SJTU 4124

这是一道非常强力的线段树题目。

首先考虑维护什么。我们用 $res_{i,k}$ 表示一个从节点 i 对应的区间中选 k 的球的总得分。显然,对于一个 节点 i , $res_{i,1}, \cdots, res_{i,10}$ 全都要维护。

然后考虑怎么把答案凑出来。对于 i , res_i 等于左儿子 res 、右儿子 res 、两个儿子 res 的卷积三者的和。这是因为从 i 对应的区间中取数,这些数要么全部来自一个儿子,要么来自左边的一部分和右边的一部分。

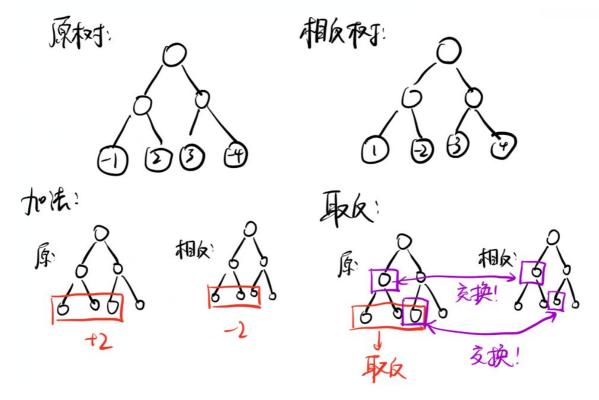
在考虑如何处理这些操作。对于加法操作,维护一个加法标记。然后比较麻烦的事情在于加法之后怎么更新 res。这里直接给出更新方法:如果对区间长度为 len 的节点 i 加了 e,那么

$$res_{i,k} = \sum_{t=0}^{k} e^{t} res_{i,k-t} inom{len - k + t}{t}$$

具体怎么来的可以手动推一下。

对于相反数操作,我们也维护一个标记。似乎可以通过规定取反和加法的顺序实现维护。但是困难之处在于对 res 的更新,直接对 res 取反是错误的!因为 res 是若干个积的和,积的符号受到乘数的奇偶性的影响!

为了解决这个问题,我们额外维护一棵**相反的树**,即一开始保存的就是和原来的树相反的值,加法操作也加相反的数。这样的话,对一个区间取相反数就等价于**将对应区间的节点和相反树上的节点进行交换**!如图所示。



