem Background

The search for a plane that has crashed in open water is a maritime search problem. A mathematical model of maritime search is inevitable to the study of the best search area and the optimal search method.

To determine the best search area, we need to build a drift model with several constraints. When we construct the model, we need to analyze forces reacting on the floating objects. To do this, our model should contain the effect of wind, current, waves, etc. In 1995, Hodgins and Mak defined leeway, which is the drift associated with wind forces on the exposed above-water part of the object. [1]

Optimal search method is a problem of search theory. Koopman devoted to search theory for stationary target that satisfy a bivariate normal distribution. [2]As a matter of fact, the target does not have to be stagnant. After the stationary target theory was established, several scientists were devoted to build a more operational model. In the 1970s, Stone studied the algorithm in searching for a moving target. In his paper, he used the Lagrange multiplier method to extend formal models with constraint to model without constraint.[3]Discenza adopted multiple rectangular to approach optimal search plan. And he stated an operational optimization of search unit arrangement.[4] With the popularization of computer and automation technology, people apply the computer to maritime search plans, enhancing the efficiency of maritime search by utilizing the powerful calculating ability and gradually accurate richer ocean environmental data. This developing process includes the computerized stage of classic search planning method and Mente Carlo based computer simulation stage. In 2001, based on the research results of Allen[5], DNMI set the leeway model aiming at drifting objects whose diameter is less than 10 meters, basing on its three-dimensional ocean environmental numerical prediction model–NORDIC4.Built by adopting Mente Carlo simulation, this model ignores the influence of waves and can provide support for SAR in open water.

2 Assumptions

In reality, the process of an air crash and the search involves great amount of unknown and unpredictable factors. Too many inputs will make our model intractable. To build a general model that applies to various situation, we need to introduce some assumptions. Below are the assumptions we take:

- **We don't consider clues from a third party(e.g.,witness).** If a witness provides a location of a drift object(e.g. ,wreckage), we can assign a plane to search.
- **The downed plane didn't disintegrate before it crashes on the water.** In fact in reality a plane might disintegrate in the air. But in order to simplify the model, we take the plane as a whole during its fall.
- **The initial speed of the wreckage is 0.** Namely, we ignore the initial speed of the plane crashed in the sea. It is not so realistic. After the plane crashed, it rushes at a speed for a while and was affected by wave. We just take it to simplify the model.

3 Model I: A Downed Plane Model

Different plane has different flight height, speed and wingspan, etc. In order to predict the crash site we use these plane parameters to build a downed plane model.

3.1 Additional Assumptions

- **The motion of the plane is horizontal linear uniform motion before it loses the power.**
- **The process of fall can be recognize as a glide, and angle of attack is 0.** During the fall of the plane, the pilot tries to remain the plane stable.
- **The plane does not crash with any object during its glide.**
- **The plane loses its power and starts to fall soon after the last communication.** Bacause common plane will contact with the ground in a high frequency.

3.2 Notations

Table 1: Notations in model I

Symbol	Definition
h	Flight height
m	Mass of the plane
g	Acceleration of gravity
V_0	Initial speed
ρ	Air density
V_x,V_y	Horizontal speed, vertical speed
S_1,S_2	Wingspan area, windward area
C_W,F_W	Lift coefficient, lift
C_L,F_L	Windage coefficient, windage
C_D,F_D	Air drag coefficient, air drag

3.3 The Forces on A Downed Plane

We analysis the force on the plane during its fall in Figure 1.

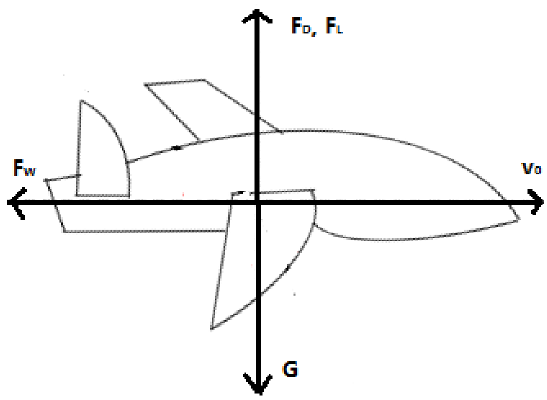

Figure 1: Force analysis

3.4 Calculation of Trajectory

Based on Figure 1, we consider C as the origin. The plane noses to the positive X direction. The direction of F_D is the positive Y direction. We build following differential equations:

$$F_W = m\frac{\mathrm{d}v_x}{\mathrm{d}t} \tag{1}$$

$$F_D + F_L - G = m\frac{\mathrm{d}v_y}{\mathrm{d}t} \tag{2}$$

$$V_x = V_0 \ \ t = 0 \tag{3}$$

$$V_y = 0 \ \ t = 0 \tag{4}$$

$$F_W = \frac{C_W S_2 \rho V_x^2}{2} \tag{5}$$

$$F_D = \frac{C_D S_1 \rho V_y^2}{2} \tag{6}$$

$$F_L = \frac{C_L S_1 \rho V_x^2}{2} \tag{7}$$

Where $(3)(4)$ are initial conditions of $(1)(2)$. Taking $(5)(6)(7)$ into $(1)(2)$, we can get the speed equations on X-axis and Y-axis. Integrating v on X-axis and Y-axis, we get the parametric equations of displacement on X-axis and Y-axis:

$$x = \int V_x(t)\mathrm{d}t$$

$$y = \int V_y(t)\mathrm{d}t$$

At this point, we get the trajectory equations of the plane which flied from A to B and crashed at D.

