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PKU Lectures – Lecture IV

→ PV Dynamics, Charney-Drazin, Modulation

– A closer look at PV.

– Momentum Theorem for Flows
⟺ PV and WMD

– Modulational Interaction and Predator-Prey Model.

$$q = PV = \nabla^2\phi + \beta y$$

fluid, local

planetary

potential vorticity

→ 2D conservative advection, according Hamiltonian system

$$\frac{dx}{dt} = -\nabla\phi \times \hat{z}$$

→ N.B.:

$$\frac{dq}{dt} = 0 +$$

$$q = \begin{cases} \nabla^2\phi & \rightarrow 2D \\ \nabla^2\phi + \beta y & \rightarrow QG \\ \ln\frac{n}{n_0} + \phi - \nabla^2\phi & \rightarrow HM \\ + \\ \vdots \end{cases}$$

⟺ obvious analogy to:

$$\frac{df}{dt} = 0 \quad , \quad \partial_t f + v\,\partial_x f + \frac{q}{m} E\,\partial_v f = 0$$

$$= C(f) \qquad H = \frac{mv^2}{2} + q\phi$$

with collisions

$$f = \langle f \rangle + \delta f$$

i.e. A few observations:

H - W system → $(\underline{B} \cdot \underline{\nabla} = 0)$

$$\frac{d}{dt} \nabla^2 \phi = + D_{\parallel} \nabla_{\parallel}^2 (\phi - n) + \nu \nabla^2 \nabla^2 \phi$$

$$\frac{d}{dt} n = + D_{\parallel} \nabla_{\parallel}^2 (n - \phi) + D_0 \nabla^2 n$$

↓
includes
$v_r \frac{d\langle n \rangle}{dr}$

n.b. $\tilde{T}_i$ complicates

i.e.
$$\frac{d}{dt} (n - \nabla^2 \phi) = 0$$

↓

$$PV = q = q_{GC} + q_{pol}.$$

n → electron / GC density

$\nabla^2 \phi$ → polarization charge (i.e. GC)
(~ ion inertia → pol. drift off surface)

$$PV = \underbrace{n - \nabla^2 \phi}$$ → conserved along orbits

Reynolds force → flux polarization charge.

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N.B.

Trade offs possible! → coupling to flows,

$$\frac{dq}{dt} = 0$$

$$q = n - \nabla^2\phi$$

$$\Rightarrow \langle \tilde{v}(\tilde{q}) \rangle = \langle \tilde{v}\, \tilde{n} \rangle - \langle \tilde{v}\, \nabla^2\tilde{\phi} \rangle$$

can transport PV via particle flux ⟺ "bad" - transport and/or via vorticity flux ⟺ "good" - ZF

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→ A Closer Look at PV and What it Means

" What is GFD [Low Freq. Dyn. Magn. Plasma] – The Fluid Dynamics of Potential Vorticity "

– Rick Salmon (P.D. insert)

Recall $\frac{\underline{\omega}}{\rho}$ frozen-in:

$$\frac{d}{dt}\left(\frac{\underline{\omega}+2\underline{\Omega}}{\rho}\right) = \left(\frac{\underline{\omega}+2\underline{\Omega}}{\rho}\right)\cdot\underline{\nabla}\,\underline{V}$$

$$\begin{cases} \underline{\omega} & \text{for } \underline{\nabla}\cdot\underline{V} = 0 \\ \underline{\omega}/\rho & \text{for } \underline{\nabla}\cdot\underline{V} \neq 0 \\ \dfrac{\underline{\omega}+2\underline{\Omega}}{\rho} & \text{with rotation.} \end{cases}$$

Why "frozen in"?

