Physics 218C

Lecture 6: Confinement Transitions, especially Predator - Prey System and L→H Transition

a.) Predator-Prey ↔ Drift Wave - Zonal Flow System.

Recall derived the coupled equations for shear flow and turbulence:

WKE for DW Action Density:

$$\frac{\partial \langle N \rangle}{\partial t} - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma \langle N \rangle - \frac{\Delta \omega_k}{N_0} \langle N \rangle^2$$

or equivalently, in terms energy

$$\frac{\partial \langle \mathcal{E} \rangle}{\partial t} = -\int d^3k \left( \frac{\partial \omega_k}{\partial k_r} \right) \underbrace{D_k}_{\to\, O(\langle \tilde{V}'^2 \rangle)} \frac{\partial \langle N \rangle}{\partial k_r}$$

$$+ \int d^3k \; \omega \underbrace{\langle C(N) \rangle}_{\gamma \langle \mathcal{E} \rangle - \frac{1}{\tau_{NL}} \langle \mathcal{E} \rangle^2} + \text{S.T.}$$

and, from Reynolds stress:

$$\partial_t |\phi_z|^2 = \underbrace{\Gamma_z \left[ \frac{\partial \langle N \rangle}{\partial k_r} \right] |\phi_z|^2}_{\text{ZF growth}} - \underbrace{\gamma_d |\phi_z|^2}_{\text{ZF drag}}$$

$$\Gamma_z |\phi_z|^2 \simeq +\langle \tilde{V}'^2 \rangle$$

Energy conservation is straight forward!

Show this:

$$\partial_t \left[ \int d^3k\, \omega \langle \mathcal{E} \rangle + \sum_z \frac{1}{2} \rho_d |\tilde{V}_z|^2 \right] = 0$$

akin to energetics in QLT.

$+ \gamma, \Delta\omega, \gamma_d$ terms.

DW ⟺ "particles"

ZF ⟺ "waves" / "Fields"

$\Rightarrow$ Coupled system for DW spectrum and ZF spectral intensity.

# Feedback Loops I

- Closing the loop of shearing and Reynolds work


- Spectral 'Predator-Prey' equations

Prey $\rightarrow$ Drift waves, $<N>$

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k\langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator $\rightarrow$ Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

→ Some FAQ's

- What of Geometry ?

What sets $\underline{J}_{pol}$ ?

- FLR $\rightarrow \rho_s^2 \nabla_\perp^2 \phi^2$

or $\downarrow$ $\downarrow$ torus

- Drifts + Particle Trapping

Length scale:

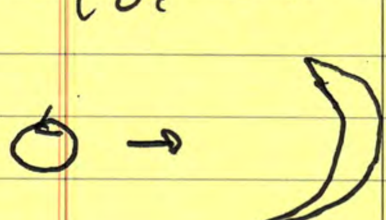
$$\delta r \sim \underbrace{V_D}_{\text{drift velocity}} \underbrace{\tau_b}_{\text{bounce time}}$$

$$\sim \frac{\rho_i v_{Ti}}{R} \frac{R\epsilon}{v_{Ti}\sqrt{\epsilon}}$$

$$\sim \sqrt{\epsilon}\, \rho_{\theta i}$$

→ poloidal gyro-radius

i.e. $\rho_{\theta i}$ sets polarization screening length



$\rho_{\theta i} \gg \rho_i$

→ enhanced screening length and inertia

For full analysis, see Rosenbluth and Hinton, 98.

→ What seeds / triggers Z.F.?

Answer: Nonlinear noise!

i.e. Can write:

$$\frac{\partial}{\partial t}\left(\nabla_\perp^2 \phi\right)_{\underline{q}} \sim -\left(\tilde{V}\cdot\nabla\nabla^2\tilde{\phi}\right)_{\underline{q}}$$

beats of DW at $\underline{k}$

$\tilde{V}_{\underline{k}}\cdot\nabla\left(\nabla^2\phi\right)_{\underline{q}-\underline{k}}$ etc.

then treat as Langevin equation, with $\tau_{c_q}$ set by coherence time of stochastically driven ZF field.