3.5 Model Testing

We can get following data of AirBus A320 to test our equations:[6]

Table 2: Data of AirBus A320

Symbol	Value
h	$10000m$
m	$68000kg$
g	$9.8m/s^2$
V_0	$240m/s$
ρ	$1kg/m^3$
S_1	$34.09m^2$
S_2	$10m^2$
C_W	0.08
C_L	1.20
C_D	0.44

Based on these data, we can get the trajectory graph of the plane in Figure 2.

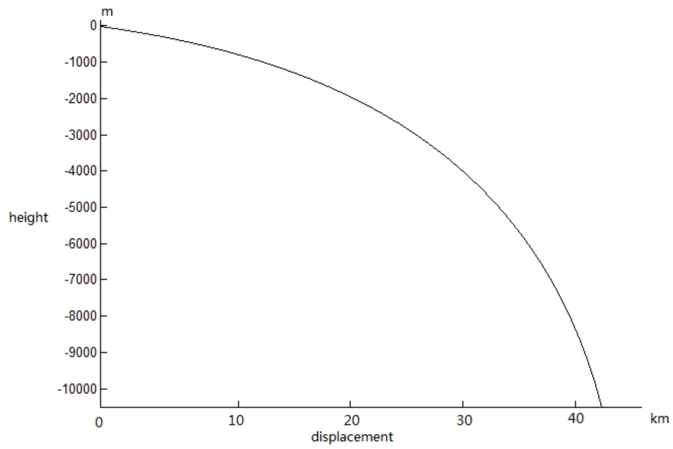

Figure 2: Trajectory of the downed plane

We can see the plane glide about 42 km and then crash. It is in accord with the data of the downed flight N106US in 2009.[7]

3.6 Model Evaluation

This model consider constant factors in the crash of a plane as flight height, route and resistance coefficient. The testing also validate the model has certain rationality. But in reality, the trajectory of a downed plane and its distance are affected by many other factors. For example the plane may hover after losing power. We should consider them otherwise.

4 Model II: A Wreckage Drift Model

In model I, we estimated the possible crash site of the plane. However, in open water such as the Atlantic Ocean, the movement of a floating object will be affected by many factors to a great extent. To determine our search area, we then build the drift model of the wreckage.

4.1 Additional Assumptions

- **We do not take boundary situations into account.** Specifically, the fragments drift to the coastline. In this situation, we just search the coastline.
- **We just consider floating wreckage.** In fact, in reality some fragments sink into the sea, but in order to simplify the model, we don't take this into consideration.
- **The influence of ballast can be neglected in order to simplify the model.**

4.2 Notations

Table 3: Notations in model II

Symbol	Definition
$\mathbf{V}_{object}$	Speed vector of the object
$\mathbf{V}_{wind}$	Speed vector of the object caused by wind
$\mathbf{V}_{current}$	Speed vector of the object caused by current
$\mathbf{V}_x, \mathbf{V}_y$	Speed vector on X-axis and Y-axis
$\mathbf{S}_{object}$	Displacement of the object
c_1, c_2	Drag coefficient of wind and current
t	Time
r	Radius of the search area
E	Error
a, b	Weight

4.3 The Forces on a Drifting Object

Wind
The drift brought about by the wind alone is termed the object's leeway.[8] The direction of this movement is downwind. This is not true in some cases. Gdynia Maritime University once did an experiment. The result was that the maximum angle of deviation can reach 70 degrees.
Ocean Current
Ocean Current plays an important part in the movement of the wreckage. Although the ocean current may not always stay in a stable condition, in a certain period of time, the direction of ocean current can remain constant basically. We can consider the direction of slippage as the direction of ocean current.
Waves
This is the most complex factor and people still havent made it clear by now. However, we can't deny the influence on the movement of the wreckage from waves.Generally, there is a stoke turbulence near the surface of the ocean whose motion trajectory is circular.[9] Its lateral shearing force has an impact on the movement of the wreckage.[10]
Coriolis Force
The movement of the wreckage will be influenced by Coriolis Force owing to the earth's rotation. This factor can be negligible compared to other factors in the short-term. It may have a cumulative impact after a long time. But we can ignore this for the searching would generally be carried out soon after the incident.
Immersion Ratio
In the research of the prediction of a container's drifting trajectory, Daniel has found that there was certain relationship between the immersion ratio and leeway speed.[11] We show it in Figure 3.

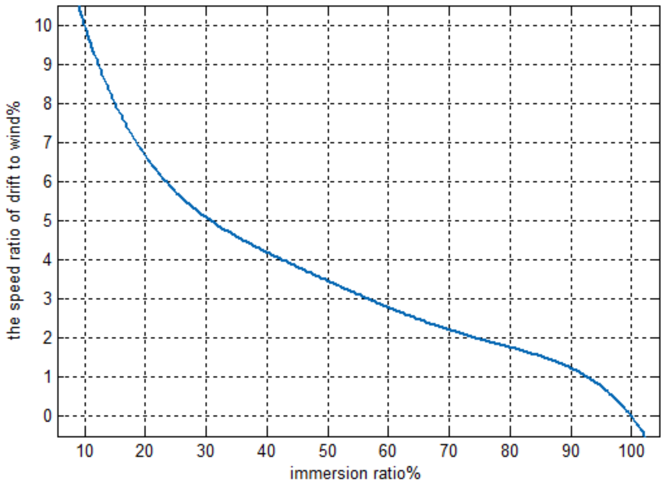

Figure 3: The relationship between the speed of drift and the immersion ratio

We can see that in Figure 3 when the immersion ratio is between $30\% - 90\%$, the drifting speed is almost in direct proportion to the immersion ratio. The immersion ratio of wreckage can be reckoned in this range. Thus, when calculating, we can simplify it as drag coefficient.