 $\delta\underline{\ell}$ ⟨ end points go with the flow

$\delta\underline{\ell}$ "frozen in"

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so $\frac{d}{dt}\delta \underline{l} = \delta \underline{l} \cdot \underline{\nabla}\, \underline{V}$

$\Rightarrow$ $\frac{d}{dt}\left(\frac{\omega + 2\Omega}{\rho}\right) = \left(\frac{\underline{\omega} + 2\underline{\Omega}}{\rho}\right) \cdot \underline{\nabla} V$

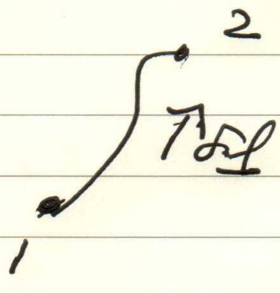
$\frac{d}{dt}\delta \underline{l} = \delta \underline{l} \cdot \nabla V$

$\rightarrow$ same equation $\Rightarrow$ $\frac{\underline{\omega} + 2\underline{\Omega}}{\rho}$ frozen in

Also observe, for:

$\psi \equiv$ passive scalar field

$\frac{d\psi}{dt} = 0$



$\frac{d}{dt}(\psi_1 - \psi_2) = 0$

$\delta\psi = \underline{\nabla}\psi \cdot \delta \underline{l}$

and

$\frac{d}{dt}(\underline{\nabla}\psi \cdot \delta \underline{l}) = 0$

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$$\delta \underline{f} \leftrightarrow \frac{\underline{\omega} + 2\underline{\Omega}}{\rho}$$

as satisfy same

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$$\frac{d}{dt}\left(\frac{(\underline{\omega}+2\underline{\Omega})\cdot\underline{\nabla}\psi}{\rho}\right) = 0$$

latitude

$$\underline{q} = \frac{(\underline{\omega}+2\underline{\Omega})\cdot\underline{\nabla}\psi}{\rho} \equiv \text{general defn of PV}$$

PV charge

depth

$Q = \int d^3x \; \underline{q}$ total PV charge

$$\frac{d\underline{q}}{dt} = 0 \rightarrow \text{PV conservation}$$

Symmetry $\leftrightarrow$ Particle re-labeling symmetry

{ Particles can be re-labeled without changing the thermodynamic state }

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"How do we get something useful out of this?"
(familiar)

→ Specialize to plasma (ions)

$$\frac{d}{dt}\left[\frac{(\underline{\omega}+2\underline{\Omega})\cdot\underline{\nabla}\psi}{\rho}\right]=0$$

For plasma, $\longrightarrow \underline{B}_0$

$$2\underline{\Omega}=\Omega_i\hat{z}$$

$$\rho=n_0(r)+\tilde{n}$$

$$\underline{\nabla}\psi=\hat{z} \qquad \text{(const. vector)}$$

$$\frac{d}{dt}\left[\frac{\omega_z+\Omega_i}{n_0(r)+\tilde{n}}\right]=0$$

⇒ includes $\nabla\cdot v\, n_0$

$$\frac{1}{n_0}\frac{d}{dt}\tilde{\omega}_z-\frac{\Omega_i}{n_0^2}\frac{d\tilde{n}_i}{dt}=0$$

$$\tilde{\omega}_z=\frac{c}{B_0}\nabla^2\phi \qquad \frac{\tilde{n}_i}{n_0}=\frac{\tilde{n}_e}{n_0}=\frac{|e|\phi}{T_e}$$

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so

$$\frac{d}{dt}\frac{c}{B_0}\nabla^2\tilde{\phi} - \Omega_0\left(\frac{\partial}{\partial t}\frac{|e|\tilde{\phi}}{T_e} + \frac{\tilde{v}_E}{n_0}\frac{\partial n_0}{\partial r}\right) = 0$$

$\Rightarrow$

$$\boxed{\frac{d}{dt}\left(\frac{|e|\tilde{\phi}}{T} - \rho_s^2\nabla_\perp^2\frac{|e|\tilde{\phi}}{T}\right) + V_*\partial_y\frac{|e|\tilde{\phi}}{T} = 0}$$

→ H-M eqn.!

→ Can develop PV much further.
See 216 (UCSD) notes, refs.

