Lecture III — Particle Transport, Flows Ambipolarity

Review — new comers.

Stochastic Field: — Island overlap

— lines $\frac{dr}{B_r} = \frac{r d\theta}{B_\theta} = \frac{dz}{B_{z0}}$

$$\nabla_\parallel f = 0$$

$$\nabla_\parallel^{(0)} \tilde{f} = - b_r \partial_r \langle f \rangle$$

$\left\{ \begin{array}{l} k_\parallel \sim \frac{l_{ac}}{\Delta_\perp} \frac{\delta B}{B_0} \\ B \cdot \nabla \quad \text{L vs NL} \end{array} \right.$

$$D_M = \langle b_r^2 \rangle l_{ac}$$

$$= \sum_k |\tilde{b}_k|^2 \pi \delta(k_\parallel)$$

'lines' → Diffusion in phase space r, θ, z.

$$l_{ac}^{-1} = |\Delta(k_\parallel)| \sim \Delta(k_\theta \Delta)/L_s$$

$$l_c^{-1} = (k_\theta^2 D_M / L_s^2)^{1/3}$$

$l_{ac} < l_c$ → $K_u < 1$.

Heat Transport

Regimes

$$l_{ac} < l_c < l_{mfp}$$

→ collisionless

$$l_{ac} < l_{mfp} < l_c$$

→ collisional

Heat ↔ test particles (assumption).

$$\boxed{\chi = v_{th} D_M}$$

From $\langle \delta r^2 \rangle \sim D_M\, l_{mfp}$ → coarse graining at l_{mfp} induces irreversibility

Must kick particle off field line.

Coarse grain every $(l_{mfp}/v_{th})^{-1} \sim \tau_c$

→

$$\frac{\langle \delta r^2 \rangle}{\tau_c} \sim D_M \frac{l_{mfp}}{\tau_c} \sim v_{th} D_M$$

Collisional Regime

$$\chi = v_{th} D_M \underbrace{\frac{l_{mfp}}{l_c}}_{\text{reduction}} \sim \langle \tilde{b}^2 \rangle^{1/2} \; !$$

Approach by hydrodynamic model.

$$\underline{q} = -\chi_{\parallel} \nabla_{\parallel} T\, \hat{b} - \chi_{\perp} \underline{\nabla}_{\perp} T$$

$$\nabla_{\parallel} = \partial_z + \tilde{\underline{b}} \cdot \underline{\nabla}_{\perp}$$

$$\underline{\nabla} \cdot \underline{q} = 0 \qquad @\,L$$

$$\underline{b} = b_z + \tilde{b}$$

$$-\chi_{\parallel} \langle \tilde{b}_r \tilde{E}_{\parallel} \rangle \; (\neq 0 \to T \text{ not const} \to Q_L)$$

$k_{\parallel} \ll 1$.

$$\langle q_r \rangle = -\chi_{\parallel} \langle \tilde{b}_r^2 \rangle \partial_r \langle T \rangle - \chi_{\parallel} \langle \tilde{b}_r \partial_z \hat{T} \rangle + \text{Triplet} - \chi_{\perp} \nabla_r \langle T \rangle$$

How get $\tilde{T}$? $\rightarrow$ $\underline{\nabla} \cdot \underline{\mathcal{Q}} = 0$

$\rightarrow$ element of self-consistency

$$\underline{\mathcal{Q}} = -\chi_{\parallel}\left[(\partial_z + \underline{b}\cdot\underline{\nabla})(T_0 + \tilde{T})(b + \tilde{b})\right]$$
$$- \chi_{\perp} \nabla_{\perp} T$$

$$\partial_z \tilde{\mathcal{Q}}_{\parallel} + \underline{\nabla}_{\perp} \cdot \underline{\tilde{\mathcal{Q}}}_{\perp} = 0$$

$$\tilde{\mathcal{Q}}_{\parallel} = -\chi_{\parallel}\partial_z \tilde{T} - \chi_{\parallel}\tilde{b}\cdot\nabla\langle T\rangle \qquad \text{say}$$

$$-\chi_{\parallel}\partial_z^2 \tilde{T} - \chi_{\perp}\nabla_{\perp}^2 \tilde{T} = \chi(\partial_z \tilde{b})\partial_r \langle T\rangle$$

$$\tilde{T}_{\underline{k}} = \frac{-\chi_{\parallel} i k_z \tilde{b}_{\underline{k}}\, \partial_r\langle T\rangle}{(\chi_{\parallel}k_z^2 + \chi_{\perp}k_{\perp}^2)}$$

