PKU Lecture III

→ "QL" Theory for Simple Drift Wave Turbulence, cont'd

- correlation times, Kubo # ↔ relation to structure.
- relaxation – transport ↔ heating constraint
  - avalanche ?

→ PV Conservation, PV Dynamics

→ Taylor Identity, and Charney-Drazin (Momentum) Theorem.

→ Relation Modulational Theory & ZF Formation

Recall:

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} - \frac{c}{B_0} \underline{\nabla}\phi \times \hat{z} \cdot \underline{\nabla} f - \frac{|e|}{m_e} E_z \frac{\partial f}{\partial v_z} = 0$$

$\frac{|e|}{m_i} \rightarrow$ ions

$$\delta f_k^c = \frac{\phi_k}{\omega - k_z v_z} \underline{L_k} \langle f \rangle$$

$$\underline{L_k} = -\frac{c}{B_0} k_\theta \frac{\partial}{\partial r} + \frac{|e|}{m_e} k_z \frac{\partial}{\partial v_z}$$

$\underbrace{\hphantom{L_k}}$ scattering operator.

so

$$\partial_t \langle f \rangle = \overset{①}{\partial_r D_{r,r} \partial \langle f \rangle} + \overset{②}{\partial_r D_{r,v} \partial_v \langle f \rangle} + \overset{③}{\partial_v D_{v,r} \partial_r \langle f \rangle} + \overset{④}{\partial_v D_{v,v} \partial_v \langle f \rangle}$$

①, ② $\rightarrow$ Momentum flux, including Π_{resid}, V

③, ④ $\rightarrow$ turbulent acceleration.

N.B. show $\Gamma_e = \Gamma_i$, in QLT.

Now, seek examine τ_{ac} for Drift Waves,

Spectral structure ⇔ Geometry ⇔ Kubo # Connection

$$D_{\perp,\perp} = \sum_{n} \frac{c^2}{B_0^2} |\phi_n|^2 k_\theta^2 \frac{i}{\omega - k_z v_z}$$

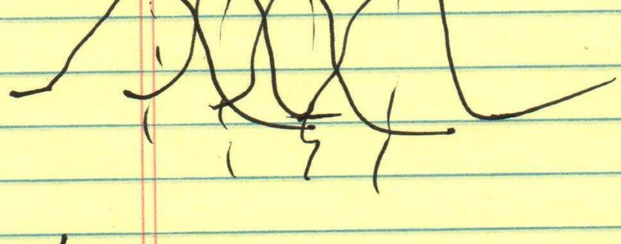
assume trial spectrum structure:

$$|\phi_n|^2 = |\phi_0|^2 \left(\frac{\Delta k_\theta}{(k_\theta - \bar{k}_\theta)^2 + (\Delta k_\theta)^2} \right) \left(\frac{\Delta k_z}{k_z^2 + (\Delta k_z)^2} \right)$$

centroid k_θ — spread — spread k_z

→ ad-hoc but useful to extract time scales.

For sheared system:



$q' > 0$

$$m = nq$$

$$dn = \frac{m}{q^2} q' \, dx$$

$$= \frac{k_\theta r q'}{q^2} dx$$

$$|\phi_M|^2 = |\phi_0|^2 \, S(k_\theta) \, F\left(\frac{r - r_0}{\Delta_S}\right)$$

↳ spectral width, spec.

$$\sum_m \sum_n \rightarrow \int dk_\theta \int \frac{k_\theta r_0 q'}{q^2} dx$$

if overlap

sum over modes sweeps spectral width

⇒ Δk_z corresponds to spectral width integration

→ can extend to ballooning space.

extent mode along line ⟺ coupling.