→ A key result: Taylor Identity

$$\langle v_y \nabla^2\phi\rangle_x = \langle \partial_x\phi\,(\partial_x^2\phi + \partial_y^2\phi)\rangle_x$$

$$= \langle \partial_x \cancel{\left(\frac{(\partial_x\phi)^2}{2}\right)}\rangle_x$$

$$+ \langle \partial_x\phi\,\partial_y^2\phi\rangle_x$$

$$= 0 + \partial_y\langle\partial_x\phi\,\partial_y\phi\rangle - \underbrace{\langle\partial_{xy}\phi\,\partial_y\phi\rangle}_{\to 0}$$

$$= \underbrace{\partial_y\langle\partial_x\phi\,\partial_y\phi\rangle_x}_{R\text{-}S} - \langle\partial_x\cancel{\left(\frac{(\partial_y\phi)^2}{2}\right)}\rangle_x$$

$$\to\ = -\partial_y\langle\tilde{v}_y\tilde{v}_x\rangle$$

vorticity (+ PV) flux = Reynolds Force.

N.B. Need 1 direction of symmetry, to eliminate odd moments.

→ key result: Reynolds force = vorticity flux.

→ Links Reynolds force to polarization charge flux

Related:

Often useful to think of flow via vorticity.

Vorticity $\leftrightarrow$ Polarization charge.

Thus, for zonal flows:

$$\underline{\nabla} \cdot \underline{J} = 0$$

$$\nabla_\perp \cdot \underline{J}_\perp + \nabla_\parallel J_\parallel + \nabla_\perp \cdot J_{ps} = 0$$

poloidal symmetry / $m=0$;

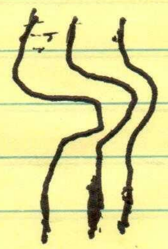
$$\partial_r \langle J_{r_{pol}} \rangle + \partial_r \left\langle \frac{\tilde{B}_r}{B_0} \tilde{J}_\parallel \right\rangle = 0 \quad \left\{ \begin{array}{l} \text{show} \\ \text{from} \\ \text{GK} \end{array} \right.$$

$$\Rightarrow \partial_t \langle \nabla_r^2 \phi \rangle = - \partial_r \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle + \partial_r \langle \tilde{b}_r \tilde{J}_\parallel \rangle$$

so

$$\underbrace{\partial_t \langle q \rangle}_{\text{total charge density}} = \underbrace{- \partial_r \langle \tilde{v}_r \tilde{\xi} \rangle}_{\text{charge flux}} + \underbrace{\partial_r \langle \tilde{b}_r \tilde{J}_\parallel \rangle}_{\text{inhomogeneous field line tilting}}$$

$\partial_r \langle \tilde{v}_r \tilde{q} \rangle \rightarrow$ 

$\partial_r \langle \tilde{b}_r \tilde{J}_\parallel \rangle \rightarrow$ 

and as $\tilde{J}_\parallel = -\nabla^2 A_\parallel$

so: $\langle \tilde{b}_r \tilde{J}_\parallel \rangle = \partial_r \langle \tilde{B}_r \tilde{B}_\theta \rangle$

Magnetic stress.
(show).

$\Rightarrow$

$$\partial_t \langle \xi \rangle = -\partial_r^2 \left[\langle \tilde{v}_r \tilde{v}_\theta \rangle - \langle \tilde{B}_r \tilde{B}_\theta \rangle \right]$$

Key point/summary for zonal structure generated by turbulence:

Zonal Flow/Jet results from:

1) inhomogeneous PV mixing

+

2) 1 direction of symmetry

N.B. { Symmetry need not be equilibrium symmetry a priori

→ Taylor Identity enables:

Key Quasilinear Flow Theorem

$\boxed{\rightarrow \text{Charney – Drazin Thm.}}$

Now, PV conservation ⇒

$$\partial_t q + \underline{v}\cdot\nabla q = \nu\nabla^2 q - \mu q$$

scale independent damping

Then, for mean:

natural quantity of interest,

$$\partial_t \langle q\rangle = -\partial_r \langle \tilde{v}_y \tilde{q}\rangle + \nu\,\partial_y^2 \cancel{\langle q\rangle}$$

$\langle q\rangle$ slow.

$$-\mu\langle q\rangle$$

friction.

so

$\langle \tilde{v}_y \tilde{q}\rangle$ is PV flux → key quantity of interest.

