PKU-HUST Lecture V

→ Nonlinear Waves II

shocks
collisionless shocks

⇒ Why?

~ Why interested in pulse dynamics, fronts etc.?

⇒ Paradigms for Avalanching, SOC (Self-Organized Criticality) Phenomena.

~ in particular: Burgers equation, Burgers turbulence are useful continuum models of Avalanching turbulence

~ Key element in Burgers ⇒

$\boxed{\text{shock}}$

Hence - discussion ...

SOC, Avalanches

"When something is trivial but interesting, I call it physics."

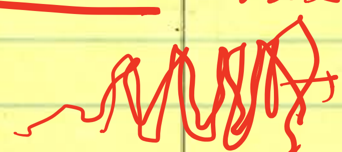
— Phil Nelson

So now;

- Basic Ideas of SOC
- Development of the Theory / Ideas
- $\boxed{\text{Hydrodynamic Models} \quad *}$
- $\boxed{\text{Burgers Turbulence}}$
  - ⇒ pulses
  - ⇒ infrared scaling.

→ Avalanches ⟷ L-H Transition

noise



Lothar Schmitz – APS

n°1

A Brief Intellectual History of 'SOC'

Physics 235 website spring 2019

- Storylines

I) $(\delta B)^2 \sim B f^{2H}$

math

II)

Hydrology

Hydrology
Characterizing Time Series

(50's) →

'Concentrated' pdf,
Intermittency
Multiplicative Processes

Answer

Random Process not difficult

H, Hurst and Holder

Lognormality,
Pareto-Levy Distributions

fractals are unifying theme

Intermittency
Fractals, Self-similarity

MW '68

(70's)

(80's)

1/f noise "universal"
→ associated with $H \to 1$

1/f Noise

SOC

BTW '87

(physical system realizing 1/f noise)

$$\frac{du}{dt} + \frac{\beta}{\tau} u = \tilde{f}$$

ㅁ1

$$\frac{dn}{dt} = (\delta_0 + \tilde{F})n - \alpha n^2 + \tilde{F}$$

- Lognormal $\leftrightarrow$ Zipf $\leftrightarrow$ 1/f related

i.e. ↳ Multiplicative Process. CLT for logs.

Zipf's Law 1949

- $$P\left(\frac{x}{\bar{x}}\right) = P(\log x)\frac{d\log x}{dx} = g\left(\frac{x}{\bar{x}}\right) d\left(\frac{x}{\bar{x}}\right)$$

↑

Probability

$x/\bar{x}$ lies in $d(x/\bar{x})$ at $x/\bar{x}$

$\log(g) = -\log f$ + variance corrections

$$\boxed{f = 1/(x/\bar{x})}$$

$P \sim c/\Delta x$

- Lognormal well approximated by power law $P \sim \frac{1}{x}$ (Zipf's law), over finite range! (Montroll '82)
- Multiplicative processes related to Zipf's law trend
- Link to 1/f noise?

L. Qi $\rightarrow$ $1/f^{\alpha}$ $\alpha \lesssim 1$

Events Prob - mult.

#1 $1/\Delta \Leftrightarrow 1/f$ shot

- 1/f Noise?

A few observations:

– Zipf and 1/f related but <u>different</u>

Zipf → $P(\Delta B) \sim 1/|\Delta B|$

$1/f$ → $\langle(\Delta B)^2\rangle_\omega \sim 1\,/\,\omega$

Both embody:

- Self-similarity → power law
- Large events rare, small events frequent → intermittency phenomena
- 1/f linked to H→1

– 1/f noise (flickers, shot...)

- Ubiquitous, suggests 'universality' → why ??
- Poorly understood, circa 80's

ii)

- N.B.: Not easy to get 1/f ...
- In usual approach to ω spectrum; ←→ (DIA, EDQNM, Dupree, Kadomtsev, Kraichnan, Krommes):

$$\langle \phi(t_1)\phi(t_2)\rangle = |\hat{\phi}|^2 e^{-|\tau|/\tau_c}$$

$1/\tau_c = ?$

$\forall\, \omega$

slow

$$\rightarrow S(\omega) = \boxed{\frac{1/\tau_c}{\omega^2 + 1/\tau_c^2}} \sim \boxed{\frac{1}{\omega^2}}$$

$\neq 1/\omega$

$1/f$

$\int dx = \int v \cdot \chi\, t$

i.e. τ_c imposes scale, but 1/f scale free !?

$1/\tau_{c_{k,\omega}} \rightarrow k^2 D_{k,\omega}$

- N.B.: Conserved order parameter may restore scale invariance
- But, consider ensemble of random processes each with own τ_c (Montroll, BTW)

Rev.:
TSH, P.D.
2018

$$S(\omega)_{eff} = \int_{\tau_{c1}}^{\tau_{c2}} \boxed{P(\tau_c)}\, \boxed{S_{\tau_c}(\omega)}\, d\tau_c$$

Probability of τ_c

- And... demand $P(\tau_c)$ scale invariant, i.e.

$$P(\tau_c) = d\tau_c/\tau_c$$

$$S(\omega) = \left. \frac{\tan^{-1}(\omega\tau_c)}{\omega} \right|_{\tau_{c1}}^{\tau_{c2}} \sim \boxed{1/\omega,}$$ recovers 1/f !