$\rightarrow$ + scattering ! $\rightarrow$ kick off line.

$$\langle q\rangle = -\left[\chi_{\parallel}\sum_{\underline{k}} \frac{\chi_{\perp}k_{\perp}^2 |b_k|^2}{(\chi_{\parallel}k_{\parallel}^2 + \chi_{\perp}k_{\perp}^2)}\right]\frac{\partial\langle T\rangle}{\partial r}$$

$\chi_{\perp\mathrm{eff}}$

need $\perp$ scattering

integrating:

$$\langle Q_r \rangle_{NL} = -(\chi_{\parallel} \chi_{\perp})^{1/2} \frac{D_M}{\Delta_{\perp}} D\langle T \rangle \Big]$$

D_B

if $\chi_{\parallel}/l_c^2 \sim \frac{\chi_{\perp}}{\Delta_{\perp}^2}$ → scale T ¿?

→ recover R.R. result.

N.B. → result not "small", though smaller than naive $\chi_{\parallel} \langle \tilde{b}^2 \rangle$.

→ good example calculation.

→ Triplet ¿

What of particles momentum ¿

→ Channels $\begin{cases} \text{heat} \\ \text{particles} \\ \text{momentum} \to e, i \end{cases}$

– Particles $\begin{cases} \text{why speak of heat transport?} \\ \text{→ test particle diffusivity calculated} \\ \text{Why not particle transport?} \end{cases}$

Note: The 'point' of stochastic fields is

$\chi_\perp \sim v_{the}\, D_M$

Other channels, → $\langle E_r \rangle$, etc. check? ↳ electron speed. → low inertia → test particles

Useful to:

→ consider kinetic eqn.

→ distinguish between:

a) self-consistent case * $\boxed{\text{relevant}}$

→ $\tilde{B}$ produced by $\tilde{J}_{\parallel}$ in plasma

(i.e. EM instabilities – Ampère's Law) also screening.

b) $\tilde{B}$ produced by external means (coil) – i.e. RMP, though plasma response (i.e. screening) significant! — test particle

Now,

$$\frac{\partial f}{\partial t} + v_{\parallel} \nabla_{\parallel 0} f + v_{\parallel} \frac{\delta B}{B_0} \cdot \nabla_{\perp} f + \cdots = 0$$

V_{+}

∂F

Particles and Flow I

~~Particle Transport~~

$\cancel{v_{\alpha}\times B} + P$

$$\frac{\partial f_e}{\partial t} + v_{\parallel}\,\underbrace{\tilde{b}\cdot\nabla}_{\frac{\delta B}{B_0}} f_e + \dots = 0$$

$$\int d^3v \qquad V_0\,\hat{n} \qquad V_{\parallel e} \sim V_0\,\hat{n} + \tilde{V}_{\parallel}\, n_0$$

$$\frac{\partial n_e}{\partial t} + n_0\,\tilde{b}\cdot\nabla\,\tilde{V}_{\parallel e} + \dots = 0$$

$$\frac{\partial n_e}{\partial t} + \tilde{b}\cdot\nabla\,\frac{\tilde{J}_{\parallel e}}{-|e|} + \dots$$

For mean,

$$\frac{\partial\langle n_e\rangle}{\partial t} + \frac{-1}{|e|}\,\partial_r\langle\tilde{b}\,\tilde{J}_{\parallel e}\rangle + \dots = 0$$

Now,

$$-\nabla_{\perp}^2\tilde{A}_{\parallel} = \frac{4\pi}{c}J_{\parallel} = \frac{4\pi}{c}\left(\tilde{J}_{\parallel e} + \tilde{J}_{\parallel i}\right)$$

$$\hat{J}_{\parallel e} = -\frac{c}{4\pi}\nabla_{\perp}^2\hat{A}_{\parallel} - \hat{J}_{\parallel i}$$

$$\frac{\partial \langle n_e \rangle}{\partial t} + \frac{I}{|e|} \partial_r \langle \tilde{b} \left(\frac{+c}{4\pi} \nabla_\perp^2 \vec{A}_\parallel \cancel{+ \vec{J}_{\parallel i}} \right) \rangle + \ldots = 0$$

how big?