→ Now, could proceed w/ QLT, with linear response $\tilde{q}$, etc.

on:

→ examine $\langle \tilde{\zeta}^2 \rangle$ → potential enstrophy balance

n.b. — $\frac{d\zeta}{dt} = 0$

pot enstroph. $\langle q^2 \rangle$ conserved.

— energy conserved.

i.e.

$$\partial_t \tilde{\zeta} + \underline{v} \cdot \nabla \tilde{\zeta} = -\tilde{v}_r \frac{d\langle \zeta \rangle}{dr}$$

$$+ \nu \nabla^2 \tilde{\zeta} \cancel{- \mu \tilde{\zeta}}$$

so

$$\partial_t \left\langle \frac{\tilde{\zeta}^2}{2} \right\rangle + \nabla \cdot \left\langle \underline{\tilde{v}} \frac{\tilde{\zeta}^2}{2} \right\rangle = -\langle \tilde{v}_y \tilde{\zeta} \rangle \frac{d\langle \zeta \rangle}{dy}$$

$$- \nu \langle (\nabla \tilde{\zeta})^2 \rangle$$

but have:

$$\partial_t \langle v_x \rangle + \mu \langle v_x \rangle = -\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle$$

↓ zonal flow

now, Taylor Identity:

$$-\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle = \langle \tilde{v}_y \nabla^2 \tilde{\phi} \rangle$$

$$= \langle \tilde{v}_y \tilde{\zeta} \rangle$$

as

$$q = \beta y + \nabla^2 \phi$$

$$\tilde{q} = \nabla^2 \tilde{\phi}$$

so

$$\partial_t \langle V_x \rangle + \mu \langle V_x \rangle = \langle \tilde{v}_y \tilde{q} \rangle$$

$\Rightarrow$

$$\partial_t \left\langle \frac{\tilde{q}^2}{2} \right\rangle + \nabla \cdot \left\langle \tilde{v} \frac{\tilde{q}^2}{2} \right\rangle = -\left[-\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle \right] \frac{d\langle q \rangle}{dy}$$

$$= -\left[\partial_t \langle V_x \rangle + \mu \langle V_x \rangle \right] \frac{d\langle q \rangle}{dy}$$

Then for $\partial\langle q \rangle / \partial y$ slowly varying in space, time:

$$\boxed{\partial_t \left\{ \frac{\langle \tilde{q}^2 \rangle}{2\, d\langle q \rangle / dy} + \langle V_x \rangle \right\} + \nabla_y \frac{\langle \tilde{v}\, \tilde{q}^2/2 \rangle}{d\langle q \rangle / dy} = -\mu \langle V_x \rangle}$$

→ Charney - Drazin Thm.

→ Consequence PE balance and Taylor identity.

→ Momentum Thm ($\sim$ QLT) for Rossby – Zonal interaction.

what does it mean?

$$\partial_t \left\{ \underset{①}{\frac{\langle \tilde{q}^2 \rangle}{2\, d\langle \xi \rangle / dy}} + \underset{②}{\langle v_x \rangle} \right\} + \partial_y \underset{③}{\frac{\langle \tilde{v}_y \tilde{q}^2/2 \rangle}{d\langle \xi \rangle / dy}}$$

$$= \underset{④}{-\mu \langle v_x \rangle}$$

① $\dfrac{\langle \tilde{q}^2 \rangle}{2\, \frac{d\langle \xi \rangle}{dy}}$?   ($\rho_0 = 1$)

$$q = \nabla^2 \phi + \beta y$$

$$\tilde{q} = \nabla^2 \tilde{\phi}$$

$$① = \sum_k \frac{(k^2)^2 |\phi_k|^2}{2\, d\langle \xi \rangle / dy}$$

$$= -\sum_k - \frac{k^2 |\phi_k|^2}{2\, \frac{k_x}{k^2} d\langle \xi \rangle / dy} \quad k_x$$