→ but what does it mean? ...

- So, circa mid 80's, need a simple, intuitive model which:
  (large events)
  – Captures 'Noah', 'Joseph' effects in non-Brownian random process (H→1) (on/off cycles)
  – Display 1/f noise

Bible Flood Noah } Conf. Classics Flood Da Yu

Bespoke

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SOC at last !

- Enter BTW '87:

Self-Organized Criticality: An Explanation of $1/f$ Noise (7000+ cites)

Per Bak, Chao Tang, and Kurt Wiesenfeld

Physics Department, Brookhaven National Laboratory, Upton, New York 11973

(Received 13 March 1987)

We show that dynamical systems with spatial degrees of freedom naturally evolve into a self-organized critical point. Flicker noise, or $1/f$ noise, can be identified with the dynamics of the critical state. This picture also yields insight into the origin of fractal objects.

Very well written paper.

- Key elements:
  - Motivated by ubiquity and challenge of 1/f noise (scale invariant)
  - Spatially extended excitations (avalanches) *

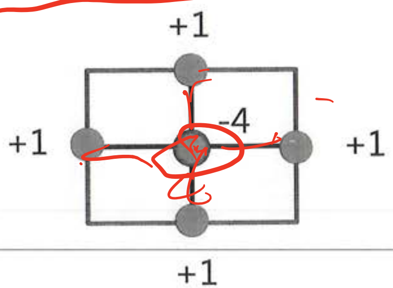
    Comment: statistical ensemble of collective excitations/avalanches is intrinsic

    Ensemble avg.
  - Evolve to 'self-organized critical structures of states which are barely stable'

    Comment: SOC state ≠ linearly marginal state!

    SOC state is dynamic

- Avalanches and Clusters:
  - BTW – 2D CA model



$Z \equiv$ occupation

$Z > Z_{crit} = K$

$Z(x, y) \to Z(x, y) - 4$

$Z(x \pm 1, y) \to Z(x \pm 1, y) + 1$

$Z(x, y \pm 1) \to Z(x, y \pm 1) + 1$

Perc trans.

avalanche – avalanche

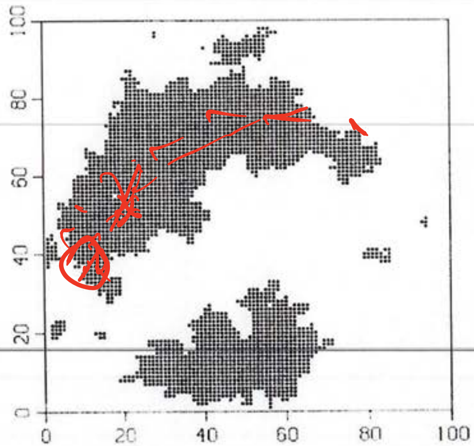


FIG. 1. Self-organized critical state of minimally stable clusters, for a 100×100 array.

  - SOC state with minimally stable clusters
  - 'Cluster' ≡ set of points reached from toppling of single site (akin percolation)
  - Cluster size distribution $D(s) \sim s^{-\alpha}$, $\alpha \sim 0.98$

➔ Zipf, again

10

1/5

- Key elements, cont'd:
  - "The combination of dynamical minimal stability and spatial scaling leads to a power law for temporal fluctuations"
  - "Noise propagates through the scaling clusters by means of a "domino" effect upsetting the minimally stable states" * [avalanche]

    Comment: space-time propagation of avalanching events *
  - "The critical point in the dynamical systems studied here is an attractor reached by starting far from equilibrium: ~~the scaling properties of the model"~~

    Comment: Noise essential to probe dynamic state *

N.B.: BTW is example of well-written PRL

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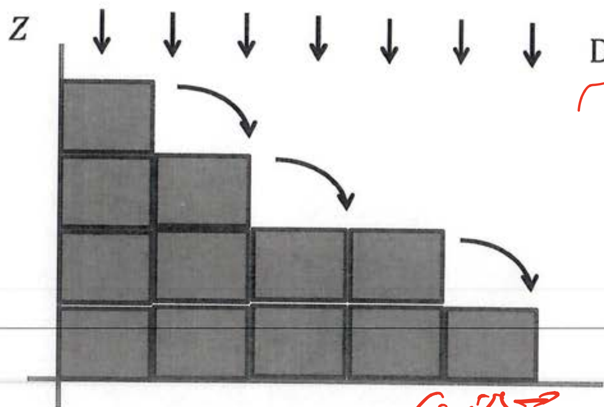
- The Classic – Kadanoff et al '89 1D driven lossy CA



Deposition → random, can profile

If $\left[\begin{array}{l} Z_i - Z_{i+1} > \Delta Z_{crt} \\ Z_{i+1} \rightarrow Z_{i+1} + N \\ Z_i \rightarrow Z_i - N \end{array}\right.$

Etc.