Now,

$$D_{\perp,2} \approx \frac{i}{\omega_k - k_z v_z + i/\tau_{ac}}$$

$$1/\tau_{ac} = \left| \, |\Delta k_\theta| \left|\frac{d\omega}{dk_\theta}\right| - v_z |\Delta k_z| \, \right|$$

→ have assumed $d\omega / dk_\parallel \to 0$

Recall 1D:

$$1/\tau_{ac} = \left| \frac{d\omega}{dk} - v \right| \Delta k$$

resonant

$$= \left| \frac{d\omega}{dk} - \frac{\omega}{k} \right| \Delta k$$

i.e. $\boxed{\text{wave-particle auto-correlation time depends on particle speed.}}$

Can see:

– electrons $\left.\begin{matrix}\text{ions near marginal}\end{matrix}\right\}$ resonant — ITG

$\omega / k_\parallel \to \omega / |k_\parallel| \leq v_{thi}$

$$1/\tau_{ac} = \left| \Delta k_\theta \frac{d\omega}{dk_\theta} - \frac{\omega}{k_\parallel} |\Delta k_\parallel| \right|$$

$$\approx \left| \Delta k_\theta \frac{d\omega}{dk_\theta} - v_{thi} |\Delta k_\parallel| \right|$$

$1/\tau_{c0} \sim |v_{The}\,\Delta k_{\parallel 0}|$ → edw.

$1/\tau_{c0} \sim |v_{Thi}\,\Delta k_{\parallel 0}|$ → ITG, near marginal (resonant case).

$1/\tau_{c0} \sim |\Delta k_{\parallel 0}\, d\omega/dk_{\parallel 0}|$ → ITG above marginal.

N.B. Ion diffusion in EDW is non-resonant.

Also:

→ Geometry / spectral structure reduces sensitivity (to dispersion), of τ_{c0}.

→ $Ku \sim \dfrac{\tau_{c0}\tilde{V}}{\Delta} < 1$ ⇒ validates ~~[illegible]~~ stochastic transport modelling.

if: $\tau_{c0}\,\omega_* < 1$

→ $Ku < 1$ → EDW
→ ~ marg. ITG

$Ku \sim 1$ for fluid ITG.

Precession Resonance: An Interesting Twist

$$D_{\perp,\perp} = \frac{c^2}{B_0^2} \sum_n \frac{n^2}{r^2} q^2 \left|\bar{\Phi}_n\right|^2 \frac{i}{\omega - \bar{\omega}_D \epsilon}$$

↑ bounce average.

as before;

$$\frac{i}{\omega - \bar{\omega}_D \epsilon} \Rightarrow \frac{1}{\left|\frac{d\omega}{dk_\theta}\Delta k_\theta - v_D \epsilon \Delta k_\theta\right|}$$

$\epsilon = \omega/\bar{\omega}_D$ for resonant particles

$$\sim \frac{1}{|\Delta k_\theta| \left[\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}\right]}.$$

$$\frac{1}{\tau_{ac}} \equiv |\Delta k_\theta| \left[\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}\right]$$

→ akin 1D → dispersion sensitive

→ precession resonance modes (CTEM, EAM) test ray

Other cases

→ precession:

$$D = \frac{c^2}{B_0^2} \sum_n \frac{n^2 q^2}{r^2} |\tilde{\phi}_n|^2 \frac{i}{\omega - \omega_D}$$

as before

$$\frac{i}{\omega - \omega_D} \sim \frac{\cancel{i}}{\cancel{i} \left| \frac{d\omega}{dk_\theta} \Delta k_\theta - V_D \epsilon \Delta k_\theta \right|}$$

$$\epsilon = \omega / \omega_D$$

$$\epsilon \sim \frac{\omega}{\omega_D} \qquad \sim \frac{1}{\left| \frac{d\omega}{dk_\theta} \Delta k_\theta - \cancel{V_D} \Delta k_\theta \frac{\omega}{k_\theta \cancel{V_D}} \right|}$$

$$\boxed{\tau_{ac} \sim \frac{1}{|\Delta k_\theta| \left| \left[\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta} \right] \right|}}$$

→ ala' 1D → dispersion sensitive.

→ CTEM tend to run hw ↑, as

τ_{ac} ↑.

N.B.: Precession resonance drives turbulence has long wave-particle correlation times.

large Ku.

⇒ ? How treat ?

— granulations ⇒ strongly correlated clusters ?
Y.K. — Ph.D., et seq.

— but still statistical ??

⇒ Useful to consider single, few coherent CTEM.

BGK solution ?