4.4 A Wreckage Drift Model

4.4.1 Classical Estimation

1.Estimate the Displacements

According to *International Aeronautical and Maritime Search and Rescue Manual*.[12] Departments like the Coast Guard often use the vector sum of wind force and current force to estimate the area of the drifting object.

$$\mathbf{v}_{object} = c_1\mathbf{v}_{wind} + c_2\mathbf{v}_{current}$$

Where c_1, c_2 are drag coefficient of wind and current respectively. Apparently we have

$$\mathbf{s}_{object} = \mathbf{v}_{object}t$$

2.Estimate the Search Area

Determining the search area is essential to a search. We consider a circular search zone. And its center is the position of the wreck after displacement estimation. We assume that radius is a function of time. It is justified for the uncertainty increases over time so we will search a larger area. We get

$$R = (\mathbf{v}_{object} + E)t$$

Where E is error. We estimate E by depending some status and wave force. We use a probability distribution model in the oval search area[13]:

$$P[R(t,\omega) < x] = \begin{cases} 0 & x < -0.16t + \mathbf{v}_{object}t + Et \\ \frac{1}{0.2t}(x + 0.16t - \mathbf{v}_{object}t - Et) & -0.16t + \mathbf{v}_{object}t + Et \leq x, \\ & x < 0.16t + \mathbf{v}_{object}t + Et \\ 1 & 0.16t + \mathbf{v}_{object}t + Wt \leq x \end{cases}$$

We can get a probability distribution graph by using the Fokker-Planck method. We show it in Figure 4.

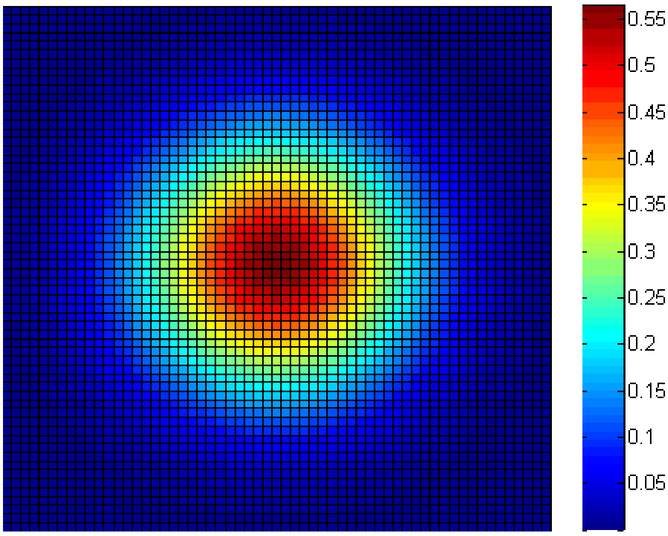

Figure 4: probability distribution graph in classical estimation

The advantage of this method in the estimation process is fairly tractable. We can predict soon after we get the relevant data.
But the situation on the sea is unpredictable. The field of wind and current data always contain errors. Accumulation of uncertainty can lead to great error. The search area is a function of quadratic time. We need to expand our search area rapidly over time. That will reduce our search efficiency.
Thus, we seek for a better model.

4.4.2 The Monte Carlo Simulation

In practice we usually cannot get sufficient accurate data for calculation. The irregular geometry of a wreckage make the computation more complex. Thus, rather than forecasting the exact trajectory of the object, a most probable area is sought.[14]
The Monte Carlo technique builds on statistics. We employ it to study complex structure or situation. We have many uncertainties in calculation of the motion of an object on the sea face. So it is an ideal method.

1.Model Establishment
Since we search for the wreckage on the sea surface, we build a two dimensional model. We use Arakawa C-grid in (X, Y, σ) coordinates of Princeton Ocean Model. We demonstrate it in Figure 5. Where x is unit length of X-axis and y is unit length of Y-axis. We define the speed of wind and current at the border as in Figure 5.[15]

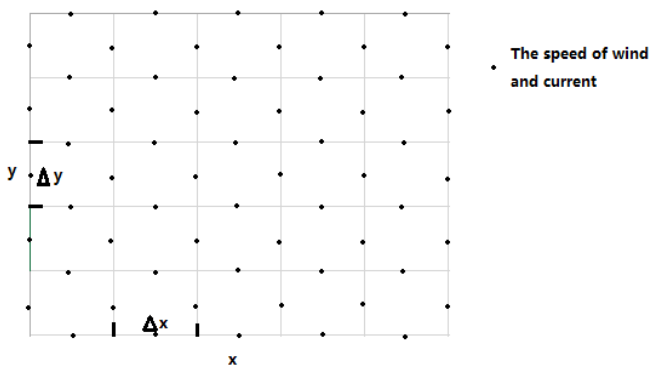

Figure 5: Grid setting in the drift model

2.Interpolation Method
Because we defined the speed at the border, we need to define the speed of any point in the grid by using the interpolation method. To solve this problem, we use the vector interpolation method. The idea of the method is presented in Figure 6.

