Wave-Particle Interaction and EP-AW Problems

→ to now: Fluid models — simple

→ Gyro & Drift Kinetics: wave-particle resonance

i.e. $$\frac{\partial g}{\partial t} + v_{\parallel}\hat{n}\cdot\nabla g + \underline{v}_D\cdot\nabla g + \frac{c}{B}\langle \underline{E}\rangle\times\underline{B}\cdot\nabla g = RHS.$$

ITG: $\omega - k_{\parallel}v_{\parallel} - \omega_{Di}$ — Landau resonance → threshold

CTEM: $\omega - \overline{\omega}_{D_e}\epsilon$ — Precession resonance → essential to drive

TIM: $\omega - \overline{\omega}_{D_i}\epsilon$ — " " ions

⇒ $\omega - kv$

key to wave-particle resonance is:

$\boxed{\text{1D Vlasov-Poisson:}}$

see
- Sagdeev & Galeev
- P.D., Itoh, Itoh
- also 218a notes Fall 2018

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}E\frac{\partial f}{\partial v} = 0$$

$$\nabla^2\phi = -4\pi n_0 q\int dv\, f \qquad -\chi_i(k,\omega)\,\phi_{k,\omega}$$

Classic Problems:

- QL, electron plasma wave, B-O-T

- CD Ion Acoustic

Others: Fluid $\perp$ dynamics + 1D along Field.
Most problems are 1D Vlasov ($\parallel$) + Fluid ($\perp$) hybrid.

Survey:

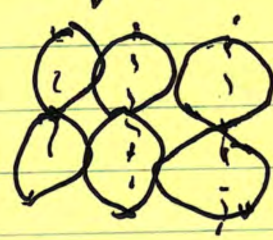
- Landau $\omega = k \cdot v$ ✓

- QL $\dfrac{\partial \langle f \rangle}{\partial t} = \dfrac{\partial}{\partial v} \dfrac{q^2}{m^2} \sum_k \dfrac{|E_k|^2 |\gamma|}{(\omega - kv)^2 + |\gamma|^2} \dfrac{\partial \langle f \rangle}{\partial v}$

(see 218a Notes Fall 2018)

Key - stochastic trajectories - irreversibility

i.e. island overlap in phase space



- $Ku < 1 \Rightarrow$ diffusion

$$Ku = \frac{q}{m} \frac{E \tau_{ac}}{\Delta v} < 1 \Rightarrow \underbrace{\omega_b \tau_{ac}}_{} < 1$$

$\boxed{1/\tau_{ac} = \left| \frac{\omega}{k} - v_{gr} \right| \Delta k}$ $\longrightarrow$ wave-particle autocorrelation time

- Energetics

$$\partial_t \left(RPKED + WED \right) = 0$$

2 energy thms $\qquad \sum_{k} \omega_k N_k = \sum_{k} \omega_k \frac{\partial \epsilon}{\partial \omega}\Big|_{\omega_k} \frac{|E_k|^2}{8\pi}$

evolving wave $\quad$ kin. dist. with CCM.

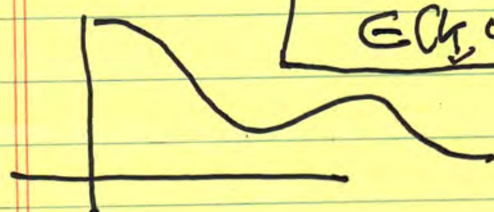
$$\partial_t \left[(RPMD) + (WMD) \right] = 0$$

$$\partial_t \left[(PKED) + (EED) \right] = 0$$

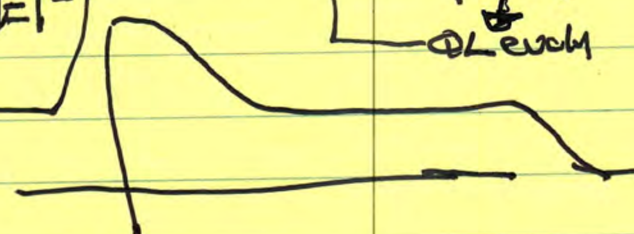
- Plateau

$$\boxed{\begin{array}{c} \text{QL } \partial\langle E\rangle/\partial t \\ \partial_t |E|^2 = 2\gamma |E|^2 \\ \epsilon(k,\omega) = 0 \end{array}}$$

Dist → modes → growth → QL evoln



→



With collisions:

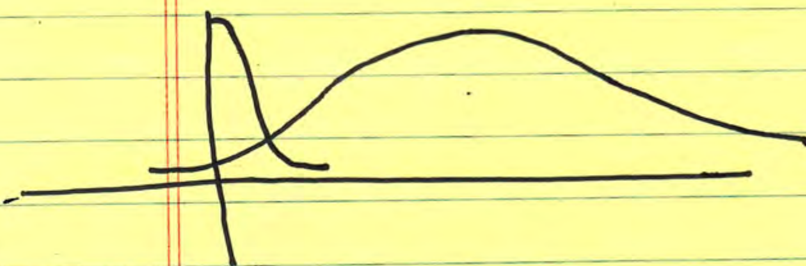
$$\partial_t \langle f \rangle = \partial_v D \, \partial_v \langle f \rangle + C(f)$$

$$1/\tau \sim \frac{D}{(\Delta v)^2} \qquad 1/\tau_c$$

2 time scales

Collisions will restore Maxwellian, etc. after τ_c. Can make cycle ala' LCO.

– Anomalous Resistivity – CDIA



see 218a notes

QL for electrons + ions.

Wave Evolution

⇒ $\boxed{\text{collisionless momentum transfer between electrons + ions} \rightarrow \text{"anomalous resistivity"}}$

⇒ $\boxed{\text{Couple to macroscopics} \rightarrow \text{Reconnection.}}$ → still open especially with spatial structure

⇒ Upshot is sitting near marginality of CDIA.

Aside: QL for electron drift wave: see 2018 NRU notes.

$$\frac{\partial \langle f\rangle}{\partial t} = \frac{\partial}{\partial r} D_{r,r} \frac{\partial \langle f\rangle}{\partial r} + \frac{\partial}{\partial v_{\parallel}} D_{v_{\parallel},r} \frac{\partial}{\partial r}\langle f\rangle$$

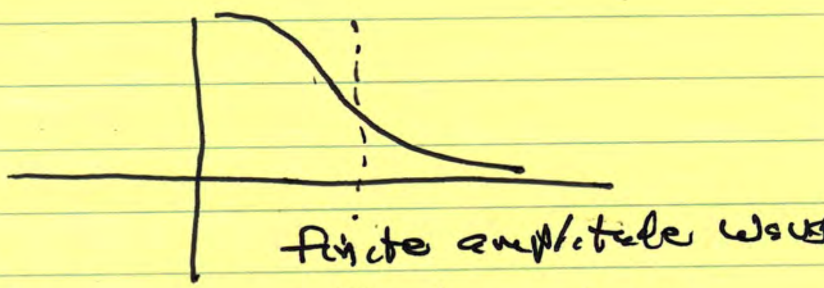
$$+ \frac{\partial}{\partial r} D_{r,v_{\parallel}} \frac{\partial}{\partial v_{\parallel}}\langle f\rangle + \frac{\partial}{\partial v_{\parallel}} D_{v_{\parallel},v_{\parallel}} \frac{\partial}{\partial v_{\parallel}}\langle f\rangle.$$

ex:

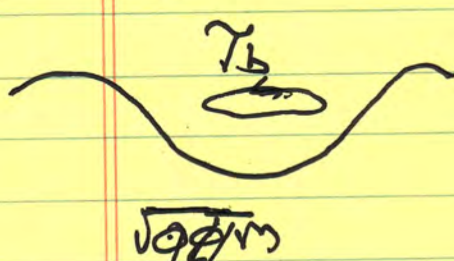
- Derive electron energy theorem.

- Use con QL equation for DWT to derive $\frac{\partial \langle v_\parallel \rangle}{\partial t}$. Remember $\langle f \rangle$ is a function of $\langle v_\parallel \rangle$. Implications for intrinsic rotation

Trapping



single, finite amplitude wave.