impurities

$$\partial_t \langle n_e \rangle + \frac{c}{4\pi|e|} \langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle + n_0 \partial_r \langle \tilde{b} \tilde{V}_{\parallel i} \rangle = 0$$

to magnetic stress, etc.

$$\nabla_{\parallel_0} v_{\parallel 0} \tilde{b}_r \tilde{n}$$

To compute $\langle \tilde{b}_r \tilde{v}_{\parallel i} \rangle$;

Need consider ion sound/parallel flow equation

$$m n \frac{dV_{\parallel i}}{dt} = -n_0 |e| \nabla_\parallel \phi - \nabla_\parallel P_i$$

For simplicity, take isothermal

$$\frac{dV_{\parallel}}{dt} = -\cancel{n} \frac{T}{m} \nabla_\parallel \left(\frac{|e|\phi}{T} \right) - \frac{T_i}{M} \frac{\nabla_\parallel n}{n}$$

$$\frac{\partial \langle n_e \rangle}{\partial t} = -\partial_r \left(\frac{c}{4\pi |e|} \langle b_r \nabla_\perp^2 A_\parallel \rangle \right) \quad \partial_r \phi \quad \partial_\theta \phi \approx \frac{c}{B} k_\theta$$

$$① = \frac{c B_0 n_0}{4\pi n_0 |e|} \left\langle \tilde{b}_r \nabla_\perp \frac{\tilde{A}_\parallel}{B} \right\rangle$$

$$= \left(\frac{|e| B_0}{M_i c} \right) \left(\frac{c^2 M_i}{4\pi n_0 |e|^2} \right) n_0 \left\langle b_r \nabla_\perp^2 \frac{\tilde{A}_\parallel}{B} \right\rangle$$

$$= n \Omega_i d_i^2 \partial_r \left\langle b_r \partial \frac{A_\parallel}{B} \right\rangle$$

Taylor identity

$$= -n_0 \Omega_i d_i^2 \partial_r \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

phasing → done

$$① = -n_0 \Omega_i d_i^2 \partial_r \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

$$= -n_0 \underbrace{\Omega_i (\rho_i^2)}_{D_B} \underbrace{\boxed{\frac{d_i^2}{\rho_i^2}}}_{} \partial_r \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

$$\frac{\cancel{c^2} \cancel{M_e} M_i}{4\pi n_0 \cancel{|e|^2} T} \frac{|\cancel{e}|^2 B_0^2}{\cancel{M_i^2} \cancel{c^2}}$$

$$= -n_0 \frac{D_B}{\beta} \partial_r \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

β ↓ ↓ phase → V_E.

! 0

$\Rightarrow$

$$\frac{\partial\langle n_e\rangle}{\partial t} + \frac{\partial}{\partial r}\left\langle \nabla_\theta A_\parallel \left(-\frac{c}{4\pi}\nabla_\perp^2 \hat{A}_\parallel - n_0 e\, V_{\parallel e}\right)\right\rangle$$

$$+ \qquad = 0$$

relevance to ZF?

so

$$\frac{\partial\langle n_e\rangle}{\partial t} = \frac{c}{4\pi}\frac{\partial}{\partial r}\overset{①}{\left\langle (\nabla_\theta \hat{A}_\parallel)(\nabla_\perp^2 \hat{A}_\parallel)\right\rangle}$$

$$+\, n_0 e \frac{\partial}{\partial r}\overset{②}{\left\langle \delta B_r\, \hat{V}_{\parallel e}\right\rangle}$$

① : <u>Taylor</u> <u>identity</u> calculation (akin $\langle v_r \nabla^2 \phi\rangle$)

$$\langle (\nabla_\theta \hat{A}_\parallel)(\partial_r^2 A_\parallel + \underbrace{\cancel{\nabla_\theta^2 A_\parallel}}_{\text{odd}})\rangle$$

$$= \langle (\nabla_\theta A_\parallel)(\partial_r^2 A_\parallel)\rangle$$

$$= \langle \partial_r(\nabla_\theta A_\parallel)(\partial_r A_\parallel)\rangle - \overset{\text{odd}}{\cancel{\langle (\nabla_\theta \partial_r A_\parallel)(\partial_r A_\parallel)\rangle}}$$

$$= \partial_r\langle (\nabla_\theta \hat{A}_\parallel)(\partial_r A_\parallel)\rangle$$

$$= -\partial_r\langle (\delta B_r)(\delta B_\theta)\rangle$$

[Magnetic stress.