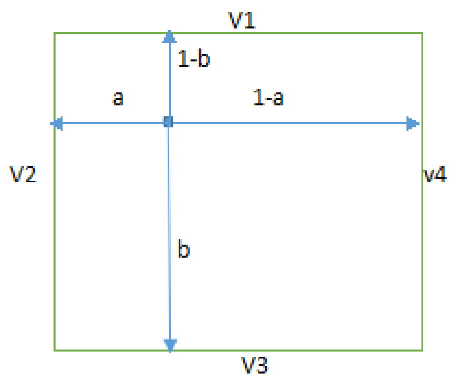

Figure 6: Vector interpolation method

Where a, b are weights. They show the position of the point in the grid. We define the coordinates of the point as (x, y). Obviously we have

$$a = x - floor(x)$$
$$b = y - floor(y)$$

Note that function floor is a downward integral function. We calculate the speed of the point using

$$\mathbf{v}_x = a\mathbf{v}_4 + (1 - a)\,\mathbf{v}_2$$
$$\mathbf{v}_y = b\mathbf{v}_1 + (1 - b)\,\mathbf{v}_3$$

Where v_x, v_y are the transverse and the vertical speed respectively.

3.Motion equation of the wreckage

We can break down the motion of the wreckage into force-caused motion(e.g.,leeway and slippage) and random motion. We denote force-caused motion as $B(t)$ and random motion as $W(t)$. We can compute $B(t)$ accurately by the interpolation method. And we regard $W(t)$ as a Wiener motion. Wiener motion is a normal process that is usually used in describing a stochastic process.

We assume $B(t)$ and $W(t)$ are independent and randomly distributed. We use $S(t)$ to represent the motion of the wreckage. We get

$$\mathrm{d}S(t) = B(t)\mathrm{d}t + C(t)\mathrm{d}W(t)$$

Where $C(t)$ is a random force field. For the convenient of numerical simulation, we use improved formulas as below:[15]

$$x_{n+1} = x_n + \frac{\mathrm{d}B(x)}{\mathrm{d}x}\Delta t + Z_1\sqrt{2B(x)\Delta t}$$

$$y_{n+1} = y_n + \frac{\mathrm{d}B(y)}{\mathrm{d}y}\Delta t + Z_1\sqrt{2B(y)\Delta t}$$

We can replace $B(x)$ with the speed from former interpolation.i.e.,$B(x) = c_1\mathbf{v}_{wind} + c_2\mathbf{v}_{current}$. c_1 and c_2 are drag coefficients. We can get them through the type of plane and experience. Δt is time step. We can valuate it according to reality. z_1 and z_2 represent standard normal distribution function.

At this point, we get the motion equation of the wreckage. After we get the equation, we can replicate particles massively. Through which we can get the probability of the wreckage occurrence area.

We simulate a graph of distribution of probability. See Figure 7.

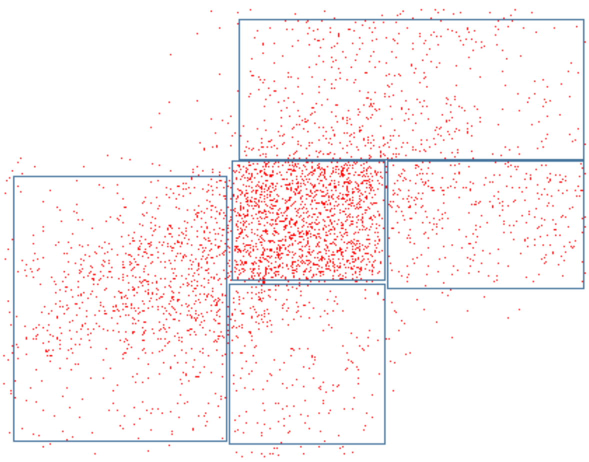

Figure 7: A graph of distribution of probability

We can observe the graph of wreckage distribution of probability. We can determine the search area and the search order by the graph. We search for the central box which has the highest probability first. Then we move to boxes around it.

4.5 Model Evaluation

To verify the stability of the model. We consider extreme situation: the speed of both wind and current are very high and in the same direction(suppose upper right). We get the result of simulation in Figure 8.

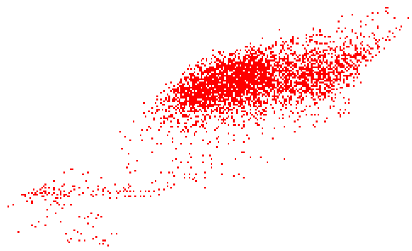

Figure 8: Simulation in extreme situation

We can see that motion of all model particles have strong directionality.i.e., the same as the direction of the wind and the current.
Therefore we can recognize this model has a good predictive power. And the construction of the model is rather simple. We just need two empirical parameters(drag coefficient of wind and current), and let computer determine the rest. Compared with classical model, it is more objective, scientific and has a strong operability.

5 Model III: Optimal Path Searching Model

In the wreckage drift model, we determine the search area and the search order. Then, we build a model for the search route and the search time. We can use the model to determine our optimized search plan.

5.1 Additional Assumptions

- **We define an 'optimized' search plan: we focus on the plan that cost the least time, then we consider the economic factor.(e.g.,the quantity of the search planes)**
- **We start our search from the area that contains maximal probability.**
- **The plane lost its power and started to fall soon after the last communication.** Bacause common plane will contact with the ground in a high frequency.
- **We assume our search plane and electronics are in a good state during the search. No misinformation will appear. And the plane has a constant speed.**

- **The wreckage is assumed to be stationary during each search.** In each subarea, the search time is rather short. In order to simplify the model, we neglect the motion of wreckage in a search for each subarea.