then

$$\frac{\partial}{\partial t}\left\langle\left(\nabla^2\tilde{\phi}\right)^2\right\rangle \sim \sum_{\underline{k}}\left(\tilde{\underline{V}}\cdot\nabla\nabla^2\tilde{\phi}\right)_{\underline{q}}\tau_{c_q}\left(\tilde{V}\cdot\nabla\nabla^2\tilde{\phi}\right)_{\underline{q}}$$

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$\langle (\nabla^2 \phi)^2 \rangle \sim D\, t$ → grow linearly in time!

and can add noise term to zonal mode growth

→ can drive flow, absent modulational instability

→ also seeds zonal density

→ see R. Singh, P.D. PPCF '21 for a complete analysis.

Zonal noise is the answer to the question of "What triggers the trigger?".

→ What limits ZF ? / Zonal Mode ?

— Tertiary instability

i.e. DW → Zonal $\xrightarrow{\text{instability}}$ Tertiary
coupling Modes
primary – secondary – Tertiary

i.e. $\nabla T \rightarrow \nabla V_{ZF} \rightarrow$ KH type

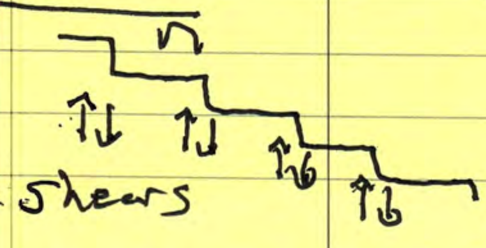
or

$\left.\begin{array}{l}\nabla n_Z \\ \nabla T_Z\end{array}\right\rangle$ Drift wave.

Tertiary controversial, especially in magnetically sheared systems.
Other? → turbulent viscosity. (Li, PD)
Key Question: How translate into effective NL ZF damping for coupled Spectra – Flow System?

→ ongoing

→ Staircase – 1D spatial pattern.



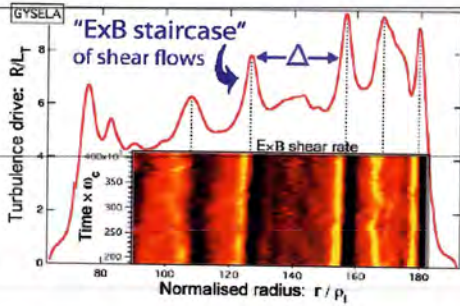

steps + shear layer pattern.
→ Bistable mixing.

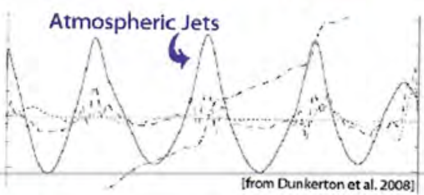
# Provocation: Staircase and Nonlocality (with G. Dif-Pradalier, et. al.)

UCSD

cea

## Analogy with geophysics: the '$\mathbf{E} \times \mathbf{B}$ *staircase*'




$$Q = -n\chi(r)\nabla T \;\Rightarrow\; \boxed{Q = -\int \kappa(r,r')\nabla T(r')\,\mathrm{d}r'}$$

- '$\mathbf{E} \times \mathbf{B}$ staircase' width $\equiv$ kernel width $\Delta$
- coherent, persistent, jet-like pattern
  ⇒ the '$\mathbf{E} \times \mathbf{B}$ staircase'
- staircase NOT related to low order rationals!

Dif-Pradalier, Phys Rev E. 2010
and many follow-ons.

# Provocation, cont'd

- The point:
  - fit: $Q = -\int dr'\kappa(r,r')\nabla T(r')$ $\quad \kappa(r,r') \sim \dfrac{S^2}{(r-r')^2+\Delta^2}$ $\rightarrow$ some range in exponent
  - $\Delta >> \Delta_c$ i.e. $\Delta \sim$ Avalanche scale >> $\Delta_c \sim$ correlation scale
  - Staircase 'steps' separated by $\Delta$ ! $\rightarrow$ stochastic avalanches produce quasi-regular flow pattern!?

    N.B.