- single wave

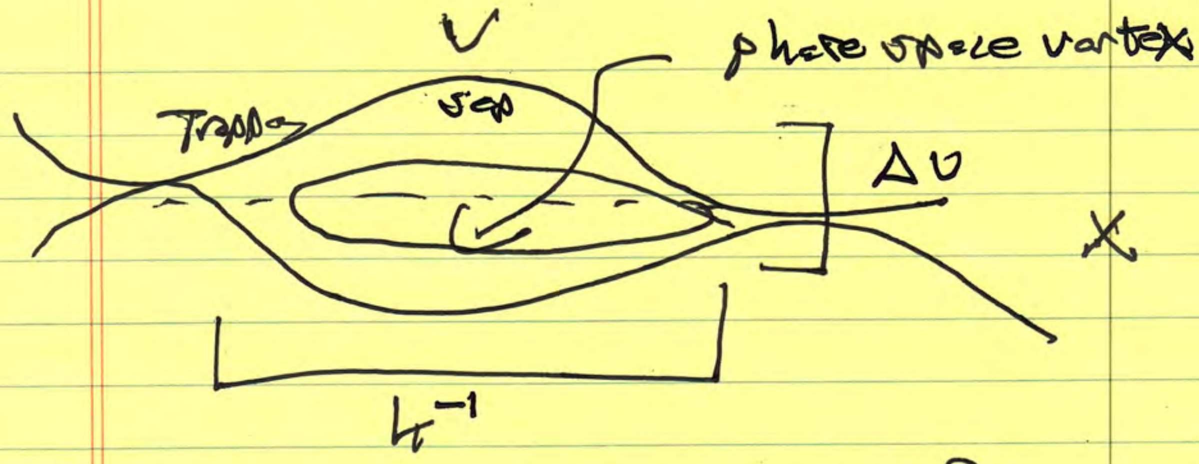
$t < \tau_b \Rightarrow$ [Landau calculation

$\gamma_L > 1/\tau_b$

or - $t > \tau_b \Rightarrow$ orbits perturbed

$\gamma_L < 1/\tau_b$

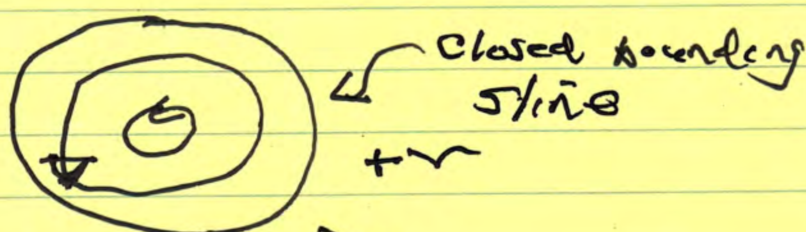
Need treat trapped orbits!



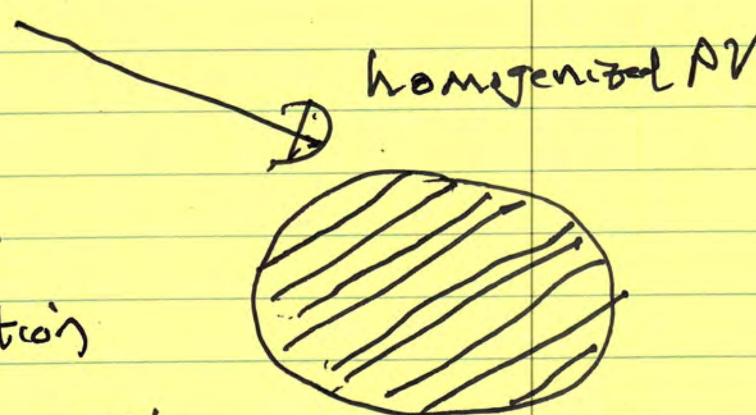
What happens in real space vortex?

Problem analogous to (PV) homogenization:

cf $\quad \frac{\partial q}{\partial t} + \nabla\phi\times\hat{z}\cdot\nabla q - \nabla\cdot\nu\cdot\nabla q = 0 \qquad q = \nabla^2\phi$



$Pe = LV/\nu \gg 1$



then: $\quad q \to$ const.

(PV) homogenization

Prandtl-Batchelor Thm.

See Rhines + Young 1982

stretch out

How

Phase mixing

Diffusion ν

+

Stretching (diff rotation)

Time scale:

$$1/\tau_{mix} \sim \left[\frac{\nu}{L^2}\left(\frac{\partial V_y}{\partial r}\right)^2\right]^{1/3} \rightarrow \left[\nu\left(\frac{\partial \Omega(r)}{\partial r}\right)^2\right]^{1/3}$$

$$\boxed{\tau_{mix} \sim \underbrace{\tau_{corr.}}_{(L/V)} Pe^{1/3}}$$

[see also 2/8b notes

NB: Coarse graining / structure of diffusive dissipation is essential.

Vlasov is fundamentally same.