Hard ← L → Lossy bndry

Grains ejected at boundary

Why of interest for MFE?

- Interesting dynamics only if $L/\Delta \cdot N \gg 1$ ↔ equivalent to $\rho_* \ll 1$ condition – analogy with turbulent transport obvious

TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains — location
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

What is SOC?

General Thoughts

KSTAR

M. Choi

ECFT

(cf: Jensen)

Avalanches

- (Constructive)

⇒ Slowly driven, interaction dominated threshold system

Classic example: sandpile

- (Phenomenological)

⇒ System exhibiting power law scaling without tuning.

contrast $T \to T_c$ etc

Special note: 1/f noise; flicker shot noise of special interest

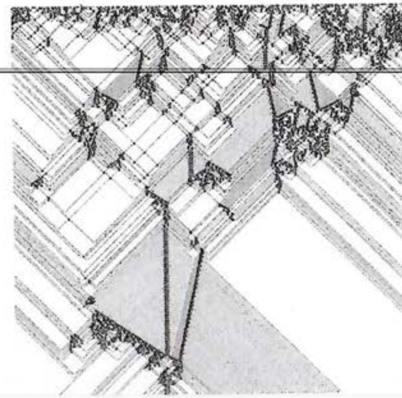
See also: sandpile

THT

N.B.: $1/f$ means $1/f^{\beta}$, $\beta \leq 1$

Candy → Fractal

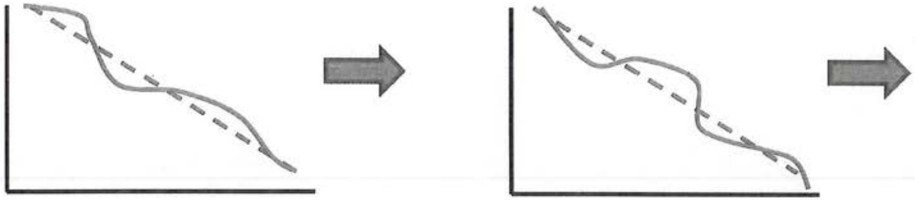
What is SOC?, cont'd

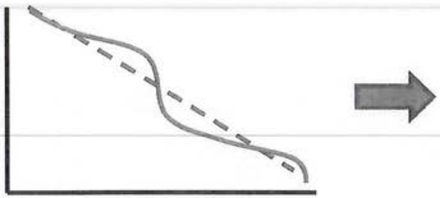


- Elements:
  - → Interaction dominated
    - Many d-o-fs { Cells / Modes }
    - Dynamics dominated by d-o-f interaction i.e. couplings
  - → Threshold and slow drive
    - Local criterion for excitation
    - Large number of accessible meta-stable, quasi-static configuration
    - 'Local rigidity' ←→ "stiffness" !?

profile stiffness

- Multiple, metastable states

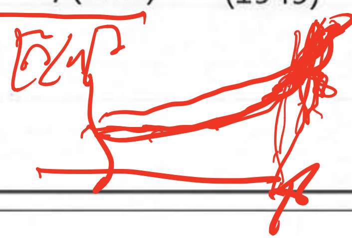


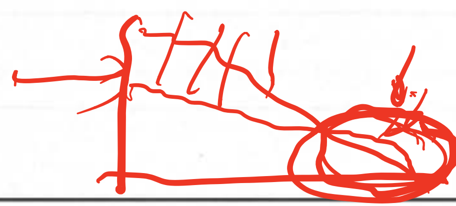


- Proximity to a 'SOC' state → local rigidity
- Unresolved: precise relation of 'SOC' state to marginal state

- Threshold and slow drive, cont'd
  - Slow drive uncovers threshold, metastability
  - Strong drive buries threshold – does not allow relaxation between metastable configurations
  - How strong is 'strong'? – set by toppling/mixing rules, box size, b.c. etc.

- Power law ←→ self-similarity
  - 'SOC' intimately related to:
    - Zipf's law: P(event) ~ 1/(size) (1949)
    - 1/f noise: S(f) ~ 1/f





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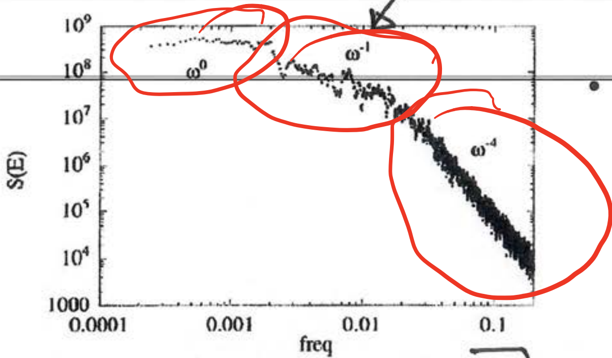
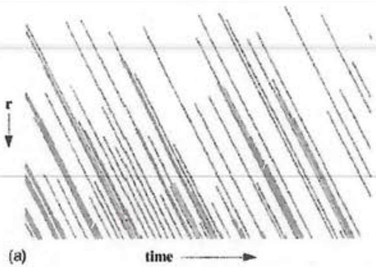
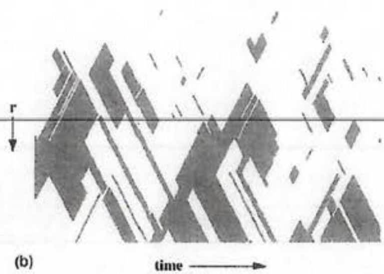
1/f Generic structure — spectra.