    - The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)
    - What IS new is the connection to stochastic avalanches, independent of geometry
  - What is process of self-organization linking avalanche scale to zonal pattern step?

    i.e. How extend predator-prey feedback model to encompass both avalanche

    and zonal flow staircase? Self-consistency is crucial!

# Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high $k_r$ ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$
    - ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work
    - ⇒ Analogous → inverse energy cascade

- Mean Field Predator-Prey Model

  (P.D. et. al. '94, DI²H '05)

$$\begin{cases} \frac{\partial}{\partial t}N = \gamma N - \alpha V^2 N - \Delta\omega N^2 \\ \frac{\partial}{\partial t}V^2 = \alpha N V^2 - \gamma_d V^2 - \cancel{\gamma_{NL}(V^2)}V^2 \end{cases}$$

Reduced Model (0D)

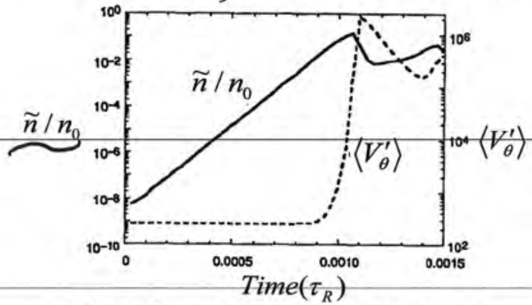
System Status

| State | No flow | Flow ($\alpha_2 = 0$) | Flow ($\alpha_2 \neq 0$) |
|---|---|---|---|
| $N$ (drift wave turbulence level) | $\frac{\gamma}{\Delta\omega}$ usual | $\frac{\gamma_d}{\alpha}$ | $\frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$ |
| $V^2$ (mean square flow) | 0 | $\underbrace{\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}}$ | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$ |
| Drive/excitation mechanism | Linear growth | Linear growth | Linear growth Nonlinear damping of flow |
| Regulation/inhibition mechanism | Self-interaction of turbulence | Random shearing, self-interaction | Random shearing, self-interaction |
| Branching ratio $\frac{V^2}{N}$ | 0 | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$ | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$ |
| Threshold (without noise) | $\gamma > 0$ | $\gamma > \Delta\omega\gamma_d\alpha^{-1}$ | $\gamma > \Delta\omega\gamma_d\alpha^{-1}$ |

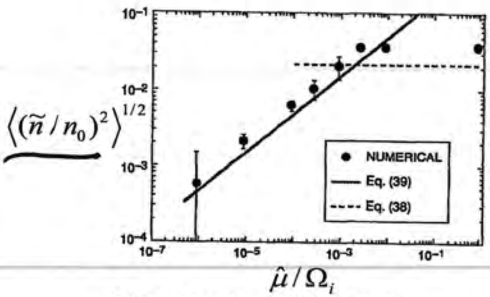
# Feedback Loops II

- Early simple simulations confirmed several aspects of modulational predator-prey dynamics

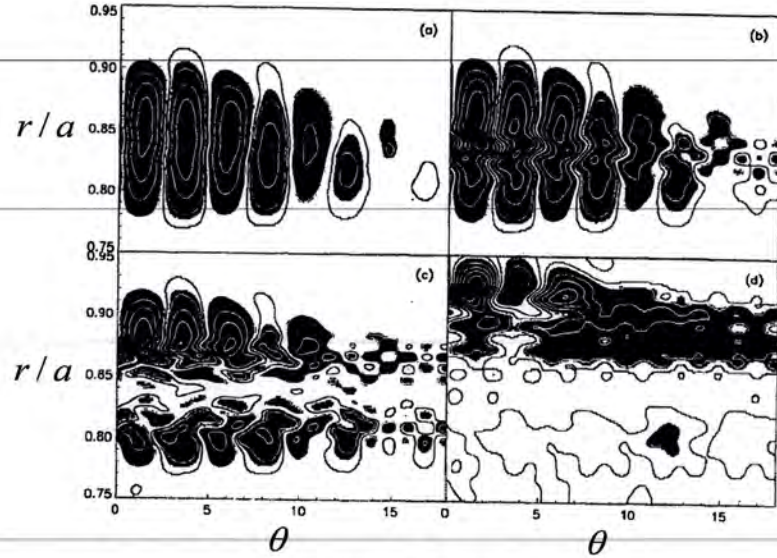
(L. Charlton et. al. '94)




Shear flow grows above critical point




'With Flow' and 'No Flow'.
Scalings of $\langle(\widetilde{n}/n_0)^2\rangle$ appear. Role of damping evident




Generic picture of fluctuation scale reduction with flow shear



$\mu \to 0 \Rightarrow$ Dimits Shift

What of Electromagnetics? – Neglected in this course....

