Equations that represent the unity

Here the unity is denoted as U.

$$U = \prod_{n=2}^{\infty} \left(\frac{n}{n+1} \cdot \frac{n^2-1}{n(n-1)} \right)$$

$$U = \sqrt{\cdots\cdots \sqrt{\sqrt{\frac{1+\sqrt{5}}{2}}}}$$

$$U = \frac{1}{2} + \frac{9801}{114818048} \cdot \sum_{n=0}^{\infty} \frac{(109n+4649) \cdot 5^{2n}}{124^n}$$

The n-th decimal digit of U, denoted as u_n can be obtained by the following equality:

$$u_n = \begin{cases} 1 + ((n \bmod 5) + 3)^2 + 2 \cdot \left(\frac{(2n)!}{(n!)^2} \bmod 5 \right) & \text{if } (n \bmod 5)! = 0 \\ 9 & \text{otherwise} \end{cases}$$