⇒ HW: Effect $E \times B$ shear on
$\mathcal{J}_{ec}$ — EDW
— Precession

$\boxed{\text{What is QL saturation for simple drift wave?}}$

$\frac{k_\theta}{m} k_z \to \frac{k_\theta}{m_0}\frac{\omega}{v_z}$ from resonance

→ Relaxation

$$L_k \langle f\rangle = \frac{k_\theta}{\Omega_e}\frac{\partial}{\partial r}\langle f\rangle + \frac{\omega_k}{v_z}\frac{\partial}{\partial v_z}\langle f\rangle$$

and

$$\partial_t \langle f\rangle = \sum_k \left[L_k \, |\phi_k|^2 \pi \delta(\omega - k_z v_z) \, L_k \right] \langle f\rangle$$

relaxn: what is QL stationary state?

$L_k \to 0$

$$-\frac{k_\theta}{\Omega_e}\frac{\partial}{\partial r}\langle f\rangle + \frac{\omega}{v_z}\frac{\partial}{\partial v_z}\langle f\rangle \to 0$$

$$\left(-\frac{k_\theta}{\Omega_e}\frac{1}{\Delta r} + \frac{\omega/v_z}{\Delta v_z}\right)\langle f\rangle \to 0$$

$$\boxed{\Delta r - \Delta\frac{v_z^2}{2}\frac{k_\theta}{\Omega_e \omega_k} = 0}$$

$$\Delta\left(r - \frac{v_z^2}{2}\frac{k_\theta}{\Omega_e \omega_k}\right) = 0$$ constraint for relaxed state.

Formally:

$$\partial_t \langle f \rangle = \left[\sum_k L_k \, |E_k|^2 \pi \delta(\omega - k_z v_z) \, L_k \right] \langle f \rangle$$

$$\partial_t \int dr \int dv_z \, \frac{\langle f \rangle^2}{2} = - \int dr \, dv_z \sum_k |E_k|^2 \pi \delta(\omega - k_z v_z) \, * \, \left(L_k \langle f \rangle \right)^2$$

$L_k \langle f \rangle = 0 \Rightarrow$ QW "plateau"

N.B. - Note this is __not__ simply flattening of density

- $L_k \to 0$ before $\frac{\partial \langle f \rangle}{\partial r} \to 0$, as heating Δv_z^2 occurs.

$$\boxed{\begin{aligned} \Delta r &= \frac{\Delta V_z^2}{2} \frac{k_\theta}{\Omega_e \omega_{*}} \\ r - \frac{V_z^2}{2} \frac{k_\theta}{\Omega_e \omega_{*}} &= \text{const.} \end{aligned}}$$

→ i.e. { any $\Delta r > 0$ displacement ⇒ heating ($\Delta V_z^2 > 0$) }

how QL saturate without $Dn \to 0$!?

→ DW expends energy on heating via Landau damping ⇒ route for non-trivial QL satn.

$\boxed{\omega \approx \omega - \omega_{*e}}$

→ $\Delta V_z^2 \sim \Delta r \frac{\cancel{\Omega_e}}{\cancel{k_\theta}} \omega_* \quad \frac{\rho_e V_{Te}^2}{L_n \cancel{\Omega_e}}$

$$\boxed{\frac{\Delta V_z^2}{V_{The}^2} \sim \frac{\Delta r}{L_n} \cancel{V_{Te}^2}}$$ → [heating - transport relation]

Lesson

- DW's ~~to~~ distort distribution function

- relevant to ITG near marginal.
- heating often overlooked!

Comment:

→ How include source ⇒ headed toward
sink

flux driven

$$\partial_t \langle f \rangle = L\, D\, L \langle f \rangle + S_0(r,t)$$

dynamics couples { transport
heating

→ What of avalanching?
- avalanche in phase space
- must evolve in r (as usual)
and V_z i.e. heating penalty for
step Δr.

