PKU - HUST Lecture Series

Lecture VII - More on Avalanches and Turbulence Spreading

① As time (and energy) in short supply:

⇒ rather a general discussion

⇒ conceptual rather than detail - intensive.

⇒ Plenty of references ....
Good area for projects ..... can discuss

② Avalanches - Review

BTW

SOC

⇒ "Noise propagates thru the scaling clusters by means of a "domino" effect upsetting the minimally stable states"

$P \sim 1/S$

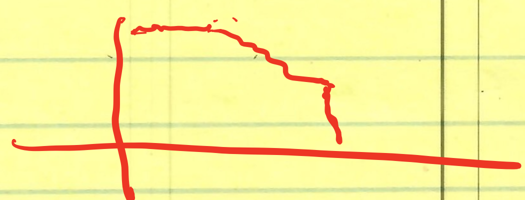
Dynamical of SOC state

⟺ Space - time propagation.

perturbative transport

⇒ Trends:

$P \sim 1/\Delta$ Zipf's Law

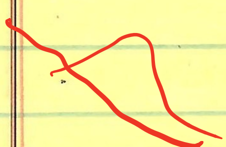


$S(\omega) \sim 1/\omega$

⇒ $\boxed{\text{We want an equation ....}}$

$\boxed{\text{JR}}$ Symmetry → $x \to -x$
$\delta P \to -\delta P$

Ginzburg-Landau



⇒ $\Gamma(\delta P)$ invariant

Cons.

so

$$\boxed{\partial_t \delta P + \partial_x \Gamma(\delta P) - D_0 \partial_x^2 \delta P = \tilde{S}}$$

⇒ Burgers!

long wavelength

$$\boxed{\partial_t \delta P + \alpha\, \partial_x \delta P^2 - D \partial_x^2 \delta P = \tilde{S}}$$

$\partial_t v + v\, \partial_x v - \nu \partial_x^2 v = 0$

(long wavelength limit)

Crank → n.o. corrections ?

coupled sandpile

So:

→ shock as model for Avalanche

→ Pulse velocity

$$\boxed{V(\delta p) \sim \alpha\,\delta p}$$

[big pulses go faster]

noisy

→ $\delta x \sim c\,t$ ("ballistic")

Whitham

Burgers Solution: $(\tilde{S} \to 0)$

iup

$$\partial_t \delta p + \partial_x \left[\alpha\,\delta p^2 - D\,\partial_x \delta p \right] = 0$$

$$\partial_t \int dx\,\delta p = 0$$

⇓

area of pulse profile invariant in time, even with D_0!

→ So, $$\boxed{Re = \frac{1}{R} \int_{-\infty}^{+\infty} \delta P \, dx}$$

$$\to \frac{1}{\nu} \int_{-\infty}^{+\infty} v \, dx \to \text{const}$$

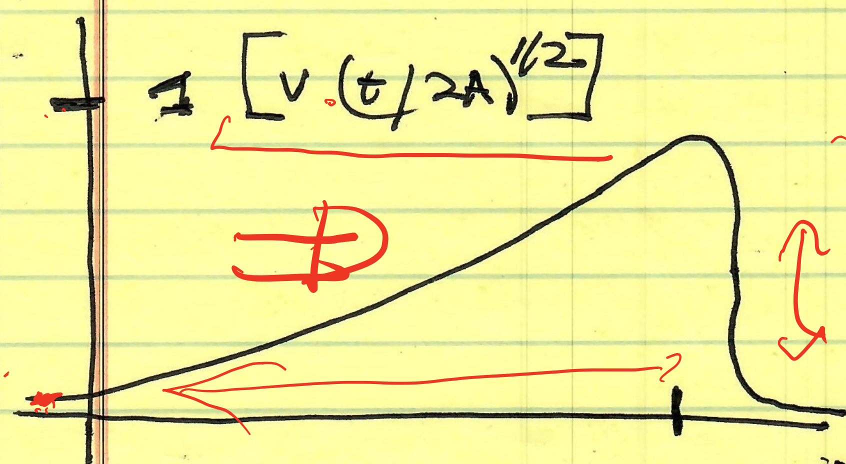
why?