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{M}E\frac{\partial f}{\partial v} = C(f) \sim \nu\frac{\partial^2 f}{\partial p^2}$$

⇒

$$\frac{\partial f}{\partial t} + \frac{\partial H}{\partial p}\frac{\partial f}{\partial x} - \frac{\partial H}{\partial x}\frac{\partial f}{\partial p} = \nu \frac{\partial^2 f}{\partial p^2}$$

action-angle

$$H = \frac{mv^2}{2} + \varepsilon\phi$$
$$= p^2/2m + \varepsilon\phi$$

$$\frac{\partial f}{\partial t} + \frac{\partial H}{\partial J}\frac{\partial f}{\partial \alpha} - \frac{\partial H}{\partial \alpha}\frac{\partial f}{\partial J} \approx \nu\frac{\partial^2 f}{\partial J^2}$$

Just like that

if closed orbits $H = H(J)$

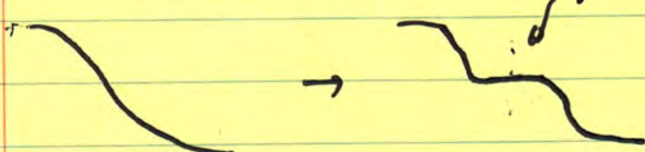
$$\frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \alpha} = \nu \frac{\partial^2 f}{\partial J^2}$$

$$\rightarrow \quad F = F_0\, e^{-\underbrace{\nu(\partial\Omega/\partial J)^2 n^2 t^3}}$$

govern time scale for decay fine structure

≈ same mixing rate! Rate

Distribution of trapped particles homogenized



plateau.

$\frac{\partial f}{\partial v} \to 0$.

"Trapping" is really saturation by phase mixing.

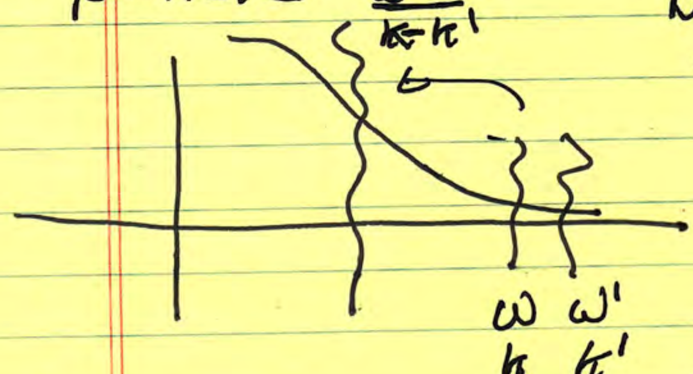
→ Both QL and Trapping are physics of the 1960's – though poorly understood by many today.

→ A bit more interesting: (though still 60's)

Nonlinear Landau Damping

- "wave-particle part" of weak turbulence theory

- resonance of 'beat wave' with particles $\frac{\omega-\omega'}{k-k'}$



DWT, Ion Acoustic
- ions non-resonant: $\frac{\omega}{k_\parallel} > v_{Thi}$

- drive resonant beats.

$$\Rightarrow \frac{i}{(\omega-\omega')-(k_\parallel-k'_\parallel)v_\parallel}\,(\underline{k}\cdot\underline{k}'\times\hat{z})^2|\phi_{\underline{k}'}|^2|\phi_{\underline{k}}|^2\,F(\underline{k},\underline{k}')$$

$$\sim \frac{\partial \mathcal{E}_k}{\partial t}$$

Wave Poll'n Compton Scattering etc

Energetics

$$\frac{\partial \langle F\rangle}{\partial t} = \frac{\partial}{\partial v}\overbrace{D_{QL}}^{O(E^2)}\frac{\partial \langle F\rangle}{\partial v} + \frac{\partial}{\partial v} D_{NLLD}\frac{\partial \langle F\rangle}{\partial v} + \cdots$$

$$\underbrace{D_{NLLD}}_{\text{beat heating}} = \underbrace{\sum_{\substack{k',\omega' \\ k,\omega}} |E_k|^2 |E_{k'}|^2 F(k,k')}_{O(E^4)} \times \underbrace{\pi\delta\big((\omega-\omega')-(k-k')v\big)}_{\text{beat wave particle resonance}}$$

N.B.: D_{NLLD} not necessarily "small" relative to D_{QL}, as primary resonance effects can be exponentially small.

N.B.: – Ion NLLD is important process for DWT. Channel for collisionless energy transfer (Zhao, PD ...).
– Potentially interesting to encompass in Predator – Prey model structure.