- "bi-variate Burgers", with
constraint on radial, $v_{\parallel}$ scattering?

and <u>Energetics</u>:

- Now DKE / QL ?

$$\partial_t \langle E_{kin} \rangle + \frac{\partial}{\partial r} Q_e - \langle E_z J_z \rangle = 0$$

radial transport

and P-Thm:

$$\partial_t W_{DW} + \partial_r S_r + \langle E_z J_z \rangle_R = 0$$

radial wave energy density flux.

$$\Rightarrow \langle E_z J_z \rangle_R = \partial_t \langle E_{kin} \rangle_R + \partial_r Q_{e\,R}$$

$\therefore$

$$\boxed{\underbrace{\partial_t \langle E_{kin} \rangle_R + \partial_t W_{DW}}_{\text{RP + wave, as before}} + \partial_r \big(Q_{e,R} + S_r \big) = 0}$$

transport

inhomog wave transport.

n.b. $S_r \longleftrightarrow Q_{e,R}$

$S_r \sim V_{gr} \mathcal{E}_{DW}$. $\rightarrow$ <u>ZF generation</u>

- Dominant Balance:

$$\partial_t \langle E_{kin} \rangle_R + \partial_r Q_{e,R} = 0$$

N.B. show: $\underbrace{\Gamma_e}_{res} = \underbrace{\Gamma_i}_{NR}$

- at edge:

$$W_{DW} \rightarrow \mathcal{E}_{kin\ parts}$$

$$S_r \Rightarrow Q_{e,R}.$$

i.e. wave radiative losses significant.

$\Rightarrow ZF\downarrow$

c.s. $$\omega = \frac{k_\theta V_*}{1 + k_\perp^2 \rho_s^2}$$

$$V_{gr} = -\frac{2\boxed{k_r k_\theta}\rho_s^2 V_*}{(1 + k_\perp^2 \rho_s^2)^2}$$

$V_{gr}\mathcal{E} \rightarrow$ [wave energy density flux]

$\updownarrow$

Reynolds stress

→ PV Mixing & G-D Thms — < QLT for Fluid DW

Closely related to Vlasov Dynamics in β-Plane / QG

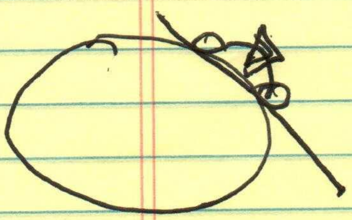
$$\frac{\partial}{\partial t}\left(\nabla^2\phi + \beta y\right) + \langle v\rangle\cdot\nabla\left(\nabla^2\phi + \beta y\right)$$

$$+\ \tilde{v}\cdot\nabla\left(\nabla^2\phi + \beta y\right) - \nu\nabla^2\left(\nabla^2\phi + \beta y\right) = 0$$

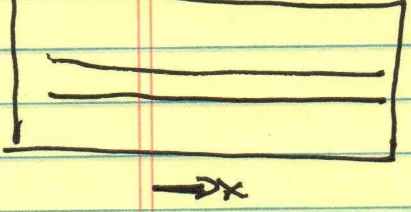
$$\partial_t \nabla^2\phi + \langle v\rangle_x \partial_x \nabla^2\phi + \tilde{\underline{v}}\cdot\nabla\nabla^2\phi$$

$$-\nu\nabla^2\nabla^2\phi = -\tilde{v}_y\left(\beta + \langle\nabla^2\phi\rangle'\right)$$

Mean vorticity



$$\zeta = \langle\zeta\rangle + \tilde{\zeta}$$



↑ y

→ x

obeys:

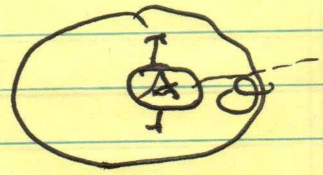
$$\partial_t \zeta + \underline{v}\cdot\nabla\zeta = +\nu\nabla^2\zeta$$

$$\int \underline{v} \cdot d\ell = \int \underline{\omega} \cdot da = \text{const.}$$

Kelvin's Thm.

