CSC420 A4

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Q1

1.) $P = 0.995$
$p = 0.7$

We know the required minimum number of trial S

$$S = \frac{\log(1-P)}{\log(1-p^k)}$$

We take $k = 4$, as we are looking for 4 minimum matches for homography.

$$S = \frac{\log(1-0.995)}{\log(1-0.7^4)} \approx 19.297$$

We need at least 20 iterations of RANSAC to fit this homography.

2.)

Fitting an affine transformation would take less iterations because affine transformation has 6 degree of freedom while homography has 8. This means that we are looking for 3 matches compare to 4, resulting in a smaller possible sample size.

Q1:

Q2

1.) $L = \vec{P}_0 + t\vec{d}$

$$\vec{p} = \begin{pmatrix} wx \\ wy \\ w \end{pmatrix} = K\vec{P} = \begin{pmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{pmatrix}$$

$$= \begin{pmatrix} fX_0 + ftd_x + p_x Z_0 + p_x td_z \\ fY_0 + p_y td_y + p_y Z_0 + p_y td_z \\ Z_0 + td_z \end{pmatrix}$$

We take the limits of wx and wy term to infinity for the vanishing point:

$$x = \lim_{t \to \infty} \frac{wx}{w} = \lim_{t \to \infty} \frac{fX_0 + ftd_x + p_x Z_0 + p_x td_z}{Z_0 + td_z}$$

$$= f\left(\frac{d_x}{d_z}\right) + p_x$$

$$y = \lim_{t \to \infty} \frac{wy}{w} = f\left(\frac{d_y}{d_z}\right) + p_y$$

The vanishing point is $\left(f\left(\frac{d_x}{d_z}\right) + p_x \ , \ f\left(\frac{d_y}{d_z}\right) + p_z \right)$.

Q2:

2.) From the hint, we know that

$\vec{n}\cdot\vec{d}=0$, which means $n_x d_x + n_y d_y = n_z d_z = 0$

$$d_x = -\frac{1}{n_x}(n_y d_y + n_z d_z)$$

We can get the line which for the vanishing point

$$\frac{f d_x}{d_z} + p_x = -\frac{f}{n_x d_z}(n_y d_y + n_z d_z) + p_x$$
$$= -\frac{f n_y d_y}{n_x d_z} - \frac{f n_z d_z}{n_x d_z} + p_x$$
$$= -\frac{f d_y}{d_z}\frac{n_y}{n_x} - p_y\frac{n_y}{n_x} + p_y\frac{n_y}{n_x} - f\frac{n_z}{n_x} + p_x$$
$$= -\frac{n_y}{n_x}\left(\frac{f d_y}{d_z} + p_y\right) + p_y\frac{n_y}{n_x} - f\frac{n_z}{n_x} + p_x$$

If we denote the vanishing point as (V_x, V_y) we get

$$V_x = \frac{-n_y}{n_x}V_y + p_y\frac{n_y}{n_x} - f\frac{n_z}{n_x} + p_x$$

Therefore we know the vanishing points are on the line

$$n_x V_x + n_y V_y = p_y n_y + p_x n_x - f n_z$$

We know that n and p terms are constants, so the points form a line.

Q3

1.) Let $l = a_1 x + b_1 y + c_1 = 0$

$l' = a_2 x + b_2 y + c_2 = 0$

be arbitrary 2D lines

$$a_2(a_1x + b_1y + c_1) - a_1(a_2x + b_2y + c_2) = 0$$

$$(a_2b_1 - a_1b_2)y = a_1c_2 - a_2c_1$$

$$y = \frac{a_1c_2 - a_2c_1}{a_2b_1 - a_1b_2}$$

we can also get $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

In homogeneous coordinate, (x, y) can be written as (xw, yw, w), take $w = a_1b_2 - a_2b_1$

we get $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xw \\ yw \\ w \end{pmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_2c_1 - a_1c_2 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Notice $l \times l' = (a_1, b_1, c_1) \times (a_2, b_2, c_2)$

$$= \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_2c_1 - a_1c_2 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = p$$

therefore, $p = l \times l'$.