这样整个问题就顺利解决了。

整个算法的时间复杂度很高,但居然还是能跑得动,也是比较神奇了。

- 1 | #include <bits/stdc++.h>
- #define INF 2000000000

```
3
    #define MAXN 131100
    #define M 100000007
 4
 5
    using namespace std;
    typedef long long 11;
 7
    int read(){
 8
        int f = 1, x = 0;
 9
        char c = getchar();
10
        while(c < '0' || c > '9'){if(c == '-') f = -f; c = getchar();}
        while(c >= '0' && c <= '9')x = x * 10 + c - '0', c = getchar();
11
12
        return f * x;
13
    }
    struct Q{
14
15
        int arr[11];
16
        Q(){
17
            for (int i = 1; i \le 10; ++i)
18
                arr[i] = 0;
19
        }
20
    };
21 | int n, m, siz;
    int fac[MAXN], inv[MAXN], invfac[MAXN];
23 | int logg[MAXN] = \{0\};
    int CC[MAXN][11] = \{0\}, tmp[11];
24
25
    bool xfs[MAXN] = \{0\}, xfs_neg[MAXN] = \{0\};
26 | int add[MAXN] = \{0\}, add_neg[MAXN] = \{0\};
27
    Q res[MAXN], res_neg[MAXN];
28
    int _a, _b, _v, opt;
29
    inline int modadd(int x, int y){
30
31
        return (x + y >= M ? x + y - M: x + y);
32
33
    inline int takexfs(int x){
34
        return (x == 0 ? 0: M - x);
35
36
   inline int C(int n, int m){
37
        return 1|| * (1|| * fac[n] * invfac[m] % M) * invfac[n - m] % M;
38
    }
39
40
   void maintain(int i){
        for (int j = 1; j \le 10; ++j){
41
42
             res[i].arr[j] = modadd(res[i << 1].arr[j], res[i << 1 |</pre>
    1].arr[j]);
43
            res_neg[i].arr[j] = modadd(res_neg[i << 1].arr[j], res_neg[i << 1</pre>
    | 1].arr[j]);
44
            for (int t = 1; t < j; ++t)
45
                res[i].arr[j] = modadd(111 * res[i << 1].arr[t] * res[i << 1 |
    1].arr[j - t] % M, res[i].arr[j]),
                 res_neg[i].arr[j] = modadd(11] * res_neg[i << 1].arr[t] *
46
    res_neg[i << 1 | 1].arr[j - t] % M, res_neg[i].arr[j]);</pre>
47
48
    }
    void modify(int id, int len, int x, int x_neg){
49
50
        for (int i = 1; i \le 10; ++i)
51
            tmp[i] = res[id].arr[i];
        tmp[0] = 1;
52
        for (int i = 1; i \le 10; ++i){
53
54
            int fin_res = 0;
55
            for (int j = 0, xx = 1; j \le i; ++j, xx = 111 * xx * x % M)
```

```
fin_res = modadd(fin_res, 111 * (111 * xx * tmp[i - j] % M) *
 56
     CC[len - i + j][j] % M);
 57
              res[id].arr[i] = fin_res;
 58
          }
 59
 60
          for (int i = 1; i \le 10; ++i)
 61
              tmp[i] = res_neg[id].arr[i];
 62
          tmp[0] = 1;
          for (int i = 1; i \le 10; ++i){
 63
 64
              int fin_res = 0;
 65
              for (int j = 0, xx = 1; j \le i; ++j, xx = 111 * xx * x_neg % M)
                  fin_res = modadd(fin_res, 111 * (111 * xx * tmp[i - j] % M) *
 66
     CC[len - i + j][j] \% M);
 67
              res_neg[id].arr[i] = fin_res;
 68
 69
     }
 70
     void pushdown(int id, int len){
 71
          if (xfs[id]){
 72
              xfs[id << 1] = !xfs[id << 1], xfs[id << 1 | 1] = !xfs[id << 1 |
     1];
 73
              xfs_neg[id << 1] = !xfs_neg[id << 1], xfs_neg[id << 1 | 1] =
     !xfs_neg[id << 1 | 1];
 74
              swap(add[id << 1], add\_neg[id << 1]), swap(add[id << 1 | 1],
     add_neg[id << 1 | 1]);
 75
              for (int i = 1; i \le 10; ++i)
 76
                  swap(res[id << 1].arr[i], res_neg[id << 1].arr[i]),</pre>
 77
                  swap(res[id << 1 | 1].arr[i], res_neg[id << 1 | 1].arr[i]);</pre>
 78
              xfs[id] = false;
 79
          }
 80
          if (add[id] != 0){
 81
              add[id << 1] = modadd(add[id << 1], add[id]);</pre>
              add[id \ll 1 \mid 1] = modadd(add[id \ll 1 \mid 1], add[id]);
 82
              add_neg[id << 1] = modadd(add_neg[id << 1], add_neg[id]);</pre>
 83
              add_neg[id \ll 1 \mid 1] = modadd(add_neg[id \ll 1 \mid 1], add_neg[id]);
 84
 85
              modify(id << 1, len, add[id], add_neg[id]);</pre>