Power spectrum of over-turnings $\langle(\Delta Z)^2\rangle_\omega$





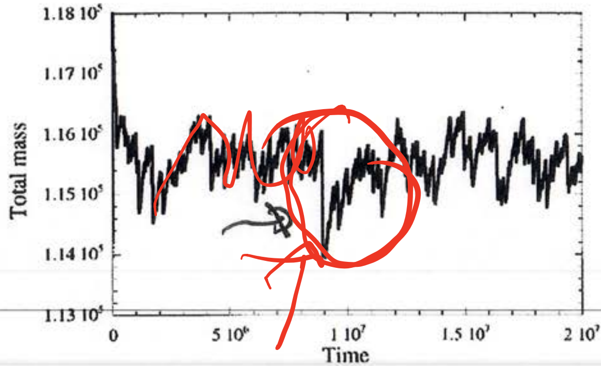
Avalanching

dark → over-turning
light → stable

- Some generic results
  - 1/f range manifest
  - Large power in slowest, lowest frequencies
  - Loosely, 3 ranges:
    - ω^0 → 'Noah'
    - 1/f → self-similar, interaction dominated ("Joseph")
    - $1/f^4$ → self correlation dominated
    - Space-time → distribution of avalanche sizes evident

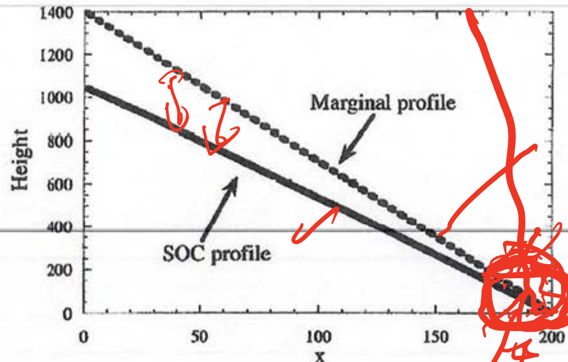
→ Outward, inward avalanching …

- **Global Structure**



- Time history of total grain content
- Infrequent, large discharge events evident

SOC vs Marginal?



with bi-stable

- SOC $\neq$ Marginal
- SOC $\rightarrow$ marginal at boundary
- Increasing N_{dep} $\rightarrow$ SOC exceeds marginal at boundary
- Transport bifurcation if bi-stable rule
- Simple argument for L-H at edge

- An Important Connection Hwa, Kardar '92; P.D., T.S.H. '95; et seq.
  - 'SOC' intimately connected to self-similarity, 'cascade' etc ultimately rooted in fluid turbulence – relate?

avalanche → cascading

And:

  - C in 'SOC' → criticality
  - Textbook paradigm of criticality (tunable) is ferromagnetic ala' Ginzburg, Landau → symmetry principle!?

$$\frac{dM}{dt} - D\nabla^2 M = -(T-T_c)aM - bM^3$$

And:

  - Seek hydro model for MFE connections

$$H = D(\nabla M)^2 + (T-T_c)a\frac{M^2}{2} + b\frac{M^4}{4}$$

MFE? → $\boxed{\text{PDE}}$

→ Fluid continuum model

→ NL waves

1a

Avalanches and Self-Organized Criticality II → Intro to Hydrodynamic Models (Avalanche Turbulence)

(aka "Flipping Burgers")

see: - FNS △
- Hwa and Kardar
- Gil + Sornette

Recall

→ SOC idea
→ Sandpile Model (CA)

Now, natural to ask:

Analogy SOC - Turbulence.

- is there a continuum model, as $l_{avalanche} \gg \Delta$? → akin granular flow

slow sand

Can one think in terms of avalanche turbulence

criticality — Ginzburg - Landau

- can one exploit symmetry in deriving SOC model, much as symmetry exploited in Ginzburg - Landau model
ie $M \to -M$ symmetry → even terms.

These bring us to the hydrodynamic theory / model of SOC

↳ N2 wave model

→ continuum model

→ valid for large scales, long time scales.