5.2 Notations

Table 4: Notations in model II

Symbol	Definition
W	Sweep width of the sensor
r	Distance between object and sensor
j	Number of subarea
a	Length of the search area
b	Width of the search area
t	Time
T	Maximal search time
v	Speed of the search plane
N	Numbers of the plane
S_n	Length of path n

5.3 Influential Factors

Electronics and Sensors on the Search Plane
Different types of search planes often use different electronics and sensors. The detective probability and detective range are various. These affect our search a lot.
The Situation of the Search Area.
The size of the search area apparently make a difference for our search time. Meanwhile, the weather of the search area(e.g.,foggy) influence our search.
Search Path
Apparently,we can not miss any area in a search. And we should search area with high probability in prior. In this prerequisite, we design a feasible route according to the graph of distribution.
An Ideal Model
Due to above factors, we build an ideal model. We need to search as fast as possible. We only consider a feasible search path in a fixed area. An ideal model may be not that in line with actual situation. We take it for its conciseness and certainty. It's a good ground work for us to consider models that contain more factors.

5.4 Search Capabilty of the Search Plane

Following Koopman, we assume the search plane moves in a line.[16] The search plane detects target on both sides. We define r as the search radius of the search plane. Due to the fact the shape of radar antenna is usually narrow, we handle the search range is a vertical line of the route of the search plane. As in Figure 9.

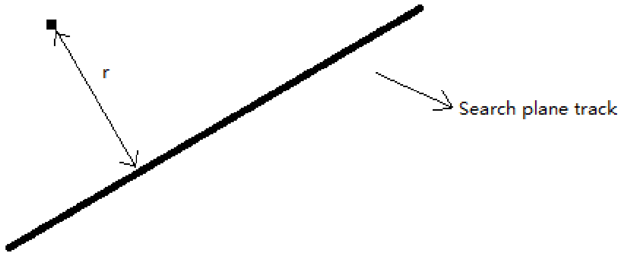

Figure 9: A graph of distribution of probability

Different search plane has a different detection probability to various r. We define $P(r)$ as the probability of the search plane detects the target in a distance of r. And the sweep width W of the sensor is defined to be the area under the lateral range function,i.e.,[17]

$$W = \int_{-\infty}^{\infty} P(r)\mathrm{d}r$$

Note that W may not be 1. The following figures show some lateral range functions of typical equipments on the search plane.

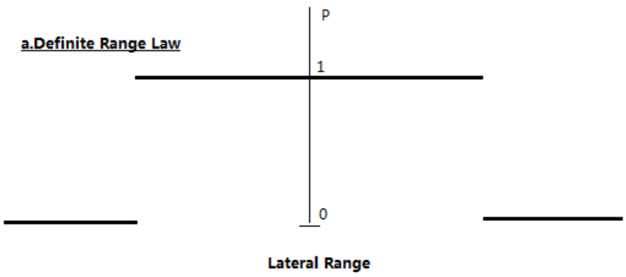

Figure 10: Lateral range example I

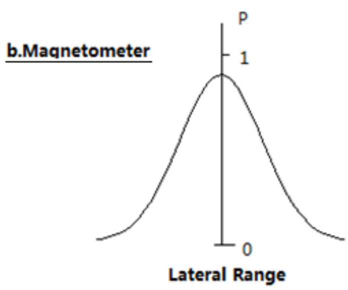

Figure 11: Lateral range example II

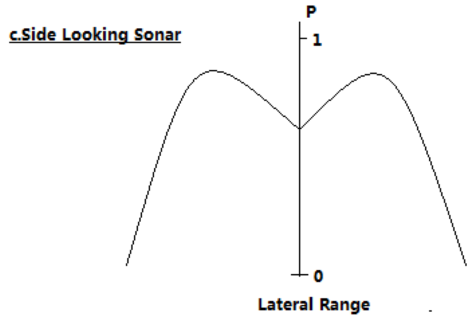

Figure 12: Lateral range example III

Apparently, we can adopt W, namely sweep width, as the detect capacity of the search plane.

5.5 Search Path

We determine the search path with the graph of wreckage distribution of probability. Here we consider two common modes: uniform distribution and gradient distribution.

Uniform Distribution
We consider the Figure 7. Particles here distribute uniformly in every rectangular. Thus we divide the search area into several equiprobable rectangular. In this circumstance, we search each rectangular uniformly. We adopt parallel path search. The figure 13 can help us demonstrate.

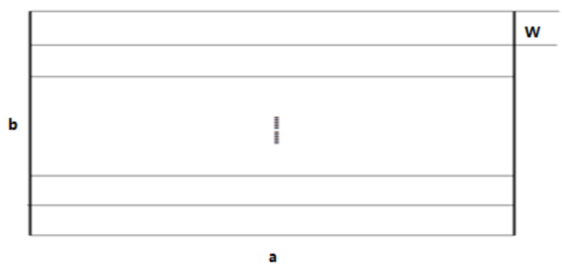

Figure 13: A possible graph of the probability distribution of the wreckage

Suppose our search area is a rectangular that has dimensions of a by b. Consider reducing the turning frequency of the search plane. We divide short side into j areas. Each area has a width of W. Recalling that W is also the sweep path of the search plane. We get

$$j = \frac{b}{W}$$

Therefore, we get the function of the maximal search time T

$$T = \frac{ja}{v}$$

Of which v is the speed of the search plane. If we send N same planes, we get

$$T = \frac{ja}{Nv}$$

Gradient Distribution
Consider following graph of probability distribution as Figure 14.