Wave action density

$$① = -\sum_{k} k_x N_k$$

→ Quasi-particle density

$$\omega = -\frac{k_x \beta}{k^2}$$

$$N_k = \mathcal{E}_k / \omega_k$$

Wave action density

Wave energy density

Thus

$$① = -WMD \equiv -\text{Pseudomomentum } (PM)$$

⟺ wave activity density

$\lim_{\phi \to \text{small}}$ Pseudomomentum = WMD

and:

$$③ = \left\langle \tilde{v}_y \frac{\tilde{q}^2}{2\,\frac{d\langle q \rangle}{dy}} \right\rangle$$

$$= + \langle \tilde{v}_y (\widetilde{-PM}) \rangle$$

⇒ obviously related to turbulence spreading → beyond QLT.

$$\partial_t \left\{ \langle v_x \rangle - k_x N \right\} - \partial_y \langle \tilde{v}_y (\mathrm{WMD}) \rangle$$
$$= -\mu \langle v_x \rangle$$

So:

→ up to ~~spreading~~ drag spreading, flow <u>locked</u> to wave momentum density, i.e.

$$\partial_t \langle v_x \rangle = \partial_t \left(\sum_k k_x N_k \right)$$

change wave amplitude ⟺ change in flow

→ no-slip theorem for fluid relative to inter-penetrating fluid of quasi-particles,

→ spreading term → linked to fluctuation envelope scale → can break local, but not global, no-slip. [Phases ?]

HW → generalize for $D, r \equiv 0$
Hasegawa - Wakatani system

Theoretical Observations:

① – theorem is general, with no truncation, though leaves enstrophy flux $\langle \nabla \tilde{q}^2 \rangle$ un-calculated

② – no linear response closure, like QL, used

③ – WMD defined beyond level of small amplitude perturbation

①–③ enabled by $\begin{cases} \text{PV conservation} \to \text{enstrophy balance} \\ \text{Taylor identity} \end{cases}$

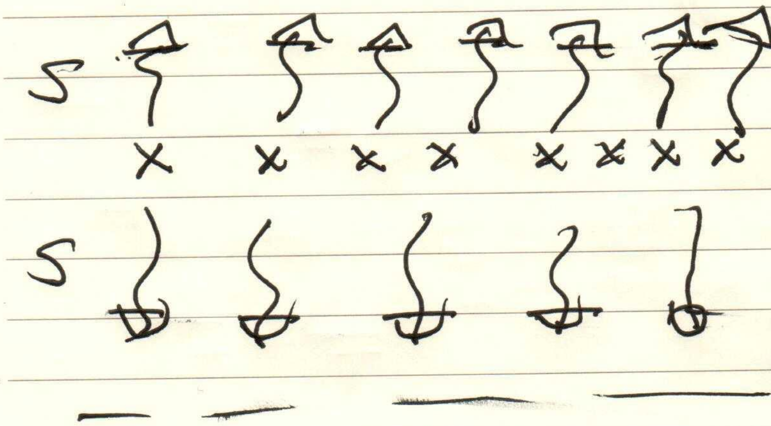
contrast to:

$$\partial_t \{ RPMD + WMD \} = 0$$

Aside → Physical Picture.

→ ZF's and "Negative Viscosity"

absorbers



$v_{gy} > 0$

→ localized source

$v_{gy} < 0$

→ outgoing wave

absorbers

Causality ⇒ selects sign v_{gy} for outgoing wave

$$\boxed{\text{n.b. irreversibility} \Rightarrow \text{outgoing wave}}$$

$$\omega = -\frac{\beta k_x}{k^2}$$

$$v_{gy} = \frac{2\beta \boxed{k_x k_y}}{(k^2)^2}$$

→ group velocity

$\text{sign}(\beta k_x k_y) \rightarrow \text{sign } v_{gy}$

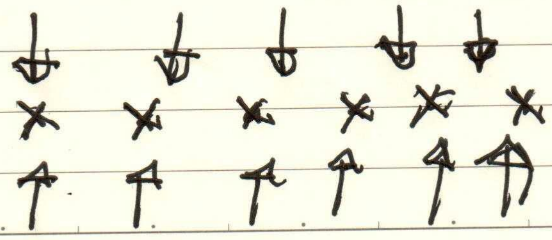
but

$$\langle v_y v_x \rangle = \boxed{-} \sum_{k} \boxed{k_x k_y} |\phi_k|^2$$

energy flux

$\beta > 0 \rightarrow$ outgoing $(v_{gy} > 0) \Rightarrow$

"incoming momentum flux"!