Q3

2) Let $p = (x_1, y_1)^T$ and $p' = (x_2, y_2)^T$

The arbitrary line (going through (p, p') should satisfy

$$\begin{cases} ax_1 + by_1 + c = 0 \\ ax_2 + by_2 + c = 0 \end{cases}$$

we have :

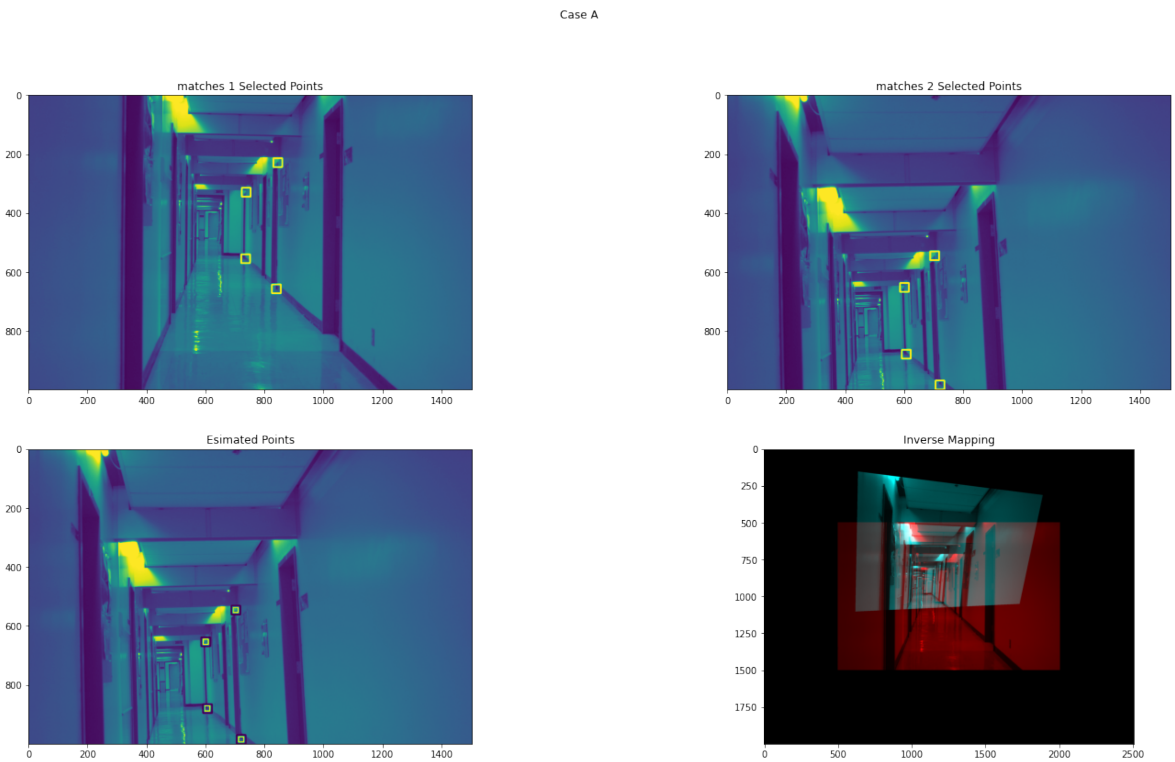
$$a = \frac{c(y_1 - y_2)}{x_1 y_2 - x_2 y_1}$$