$L \gg l_0 > \Delta$

$\tau_{conf} \gg T \gg \tau_{tr}$

steps

Choice → FSTAR

w/ zero-noise edge symm. NE

3

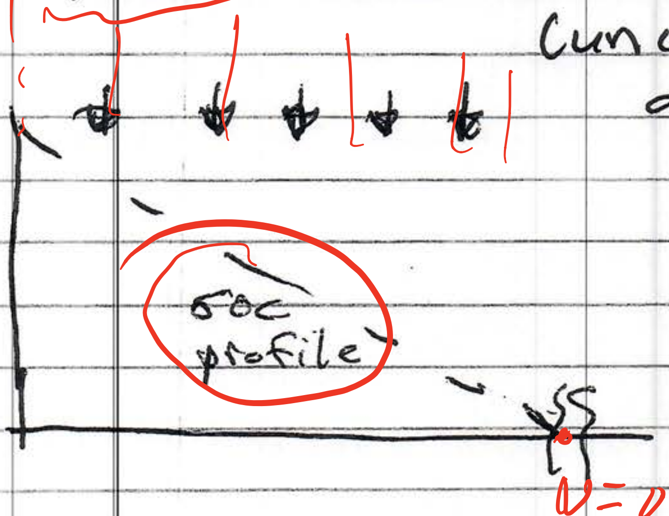
→ Basics

Consider:

→ box with ejecting bndry on RHS, accumulating bndry on LHS

→ SOC profile, TBD n.b. what is SOC profile?

→ noise → random rain (uncorrelated, correlated drops irrelevant.)



→ Now, consider deviations from SOC profiles, i.e.

→ bumps, "blobs"

→ voids, "holes"

(no self-binding mechanism).

3

Conservative Process

→ Also assume conservation of "stuff" in the profile up to boundary losses and noise source. Call stuff ρ.
(could represent pressure)

→ Idea is to describe dynamics of deviation from SOC state

i.e. $\rho = \rho_{soc} + \delta\rho$

ρ_{soc}: formally posited, not calculated.

→ but evolve only deviation
→ only small deviation theory

∴ have

$$\boxed{\partial_t \delta\rho + \partial_x \left[\Gamma(\delta\rho) - D_0 \partial_x \delta\rho \right] = S}$$

- $\Gamma(\delta\rho)$ is flux induced by deviation from SOC state
- obviously, ρ conserved so $\delta\rho$ evolves via $\nabla \cdot \Gamma$ only
- background diffusion posited.
- can generalize to higher dimensions. See Hwa + Kardar.

— $\Gamma(\delta P) \to 0$ as $\delta P \to 0$

$\delta P \to 0$ as $\tilde{S} \to 0$.

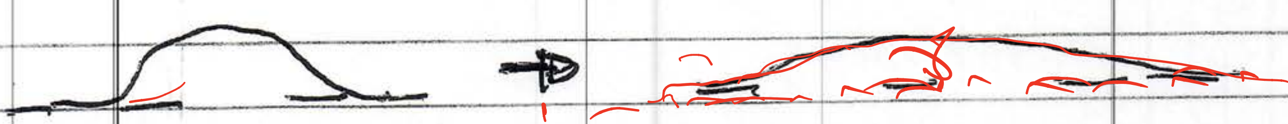
— How constrain $\Gamma(\delta P)$? → $\boxed{\text{Symmetry}}$!

in spirit of Ginzburg / Landau prescription.

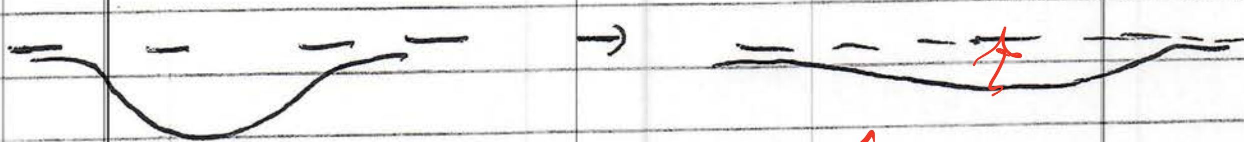
$H(m)$

Now, consider:

level

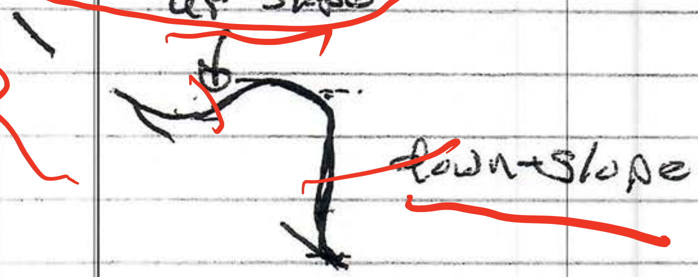


blob spreads out, conserving area.



likewise void.

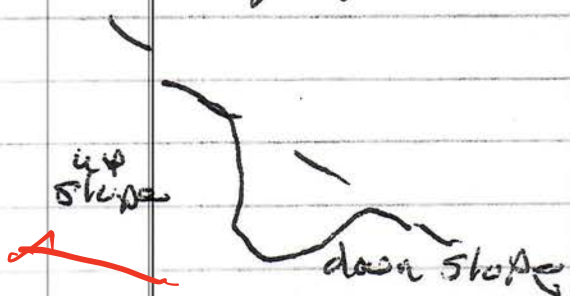
Now, if left-right symmetry broken by $\nabla p_{soc} \neq 0$ (instantaneous up slope



bumps →
greater (extent, steepness) on down-slope

⇒ bumps / local excesses propagate down gradient, to right

Necessarily;



void →
greater (extent, steepness) on up-slope than down slope

⇒ voids / local deficits propagate up gradient, to left.