So (see Whitham), pulse shape:

i.e. $F = c_0 + A\delta(x)$

↓

impulse



$(2At)^{-1/2}$

A cons.

i.e. $v \sim \frac{x}{t}$ $\quad \boxed{0 < x < 2\sqrt{At}}$

details noise matter

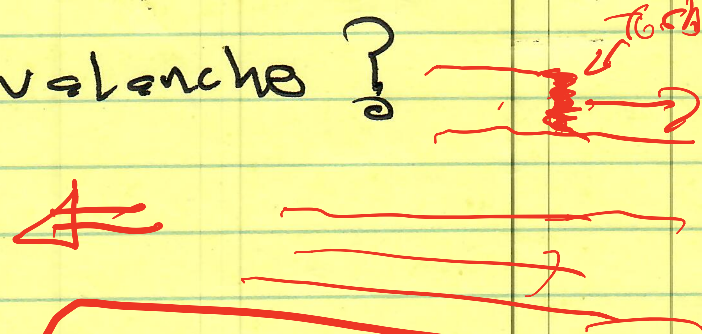
but

Where's the Physics?

Direction

③ → Can we learn from REAL avalanches?

What's a real avalanche?



→ Flood waves

→ Flood wave ⊃ Kinematic Waves

"Kinematic" waves: (Lighthill + Whitham)

$$\partial_t \rho + \partial_x (V(\rho)\rho) - \nu \partial_x^2 \rho = 0$$

↓ $V(\rho)$ specified ↘ regularization

$$\partial_t \rho + \partial_x Q(\rho) = 0$$

$$Q(\rho) = V(\rho)\rho$$

→ prop speed: (sgn v)

$$\partial_t \rho + \partial_x Q(\rho) = 0$$

$$\partial_t \rho + \frac{\delta Q}{\delta \rho} \partial_x \rho = 0 \rightarrow c(\rho)$$

$$c(\rho) = \frac{\delta Q}{\delta \rho}$$

$V_g \neq s(\rho)$

$$\boxed{c(\rho) = V(\rho) + \rho \frac{\partial V(\rho)}{\partial \rho}}$$

$c(\rho)$ ↑ ↓

of course – $c(\rho)$ increasing / decreasing with ρ

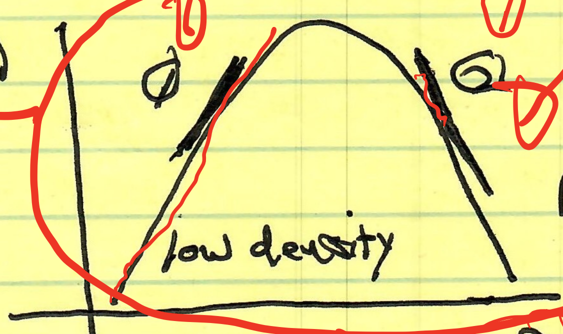
⇒ shock structure.

Classic: Traffic Flow; ρ = car density

$V(\rho)$ ⇒ [see Helbing, RMP]

$\boxed{V(\rho)}$

$Q(\rho)$



Whitham

left → empirical

Fluidization

① $\delta Q/\delta\rho > 0$ → usual

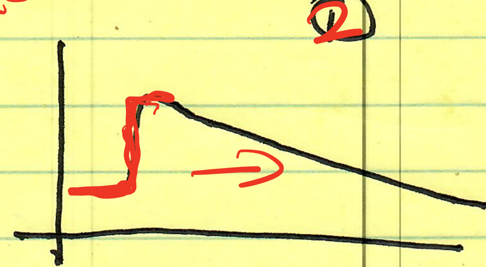
⇒ overtaking, forward shock

② $\delta Q/\delta\rho < 0$ → backward shock

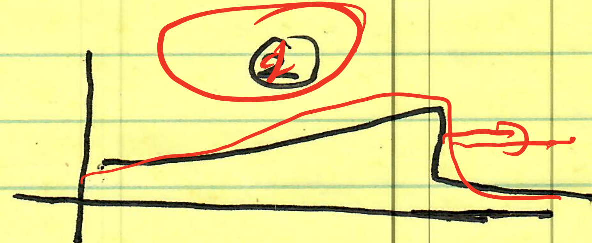
high density goes slower

$\boxed{V(\rho)\ ?}$

$\frac{\delta Q}{\delta \rho} < 0$ $\quad c < \rho$

→ 

not



① as if coming upon toll booth

→ Beyond Kinematic:

$$\partial_t \rho + \partial_x Q = 0$$

$$\begin{cases} Q = Q(\rho) - \nu\, \partial_x \rho \\ V = V(\rho) - \nu/\rho\ \rho_x \end{cases}$$

now:

$$\partial_t \rho + \partial_x (\rho V) = 0$$

Dyn. shock → [illegible] dynamics

gas dyn P Crooll

where:

$$\partial_t v + v\partial_x v = -\frac{1}{\tau}\left[v - \bar{V}(\rho) + \frac{\nu}{\rho}\rho_x\right]$$

i.e. v relaxes to $\bar{V}(\rho)$

(distribution) ?