$$\omega \to \underline{\omega} + 2\,\underline{\Omega}$$

$$C = \int d\underline{a} \cdot (\underline{\omega} + 2\underline{\Omega})$$

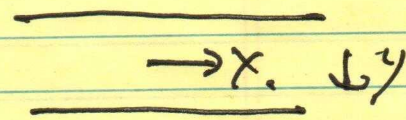


$$\dot{C} = 0 \Rightarrow$$

$$A \frac{d\omega}{dt} = -2\Omega \sin\theta_0 \frac{dA}{dt}$$

$$\frac{d\omega}{dt} = \frac{-2\Omega}{A} \sin\theta_0 \frac{dA}{dt}$$

$$= -2\Omega \frac{d\theta}{dt} \sin\theta_0$$



$$= -\frac{2\Omega}{R} \left(R\frac{d\theta}{dt} \right) \sin\theta_0$$

$$= -\beta\, V_y$$

$$\omega = \nabla^2 \phi \qquad \underline{V} = -\frac{\nabla p}{2\Omega} \times \hat{z}$$

$$q = PV = \nabla^2\phi + \beta y$$

fluid, local

planetary

potential vorticity

→ 2D conservative advection, according Hamiltonian system

$$\frac{dx}{dt} = -\nabla\phi \times \hat{z}$$

→ N.B.:

$$\frac{dq}{dt} = 0 +$$

$$q = \begin{cases} \nabla^2\phi & \to 2D \\ \nabla^2\phi + \beta y & \to QG \\ \ln\frac{n}{n_0} + \phi - \nabla^2\phi & \to HM \\ + \\ \vdots \end{cases}$$

⟺ obvious analogy to:

$$\frac{df}{dt} = 0 \quad , \quad \partial_t f + v\,\partial_x f + \frac{q}{m} E\,\partial_v f = 0$$

$$= C(f) \qquad H = \frac{mv^2}{2} + q\phi$$

with collisions

$$f = \langle f \rangle + \delta f$$

→ A key result: Taylor Identity

$$\langle v_y \nabla^2\phi\rangle_x = \langle \partial_x\phi \left(\partial_x^2\phi + \partial_y^2\phi\right)\rangle_x$$

$$= \langle \partial_x \cancel{\left(\frac{(\partial_x\phi)^2}{2}\right)}\rangle_x$$

$$+ \langle \partial_x\phi\, \partial_y^2\phi\rangle_x$$

$$= 0 + \partial_y \langle \partial_x\phi\, \partial_y\phi\rangle - \underbrace{\langle \partial_{xy}\phi\, \partial_y\phi\rangle}_{\to 0}$$

$$= \underbrace{\partial_y \langle \partial_x\phi\, \partial_y\phi\rangle_x}_{\text{R-S}} - \langle \partial_x \cancel{\left(\frac{(\partial_y\phi)^2}{2}\right)}\rangle_x$$

$$\to = -\partial_y \langle \tilde{v}_y \tilde{v}_x\rangle$$

vorticity (+ PV) flux = Reynolds Force.

N.B. Need 1 direction of symmetry, to eliminate odd moments.

→ key result: Reynolds force = vorticity flux.

→ Links Reynolds force to polarization charge flux i.e. A few observations:

H - W system → $(\underline{\nabla}\cdot\underline{V}=0)$

$$\frac{d}{dt}\nabla^2\phi = -D_{\parallel}\nabla_{\parallel}^2(\phi - n) + \nu\nabla^2\nabla^2\phi$$

$$\frac{d}{dt}n = +D_{\parallel}\nabla_{\parallel}^2(n-\phi) + D_0\nabla^2 n$$

includes $V_r \frac{d\langle n\rangle}{dr}$

n.b. $\tilde{T}_i$ complicates

i.e.

$$\frac{d}{dt}(n + \nabla^2\phi) = 0$$

$$PV = q = q_{GC} + q_{pol}$$

n → electron / GC density

$\nabla^2\phi$ → polarization charge (i.e. GC) (~ ion inertia → pol. drift off surface)

$$PV = \underbrace{n + \nabla^2\phi}$$ → conserved along orbits

Reynolds force → flux polarization charge.

Related:

often useful to think of flow via vorticity.

Vorticity $\leftrightarrow$ Polarization charge.