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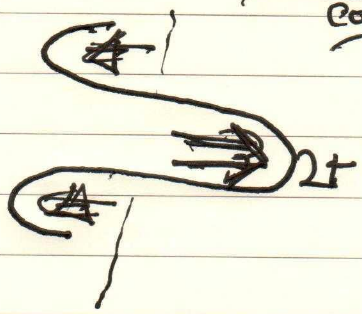
Momentum conservation



absorption → builds till flow modifies dispersion sufficiently

k_y change

or

Pred-Prey → feedbacks on source (instability)

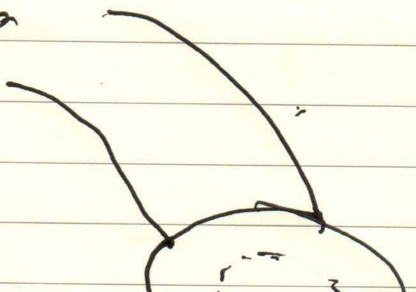
momentum transported up gradient → not a simple Fick's Law.

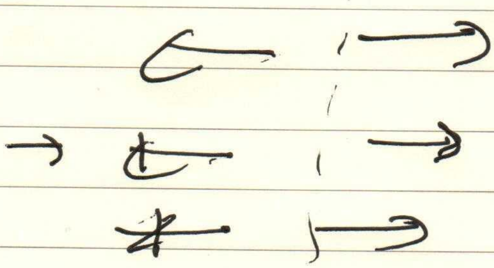
"... the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into that region."

— Isaac Held.

⇒ applies to Plasma, with obvious transcription

i.e





etc. ⇒ V_{gr}

⇒ V_E' modifies V_{gr}.

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→ While we're here:

Drift Wave - Zonal Flow Interaction:

Modulational Instability and the

Predator-Prey Model

- Have established connection between zonal flow and pseudo-momentum / WMD.

- Of course, coupling of WMD + flow must extract energy from turbulence ⇒ feedback loop for flow on turbulence.

- Structure closely analogous to theory of weak Langmuir turbulence

Plasma Wave ⟷ Drift Wave

↕ ↕

Ion Acoustic Mode (finite frequency) — Zonal Flow. (→0 frequency)

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Recall: Langmuir wave envelope

$$i\omega_0 \partial_t \mathcal{E} = \frac{\omega_{p0}^2}{2}\frac{\delta n}{n_0}\mathcal{E} - \alpha V_{The}^2 \nabla^2 \mathcal{E}$$

refraction in density perturbation

diffraction

$$\partial_t^2 \frac{\delta n}{n_0} - C_s^2 \nabla^2 \frac{\delta n}{n_0} = \nabla^2 \left(\frac{|\mathcal{E}|^2}{8\pi n m_i} \right)$$

radiation pressure.

treating plasma waves at level of eikonal theory ⇒ rays, can write:

$$\frac{\partial N}{\partial t} + \underline{v}_{g} \cdot \underline{\nabla} N - \frac{\partial}{\partial \underline{x}}(\omega) \cdot \frac{\partial N}{\partial \underline{k}} = 0$$

i.e.

$$\frac{dN}{dt} = 0$$ → statement of conservation of wave action density

Conservation Action ⟺ Phase Symmetry

$N = \frac{\mathcal{E}}{\omega_k}$ ⇒ population density in $\underline{x}, \underline{k}$
→ i.e. phase space density