Resonance Broadening Theory

→ Nonlinear response, in turbulence

→ Consider scattering in response

i.e. Landau

$$f \sim E_k \frac{\partial \langle f \rangle}{\partial v} \int_0^\infty d\tau \exp\left[i(\omega - kv)\tau\right]$$

$$\rightarrow E_k \frac{\partial \langle f \rangle}{\partial v} \int_0^\infty d\tau \exp\left[i(\omega - kv)\tau + i\underset{\uparrow}{\underline{k}\cdot\delta\underline{r}(-\tau)}\right]$$

excursion from upo

$$\approx E_k \frac{\partial \langle f \rangle}{\partial v} \int_0^\infty d\tau \exp\left[i(\omega - kv)\tau\right] \langle \exp[ik\cdot\delta r(-\tau)]\rangle_{avg.}$$

but v scatt. $\delta r = \int^{\tau} d\tau' \, \delta v$

as $\dfrac{dv}{dt} = \dfrac{q}{m} E$

$$f \sim E_k \frac{\partial \langle f \rangle}{\partial v} \int_0 d\tau \, e^{i(\omega - kv)\tau} \left\langle e^{ik\int^{\tau} d\tau' \delta v(-\tau')} \right\rangle$$

$$\left\langle 1 + \text{L.T.} - k^2 \frac{\tau^3}{3} D_v \right\rangle$$

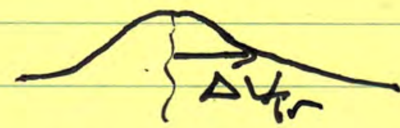
$D_v \equiv$ scattering in velocity space
(taken QL)

so

$$f \sim E_k \frac{\partial \langle f \rangle}{\partial v} \int_0^\infty d\tau \, \exp\left[i(\omega - kv)\tau - \frac{k^2 D_v \tau^3}{3} \right]$$

so

$\delta(\omega - kv) \rightarrow$



broadened resonance

$$\boxed{\Delta v_{tr} = \frac{1}{k}\left(k^2 D_v\right)^{1/3}}$$

broadened resonance width

$$\boxed{1/\tau_c = \left(k^2 D_v\right)^{1/3}} \longrightarrow \boxed{\text{orbit decorrelation rate}}$$

scattering rate.

need $\tau_{ac} \leq \tau_c$, generally.

$$D_v = \sum_{k,\omega} \frac{q^2}{m^2} |E_{k,\omega}|^2 \underbrace{R(\omega - kv)}_{\text{broadened resonance function}}$$

- $1/\tau_c$ is decorrelation rate response.
- Δv_{tr} gives resonance mods.

Can approach as renormalization:

$$-i(\omega-kv)f_{k,\omega}+\frac{q}{m}\frac{\partial}{\partial v}\sum_{k',\omega'}E_{-k',-\omega'}\,\delta f_{k'+k,\omega'+\omega}=\frac{-q}{m}E_k\frac{\partial\langle f\rangle}{\partial v}$$

crank as usual:

$$NLT \Rightarrow -\underbrace{\frac{\partial}{\partial v}D_{k,\omega}\frac{\partial}{\partial v}f_{k,\omega}}_{\text{orbit scattering}}+\underbrace{\frac{\partial}{\partial v}b_{k,\omega}(v)\,E_{k,\omega}}_{\text{backgnd renorm.}}$$

$$-i(\omega-kv)\delta f-\frac{\partial}{\partial v}D_{k,\omega}\frac{\partial}{\partial v}\delta f=\frac{-q}{m}E_k\left[\frac{\partial\langle f\rangle}{\partial v}+\frac{\partial}{\partial v}b_{k,\omega}(v)\right]$$

$$\sim\sum_{k',\omega'}|E_{k',\omega'}|^2\frac{i}{\omega+\omega'-(k+k')v}$$

[investigate effect.

if: $\omega'>\omega$, $k'>k$ $\quad D_{k,\omega}=D_{QL}$

Markovian limit

but even if not, on resonance:

$$\omega=kv.$$

$$D_{k,\omega}=D_{QL}.$$

→ This brings us to granulations $\left\{ \begin{array}{l} \text{Clumps} \\ \text{Holes} \end{array} \right.$

['70's and '80's]

— Till now: all fluctuations are waves

i.e. $\omega = \omega(k)$.

what of phase space structures — analogue of eddys and vortices.

