ASTP-720 Homework 3

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Github Link: `https://github.com/zrd7527/ASTP720.git`

1 Problem 1

To start this problem I copied my matrix class and all of the included functions from the last homework over to this homework. I solved all of the operations in the main function by hand then used assert statements which were true if the output of each function was equal to the values I got when solving by hand. No errors were encountered meaning the functions work properly.

2 Problem 2

This problem involved many different pieces. First I used the data and function given on MyCourses to read off the Einstein A_{ul} coefficients and create a matrix with each coefficient in the row/column of the respective energy level from/to transition. This also allowed me to make a matrix of the B_{ul} coefficients using Equation 1 so I made both matrices in one function. Then using Equation 2 and my transpose function in the matrix class, I was able to make the B_{lu} matrix from the B_{ul} matrix. Frequencies were fround using Equations 3 and 4. Next the entire coefficient matrix described in class was created using the 3 coefficient matrices already found. A 1D array for the right hand side of the matrix equation $\bar{\bar{A}}\vec{x} = \vec{B}$ was initialized with values of 10^{-6} instead of zeros to avoid a zero solution. For a range of temperatures between 100 kelvin and 10100 kelvin, the coefficient matrix was built then solved using an LU decomposition. I used the LU decomposition function from my matrix class and created 2 new functions, lower solver and upper solver. These functions find the solutions to their respective matrix equations $\bar{\bar{L}}\vec{y} = \vec{B}$ and $\bar{\bar{U}}\vec{y} = \vec{x}$, where $\bar{\bar{A}}$, $\vec{x}$, and $\vec{B}$ are from the matrix equation being solved but $\bar{\bar{L}}$ and $\bar{\bar{U}}$ are the LU decomposition of matrix A. The final plot is shown in Figure 1.

$$A_{ul} = \frac{2h\nu^3}{c^2} B_{ul} \tag{1}$$

$$B_{lu} = \frac{g_u}{g_l} B_{ul} \tag{2}$$

$$dE = -13.6eV\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \tag{3}$$

$$E = \frac{hc}{\lambda} = 1240eVnm \tag{4}$$

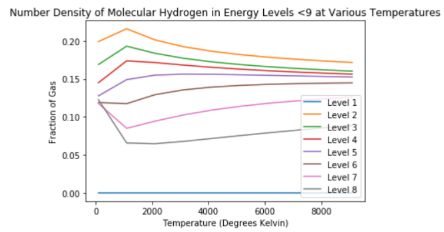

Figure 1

3 Problem 3

I wrote 2 sets of ODE solver functions, one set handles 2 variables and the other set handles 1. The first set is in the first cell of the ASTP720-HW3-2 notebook. This set handles the 2 variable ODEs and is tested in problem 4. The first set uses the methods described in class but repeat the method for the second variable. The second set is in the next cell of the same file and handles the single variable ODEs like in problem 5. This set of functions also uses the methods described in class. The time range taken by both sets of functions is a tuple so any input list must be pairs of times. The step size is simply hardcoded into each function but could easily be turned into a user input.

4 Problem 4

In the main of the first cell of ASTP720-HW3-2 (the set of ODE solvers for 2 variables), I ran each ODE solver with scipy's damped pendulum example. The damped pendulum function was copied from the webpage of the link given in the homework. I ran each for the time range 0 to 10 seconds with a step size of 0.01 second and the same initial conditions given in the link. Each solver recreated the example in the link very well as can be seen in figures 2-4.

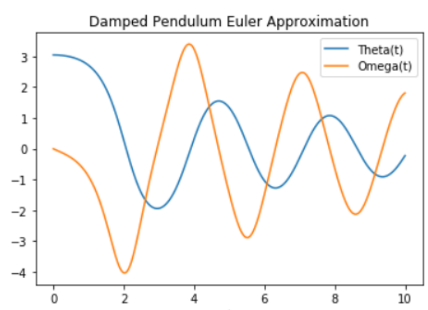

Figure 2

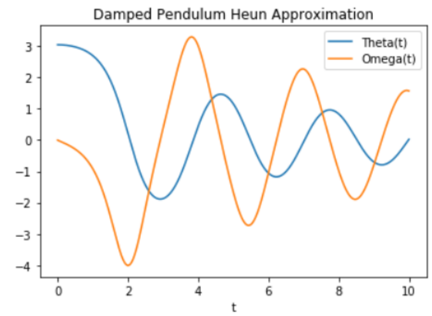

Figure 3

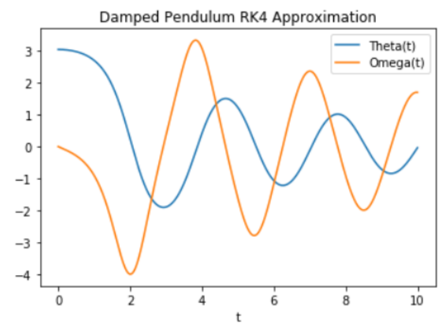

Figure 4

5 Problem 5

In the main of the second cell of ASTP720-HW3-2 (the set of ODE solvers for 1 variable), I ran each ODE solver with the stiff equation example given in the homework. The stiff equation function was written separately in the same cell with lambda hardcoded as 10 but could easily be changed to an input or range of values. I ran each solver for the same time range as in problem 4 with a step size of 0.01 second and the initial conditions given in the homework description. The solvers all seemed to diverge from the actual solution of the function rapidly and none seemed to diverge more than the others as can be seen in figures 5-7. This could be due to the time range or step size. An increase in either may change the results.

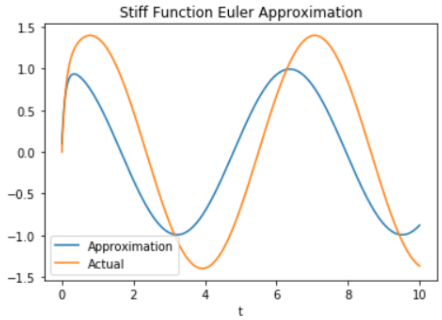

Figure 5

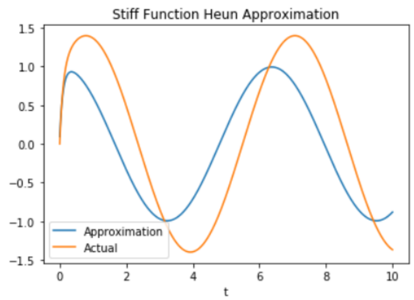

Figure 6

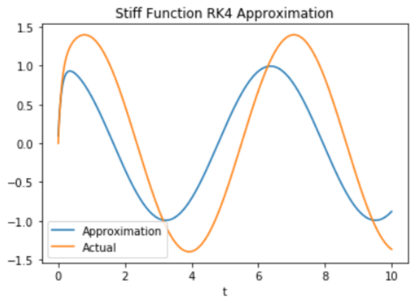

Figure 7