 86
              modify(id << 1 | 1, len, add[id], add_neg[id]);</pre>
 87
              add[id] = add_neg[id] = 0;
 88
          }
 89
     }
 90
     void update(int id, int 1, int r){
          if (1 > b \mid | r < a) return;
 91
 92
          if (1 \ge a \& r \le b)
              if (opt == 1){
 93
 94
                  // add
 95
                  add[id] = modadd(add[id], _v);
 96
                  int neg_v = takexfs(_v);
 97
                  add_neg[id] = modadd(add_neg[id], neg_v);
 98
                  modify(id, (r - l + 1), v, neg_v);
 99
              }else {
100
                  // takes minus
101
                  swap(add[id], add_neg[id]);
102
                  xfs[id] = !xfs[id];
103
                  xfs_neg[id] = !xfs_neg[id];
104
                  for (int i = 1; i \le 10; ++i)
105
                      swap(res[id].arr[i], res_neg[id].arr[i]);
106
              }
107
              return ;
108
```

```
int mid = (1 + r) >> 1;
109
110
         pushdown(id, mid - 1 + 1);
111
         if (a \ll mid) update(id \ll 1, 1, mid);
112
         if (b > mid) update(id << 1 | 1, mid + 1, r);
113
         maintain(id);
114
     }
115
     Q query(int id, int 1, int r){
116
         if (1 > b \mid\mid r < a) return Q();
117
         if (1 \ge a \& r \le b) return res[id];
118
         int mid = (1 + r) >> 1;
119
         pushdown(id, mid - 1 + 1);
120
         if (_b <= mid) return query(id << 1, 1, mid);</pre>
121
         if (a > mid) return query(id << 1 | 1, mid + 1, r);
         Q logo = query(id << 1, 1, mid), rres = query(id << 1 | 1, mid + 1,
122
     r);
123
         Q curres;
124
         curres.arr[1] = modadd(lres.arr[1], rres.arr[1]);
         for (int j = 2; j \le 10; ++j){
125
126
             curres.arr[j] = modadd(lres.arr[j], rres.arr[j]);
127
             for (int t = 1; t < j; ++t)
                  curres.arr[j] = modadd(111 * lres.arr[t] * rres.arr[j - t] %
128
     M, curres.arr[j]);
129
         }
130
         return curres;
131
     void init(){
132
         n = read(), m = read();
133
         for (siz = 1; siz < n; siz <<= 1);
134
135
         for (int i = 1; i \le n; ++i){
136
             int a = read();
137
             if (a < 0) a += M;
138
              res[i + siz - 1].arr[1] = a;
139
              res_neg[i + siz - 1].arr[1] = takexfs(a);
140
         }
141
         for (int i = siz - 1; i >= 1; --i)
142
             maintain(i);
143
         // build C
144
         fac[1] = inv[1] = invfac[1] = 1;
145
146
         fac[0] = invfac[0] = 1;
         for (int i = 2; i \le siz; ++i)
147
              inv[i] = 111 * (M - M / i) * inv[M % i] % M,
148
149
             fac[i] = 111 * fac[i - 1] * i % M,
             invfac[i] = 1]] * invfac[i - 1] * inv[i] % M;
150
151
         CC[0][0] = 1;
152
         for (int i = 1, j = 1; i \le siz; i \le 1, ++j)
153
             logg[i] = j;
154
              for (int u = i; u >= i - 10 \&\& u > 0; --u)
                  for (int t = min(10, u); t >= 0; --t)
155
156
                      CC[u][t] = C(u, t);
157
         }
158
     void solve(){
159
160
         while (m--) {
161
             opt = read();
162
             if (opt == 1){
163
                  _a = read(), _b = read(), _v = read();
164
                 if (_v < 0) _v += M;
```

```
165
                 update(1, 1, siz);
166
             }
             if (opt == 2){
167
                 _a = read(), _b = read();
168
                 update(1, 1, siz);
169
170
             }
171
             if (opt == 3){
                 _a = read(), _b = read(), _v = read();
172
173
                 Q curres = query(1, 1, siz);
                 printf("%d\n", curres.arr[_v]);
174
175
             }
176
         }
177
178 | int main(){
179
         init();
180
         solve();
181
         return 0;
182 }
```