Important:

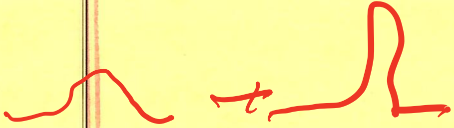
$1/\tau$ → rate of relaxation

τ ≡ driver reaction time

$\tau = \tau[\quad]$ — none → short; lots → long

Note: Another appearance of the mysterious time delay (recall LCO). !

Now, if linearize:



$\nu \to \nu \to (V_0 - c_0)^2 \tilde{T}$

$\downarrow$

$$\partial_t \tilde{r} + c_0 \frac{\partial \tilde{r}}{\partial x} = \left[\nu - (V_0 - c_0)^2 \tilde{T}\right] \frac{\partial^2 \tilde{r}}{\partial x^2}$$

Long $\tilde{T}$ $\to$ $\nu - (V_0 - c_0)^2 \tilde{T} < 0$ $\to$

negative diffusion instability

$\Rightarrow$ $\boxed{\text{Jam}}$ $\leftrightarrow$ clustering

Time Delay of interest!

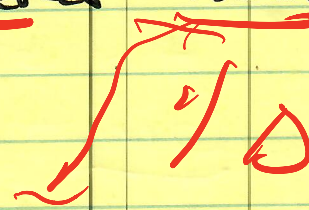
$\boxed{\text{(excess delay time} \Rightarrow \text{instability} \Leftarrow \text{again)}}$

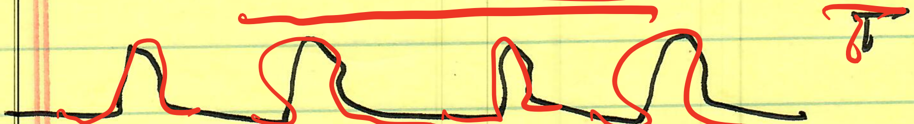
N.B. In avalanches (heat):

$\bar{V} \Rightarrow \langle Q \rangle$ – mean heat flux (QL)

$V \Rightarrow Q$ – instantaneous heat flux

$\Rightarrow$ heat flux jams $\Rightarrow$





Jamitons

Layers, staircase, etc.

Kosuga, P.D, Gurcan PRL (2012)

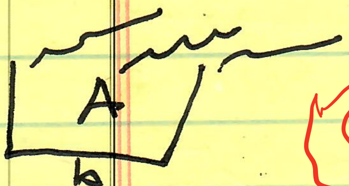
+ PoP

$1/\tau \to 1/\tau_c$

Key: τ ? → set by time dynamics of heat flux response.
(c.f. Yan Qinghao, PD ongoing)

τ → large near marginal, etc.

→ Flood Waves:



channel

cross sectional area
riverbed

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$Q = Q(A, x)$$

$$\Rightarrow \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial A}\frac{\partial A}{\partial x} = -\cancel{\frac{\partial Q}{\partial x}}$$

const. flux

$$c = \partial Q/\partial A = 1/b\, \frac{\partial Q}{\partial h}$$

$$V = Q/A$$

if neglect RHS:

Flood wave

&

avalanche

$$\partial_t A + c(A)\, A_x = 0$$

→ Kinematics.

hydraulics

shallow water theory

h, v

h

→ Beyond kinematics:

$$\partial_t h + \partial_x (hv) = 0$$

mom. eqtn.   stress

gravity

$$\partial_t (hv) + \partial_x \left(hv^2 + \tfrac{1}{2} g' h^2 \right) = g\, h\, S - C_F v^2$$

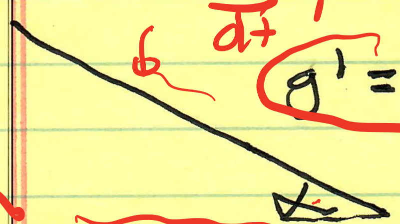
$\frac{d}{dt}$ P P

friction

C_F

hydraulics

$$g' = g \cos\alpha$$



$\alpha \neq 0$

→ downhill avalanche of fluid ..