$$b = \frac{c(x_1 - x_2)}{x_2 y_1 - x_1 y_2}$$

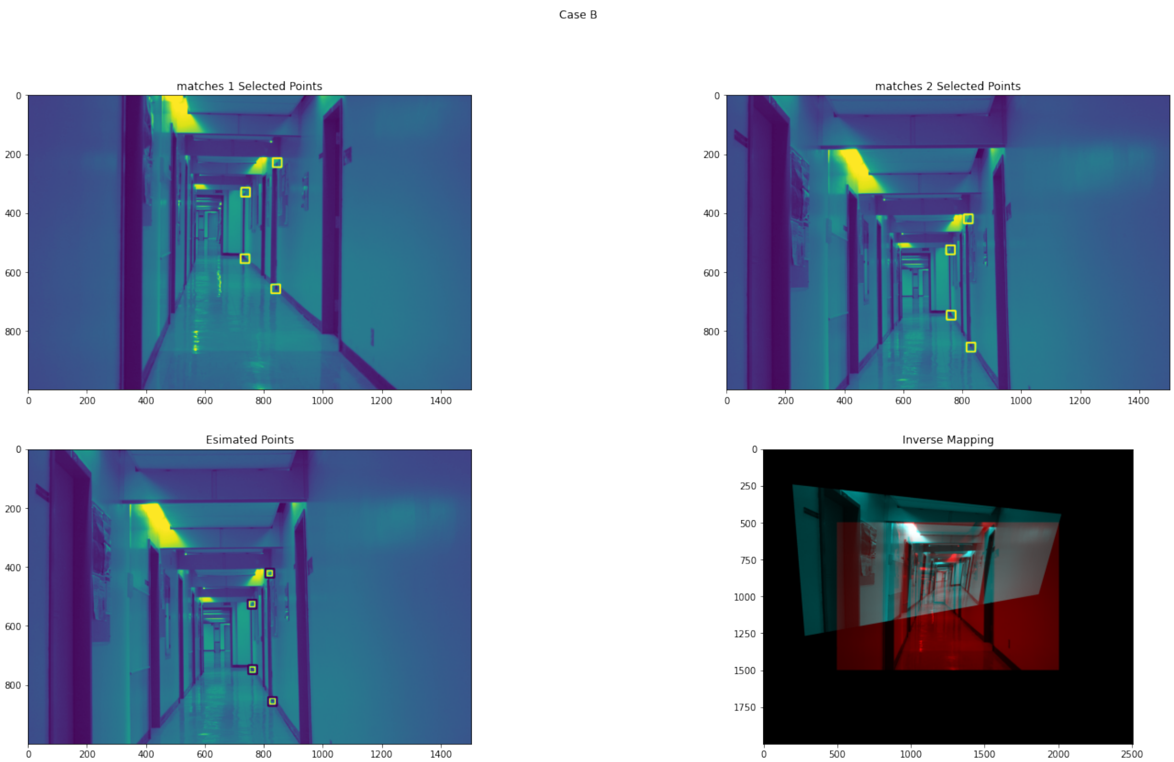
We take the homogenous coordinate and get