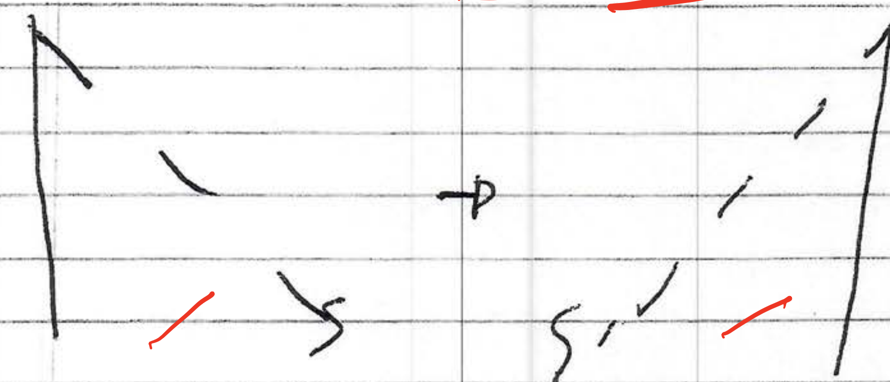
→ Both criteria local,

→ Both criteria common sense.

Now, observe:

① reflection $x \to -x$

i.e.



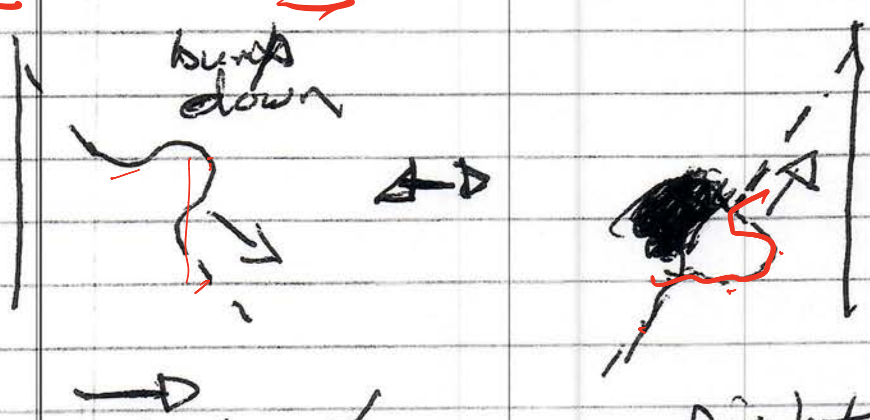
2 changes

$x \to -x$

$\delta p \to -\delta p$

and

② bump ↔ hole interchange



void up

→ right ✓ right → ✓

same flux direction!

Extended HW: 2 channels

+ up-grad. one channel relaxation

→ This brings us to the principle of joint reflection symmetry!

$$\Gamma\Big|_{\substack{x\to -x \\ \delta p \to -\delta p}} = \Gamma$$

constraint

[akin $\nabla\cdot\underline{\Gamma}$ invariant under direction flip]

This constrains the form of $\Gamma(\delta p)$!

$\tau_{step} < \tau < \tau_{conf}$

How?

$\Delta < \ell < L$

N.B.: - Full flux is complicated.

- seek flux in large scale, long time limit ⇒ smoothest form

interested in long time, large scale > biggest non-trivial

low ω, low k

so have

$$\partial_t \delta p + \partial_x\left[\Gamma(\delta p) - D_0 \partial_x \delta p\right] = \tilde{S}$$

$\Gamma(\delta p)$ must satisfy joint reflection symmetry.

Then formally:

$$\Gamma(\delta P) = \sum_{\substack{m,n,\\ \xi,r,\alpha}} \Big\{ A_n (\delta\rho)^n + B_m (\partial_x \delta\rho)^m + D_\alpha (\partial_x^2 \delta\rho)^\alpha + C_{\xi,r} (\delta\rho)^\xi (\partial_x \delta\rho)^r + \Big\}$$

① ② ③ ④

JRS ≡ joint reflection symmetry

① $n=1$ violates JRS

$① \cong \alpha\, \delta\rho^2 + h.o.t.$

$\alpha > 0$

② $m=1$ OK

$m=2$ OK

$m=2$

diffusion

$② \cong -D\, \partial_x \delta\rho + \boxed{h.o.t.}$

$D > 0$ (well behaved)

③ $\alpha = 1$ violates JRS

$\alpha = 2$ too fine scaled,

ignore..