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# Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

## Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla\phi \times \hat{z} \cdot \nabla\nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + \underbrace{B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla\nabla^2 A} + \tilde{f}$$

$$\partial_t A + \nabla\phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega\beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

# Progress II, cont'd

## Observation re: What happens?

- Turbulence $\rightarrow$ stretch field $\rightarrow$ $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$ (ala Zeldovich)
- Cascades : - forward or inverse?
  - MHD or Rossby dynamics dominant !?
- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v}\tilde{q} \rangle$ $\longrightarrow$ net change in charge content due PV/polarization charge flux

Now $\frac{dQ}{dt} = -\int dA \left[ \langle \tilde{v}\tilde{q} \rangle - \langle \tilde{B}_r \tilde{J}_\parallel \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\}$ $\longrightarrow$ Reynolds mis-match

PV flux    current along tilted lines    New Player $\longrightarrow$ vanishes for Alfvenized state

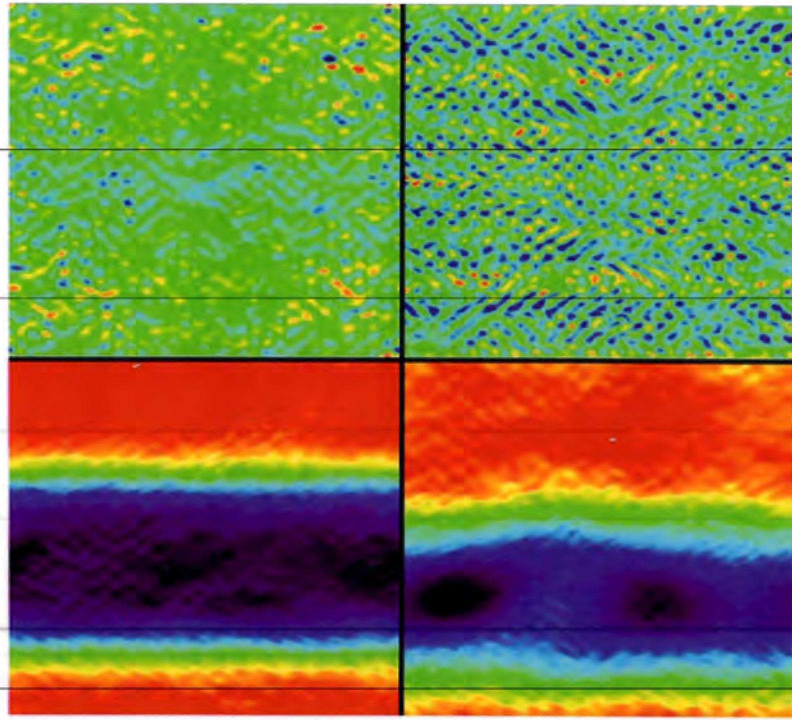
Taylor: $\langle \tilde{B}_x \tilde{J}_\parallel \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

# Progress II, cont'd

- **With Field**

$B_0 = 10^{-1}$

$B_0 = 10^{-2}$



$B_0 = 0$

$B_0 = 10^{-3}$

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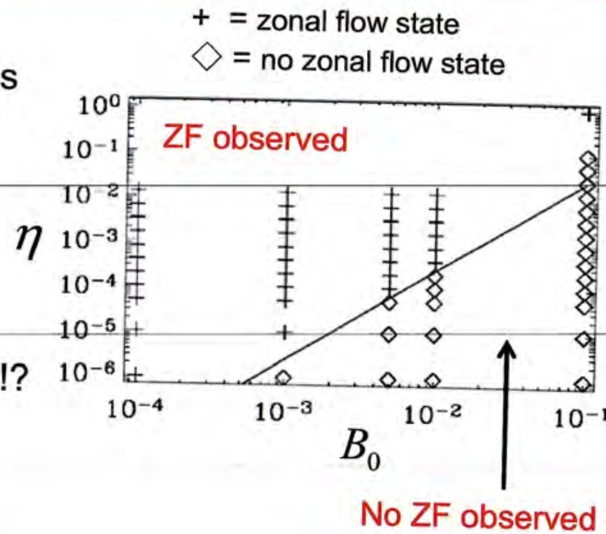
# Progress II, cont'd