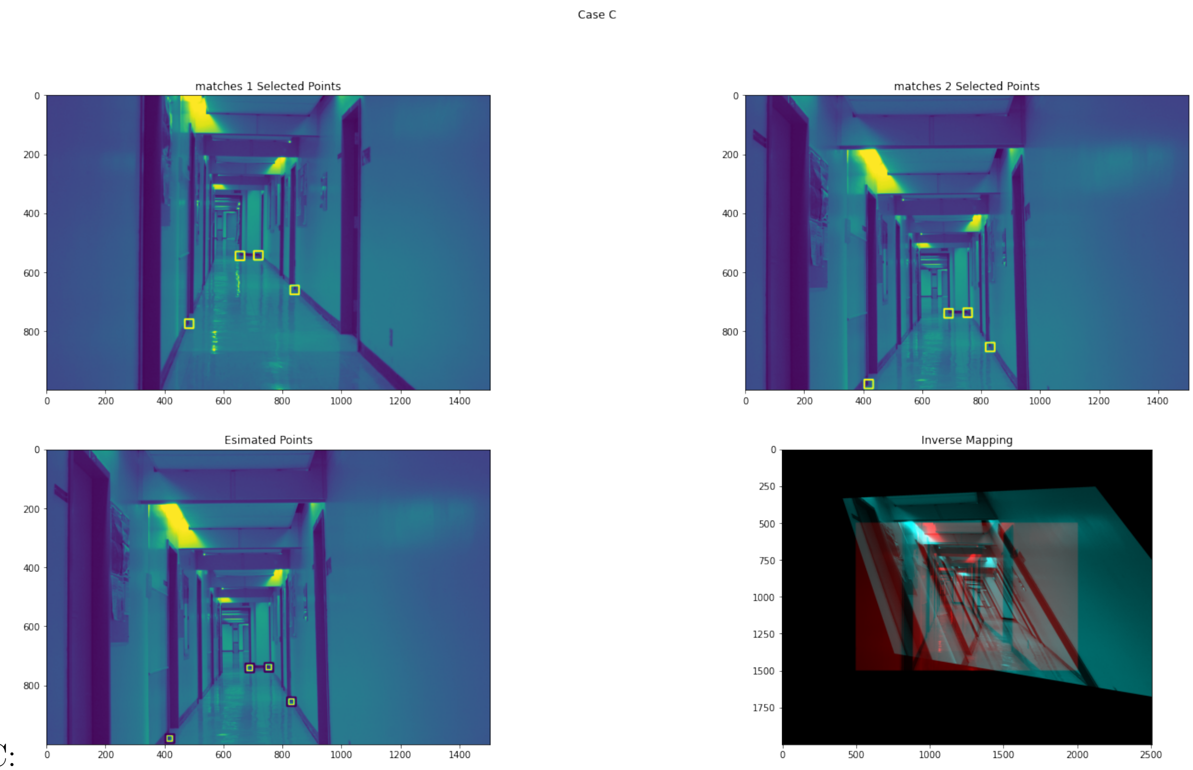
$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{c(y_1 - y_2)}{x_1 y_2 - x_2 y_1} \\ \frac{c(x_1 - x_2)}{x_2 y_1 - x_1 y_2} \\ c \end{pmatrix} = \begin{pmatrix} y_1 - y_2 \\ x_2 - x_1 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}$$

4: A:


Case A
matches 1 Selected Points
matches 2 Selected Points
Esimated Points
Inverse Mapping

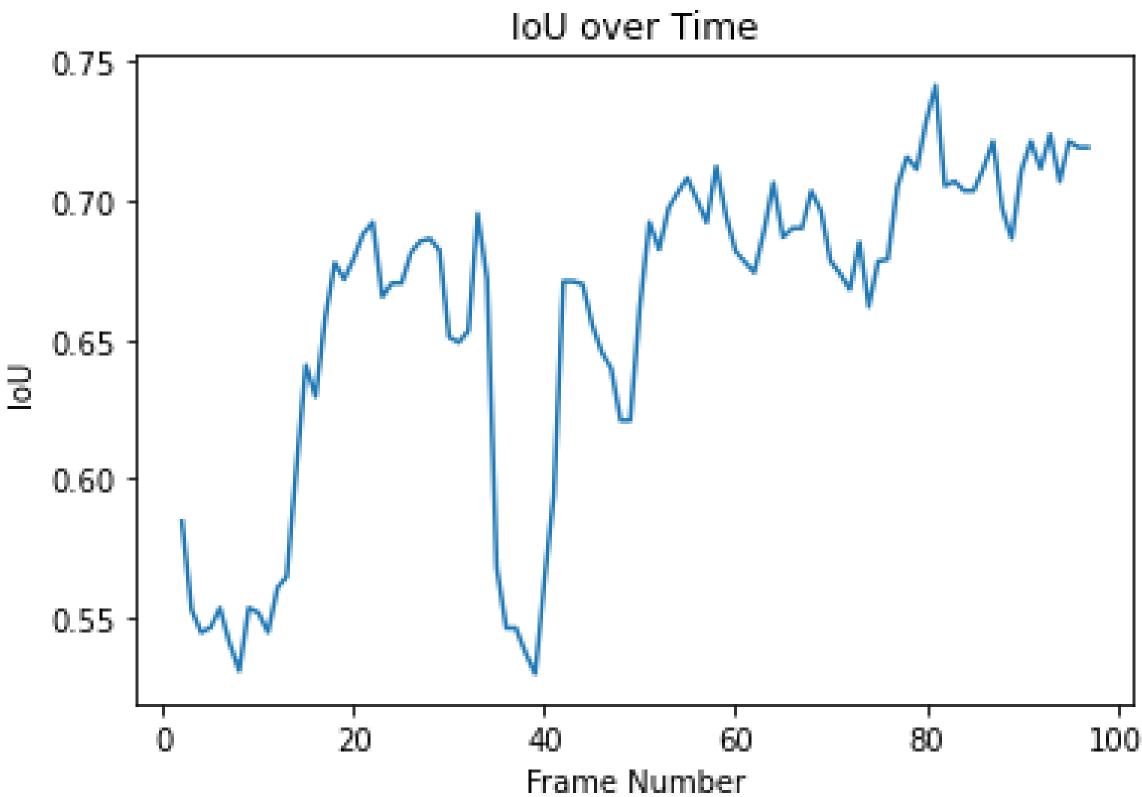
B:


Case B
matches 1 Selected Points
matches 2 Selected Points
Esimated Points
Inverse Mapping

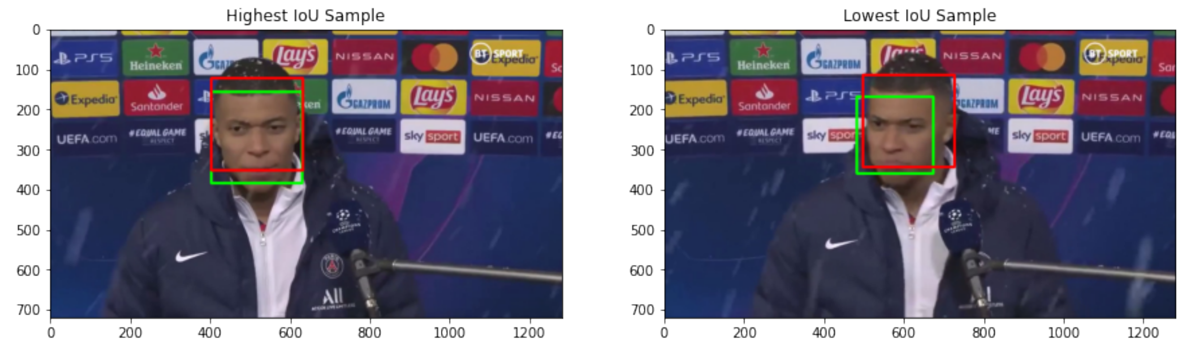
C: 
Case C
matches 1 Selected Points
matches 2 Selected Points
Esimated Points
Inverse Mapping

