

SAT-based Sudoku Solving

CSC 320 Summer 2018

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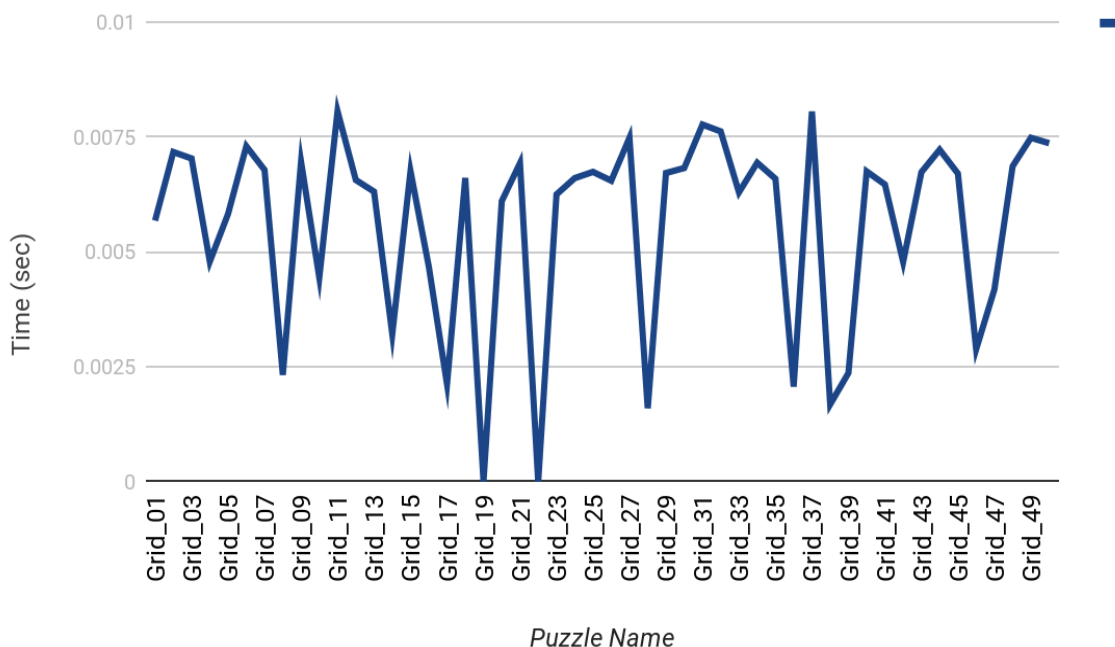
1 Project Overview

The following software project was created to explore the NP hard problem of solving sudoku puzzles, and the conversions between the various encodings and their impact on computational solving time. Two main programs were written for the project, as well as three extended tasks. The first main program used python to convert a sudoku puzzle in a specified text format, into a CNF formula suitable for the miniSAT SAT solver. The second took the output from the SAT solver and converted to a more human readable sudoku format. The extended tasks chosen were hard inputs, the use of multiple encodings for sudoku problems, and the use of other SAT solvers (for this project we used walkSAT). This report outlines the main components of both main tasks as well as the extended ones and was completed for CSC 320 at the University of Victoria. The scripts, inputs, outputs, read me, and make file referenced are all included within the containing folder.

2 Sudoku to SAT

The following table displays the time taken to solve each puzzle found at projecteuler.net/project/resources/p096_sudoku.txt.

Time Required to Solve Each Puzzle



Puzzle Name	Time to Solve (Sec)
Grid_01	0.005682
Grid_02	0.007173
Grid_03	0.007033
Grid_04	0.004792
Grid_05	0.005828
Grid_06	0.0073
Grid_07	0.006783
Grid_08	0.002326
Grid_09	0.006963
Grid_10	0.004443
Grid_11	0.008062
Grid_12	0.006566
Grid_13	0.006312
Grid_14	0.003213
Grid_15	0.006758
Grid_16	0.004687
Grid_17	0.002127
Grid_18	0.00661
Grid_19	0
Grid_20	0.006105
Grid_21	0.006931
Grid_22	0
Grid_23	0.006251
Grid_24	0.006606
Grid_25	0.006741
Grid_26	0.00655
Grid_27	0.007483
Grid_28	0.001601
Grid_29	0.006717