6.17.
&17

→ substantial ($\sim D_B / \beta$)

→ envelope.

$$n_0 \frac{\Omega_B}{c_s^3} \frac{\partial^3}{\partial r^2} \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

phase] tilt.

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$$\frac{\partial \langle n_e \rangle}{\partial t} = -\frac{c}{4\pi} \partial_r \langle (\delta B_r)(\delta B_\theta) \rangle$$

→ magnetic stress

$$+ n_0 k \partial_r \langle \delta B_r \hat{V}_{\parallel i} \rangle$$

→ magnetic "flutter" of parallel ion flow.

Points:

— no explicit dependence on small electron inertia; i.e. $1/m_e$

— relation of magnetic stress to electron particle transport due stochastic fields

i.e. suggests effect on zonal flows, as zonal flows are fundamentally charge transport effect.

— fluctuating parallel ion flows + magnetic tilt ⇒ change electron density. (see Chernikov). show

How get phase?

$$\underline{b} = \nabla A \times \hat{z}$$

$$\begin{matrix} \partial_x A & \partial_y A & 0 \\ 0 & 0 & \hat{z} \end{matrix}$$

Field Correlations

$$\langle b_r b_\theta \rangle = -\langle \partial_y A \, \partial_x A \rangle$$

$$A e^{i\psi}$$

$$= -\sum_{\underline{k}} -ik_\theta \, ik_r |A_k|^2$$

$$\hat{A} = \frac{A}{B_0}$$

$$= -\sum_{\underline{k}} \boxed{k_\theta k_r} |\hat{A}_k|^2$$

A_k

What is k_r? $-i\partial_r A_k \sim \partial_r \phi$.

Now k_r 'tilts' in $E\times B$ shear:

$$\frac{d}{dt} k_r = -\frac{\partial}{\partial r}(k_\theta V_E) \quad , \quad k_r = k_r^{(0)} - k_\theta V_E' \tau$$

$$k_r = -k_\theta V_E' \tau + k_r^{(0)}$$

$$\langle b_r b_\theta \rangle = -\sum_{\underline{k}} k_\theta k_r |A_k|^2$$

other.

$$= -\sum_{\underline{k}} k_\theta (k_r^{(0)} - k_\theta V_E' \tau_c) |A_k|^2$$

Tilting of A, $\underline{B}$ so τ_c is coherence time (in shear) of magnetic perturbation.

simple

Perturbation is in plasma, so must satisfy:

$$\frac{\partial A}{\partial t} + \langle V_E \rangle \cdot \nabla A = \langle B \rangle \cdot \nabla \phi + D \nabla^2 A$$

shearing — linear term

what is D?

$\eta \to$ collisional η

$D_T \to$ turbulence

$D_\eta \to$ stochasticity

Really, A in plasma satisfies

$$\frac{\partial A}{\partial t} + V_E(r)\,\partial_\theta A = \langle B \rangle \cdot \nabla \phi + \eta \nabla^2 A$$

$$+ \tilde{V} \cdot \nabla A$$

ES turbulence will scatter A.

$\Rightarrow D_T$

so

$$\partial_t A + V_E\, \partial_y A - D_T \nabla_\perp^2 A = 0$$

is working model.

shearing coordinate

$b \quad -i\nabla_y\varphi = k_x(t)$

if $A = \sum_k e^{ik_y y} e^{ik_x(t)x} a_k$

$$\frac{\partial A_k}{\partial t} = i\dot{k}_x(t) x a_k + \dot{a}_k + ik_y V_E' x \, a_k + D_T\left(k_x(t)^2 + k_y^2\right) a_k$$

$k_x = k_x^0 - V_E' k_y x t$

cancel shearing

$$\dot{k}_x = -V_E' k_y x \, a_k$$

$$i\dot{k}_x x\, a = -iV_E' k_y \cancel{x}\, a_k$$

so

$$\partial_t a_k + D_T\left(k_y^2 V_E'^2 \tau^2 + k_y^2\right) a_k = 0$$

$\Rightarrow$

$$a_k = a_k(0)\, e^{-\frac{k_y^2 V_E'^2}{3} D_T \tau^3} e^{-D_T k_y^2 \tau}$$

so

$$1/\tau_{c_k} = \left(\frac{k_\theta^2 V_E'^2}{3} D_T\right)^{1/3}$$

so

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle = \sum_k k_\theta^2 |A_k|^2 V_E' \tau_{c_{k}}$$