(4) $q=1$, $r=1$ violates JRS

so, to lowest order in roughness:

$$\boxed{\partial_t \delta P + \partial_x\left[\alpha\, \delta P^2 - D\, \partial_x \delta P\right] - D_0\, \partial_x \delta P = S}$$

α, D are constants to be specified, as a, b in σ-L theory are

Burgers → shock

Re-absorb D_0 into D:

$$\boxed{\partial_t \delta P + \partial_x\left[\alpha\, \delta P^2 - D\, \partial_x \delta P\right] = \tilde{S}}$$

Burgers eqn

$\partial_t V + V \partial_x V - \nu \partial_x V = \tilde{S}$

- hydro model limit is noisy Burgers $\partial_t V + V \partial_x V - \nu \partial_x^2 V = \tilde{S}$
- exactly solvable for $\tilde{S} = 0$
- basic solution structure is shocks! (shocks produce entropy)

- shocks ahin avalanches, ...
  shock trains

Now, seek long wavelength approximation to nonlinear flux

i.e. $\left[\partial_x(\alpha\,\delta P^2)\right]_k \rightarrow d_k\,\delta P_k$

$\cong \chi k^2\,\delta P_k$

turbulent viscosity

$\Delta(\nabla P) \sim \alpha\,\delta P\,\delta P$

$\frac{1}{L_p}$ $\alpha\,\delta P^2$

$\delta P \sim \nabla P\,\Delta x$

N.B. $\alpha\,\delta P^2 \longleftrightarrow D_T(\nabla P)\,\delta P$

$D(\nabla P)$

δP

critical gradient

Threshold

$\longrightarrow -D(\nabla P - \nabla P_{crit})$ $\nabla\delta P$ P_{crit}

"critical gradient"

δP

$I \sim \delta P$

DRS

[clear correspondence to expected QL expression for flux, with threshold.

Threshold

i.e. $\delta P^2 \rightarrow -D(\nabla P - \nabla P_{crit})\nabla\delta P$

Now,

$$N_{\substack{k\\ \omega}} = \left[\partial_x(\alpha\, \delta\rho^2)\right]_{k,\omega} \xrightarrow{\text{renormalized NL}} \nu k^2 \delta\rho_{\substack{k\\ \omega}}$$

$$= ik\alpha \sum_{k',\omega'} \delta\rho_{\substack{-k'\\ -\omega'}}\, \delta\rho_{\substack{k+k'\\ \omega+\omega'}}$$

on spirit QL.

$$\cong ik\alpha \sum_{k',\omega'} \delta\rho_{\substack{-k'\\ -\omega}}\, \delta\rho^{(2)}_{\substack{k+k'\\ \omega+\omega'}}$$

where:

nonlinear scrambling of coupling time

$$\left[-i(\omega+\omega') + (k+k')^2 D_0 + (k+k')^2 \nu_{k}\right] \delta\rho^{(2)}$$

$$= -i\alpha(k+k')\, \delta\rho_{\substack{k'\\ \omega'}}\, \delta\rho_{\substack{k\\ \omega}}$$

and substituting gives:

$$\boxed{N_{k,\omega} = \nu k^2 \delta\rho_{k\omega}}$$

where:

strong turb → ν_T, D_T

WTT → $\delta(\;)$, ν_{coll} ...

for $k, \omega \to 0$; $\left\{ \begin{array}{l} \text{long, smooth} \\ \text{slow} \quad \text{light} \end{array} \right.$

$$\nu_T \cong \sum_{k',\omega'} |\delta P_{k',\omega'}|^2 \frac{k'^2 \nu_T}{\left[\omega'^2 + (k'^2 \nu_T)^2\right]}$$

where neglected ν_0 relative to ν_T.
Note recursive defn ν: ν_T, ν_T, scale

Now, need related $\delta P_{k',\omega'}$ to noise

(i.e. k', ω' → high freq, short wavelength

modes excited). This must also

include nonlinear response, self-consistently

so   drop sub T

$$\left(-i\omega' + k'^2 \nu\right) \delta P_{k',\omega'} = \tilde{S}_{k',\omega'}$$

so

$$\nu = \alpha^2 \sum_{k',\omega'} \frac{|\tilde{S}_{k',\omega'}|^2}{(k'^2\nu)^3} \frac{1}{\left[1+(\omega'/\nu k'^2)^2\right]^2}$$

$\emptyset$ $\emptyset$

$$\sum_{k',\omega'} = \int_{k_{min}}^{\infty} dk' \int d\omega'$$

'noise color' in space time significant.

and

$$|\tilde{S}_{k,\omega}|^2 = S_0'^2 \Rightarrow \text{white noise}$$

conserved order param.

$$\frac{\partial \delta p}{\partial t} = -\nabla \cdot \Gamma$$

$\Rightarrow$

const

$$\nu = \frac{c_1 \alpha^2 S_0^2}{\nu^2} \int_{k_{min}}^{\infty} \frac{dk'}{k'^4} \sim 1/k_{min}^3$$

$k^2 \nu \neq$

$\emptyset$

$\rightarrow$ infrared divergence?!