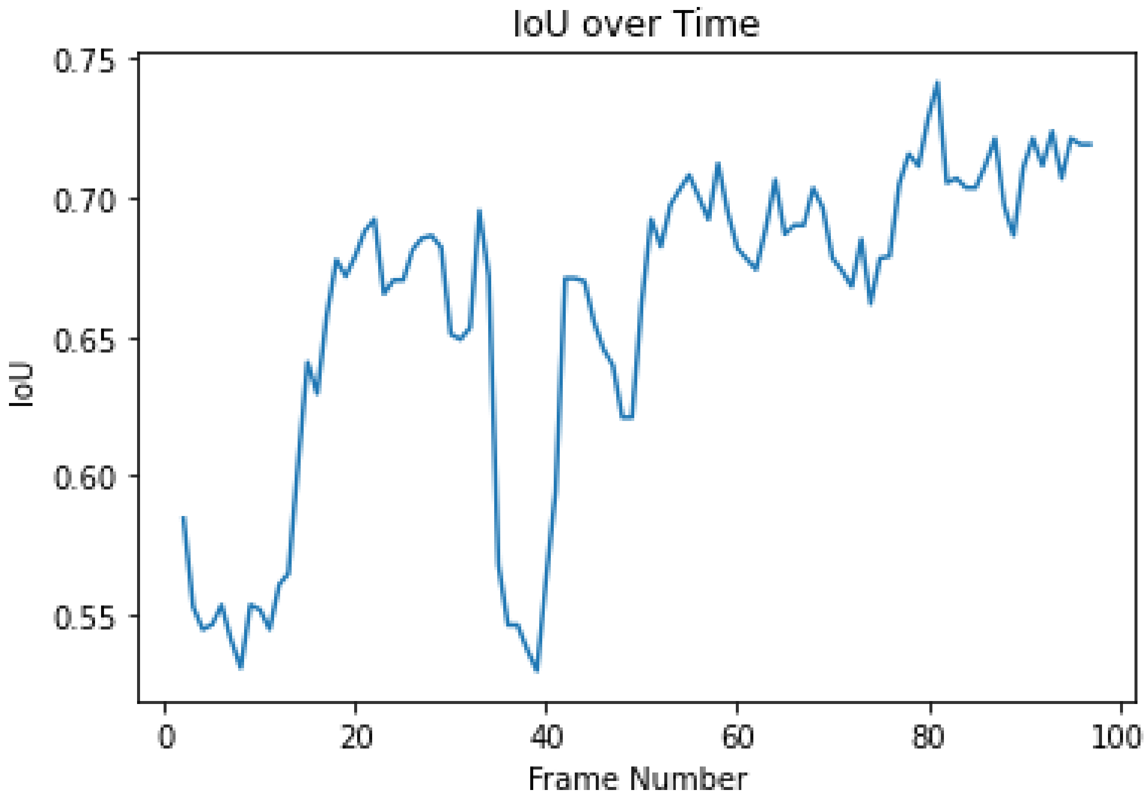
Q4 1: Homography matrices: A: [7.46343415e-01 -5.28880844e-02 -2.39607419e+01 -1.74322083e-01
7.30520701e-01 4.40246222e+02 -1.45210989e-04 -1.36808324e-04
1.00000000e+00]
B: [2.59139628e-01 -1.10259738e-01 4.32591244e+02 -1.97159318e-01
6.40452876e-01 3.40230044e+02 -2.35890575e-04 -1.59040357e-04
1.00000000e+00]
C: [1.52736403e+00 -3.64181964e-01 8.17429411e+01 1.96751154e-01
1.49134479e+00 6.33987977e+00 2.63688598e-04 2.02293139e-04
1.00000000e+00]

Q5: 1. Viola Jones is more accurate than mean shift as it is using a learned facial recognition whereas mean shift is just heuristic based on the histogram.
All of the frames were above 50 % IoU.
2. All of the frames were above 50 % IoU in this experiment too.


IoU over Time
0.75
0.70
0.65
0.60
0.55
IoU
0
20
40
60
80
100
Frame Number

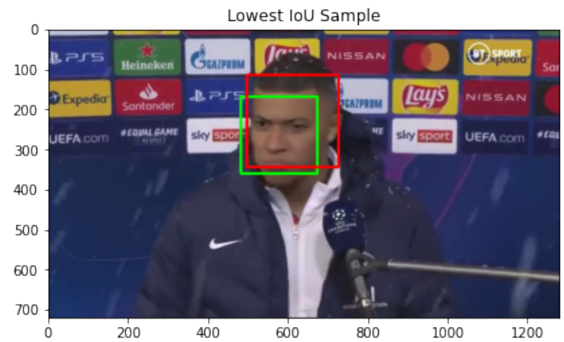
Q5.1:


Highest IoU Sample
0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200
Lowest IoU Sample
0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200


IoU over Time
0.75
0.70
0.65
0.60
0.55
IoU
0
20
40
60
80
100
Frame Number

Q5.2


Highest IoU Sample


Lowest IoU Sample