Grid_30	0.006825
Grid_31	0.00777
Grid_32	0.007621
Grid_33	0.006297
Grid_34	0.00694
Grid_35	0.006591
Grid_36	0.002071
Grid_37	0.008056
Grid_38	0.001686
Grid_39	0.002369
Grid_40	0.006751
Grid_41	0.006467
Grid_42	0.004782
Grid_43	0.006736
Grid_44	0.007224
Grid_45	0.006706
Grid_46	0.002884
Grid_47	0.004185
Grid_48	0.006865
Grid_49	0.007483
Grid_50	0.007365
Sum	0.281317
Average	0.005626

3 SAT to Sudoku

The following text contains the solved puzzles from projecteuler.net/project/resources/p096
sudoku.txt:

Grid 1

```
483921657
967345821
251876493
548132976
729564138
136798245
372689514
814253769
695417382
```

Grid 2

```
245981376
169273584
837564219
976125438
513498627
482736951
391657842
728349165
654812793
```

Grid 3

```
462831957
795426183
381795426
173984265
659312748
248567319
926178534
834259671
517643892
```

Grid 4

```
137256849
928314567
465897312
673542981
819673254
542189736
256731498
391428675
784965123
```

Grid 5

```
523816749
784593126
691472835
239145687
457268913
168937254
342789561
915624378
876351492
```

Grid 6

```
176923584
524817639
893654271
957348162
638192457
412765398
265489713
781236945
349571826
```

Grid 7

```
143986257
679425381
285731694
962354178
357618942
418279563
821567439
796143825
534892716
```

Grid 8

```
487156932
362498751
915372864
846519273
593724186
271863549
124685397
738941625
659237418
```

Grid 9

```
814976532
659123478
732854169
948265317
275341896
163798245
391682754
587439621
426517983
```

Grid 10

761928453
925743168
438615927
357461289
894372615
216589374
689154732
142837596
573296841

Grid 11

976125438
158436927
423879156
234761895
867952314
519384762
782513649
395647281
641298573

Grid 12

962341758
148975623
573268149
321694875
487512936
695837412
834726591
216459387
759183264

Grid 13

397681524
645279813
218534976
823956741

169742358

754318692
472893165
531467289
986125437

Grid 14

639218457
471539268
825674139
564823791
793451826
218796345
352987614
186345972
947162583

Grid 15

697128345
428635197
315479682
531246978
286397451
974581263
149852736
752963814
863714529

Grid 16

361725948
587964213
492831657
638259471
174683592
259147836
746392185
923518764
815476329

Grid 17

359867124
648312597
712549836
876924351
524731968
193685472
931476285
465298713
287153649

Grid 18

786945312
219863457
534271869
165482973
327619548
498537126
951728634
842356791
673194285

Grid 19

743512986
589346217
126987345
934251768
671498532
852763491
398675124
417829653
265134879

Grid 20

782614359
439825176
651937428

293471865
568392714
147568293
326749581
975183642
814256937

Grid 21

428531796
365947182
971268435
214896573
697453218
583172964
849615327
752389641
136724859

Grid 22

425781936
178369524
369524187
894157362
652843791
713692845
987216453
536478219
241935678

Grid 23

348267951
571943628
269185374
697351482
123874596
854629137
415798263
982436715

736512849

Grid 24

124986735
867435912
395712684
478359261
259861347
631274598
712698453
983547126
546123879

Grid 25

361524789
789361425
524879361
893157642
412683597
657942138
148796253
235418976
976235814

Grid 26

581479263
329156847
647328159
956731428
238964571
714582936
172695384
893247615
465813792

Grid 27

387256419
469781325

512439867
123548976
758963241
694127583
835674192
271895634
946312758

Grid 28

345871269
279653184
861429537
197346852
452718396
683592741
738264915
516937428
924185673

Grid 29

235761489
419328576
867549213
746135928
521896734
983472651
394287165
652913847
178654392

Grid 30

298175643
657394128
134286579
821649735
573821496
469753281
312468957

785912364
946537812

Grid 31

761543289
832791645
549628137
374215968
128936574
695487321
417369852
953872416
286154793