→ clumps, holes

Clump (1D): eddy $\Delta v_T, \lambda$.

$$\delta f = f^c + \tilde{f}$$

f^c: coherent response to ϕ (QL)

$\tilde{f}$: incoherent

Describe by 2pt eqn:

· can calculate $\langle \tilde{f}^2 \rangle$

mode - mode coupling

$$\left(\partial_t + v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2} \right) \langle \delta f^2 \rangle + \frac{q}{m} \frac{\partial}{\partial v_1} \langle E_1 \delta f(1) \delta f(2) \rangle$$

relaxation → not QL.

$$+ \frac{q}{m} \frac{\partial}{\partial v_2} \langle E_2 \delta f(1) \delta f(2) \rangle = - \frac{q}{m} \langle E \delta f(2) \rangle \frac{\partial \langle f \rangle}{\partial v_1}$$

$$- \frac{q}{m} \langle E \delta f(1) \rangle \frac{\partial \langle f \rangle}{\partial v_2}$$

x_-, v_- :

$$\frac{\partial}{\partial t}\langle \delta f^2\rangle + v_- \frac{\partial}{\partial x_-}\langle \delta f^2\rangle$$

$$+ \frac{q}{m}\left\langle (E(1)-E(2))\frac{\partial}{\partial v_-}\delta f(1)\,\delta f(2)\right\rangle = -\frac{q}{m}\langle E\,\delta f\rangle \frac{\partial\langle f\rangle}{\partial v}$$

$\downarrow$

close w/ QL

$$\partial_t\langle\delta f^2\rangle + v_-\frac{\partial}{\partial x_-}\langle\delta f^2\rangle - \frac{\partial}{\partial v_-}D_{rel}\frac{\partial}{\partial v_-}\langle\delta f^2\rangle$$

$$= -\frac{q}{m}\langle E\,\delta f\rangle\frac{\partial\langle f\rangle}{\partial v}$$

LHS is QL ideology to 2 pt. → stoch accel.

Relative separation

→ defines τ_{cL} — "clump lifetime".

N.B.: Used in shearing calculations, (Big noise, standard physics ----)

But RHS is interesting part.

$$\delta f = f^c + \tilde{f}$$

So

$$-\frac{q}{m}\langle E(f^c+\hat{f})\rangle \frac{\partial\langle f\rangle}{\partial v} = D_{QL}\left(\frac{\partial\langle f\rangle}{\partial v}\right)^2 + -\frac{q}{m}\langle E\hat{f}\rangle \frac{\partial\langle f\rangle}{\partial v}$$

$$\sim D_{QL}\left(\frac{\partial\langle f\rangle}{\partial v}\right)^2 + \underbrace{F}_{\text{drag.}}\frac{\partial\langle f\rangle}{\partial v}$$

RHS $\Leftrightarrow$ $$\frac{\partial\langle f\rangle}{\partial t} = -\frac{\partial}{\partial v}\left\langle \frac{q}{m} E(\underbrace{\delta f}_{f^c,\ \tilde{f}})\right\rangle$$

$$= -\frac{\partial}{\partial v}\left[-D_{QL}\frac{\partial\langle f\rangle}{\partial v} + \underbrace{F}_{\text{Drag, } \tilde{f}}\right]$$

like macroparticle.

Lenard-Balescu structure replaces QL structure.

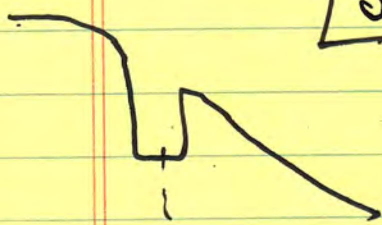
Here granulation is counterpart of test particle.

→ $\boxed{\text{Drag enables new route to relaxation ~~to~~ nonlinear instability}}$ - ...

— something fundamentally different

– Related : Hole → localized phase space structure (coherent)

[See PKU Lectures 2018]



$$\tilde{f} = -\frac{\Delta V}{\epsilon(kV)} (\ )$$

[Fundamentally different from wave.]

~ Screened Jeans equilibrium

↳ $-4\pi G\rho_0 + k^2 c_s^2 = \omega^2$

~ ΔV dispersion $(\omega - kV_0)^2 = k^2 \Delta V^2 + \text{"}G\text{"}\frac{\Delta V \tilde{f}}{\epsilon}$

~ localized phase space structure.

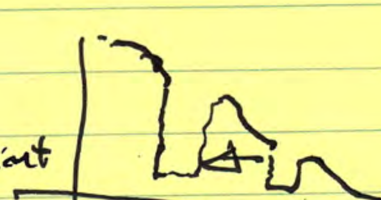
~ <u>self-bound</u> – electrostatic self-force holds together

– akin to gravity.