- Control Parameters for $\tilde{\vec{B}}$ enter Z.F. dynamics

  ~~Like RMHD~~ Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m$ $\longrightarrow$ origin of $B_0^2 / \eta$ scaling !?




- Further study $\rightarrow$ differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution

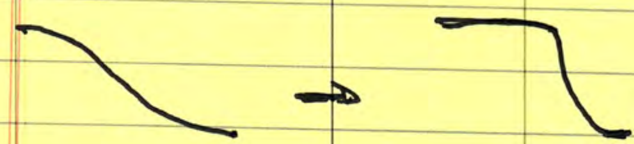
## b.) L–H Transition

{ Converging from Wagner '82, but not yet fully converged

Phenomenology:

⇒ LH characterized by:

→ edge gradient steepening

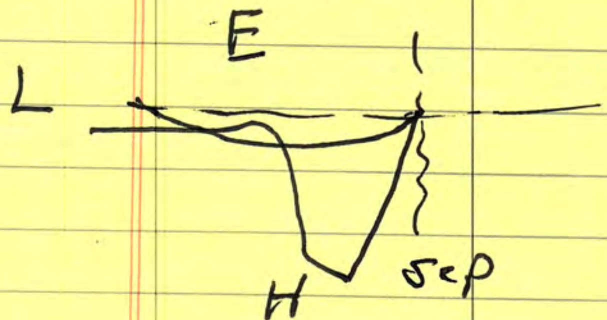


→ pedestal formation

→ confinement improvement

→ Fluctuation (low k) drop ↓
High k's persist in pedestal.

→ Increase in E×B shear, formation of $E_r$ well



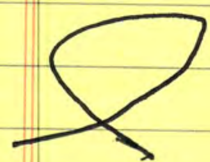

barrier can develop from inner, outer layer. (Schmitz 2021)

⇒ Power threshold ($P_{crit}$)

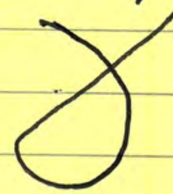
— critical to ITER

⇒ recall $P_{crit}$ — ∈ ⊕ Sawtooth HPP

⇒ $\nabla B$ drift asymmetry



vs.



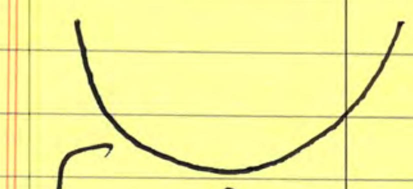
$B_T \odot$

Lower $P_{crit}$ Higher $P_{crit}$

(unclear) cf. Fedorczak et al. 2012

→ $P_{Thresh}$ — major concern ITER

$P_{crit}$



$n$

related LOC-SOC

why? ⇒ coupling to ions

i.e. $\nabla P_i \sim E_r$

increased $n$ assures stronger electron-ion coupling $\sim n T \nu_E (T_e - T_i)$

Some evidence $P_{Th} \sim n B_T$

Current ? ?

→ isotope ! — lower in D than H, etc.

<u>Relation</u> to <u>microphysics</u> <u>unclear</u>.

→ Other points:

- universal to all heating method

- limiter and diverted plasmas, but:

  → <u>never</u> in outside limiter

→ $P_{Th}$ higher for limiter plasma.