Thus, for zonal flows:

$$\underline{\nabla} \cdot \underline{J} = 0$$

$$\nabla_\perp \cdot \underline{J}_\perp + \nabla_\parallel J_\parallel + \nabla_\perp \cdot J_{PS} = 0$$

poloidal symmetry / $m=0$;

$$\partial_r \langle J_{r\,pol} \rangle + \partial_r \left\langle \frac{\tilde{B}_r}{B_0} \tilde{J}_\parallel \right\rangle = 0 \quad \left\{ \begin{matrix} \text{show} \\ \text{from} \\ \text{GK} \end{matrix} \right.$$

$$\Rightarrow \partial_t \langle \nabla_r^2 \phi \rangle = - \partial_r \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle + \partial_r \langle \tilde{b}_r \tilde{J}_{\parallel} \rangle$$

so

$$\underbrace{\partial_t \langle q \rangle}_{\text{total charge density}} = - \partial_r \underbrace{\langle \tilde{v}_r \tilde{\xi} \rangle}_{\text{charge flux}} + \partial_r \underbrace{\langle \tilde{b}_r \tilde{J}_\parallel \rangle}_{\text{inhomogeneous field line tilting}}$$

$\partial_r \langle \tilde{v}_r \tilde{q} \rangle \rightarrow$ 

$\partial_r \langle \tilde{b}_r \tilde{J}_\parallel \rangle \rightarrow$ 

and as $J_\parallel = -\nabla^2 A_\parallel$

so: $\langle \tilde{b}_r \tilde{J}_\parallel \rangle = \partial_r \langle \tilde{B}_r \tilde{B}_\theta \rangle$

Magnetic stress
(show).

$\Rightarrow$

$$\partial_t \langle \varepsilon \rangle = -\partial_r \left[\langle \tilde{v}_r \tilde{v}_\theta \rangle - \langle \tilde{B}_r \tilde{B}_\theta \rangle \right]$$

Key point/summary for zonal structure generated by turbulence:

Zonal Flow/Jet results from:

1) inhomogeneous PV mixing

+

2) 1 direction of symmetry

N.B. { Symmetry need not be equilibrium symmetry a priori

→ Taylor Identity enables:

Key Quasilinear Flow Theorem

→ $\boxed{\text{Charney - Drazin Thm.}}$

Now, PV conservation ⇒

$$\partial_t q + \underline{v}\cdot\nabla q = \nu\nabla^2 q - \mu q$$

scale independent damping

Then, for mean:

natural quantity of interest,

$$\partial_t \langle q\rangle = -\partial_y \langle \tilde{v}_y \tilde{q}\rangle + \cancel{\nu \partial_y^2 \langle q\rangle} - \mu \langle q\rangle$$

$\langle q\rangle$ slow.

so

$\langle \tilde{v}_y \tilde{q}\rangle$ is PV flux → key quantity of interest.

→ Now, could proceed w/ QLT, with linear response $\tilde{q}$, etc.

<u>on</u>:

→ examine $\langle \tilde{\zeta}^2 \rangle$ → potential enstrophy balance

<u>i.e.</u>

$$\partial_t \tilde{\zeta} + \underline{v} \cdot \nabla \tilde{\zeta} = -\tilde{v}_r \frac{d\langle \zeta \rangle}{dr} + \nu \nabla^2 \tilde{\zeta} - \mu \tilde{\zeta}$$

<u>so</u>

$$\partial_t \left\langle \frac{\tilde{\zeta}^2}{2} \right\rangle + \underline{\nabla} \cdot \left\langle \underline{\tilde{v}} \frac{\tilde{\zeta}^2}{2} \right\rangle = -\langle \tilde{v}_y \tilde{\zeta} \rangle \frac{d\langle \zeta \rangle}{dy} - \nu \langle (\nabla \tilde{\zeta})^2 \rangle$$

but have:

$$\partial_t \langle v_x \rangle + \mu \langle v_x \rangle = -\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle$$

↓ zonal flow

now, Taylor Identity:

$$-\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle = \langle \tilde{v}_y \nabla^2 \tilde{\phi} \rangle$$

$$= \langle \tilde{v}_y \tilde{\zeta} \rangle$$

as $\quad q = \beta y + \nabla^2\phi$

$$\tilde{q} = \nabla^2\tilde{\phi}$$

so

$$\partial_t \langle V_x\rangle + \mu\langle V_x\rangle = \langle \tilde{v}_y \tilde{q}\rangle$$