$$= \sum_k |\tilde{v}_{\theta_k}|^2 V_E' \tilde{\tau}_{c_{k}}$$

$$\langle k_r k_\theta \rangle = \sum_k |k_{r,k}|^2 \, V_E' \left(\frac{3}{k_\theta^2 V_E'^2 D_T} \right)^{1/3}$$

$$= \underbrace{\sum_k |k_{r,k}|^2}_{\text{intensity}} \; \underbrace{\frac{V_E'}{V_E'^{2/3}}}_{\substack{\text{sgn } V_E' \\ \text{weak dep.}}} \left(\frac{3}{k_\theta^2 D_T} \right)^{1/3} \rightarrow \text{To weak.}$$

$$\cancel{= \text{sgn } V_E' \sum_k |k_{r,k}|^2 \left(V_E'^2\right)^{1/3}}$$

$$\text{sgn } V_E' \sum_k |k_{r,k}|^2 \, |V_E'|^{1/3} \left(3 / k_\theta^2 D_T \right)^{1/3}$$

N.B. For external field, key is to calculate:
- δB_r in plasma
- $\tilde{v}_{\parallel e}$ in plasma

$$\Rightarrow \delta B_{r,\text{in}} \sim \frac{\delta B_{ext}}{\epsilon}$$

→ Re: zonal flows:

Point: Magnetic wandering impacts zonal flows via charge

Recall: - fluctuations → DW model i.e. H-M, etc.

- zonal mode: charge separation, radially - ambipolarity breaking

$\underline{\nabla} \cdot \underline{J} = 0$

usual e.s.; $\nabla_r J_{r\,pol} = 0$

⇒

$$\underbrace{\frac{\partial}{\partial t} \nabla_\perp^2 \phi}_{\text{linear pol. drift}} + \underbrace{\underline{V} \cdot \underline{\nabla}\, \nabla^2 \phi}_{\text{NL pol. drift}} = 0$$

so, for $\langle\ \rangle \rightarrow$ zonal symmetry

$\left[\int \text{ions} - \int \text{electrons} \right.$

$$\frac{\partial}{\partial t}\langle \nabla_r^2 \phi \rangle + \partial_r \langle \tilde{V}_r \nabla_r^2 \tilde{\phi} \rangle + \dots = 0$$

↓ polarization charge Q_{pol}.

↓
- flux of polarization charge (i.e. net difference ions, electrons)
- relate to Reynolds stress by Taylor identity
- compute via modulation

so, can view:

$$\partial_t \langle Q_{pol} \rangle + \partial_r \langle \tilde{V}_r Q_{pol} \rangle = 0$$

$$\rightarrow \Gamma^{+}_{charge} = \left(\langle \tilde{V}_r \tilde{n}_{i,GC} \rangle + \langle \tilde{V}_r \tilde{Q}_{pol} \rangle \right) |e| - \left(\langle \tilde{V}_r \tilde{n}_{e,GC} \rangle \right) |e|$$

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$\tilde{\hat{n}}_i = \hat{n}_e \Rightarrow \Gamma_{charge} = 0$ ✓.

$n_{i\,gc} + n_{i\,pol}$

But, magnetic perturbations ~~[illegible]~~ induce new means charge transport

$\Rightarrow \underline{\nabla} \cdot \underline{J} = 0$

$\nabla_\perp \cdot \underline{J}_{pol} + \underline{\nabla_\parallel J_\parallel} = 0$

$\nabla_\parallel = \nabla_\parallel^{(0)} + \frac{\delta \underline{B}}{B_0} \cdot \nabla_\perp$

$\frac{\partial}{\partial t}\langle \nabla^2 \tilde{\phi} \rangle + \langle \tilde{\underline{v}} \cdot \nabla \nabla^2 \tilde{\phi} \rangle = \langle \tilde{\underline{B}} \cdot \nabla \tilde{J}_\parallel \rangle$

obvious from 2D MHD

ion pol. charge transport
$\Rightarrow$ advection

current flow along tilted lines

(n.b. current: electrons
~~≠~~ ions)

so

$$\partial_t \langle \nabla^2 \phi \rangle = -\partial_r \left\{ \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle - \langle \underbrace{\tilde{B}_r \tilde{J}_\parallel}_{\text{electron particle}} \rangle \right\}$$

so current flow along tilted lines acts to affect charge balance

⇒ enters effect in Z.F.