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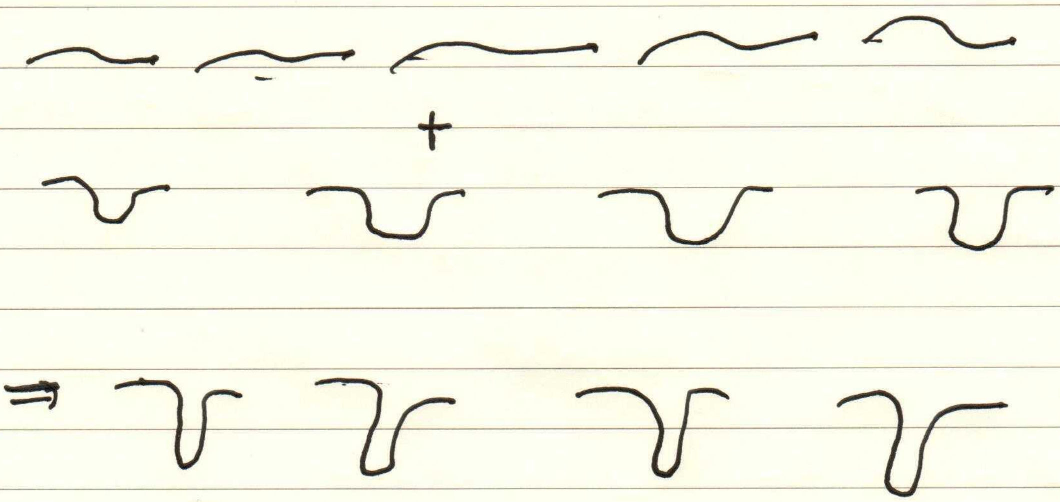
Modulation:



i.e. plasma wave gas unstable to refracting density perturbations

→ perturbations grow

→ plasma wave ensemble depleted.

To calculate:

– I A W growth rate

→ field energy density

$$\frac{|E|^2}{8\pi} = \left(\frac{1}{2}\right) W_k$$

$$= \int \left(\frac{1}{2}\right) \omega_k N_k \, dk$$

modulated WAD

then

$$\frac{\partial^2}{\partial t^2}\frac{\delta n}{n_0} - c_s^2 \nabla^2 \frac{\delta n}{n} = \nabla^2\left(\frac{1}{M_i}\int dk\, \omega_k \tilde{N}_k\right)$$

9/9. [illegible]

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Where:

$\underline{k}$, ω_k → plasma wave

$\underline{q}$, Ω → modulation field

$\tilde{N}_{\underline{q},\Omega}$

↓

modulated WAD

straightforward linearization:

$$\tilde{N}_{\underline{q},\Omega} = \frac{-\underline{q}}{\Omega - \underline{q}\cdot \underline{v}_{gr}} \, \omega_{pe_0} \frac{\tilde{n}_{\underline{q},\Omega}}{2n_0} \cdot \frac{\partial \langle N \rangle}{\partial \underline{k}}$$

where

$$\frac{1}{\Omega - \underline{q}\cdot \underline{v}_{gr}} = \frac{P}{\Omega - \underline{q}\cdot \underline{v}_{gr}} - i\pi\,\delta(\Omega - \underline{q}\cdot \underline{v}_{gr})$$

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so

$$\partial_t^2 \frac{\tilde{n}}{n_0} - c_s^2 \nabla^2 \frac{\tilde{n}}{n_0} = \nabla^2 \left(\frac{1}{m_i} \int dk\, \omega_{n_0}\, k \cdot \frac{-\underline{q}\, \omega_{pe_0}}{\Omega - \underline{q} \cdot \underline{v}_{gr}\, 2 n_0} \frac{\tilde{n}_q}{} \cdot \frac{\partial \langle N \rangle}{\partial \underline{k}} \right)$$

$\Rightarrow$

$$\Omega^2 = q^2 c_s^2 + \frac{q^2 \omega_{pe}^2}{2 m_i} \int d\underline{k}\; i\pi \delta(\Omega - \underline{q} \cdot \underline{v}_{gr})\, \underline{q} \cdot \frac{\partial \langle N \rangle}{\partial \underline{k}}$$

instability of IAW if

$$\frac{\partial \langle N \rangle}{\partial \underline{k}} \cdot \hat{q} > 0 \quad \text{at:} \quad \Omega = \underline{q} \cdot \underline{v}_{gr}$$

i.e. population inversion !