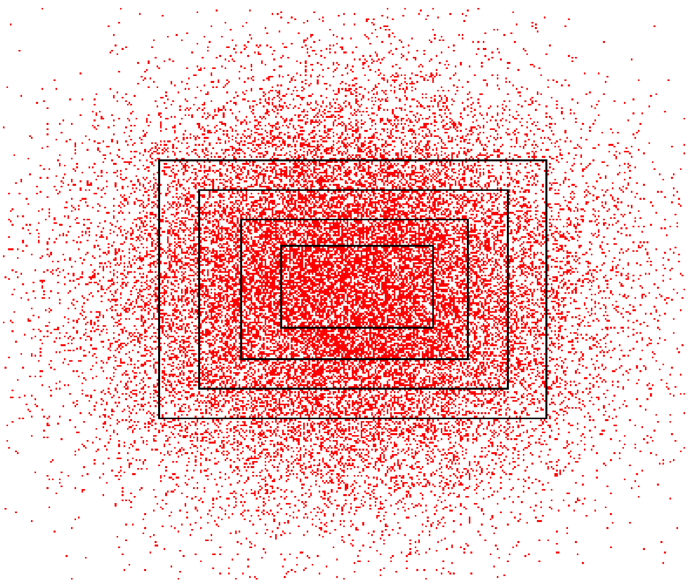

Figure 14: Lateral range example III

Clearly, we should search progressively from the center to the surrounding. Compared with parallel path search, this method is more conductive to find the wreckage. Thus, we adopt the spiral search. We demonstrate it with Figure 15.

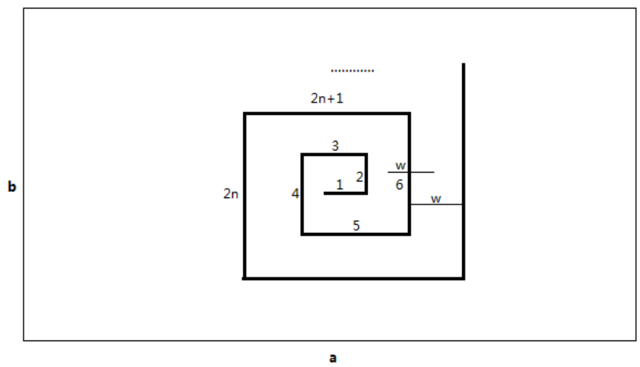

Figure 15: A spiral search

Here, W is the sweep width. In order to cover every corner of the search area, without blind sector. The width between every odd-numbered path is W. And the width between even-numbered path is the same. We denote S_n as the length of path n.

$$S_n = \begin{cases} S_1 + \frac{n-1}{4}W & n = 1,3 \\ W & n = 2 \\ S_{n-2} + W & n > 3 \end{cases}$$

According the equations, we can determine the search path(e.g.where to turn). S_1 can be 0 or a reasonable value. Because both the area of the search and the speed of the search plane are constant, the maximal search time equals to that of parallel path search.i.e.,

$$T = \frac{ab}{Wv}$$

5.6 Detection Function

In a search problem, we focus on the time we cost to detect the target. So we define detection function:

$$b(t) = probability\ of\ detecting\ the\ target\ after\ time\ t$$

Knowing the detection function can help us to evaluate the detection capability of our plan. For our parallel search, it is a linear process. The probability in each subarea is the same. And obviously we have $b(t) = 0$, $b(t) = 1$. So we get

$$b(t) = \frac{Nvt}{ja}$$

We present it in Figure 16.

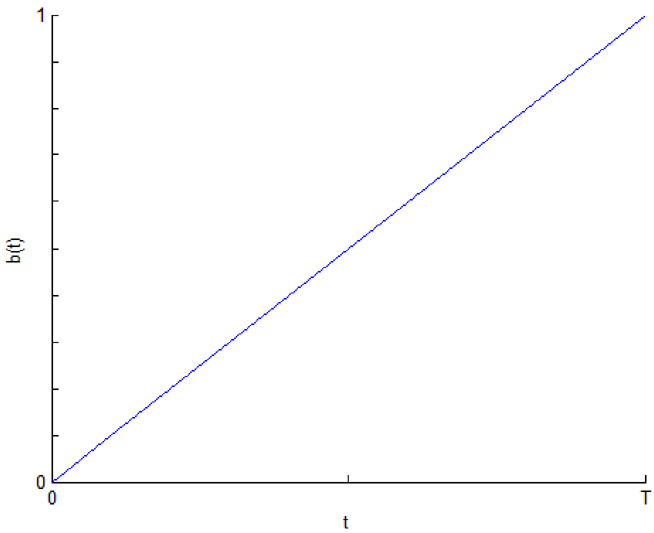

Figure 16: Detection function I

For our spiral search, the detection probability is even changing. And b(t) satisfy following relation before $T/2$.

$$\frac{\mathrm{d}B(t)}{\mathrm{d}t} = F$$

Where F is a probability distribution function(e.g.Gaussian function). The graph of b(t) is shown in Figure 17.

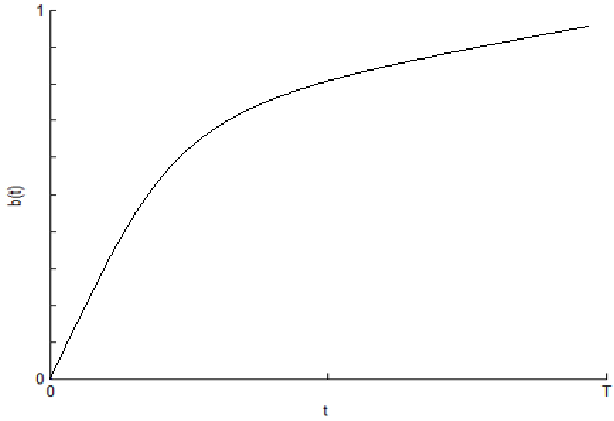

Figure 17: Detection function II

We can see that we have a large chance to find the wreckage of the plane.