Note:

- sign not clear apriori
- for Alfvenic fluctuations → damps
  i.e. acts cancel e.s.
- obviously related to electron particle transport.

- can recover $\langle \tilde{v}_r \tilde{v}_\theta \rangle$ vs $\langle \tilde{B}_r \tilde{B}_\theta \rangle$
  competition

For $\langle \hat{b}_r \tilde{v}_{\parallel} \rangle$?

$\underline{IV.\ X.}$

Try ala Finn:

$\langle \tilde{b}_r \tilde{v}_{\parallel} \rangle$

v, n is static stochastic field.

$\langle \hat{b}_r \tilde{p} \rangle$

$$\frac{\partial \tilde{v}_{\parallel}}{\partial t} = -c_s^2 \nabla_{\parallel}^{(0)} \frac{\tilde{n}}{n_0} - \frac{c_s^2 \tilde{b}_r \partial_r \langle n \rangle}{n_0}$$

$$\frac{\partial \tilde{n}}{n_0 \partial t} = -n_0 \nabla_{\parallel}^{(0)} \tilde{v}_{\parallel} - \tilde{b}_r \partial_r \langle v_{\parallel} \rangle$$

static stoch

$\nabla_{\parallel} \tilde{n} = 0$

$\nabla_{\parallel} v_{\parallel} = 0$

∂_t

$$\partial_t \frac{\tilde{v}_{\parallel}}{c_s} = -c_s \nabla_{\parallel}^{(0)} \frac{\tilde{n}}{n_0} - c_s \tilde{b}_r \frac{\partial_r \langle n \rangle}{n_0}$$

$$\partial_t \frac{\tilde{n}}{n_0} = -c_s \nabla_{\parallel}^{(0)} \frac{\tilde{v}_{\parallel}}{c_s} - \tilde{b}_r c_s \partial_r \langle v_{\parallel} \rangle$$

$$f_{\pm} = \frac{\tilde{v}_{\parallel}}{c_s} \pm \frac{\tilde{n}}{n}$$

$$\partial_t \delta v_{\parallel} = -c_s \nabla_{\parallel}^{(0)} \delta n - c_s \tilde{b}_r \frac{\partial_r \langle n \rangle}{n_0}$$

$$\partial_t \delta n = -c_s \nabla_{\parallel}^{(0)} \delta v_{\parallel} - \tilde{b}_r c_s \partial_r \langle v_{\parallel} \rangle$$

$$\partial_t f_+ + c_s \nabla_{\parallel}^{(0)} f_+ = -c_s \tilde{b}_r \frac{\partial_r \langle n \rangle}{n_0} - \tilde{b}_r c_s \partial_r \langle v_{\parallel} \rangle$$

$$\cancel{\partial_t} f_- = + c_s \nabla_\parallel^{(d)} f_- - c_s \tilde{b}_r \partial_r \frac{\langle n \rangle}{n_0} + \tilde{b}_r c_s \partial_r \langle v_\parallel \rangle$$

$+ c_s \nabla_\parallel f_-$

static

$$\hat{f}_+ = - \int d\ell \left[\tilde{b}_r \partial_r \frac{\langle n \rangle}{n_0} + \tilde{b}_r \partial_r \langle v_\parallel \rangle \right]$$

$$\hat{f}_- = + \int d\ell \left[\tilde{b}_r \partial_r \frac{\langle n \rangle}{n_0} - \tilde{b}_r \partial_r \langle v_\parallel \rangle \right]$$

$$f_+ = \frac{v_\parallel}{c_s} + \delta n$$

$$f_- = \frac{v_\parallel}{c_s} - \delta n$$

$$\frac{\tilde{v}_\parallel}{c_s} = \frac{f_+ + f_-}{2} = - \int d\ell\, \tilde{b}_r \partial_r \frac{\langle v_\parallel \rangle}{c_s}$$

$$\delta n = \frac{1}{2}(f_+ - f_-) = - \int d\ell\, \tilde{b}_r \partial_r \frac{\langle n \rangle}{n_0}$$

checks: $\tilde{v}_\parallel$ → flow coupling to ~~particles~~ density

δn → density coupling to flow (residual stress)

So

$$\frac{\partial \langle v_\parallel \rangle}{n_0 \, \partial t} = \ldots + \partial_r \langle \tilde{v}_r \tilde{V}_\parallel \rangle$$

$$= \partial_r \left[- D_{QL} \, \partial_r \langle V_\parallel \rangle \right]$$

$$= \partial_r \left[- \underbrace{D_{QL} c_s} \, \partial_r \frac{\langle V_\parallel \rangle}{c_s} \right]$$

transp rate coeff $\sim c_s D_{QL}$

but not a particle diffn.