Whitham

Whitham

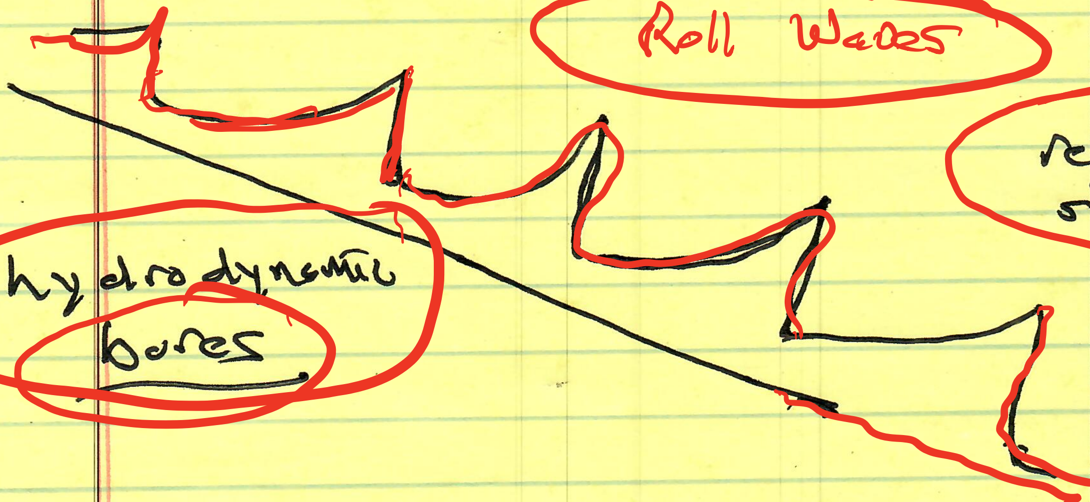
→ $\partial_t h + \partial_x (hv) = 0$

and RHS balance

$$v \cong \left(\frac{g' S}{C_F}\, h \right)^{1/2} \to v_0$$

shocks

⇒ roll waves → type of kinematic wave



Can go unstable and jam for

$c_0 < c_-$, $c_0 > c_+$

$c_+ = V_0 + \sqrt{g h_0}$

$c_0 = \frac{3 V_0}{2}$

$c_- = V_0 - \sqrt{g h_0}$

→ $V_0 < 2\sqrt{g h_0}$

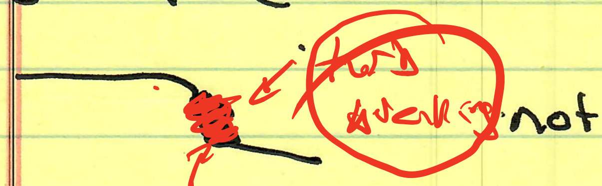
⇒ $S < 4 c_f$

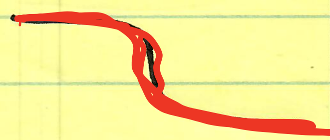
Model =
here + turb visc

→ Promising model. ...

→ See $\boxed{\text{Balmforth}}$, et. al. 2004

Incorporates turbulent mixing of bore face





Shocks: [ ]

$\dot{S} > 0$

highthill

[ ]

13 ves

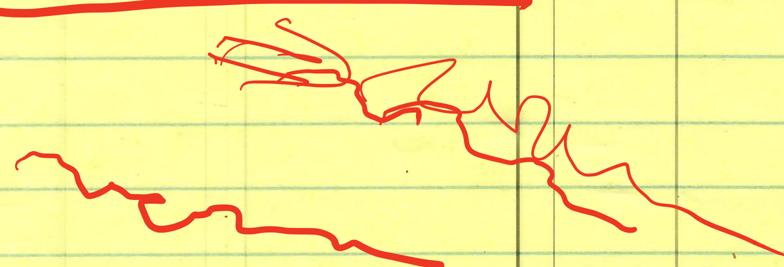
$\dot{E} < 0$

$\dot{\Sigma}_T > 0$

→ Interesting problem:

Noisey flood wave model of Avalanche.