$\rightarrow$ energy decays from plasma waves to IAW.

$\rightarrow$ $\Omega_{real} \approx q c_s$ as usual.

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What of Plasma Waves?

$\langle N \rangle$ evolves!

$$\frac{\partial \langle N \rangle}{\partial t} = \frac{\partial}{\partial \underline{k}} \cdot \left\langle \frac{\partial \tilde{\omega}}{\partial \underline{x}} \tilde{N} \right\rangle$$

can approximate as in QL

$\Rightarrow$

$$\frac{\partial \langle N \rangle}{\partial t} = \frac{\partial}{\partial \underline{k}} \cdot \underline{\underline{D}}_k \cdot \frac{\partial \langle N \rangle}{\partial \underline{k}}$$

- Quasi-particle diffusion equation
- "Induced Diffusion"

For energy

$$E = \int d\underline{k} \;\; \omega \langle N \rangle$$

$$\frac{dE}{dt} = -\int \frac{\partial \omega}{\partial \underline{k}} \cdot \underline{\underline{D}}_k \cdot \frac{\partial \langle N \rangle}{\partial \underline{k}}$$

sign v_{gr} enters!

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sign $\frac{\partial \omega}{\partial k}$ $\frac{\partial \langle N \rangle}{\partial k}$ sets net

gain / loss energy from plasmons.

$$\boxed{\underline{\underline{D}}_k = \pi \left(\frac{\omega_p^2}{2n_g}\right) \sum_{\underline{q},\Omega} |\tilde{n}_{q,\Omega}|^2 \frac{i}{\Omega - \underline{q}\cdot v_{gr}} \underline{q}\,\underline{q}}$$

- physics is stochastic refraction

- requires: ray chaos

i.e. overlap of phase speed ⟷ group speed resonances (Q-P)

- energetics Plasmon Q-P energy vs IAW energy.

- structure very impt.

For plasma waves $\frac{\partial \omega}{\partial k} > 0$, so

$\frac{\partial \langle N \rangle}{\partial k} >$ for decay.

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– how related to wave interaction,

$$\frac{i}{\omega_{\underline{k}+\underline{k}'} - \omega_{\underline{k}'} - \omega_{\underline{k}}} \qquad |\underline{k}| < |\underline{k}'|$$

$$= \frac{i}{\cancel{\omega_{\underline{k}'}} + \underline{k} \cdot \frac{\partial \omega}{\partial \underline{k}'} - \cancel{\omega_{\underline{k}'}} - \omega_{\underline{k}}}$$

$$= \frac{i}{\underline{k} \cdot v_{gr}^{(\underline{k}')} - \omega_{\underline{k}}} \quad \checkmark$$

To Zonal Flow + DW!

– key points:

① $\frac{\partial \omega}{\partial \underline{k}} < 0$ so $\frac{\partial N}{\partial \underline{k}} < 0$ for depletion DW.

② $$N = \frac{(1+k_\perp^2 \rho_s^2)\,|\phi_{\underline{k}}|^2}{k_\theta v_* / (1+k_\perp^2 \rho_s^2)} = \frac{\mathcal{E}}{\omega_{\underline{k}}}$$

$$N = \frac{\Omega}{k_\theta v_*}$$

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but $$\frac{dy}{dt} = -\frac{\partial}{\partial \underline{x}}\left(\underline{k}\cdot\underline{V} + \cancel{\omega}\right)$$

$$= -\frac{\partial}{\partial \underline{x}}\left(k_\theta V_\theta(r)\right)$$

$$\Rightarrow \quad \frac{dk_r}{dt} = -\frac{\partial}{\partial r}\left(k_\theta V_\theta(r)\right)$$

i.e. k_θ constant

Action Conservation $\leftrightarrow$ Potential Enstrophy Conservation in Shearing Field