$\rightarrow \nabla \langle V_\parallel \rangle$ driven pinch

N.B. :

$$\frac{\partial \langle v_\parallel \rangle}{\partial t} = \partial_r [V \langle n \rangle] \qquad V = -D_{QL} c_s \nabla \frac{\langle V_\parallel \rangle}{c_s}$$

- Rate is $c_s D_{QL} / L^2$ but fluxes not diffusive,

- Residual stress ✓ $\nabla \langle V_\parallel \rangle$ drives flux.

Physics ?!

$V_\parallel \frac{\delta B}{B} \delta f$

$\langle V_\parallel \rangle$

- Finn paper seriously misleading!
  Discusses Elsasser fluxes,
  not physical fluxes.

- Can check E-field:

$$\left(-i\omega + ik_\parallel C_s + ik_\theta V_E\right) f_+ = S_+$$

$$\left(-i\omega - ik_\parallel C_s + ik_\theta V_E\right) f_- = S_-$$

$\Rightarrow V_E$ acts asymmetrically on Elsasser populations! $\Rightarrow$ splits

$\Rightarrow \tilde{\tau}_c$ now set by:

$\rightarrow |\Delta k_\parallel| C_s \sim \dfrac{C_s}{l_{ac}}$

dispersion of sound waves

$\Downarrow$ in bundle, stochastic fields

and

$\rightarrow |k_\theta V_E' \Delta_x| \simeq V_E' k_\theta \Delta_x$.

dispersion due shear differt. rotation.

ExB shear can easily set $\tilde{\tau}_c$ in realistic case. $\Rightarrow$ loses R-R relation..

$$\frac{k_\theta V_E' \Delta x}{C_s |\Delta k_\parallel|} \sim L_s \frac{k_\theta V_E' \Delta x}{C_s |k_\theta \Delta x|}$$

$$\sim L_s \frac{V_E'}{C_s} \qquad k_\parallel = \frac{k_\theta x}{L_s}$$

$\bar{s}/R\varepsilon$

$$\sim \frac{R\varepsilon}{\hat{s}} \frac{V_E'}{C_s} \qquad \text{unclear}$$

$\tilde{T}_c$ by both processes!

etc

$$f_+ = \frac{-i}{k_\parallel C_s + k_\theta V_E} S_+ \quad \to -\pi \delta(k_\parallel C_s + k_\theta V_E) S_+$$

$$f_- = \frac{-i}{-k_\parallel C_s + k_\theta V_E} S_- \quad \to -\pi \delta(k_\theta V_E - k_\parallel C_s) S_-$$

$$\frac{v_\parallel}{C_s} = \frac{1}{2}(f_+ + f_-)$$

etc

$$\frac{\tilde{n}}{n_0} = \frac{1}{2}(f_+ - f_-)$$

Others:

see
→ Che et al
→ Lazarian + V.

→ electron momentum transport

→ "electron viscosity" / "hyper-resistivity"

i.e.

$$E_{\parallel} = \eta J_{\parallel} - \nabla_{\perp} \cdot \mu \cdot \nabla_{\perp} J_{\parallel}$$

(?!)

$E \times B \cdot \nabla J_{\parallel}$ flow

$$\frac{c^2}{\omega_{pe}^2} V_{Th} \Delta_M$$ Kov.

is this plausible?

→ Reconnection rate

i.e. Sweet-Parker:

$$V \sim V_A \frac{\Delta}{L}$$

$$\sim 1/S^{1/2} V_A$$

$$= V_A / \sqrt{R_m}$$

Electron viscosity:

$$V \sim V_A / (R_{m_\mu})^{1/4}$$

→ Physics of Hyper-resistivity is essential to ELM crash.

→ One take: Drake '95

ITG : PSFI :: ETG : ∇J-driven

$\eta = \eta(\nabla J)$ → nonlinearity !

⇒ Good topic for further research.