$\Rightarrow$ why? $\Rightarrow$ – conserved order parameter (flux form) $\partial_x \Gamma$

– slow modes $1/\tau_c \sim k^2 \nu \rightarrow 0$.

criticality

$\boxed{\text{soft modes}}$

$\sim k^3$

slow modes → damping drops

$$\gamma \sim -k^2 \nu$$

$$\downarrow$$

$$\to 0 \text{ as } k \to 0$$

weak noise + tiny decay ⇒ strong intensity

weakly damped modes

⇒ general point: weakly damped modes dangerous if any excitation available.

$\Lambda = 0$

So

$$\nu_T = \left(C_1 \alpha^2 S_0^2 \int_{k_{min}}^{\infty} \frac{dk'}{k'^4} \right)^{1/3}$$

$$\simeq \left(C_1 \alpha^2 S_0^2 \right)^{1/3} k_{min}^{-1}$$

$$\boxed{\Rightarrow \nu_T \text{ depends explicitly on cut-off scale.}}$$

Now meaning? → [what is physics message of inferred divergence?]

$\boxed{k_{min}^{-1} \equiv \delta l}$ → scale being observed

$\delta l' \sim \delta l$ → scatterers

so

$v_T \sim v_{T,0}\, \delta l$ {v_T grows with scale of interest}

but v_T is diffusion ⇒

$\frac{d}{dt}\langle \delta l^2 \rangle \sim v_T$, but

$\boxed{\text{ballistic}}$

$\delta l^2 \sim v_{T_0}\, \delta l\, t$

⇒ $\boxed{\delta l \sim v_{T,0}\, t}$ ⇒ δl pulse propagates ballistically, not diffusively.

$\boxed{\text{pulse}}$.

→ infrared divergence ultimately identifies uncovers ballistic propagation

⟺ conserved order param. is key

→ supported by scaling analysis

see { Hwa $\rightarrow$
FNS $\rightarrow$

→ if 2D, anisotropic pile:

$$\partial_t \delta P + \partial_{\parallel}\underbrace{\left\{ \alpha\, \delta P^2 - D\,\partial_{\parallel} \delta P \right\}}_{\text{Cons. Order Param}} = \cancel{\mp}\, \nu_0 \nabla_\perp^2 \delta P + \tilde{S}$$

$$\partial_{\parallel} = \frac{\nabla P}{|\nabla P|} \cdot \nabla \quad \rightarrow \text{derivative parallel to pile gradient of surface.}$$

Avalanche SOC → Hydro limit model — Burgers → NL waves — Anl. $\nu_T \sim k_{min}^{-1}$, $\boxed{\delta\ell \sim t}$

see refs for more.

see also: $\boxed{\text{Gil + Sornette}}$

I and P eqn.

aggregation

- More on Burgers/hydro model (mesoscale)
  - Akin threshold scattering
  - $V \sim \alpha\, \delta P$ relation → bigger perturbations, faster, over-take
  - Extendable to higher dimensions
  - Cannot predict SOC state, only describe dynamics about it. And α, D to be specified
  - $\langle \delta P \rangle$? → corrugation (!?)
  - Introducing delay time → traffic jams, flood waves, etc (c.f. Whitham; Kosuga et al '12)

$\partial_t \delta P + \alpha\, \delta P \partial_x \delta P$ . . .

$\partial_t v + v \partial_x v$

jams

ने।

- Avalanche Turbulence

  – Statistical understanding of nonlinear dynamics → renormalization

  – Conserved order parameter

  $\partial_x (\alpha \delta P^2) \rightarrow \nu_T k^2 \delta P_k$

  $$\nu_T \approx \left(\alpha^2 S_0^2 \int_{k_{min}}^{0} dk/k^4\right)^{1/3} \rightarrow (\alpha^2 S_0^2)^{1/3} k_{min}^{-1}$$

  $$\sim (\alpha^2 S_0^2)(\delta l)$$

  Infrared divergence
  due slow relaxation

  – $(\delta l)^2 \sim \nu_T \delta t \;\; \rightarrow \;\; \delta l \sim \delta t$

    - H → 1
    - 'Ballistic' scaling

- Infrared trends ←→ non-diffusive scaling, recover self-similarity
- Amenable to more general analyses using scaling, RG theory
- Pivotal element of 'SOC' theory as connects 'SOC' world to turbulence world, and enables continuum analysis

$$\partial_t \delta P + \partial_x U \delta P + \cdots$$

$\boxed{\text{Aggregation}}$

Dis. - P.

Tok

α

$\boxed{\alpha[X]}$ δP

$(1/L_T - 1/L_{crit})$

Stiffness

$V \sim \delta \epsilon \, f$

$$Q = -\chi \left(\boxed{1/L_T - 1/L_{T,crit}} \ldots \right) \nabla T$$

$D, \mathcal{C}$ = ρ τ

$\chi = \chi_0 \, \Theta \left(1/L_T - 1/L_{crit} \right)$

Threshold
+
radial dressing