Grid 32

132649785
758213649
964785123
543897216
276531894
891426537
619378452
327154968
485962371

Grid 33

698173542
354628179
172549368
531897426
946312857
827456913
765931284
213784695
489265731

Grid 34

852716943

197843652
463925187
278634591

645179328
931582476
786491235
314258769
529367814

Grid 35

453218796
629753481
178496532
796582314
314967825
285134679
542879163
937641258
861325947

Grid 36

516289347
849173256
732465918
698317524
327954861
154826739
961732485
275648193
483591672

Grid 37

945681723
781234965
326759184
269175348
138942576
574863219

457326891
612598437
893417652

Grid 38

365942871
128756493
974813562
819435627
537268149
642179358
296384715
753691284
481527936

Grid 39

134587296
278169354
695234817
359816472
821473569
746925183
917348625
462751938
583692741

Grid 40

193672485
462358971
785914623
538296714
674135298
219487356
826741539
941523867
357869142

Grid 41

814976532
659123478
732854169
948265317
275341896
163798245
391682754
587439621
426517983

Grid 42

384567921
126439785
759821346
563798214
847312659
912645873
231974568
495286137
678153492

Grid 43

469158372
712463859
538297641
927634518
385719426
146582793
653941287
294876135
871325964

Grid 44

316549278
987321645
452678931

594236817
238417569
671985324
845162793
129753486
763894152

Grid 45

586127943
723469851
491853267
135974628
279618534
648532179
917246385
352781496
864395712

Grid 46

954213687
617548923
832796541
763851294
128974365
549362178
281637459
475129836
396485712

Grid 47

159743862
276589431
348612759
624978315
917235684
583164297

435821976
861497523
792356148

Grid 48

861357294
597482361
432619785
916275843
358964127
274138956
789541632
143826579
625793418

Grid 49

294863517
715429638
863751492
152947863
479386251
638512974
986134725
521678349
347295186

Grid 50

351286497
492157638
786934512
275469183
938521764
614873259
829645371
163792845
547318926

4 Extended Tasks

1 Hard Inputs

For the first extended task of reading in hard inputs of the form shown below (from <http://magictour.free.fr/top95>), a file called sat2sudHI.py was created took the given hard input, converted it to minimal encoding generated the CNF formula suited as input to the miniSAT. The runtime for these puzzles are shown in the following table.

```
4.....8.5.3.....7.....2.....6.....8.4.....1.....6.3.7.5..2.....1.4.....
52...6.....7.13.....4..8..6.....5.....418.....3..2...87.....
6.....8.3.4.7.....5.4.7.3..2.....1.6.....2.....5.....8.6.....1....
48.3.....71.2.....7.5...6.....2..8.....1.76...3.....4.....5....
....14....3....2...7.....9...3.6.1.....8.2....1.4....5.6.....7.8...
```