$\Rightarrow$

$$\partial_t \left\langle \frac{\tilde{q}^2}{2}\right\rangle + \nabla\cdot\left\langle v\frac{\tilde{q}^2}{2}\right\rangle = -\left[-\partial_y\langle \tilde{v}_y\tilde{v}_x\rangle\right]\frac{d\langle q\rangle}{dy}$$

$$= -\left[\partial_t\langle V_x\rangle + \mu\langle V_x\rangle\right]\frac{d\langle q\rangle}{dy}$$

Then for $\partial\langle q\rangle/\partial y$ slowly varying in space, time:

$$\boxed{\partial_t\left\{\frac{\langle\tilde{q}^2\rangle}{2\frac{d\langle q\rangle}{dy}} + \langle V_x\rangle\right\} + \nabla_y \frac{\langle v\tilde{q}^2/2\rangle}{d\langle q\rangle/dy} = -\mu\langle V_x\rangle}$$

→ Charney-Drazin Thm.

→ Consequence PE balance and Taylor identity.

→ Momentum Thm ($\sim$ QLT) for Rossby - Zonal interaction.

what does it mean?

$$\partial_t \left\{ \frac{\langle \tilde{q}^2 \rangle}{2\, d\langle \bar{q} \rangle / dy} + \langle V_x \rangle \right\} + \partial_y \frac{\langle \tilde{V}_y \tilde{q}^2 / 2 \rangle}{d\langle \bar{q} \rangle / dy}$$

$$= -\mu \langle V_x \rangle$$

① $\dfrac{\langle \tilde{q}^2 \rangle}{2 \dfrac{d\langle \bar{q} \rangle}{dy}}$? ($\rho_0 = 1$)

$$q = \nabla^2 \phi + \beta y$$

$$\tilde{q} = \nabla^2 \tilde{\phi}$$

$$① = \sum_k \frac{(k^2)^2 |\phi_k|^2}{2\, d\langle \bar{q} \rangle / dy}$$

$$= -\sum_k - \frac{k^2 |\phi_k|^2}{2 \frac{k_x}{k^2} d\langle \bar{q} \rangle / dy} \quad k_x$$

Wave action density

$$① = - \sum_{k} k_x N_k$$

→ Quasi-particle density

$$N_k = \mathcal{E}_k / \omega_k \qquad \omega = \frac{-k_x \beta}{k^2}$$

Wave action density — Wave energy density

Thus

$$① = -WMD \equiv - \text{Pseudomomentum } (-PM)$$

$\Leftrightarrow$ wave activity density

$\lim_{\phi \to \text{small}}$ Pseudomomentum = WMD

and:

$$③ = \left\langle \tilde{V}_y \frac{\tilde{q}^2}{2 \frac{d\langle q \rangle}{dy}} \right\rangle$$

$$= + \langle \tilde{V}_y (-\tilde{PM}) \rangle$$

⇒ obviously related to turbulence spreading → beyond QLT.

so

QQ

$$\partial_t \left\{ \langle v_x \rangle - k_x N \right\} - \partial_y \langle \tilde{v}_y (\widetilde{WMD}) \rangle$$

$$= - \mu \langle v_x \rangle$$

So:

→ up to ~~spreading~~ drag, flow locked to wave momentum density, i.e.

$$\partial_t \langle v_x \rangle = \partial_t \left(\sum_k k_x N_k \right)$$

change wave amplitude ⟺ change in flow

→ no-slip theorem for fluid relative to inter-penetrating fluid of quasi-particles.

→ spreading term → linked to fluctuation envelope scale → can break local, but not global, no-slip. [Phases ?]

HW → generalize for $D, r \equiv 0$
Hasegawa - Wakatani system

Theoretical Observations:

① – theorem is general, with no truncation, though leaves enstrophy flux $\langle \nabla \tilde{q}^2 \rangle$ un-calculated

② – no linear response closure, like QL, used

③ – WMD defined beyond level of small amplitude perturbation

①–③ enabled by $\left\{ \begin{array}{l} \text{PV conservation} \rightarrow \text{enstrophy balance} \\ \text{Taylor identity} \end{array} \right.$

contrast to:

$$\underbrace{\partial_t \{ RPMD + WMD \}}_{} = 0$$