5.7 Practical Model

In a real life search, the situation is not that optimistic as in the ideal model. The greatest uncertainty is the situation of the search area. For example, a heavy fog affects visibility which will influence the speed of the search plane. Rain and snow weather may narrow the sweep width. Luckily, Koopman gave a classical and practical search probability function:

$$b(t) = 1 - exp(-Wvt/A)$$

We call it random search formula. This model is proved to be practical in many cases. The graph of $b(t)$ is in Figure 18.

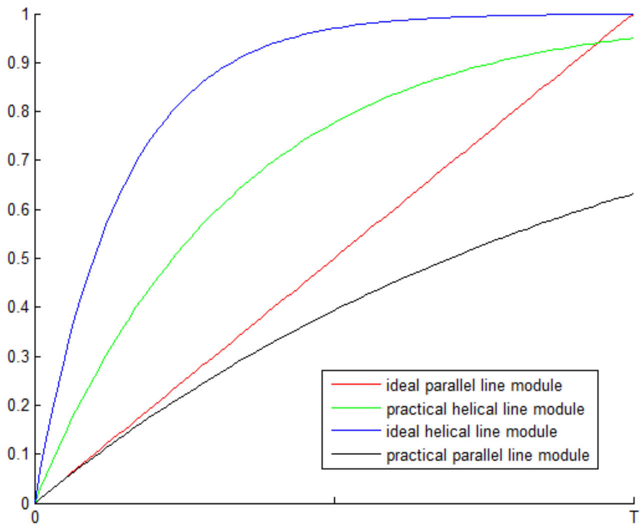

Figure 18: The graph of random search formula

The graph shows search probability function curve of different search modes in different models. Apparently, this practical model is more realistic. It also implies we may not detect the wreckage even in a long time. That accord with our experience. Through this graph, we can estimate the time of the wreckage detection. It is guided for the whole search.

5.8 Model Evaluation

Our search path model analyze two common wreckage probability distribution graphs. We design different path to their characteristics. Our model is operational and instructive. But for simplification, our model is a stagnant search model. We didn't discuss the motion of wreckage in our search. Thats the shortage of our model.

6 Application: An Example

We combine Model 1, Model 2 and Model 3 together to build a complete model, which can help to estimate the crash site from the location where the we lost communication with the plane. Then we can determine the search area and the search path. We now simulate the process. (Data are just used for simulation)

On March 4th, 2017, an Airbus A320-211 from Los Angeles to Sydney by way of Hawaiian Islands lose the communication. Its original flight range is $12,118$ kilometers. A south wind which has a speed of $5m/s$ blows near the Hawaiian Islands. The plane loses communication four hours and ten minutes after its departure. This is demonstrated in Figure 19.

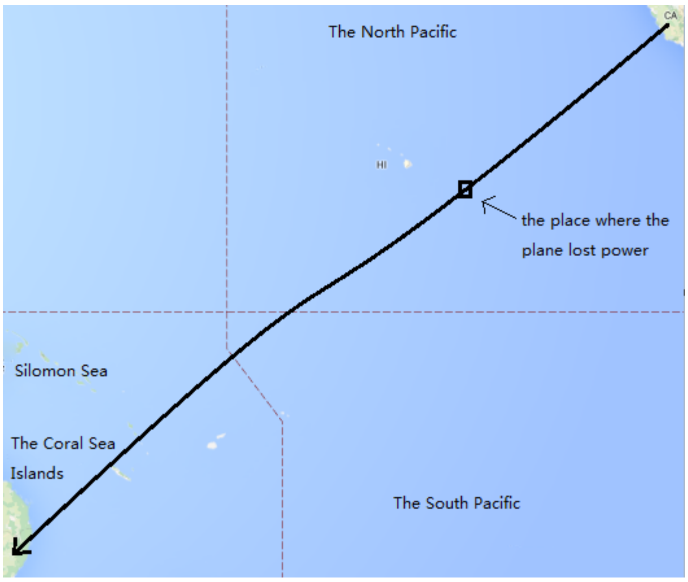

Figure 19: The route of the Airbus A320-211

We can easily calculate the distance between the point C and the initial point A. $s = 15000s \times 240m/s = 3600km$. Based on the flight route, we know that C is located at $16.35N, 148.32$W. According to Model I, we can quickly get the probable crash site D at

$16.05N, 148.58W$. We firstly consult residents living on the Hawaiian Islands. Unfortunately, they havent noticed any plane or wreckage passing by. Then we can only use the wreckage drift model to estimate possible areas. To begin with, taking current and wind direction into consideration, we know that here exists equatorial counter current on the Pacific Ocean. We demonstrate it in Figure 20.