→ Phase space holes observed.

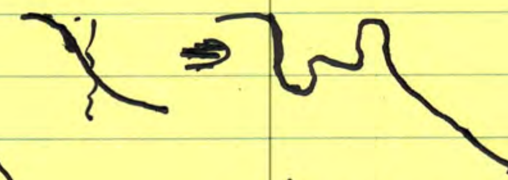
Related to double layers, in space.

→ Grow via motion up gradient



f conserved.

Move by momentum exchange..

→ holes and "clump" → 
(anti-hole, "phase space blob")

clump-hole pair

B + B

is this just distortion of f.

→ Related:

Hole is "most probable" localized structure of given mass, energy, momentum.

constrained variation ⟺ Dupree '82
after $\boxed{\text{Lynden-Bell '67}}$ "Violent Relaxation" ← Read this!

see PHU '78 and UCSD Disks W'22 Lect 11, 9, 1

→ Holes can tap free energy (grow) when waves damped.

cf Laser, PD

→ Ok, so what of EP transport?

- EP drives AE, EPM.
  High frequency, relative DW.

- Huge zoology of AE:

TAE　　　　EPM
RSAE　　　　⋮
BAE　　　　hot precession resonance (similar TIM / Dermet)
GAE
⋮

see: Chen + Zonca RMP for infinite crank.

EP driven, inverse Landau.

In China
- ZJU for linear theory
- SWIP for mode identification

but what happens nonlinearly?

⇒ EP transport?　　N.B. GZ paper - no physics

and how EPs interact with thermals.

→ 3 approaches:

- B & B Model — reduced model of AE + particle. Maps to 1D.
  Berk–Breizmann

  Comment: – Single wave
  – nice, but "well formed." — what is left?

- Simulations → Multi-Scale (Mazzi, Garcia, Lin grp)

  Picture of: — AE's high freq.
  how interact? — Thermals DW low freq.
  $\boxed{\text{Sources critical.}}$ — Zonal Zero freq.

- Stochastic Scattering → ITG scatter EPs, dephase AE response.
  QL + Resonance Broadening theory.
  [Vinicius Duarte, Gorelenkov]

a)
B + B Model

Lesur, PD 2013

→ Claim that EP beam + wave maps to 1D plasma

collnl effects

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}E\frac{\partial f}{\partial v} = -\gamma_a \delta f + \frac{\nu_f^2}{k}\frac{\partial \delta f}{\partial v} + \frac{\nu_d^3}{k^2}\frac{\partial^2 \delta f}{\partial v^2}$$

thermal damping

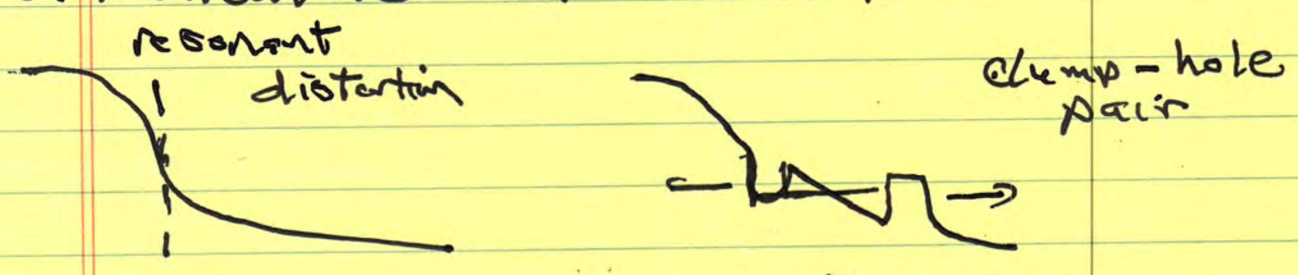
$$\frac{dZ}{dt} = -\frac{m\omega_p^3}{4\pi q n_0}\int f e^{-i\varphi}\, dx\, dv - \gamma_d Z$$

$\varphi = kx - \omega t$

$E = Z(e^{i\varphi} + cc)$

key: f is single wave.

→ can calculate δf evolution



velocity evolves.

∴ time evolving frequency shift in spectral components — i.e. $\boxed{\text{"chirp"}}$

$\delta\omega(t)$

- B+B model successful in explaining observed chirping.