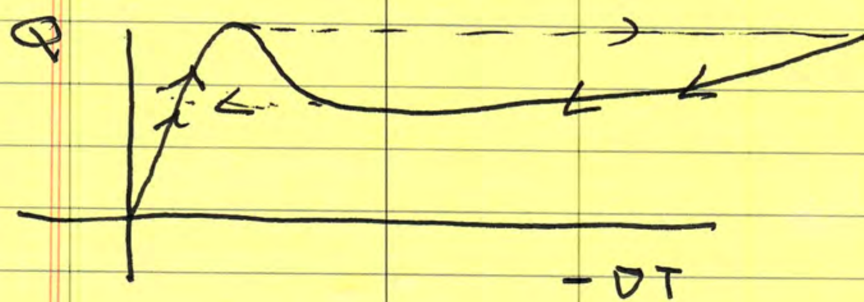
→ observed in stellarators

W7AS, TJ II, LHD

→ RFP ? → QSH ?!

$P_{OH}$ big ! → Boundary

→ Hysteresis happens




$P_{LH} > P_{HL}$.

Poorly understood → very important.

→ $P_{LH} \uparrow$ in RMP plasmas

important for ELM mitigation.

How ?  Trigger + $\nabla P$ Feedback

Energy to Shear Flow (ZF)

↓

$\nabla P$ steepen

↓

Turbulence collapses

Implications for transition thresh, if any ?

→ Multi-step

⇒ Many examples. DIII-D, EAST, TJ-II, AUG, HL-2A, Textor, ....

⇒ A few dissenters .... not many

JFT-2M, AUG, HL-2A

→ ? Orbit loss ?

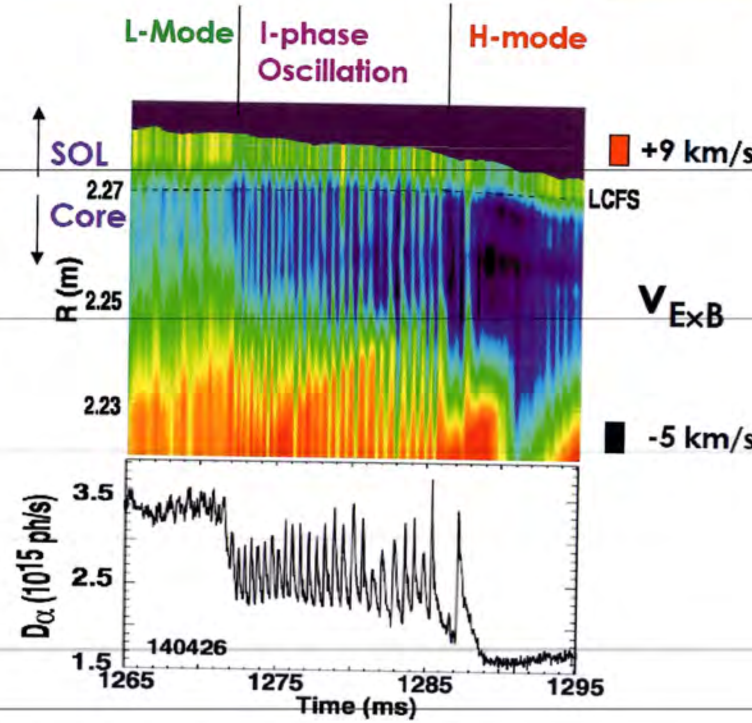
→ Is there a unique route to transition ?

# The Oscillating Flow Layer Widens Radially (Frequency Decreases) - Steady Flow after Final H-Mode Transition

A weak ExB flow layer exists in L-mode (L-mode shear layer)

At the I-phase transition, the ExB flow becomes more negative first near the separatrix, flow layer then propagates inward

The flow becomes steady at the final H-mode transition (after one final transient)