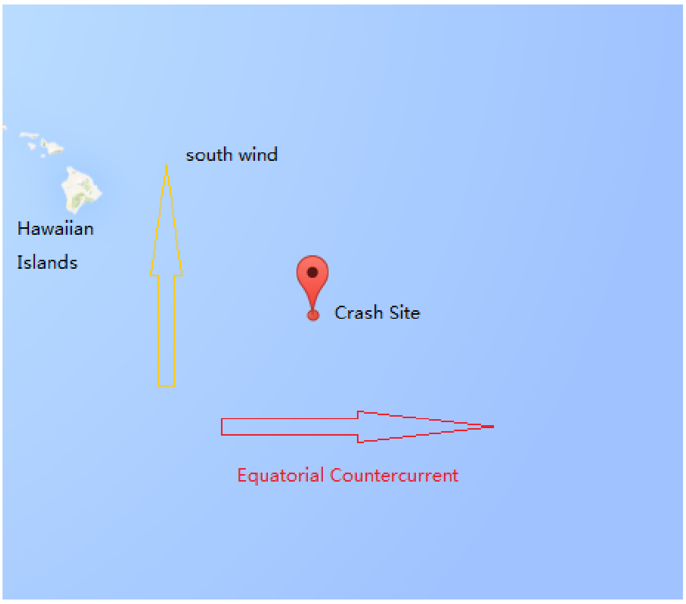

Figure 20: Wreckage drift model

We can use Arakawa-C grid to interpolate for wind speed and water velocity. Then we use Monte Carlo technique to simulate. Assuming that we begin the search one day later, we can set the time step dt as one second and particle iterations as $86,400$ times(apparently there are $86,400$ seconds a day). Then we get the probability distribution graph as Figure 21 and define the search range.

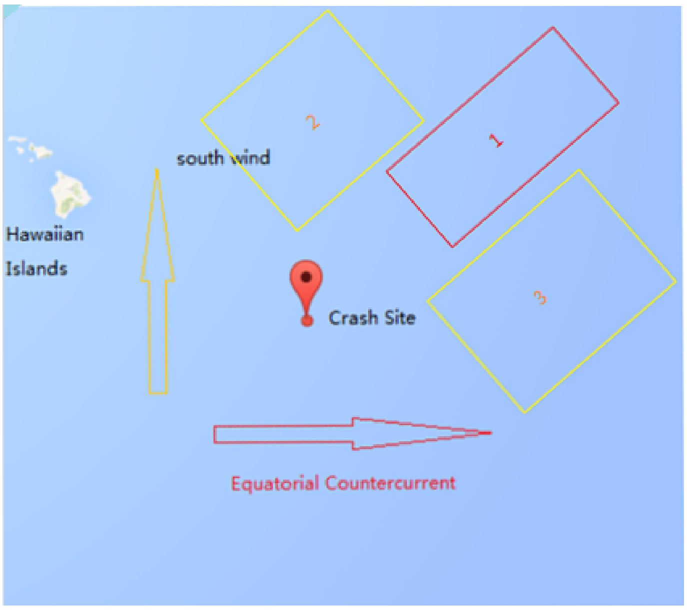

Figure 21: Search areas

The red zone stands for area with high probability and the yellow zone stands for area with low probability. Particles distribution is uniform in each zone. Then as for the uniform probability distribution graph, we can adopt parallel search pattern. According to optimal search path model, its search probability function meets the condition that $b(t) = 1 - exp(-Wvt/A)$.
Referring to *National Search and Recue Manaul*, we find that the visual sweep width is 7km, and our search plane flies at $250km/h$. So if $t = 2A/Wt = 11.1\ hours$, we have a probability of $b(11.1) = 0.86$ to successfully find the wreckage. This is an optimistic estimation. That is to say, we have a high probability to find the wreckage within 11 hours. Then, we can send fleets to salvage the black box there.

7 Conclusions

Aiming to approach the problem of a search for the lost plane, we build a three stage model. Note that there are many stochastic factors involved in a search, we start from the basic. In each stage, we construct a fundamental model first. Then we discuss uncertainties gradually to extend our model. Based on former studies, we seek for a better method that is suitable for various situations. In our test, our model is sensitive to the variation of inputs. It is also stable in the extreme situation. We give an example of applying our model to search a plane that crashed in the way to Sydney from Los Angeles. The result is we can make an efficient search plan using our model.

8 Strengths and Weaknesses

8.1 Strengths

In general, our calculation results are consistent with that from our mathematic models. Namely, we can come to reliable conclusions which means our models can be applied to search the lost plane crashed in open water to some extent. Furthermore, our models are concise, which can be easily understood. Last but not least, as we have carefully tested the effectiveness and feasiblility, results of our models can be trusted.

8.2 Weaknesses

Model I is restricted by limited data. And we define several restrictions to simplify simulation process, which certainly introduces uncertainty. Some factors like flight posture during the fall are neglected. For the lack of time, we don't study the motion of the wreckage during each time of a search.

9 Future Works

In the future work, we can focus on optimizing the model of a downed plane. We can include changes in air density, bank angle of aircraft, angle of attack and so on. This can make the model more accurate. Meanwhile we can consider optimizing the search path model for a moving wreckage during our search. Or more likely, there may be some survivors moving towards the bank actively, which we didn't take into consideration in our model. These should be attached much attention in our future work.

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Appendices

Appendix A Non-technical paper

First, we feel deeply sorry for this aircraft accident. Meanwhile, we express sincere apologies to relatives of passengers. We have asked relevant experts to predict on the possible crash sites which will take one day at most. Then we will define the searching area based on these sites. Soon, our maritime sector will send professional SAR team to arrange rational SAR plans according to the conditions of the searching area so that they can search faster. This time we will designate two advanced P-8A Poseidons and three P-3C Orions to make efficient and quick investigations on marine conditions of target areas. Recent steady currents and good weather conditions are very favorable for our investigation work. We will also specially send some people to consult residents who live on islands along the air route and expect some useful clues for our searching. We have confidence that we will find the wrecks in three or four days. Then we will have professional searching fleets and submarines look for the black box underwater. And by then we shall announce related details about the aircraft accident as soon as possible.