$\tilde{S}$



Model avalanche

$z(X)$

2018

Jnl Kor. Phys. Soc.

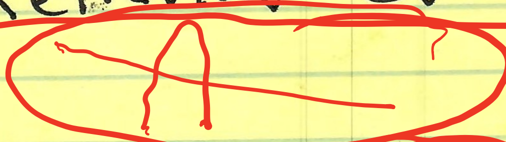
→ Turbulence Spreading

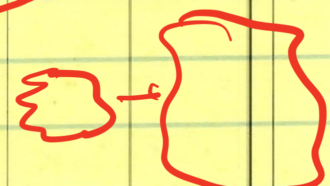
see [Hahm, P.D.
review + refs
therein

– why?

– ubiquity of turbulence, apart
ITB, ETB

– penetration of stable zone





– Old idea – $k-\epsilon$ / l models

$K \equiv$ turbulence energy / spreading

$K(x,t) \approx \langle \tilde{v}^2 \rangle$

$$\partial_t K + \langle v \rangle \cdot \nabla K - \nabla \cdot \sqrt{K}\, l \cdot \nabla K = P_{Re} - \frac{K^{3/2}}{l} + \ldots$$

Reynolds

$\epsilon \to$ EGR

"spreading" = spreading of turbulence by self-scattering

↔ "Townsend" → entrainment

Struct Turb Shear Flow

→ obviously,

$$\Gamma = -\sqrt{\varepsilon}\, \ell\, \nabla \varepsilon$$

$\Updownarrow$ related

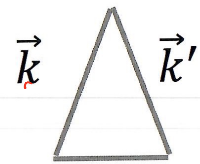
$$\Gamma = -\alpha\, \delta p^2$$

overturning, pulse propagation

⇒ turbulence propagated.

So: what is the relation / difference between turbulence spreading and avalanching ?

- **Turbulence Spreading vs Avalanching**
  - Both: (non-Brownian) radial propagation of excitation
  - Avalanching:
    - via overturning and mixing of neighboring cells
    - Coupling via $\nabla\langle P\rangle$
    - $\partial_t \delta P \sim \partial_x(\alpha \delta P^2)$

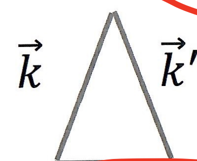


$\vec{q} = q\hat{r}$

- corrugation

Pulse

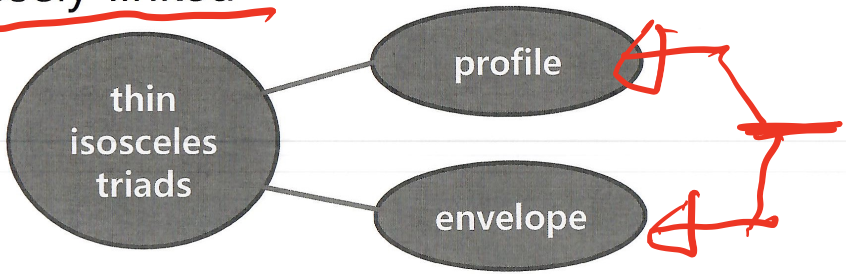
  - Turbulence spreading (t.s. by T.S.)
    - via spatial scattering due nonlinear coupling
    - Couple via turbulence intensity field
    - Usually $\partial_t I \sim \partial_x(D_0 I \partial_x I)$



$\vec{q} = q\hat{r}$

- envelope

- **Bottom Line:**
  - Very closely linked



  - ~ impossible to have one without other
  - t.s. can persist in strong driven, non-marginal regimes
  - Which effect more dramatic is variable → specifics?
  - Controversy sociological (or sociopathic)…

Physics + NLD → MFE

→ The NL Proto-type: Nonlinear Diffusion

Insect dispersal

J. D. Murray

Mathematical Biology

$$\frac{\partial n}{\partial t} = -\nabla\cdot\underline{J}$$

$$\underline{J} = -\boxed{D(n)}\,\boxed{\nabla n}\,, \qquad \boxed{\frac{\partial D}{\partial n} > 0.}$$

so 1D:

$$\frac{\partial}{\partial t}T = \frac{\partial}{\partial x}D_0\partial_x T$$

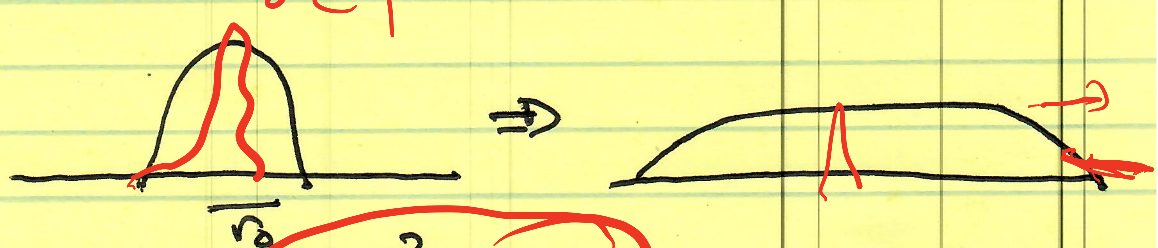
$$\boxed{\frac{\partial n}{\partial t} = D_0\frac{\partial}{\partial x}\left[\boxed{(n/n_0)^m}\,\frac{\partial n}{\partial x}\right]}$$

NL Heat Conduction

Point: $n\to 0 \Rightarrow D\to 0$

$\delta(\,)$

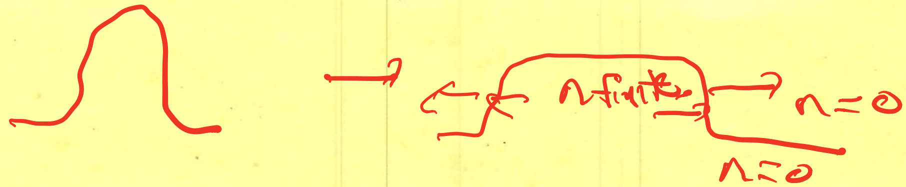
→ Front:



not $\dfrac{e^{-x^2/Dt}}{\sqrt{Dt}}$

NL → Front

not Ballistic?



n^M

Can solve exactly:

$$n(x,t) = n_0[\lambda(t)]^{-1}\left[1-\left\{\frac{x}{r_0\lambda(t)}\right\}^2\right]^{1/M}$$

$|x| < r_0\lambda(t)$

$$= 0, \quad |x| > r_0\lambda(t)$$

$$\lambda(t) = (t/t_0)^{1/2+M}$$

$$r_0 = \frac{Q\Gamma(1/M+3/2)}{\{\pi^{1/2} n_0 \Gamma(1/M+1)\}}$$

$$t_0 = r_0^2 M / 2D_0(M+2)$$

Front: $x \sim r_0\lambda(t)$

$$\sim r_0(t/t_0)^{1/2+M}$$

(slower than diffusion)

$$x \sim \sqrt{D_0 t}$$

$\partial_t n - \partial_x D_0 n\, \partial_x n = \gamma$ ...

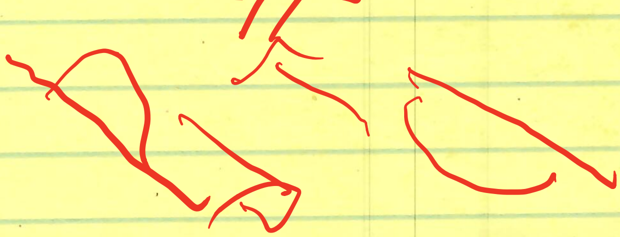
→ Other issues:

- with growth → back to Fisher!
  See Gurcan, et.al. 2005
- subcritical? → see Heinonen, P.D. 2018.
- local or non-local → See Yan, P.D. in preparation...

→ Spreading is mysterious, oft-invoked as a "deus ex machina" (god ... machine).

→ Not easy to understand by simulation

→ Should use theory to clarify!



E_r^* → Idomura

Avalanching → profile relaxation

turb spot → no profile

Boedo

E_r'

→ SOL

Blob

Probe

Pan Ou

Mo Choi → JRSD

Y. Xu

NF 2018-24

Driver → Avalanches observed

Sim exper.

$\tau < \frac{L^2}{D}$

over threshold

→ SOC (Sandpile) example spot → avalanches

Model

→ Avalanches more general than SOC

$P \sim 1/\Delta$

→ Physical System Avalanching

~~JRST~~ Burgess

— Flood wave — physics — kinematic waves

→ 3

$\partial_t \theta + \partial_x (u\theta/\theta)$

turbulent

Bone

$U(\varphi)$