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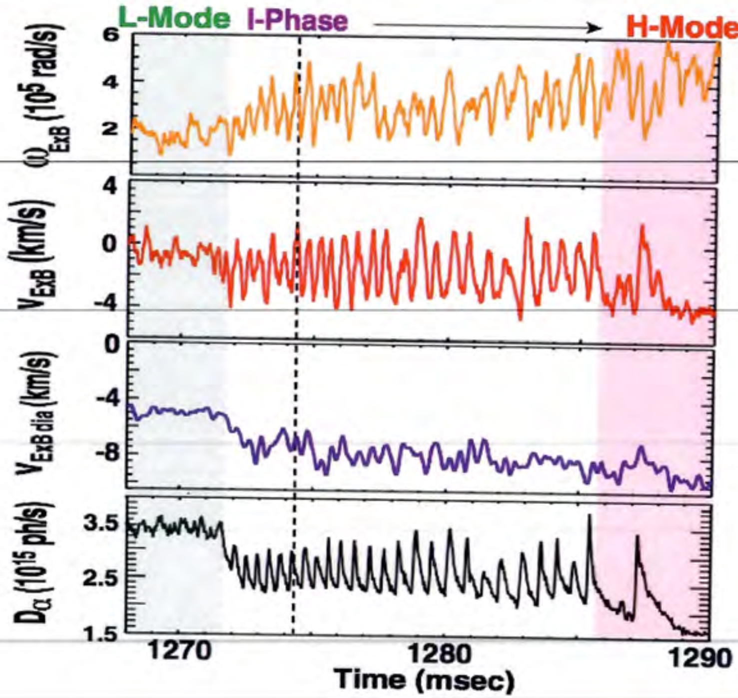
## During the I-phase, the Mean Shear $\langle\omega_{ExB}\rangle$ Increases with Time and Eventually Dominates

Outer layer Shearing Rate (Mean flow+ ZF)

ExB Flow from DBS (includes ZF)

Diamagnetic component of ExB flow (from ion pressure Profile)

R~2.265m

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# Feedback Loops III

- $\nabla P$ coupling
  - $\gamma_L$ drive
  - $\langle V_E \rangle'$

i.e. $\gamma = \gamma(\nabla P)$

$$\partial_t \varepsilon = \varepsilon N - a_1 \varepsilon^2 - a_2 V^2 \varepsilon - a_3 V_{ZF}^2 \varepsilon$$

$$\partial_t V_{ZF} = b_1 \frac{\varepsilon V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF}$$

$$\partial_t N = -\underset{\text{transport}}{c_1 \varepsilon N} - c_2 N + \underset{\text{source, flux}}{Q}$$

$\varepsilon \equiv$ *DW energy*

$V_{ZF} \equiv$ *ZF shear*

$N \equiv \nabla\langle P \rangle \equiv$ *pressure gradient*

$V = dN^2$ *(radial force balance)*

- Simplest example of 2 predator + 1 prey problem (E. Kim, P.D., 2003)

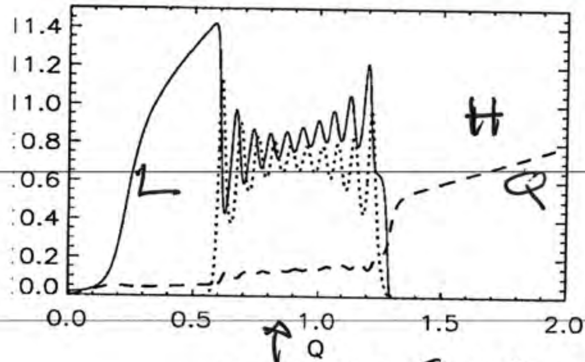
  i.e. prey sustains predators, predators limit prey } usual feedback

  now: { 2 predators ( *ZF*, $\nabla\langle P \rangle$ ) compete; $\nabla\langle P \rangle$ as both drive and predator

- Relevance: LH transition, ITB
  - Builds on insights from Itoh's, Hinton
  - ZF $\Rightarrow$ triggers
  - $\nabla\langle P \rangle \Rightarrow$ 'locking in'

Multiple predators are possible

# Feedback Loops III, cont'd

- 


Solid - $\mathcal{E}$

Dotted - $V_{ZF}$

Dashed - $\nabla\langle P\rangle$

— simple, useful model of L-H transition,

— turbulence extinguished on H,

- Observations:
  - ZF's trigger transition, $\nabla\langle P\rangle$ and $\langle V\rangle$ lock it in
  - Period of dithering, pulsations .... during ZF, $\nabla\langle P\rangle$ oscillation as Q ↑
  - Phase between $\mathcal{E}$ , $V_{ZF}$, $\nabla\langle P\rangle$ varies as Q increases
  - $\nabla\langle P\rangle \Leftrightarrow$ ZF interaction $\Rightarrow$ effect on wave form

extended.