- B+B model limited to single, coherent wave interaction

$\boxed{\text{Extend?}}$

b) Stochastic Scattering [Duarte, Gorelenkov]

Motivation : Chirps ↑ when χ_{th} ↓
(Collins +, DIII-D)

⇒ hypothesize that AE resonance occurs in background of ITG turbulence.

⇒ Calculate AE response, evolution f_{hot} in ITG background.
- QL
- RBT

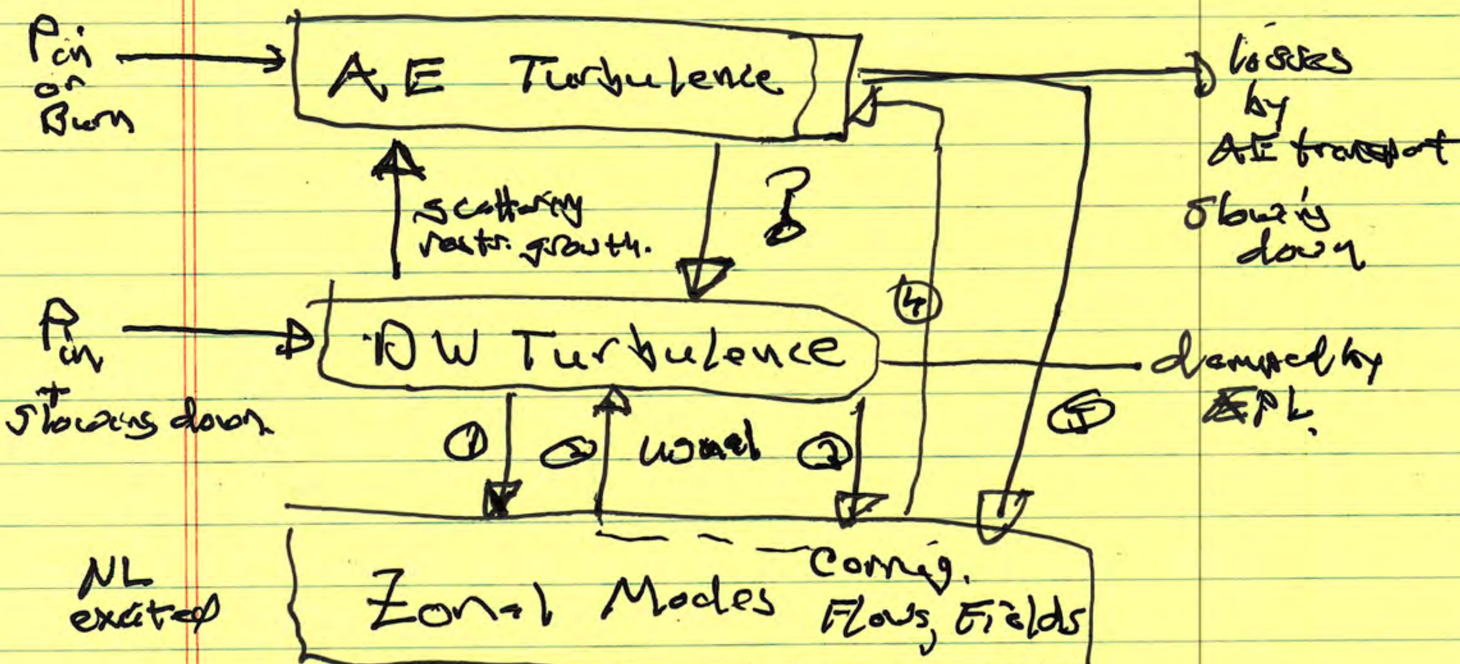
What is level of thermal turbulence to scatter EP? Claim not much needed ....

⇝ begins to address cross effects. But only part of story. $\boxed{\text{Relevent}}$

c) Multiscale

- Simulations note effect of hots, AE's on thermals.

- begs for extended predator-prey:



- branching ratio $P_{in\,hot} / P_{in\,th}$ is important.

- hots can be active (AE's) or passive (sink energy from DW's).

- ZF's, corrugations affect both
- Look for bifurcation in

  $\mathcal{E}_{AE} / \mathcal{E}_{DW}$ ratio.

- includes D+G ⇒ scatt. [DW] restrict AE growth.

- need address simulation

⇒ heads for reduced model.

⇒ Goal is insight, then transport model for EP's.

→ Extended predator-prey model seems like useful next step.

Look to predict bifurcations.