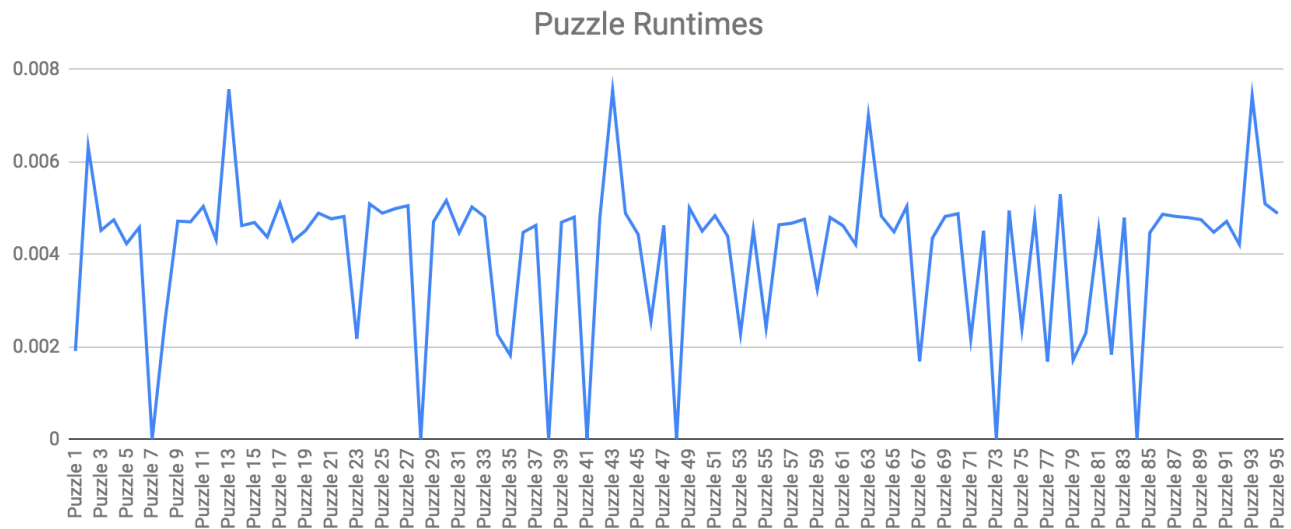
Hard Inputs Format

Puzzle Name	Time to Solve (Sec)
Puzzle 1	0.001907
Puzzle 2	0.006328
Puzzle 3	0.00452
Puzzle 4	0.00475
Puzzle 5	0.004232
Puzzle 6	0.004584
Puzzle 7	0
Puzzle 8	0.002541
Puzzle 9	0.00472
Puzzle 10	0.004705
Puzzle 11	0.005036
Puzzle 12	0.004318
Puzzle 13	0.007569
Puzzle 14	0.004622
Puzzle 15	0.00469

Puzzle 16	0.004378
Puzzle 17	0.005104
Puzzle 18	0.004286
Puzzle 19	0.004518
Puzzle 20	0.004893
Puzzle 21	0.00477
Puzzle 22	0.004821
Puzzle 23	0.002174
Puzzle 24	0.005092
Puzzle 25	0.004892
Puzzle 26	0.004989
Puzzle 27	0.005054
Puzzle 28	0
Puzzle 29	0.004709
Puzzle 30	0.005166
Puzzle 31	0.004468
Puzzle 32	0.005027
Puzzle 33	0.004813
Puzzle 34	0.002265
Puzzle 35	0.001816
Puzzle 36	0.004477
Puzzle 37	0.004627
Puzzle 38	0
Puzzle 39	0.004695
Puzzle 40	0.004807
Puzzle 41	0
Puzzle 42	0.004794
Puzzle 43	0.00755

Puzzle 44	0.004891
Puzzle 45	0.004431
Puzzle 46	0.002562
Puzzle 47	0.004626
Puzzle 48	0
Puzzle 49	0.005004
Puzzle 50	0.004502
Puzzle 51	0.004839
Puzzle 52	0.004392
Puzzle 53	0.002281
Puzzle 54	0.004529
Puzzle 55	0.002412
Puzzle 56	0.004641
Puzzle 57	0.004675
Puzzle 58	0.004763
Puzzle 59	0.003244
Puzzle 60	0.004801
Puzzle 61	0.004624
Puzzle 62	0.004215
Puzzle 63	0.007005
Puzzle 64	0.00483
Puzzle 65	0.004486
Puzzle 66	0.005039
Puzzle 67	0.001685
Puzzle 68	0.004352
Puzzle 69	0.004821
Puzzle 70	0.004881
Puzzle 71	0.002172

Puzzle 72	0.00451
Puzzle 73	0
Puzzle 74	0.004948
Puzzle 75	0.002389
Puzzle 76	0.00478
Puzzle 77	0.001682
Puzzle 78	0.005303
Puzzle 79	0.001711
Puzzle 80	0.002298
Puzzle 81	0.004539
Puzzle 82	0.001833
Puzzle 83	0.004791
Puzzle 84	0
Puzzle 85	0.004471
Puzzle 86	0.004867
Puzzle 87	0.004822
Puzzle 88	0.004797
Puzzle 89	0.004752
Puzzle 90	0.004479
Puzzle 91	0.004713
Puzzle 92	0.004213
Puzzle 93	0.007431
Puzzle 94	0.005098
Puzzle 95	0.004888
Average	0.00407078



Hard Input Puzzle Run Times

2 Multiple Encodings

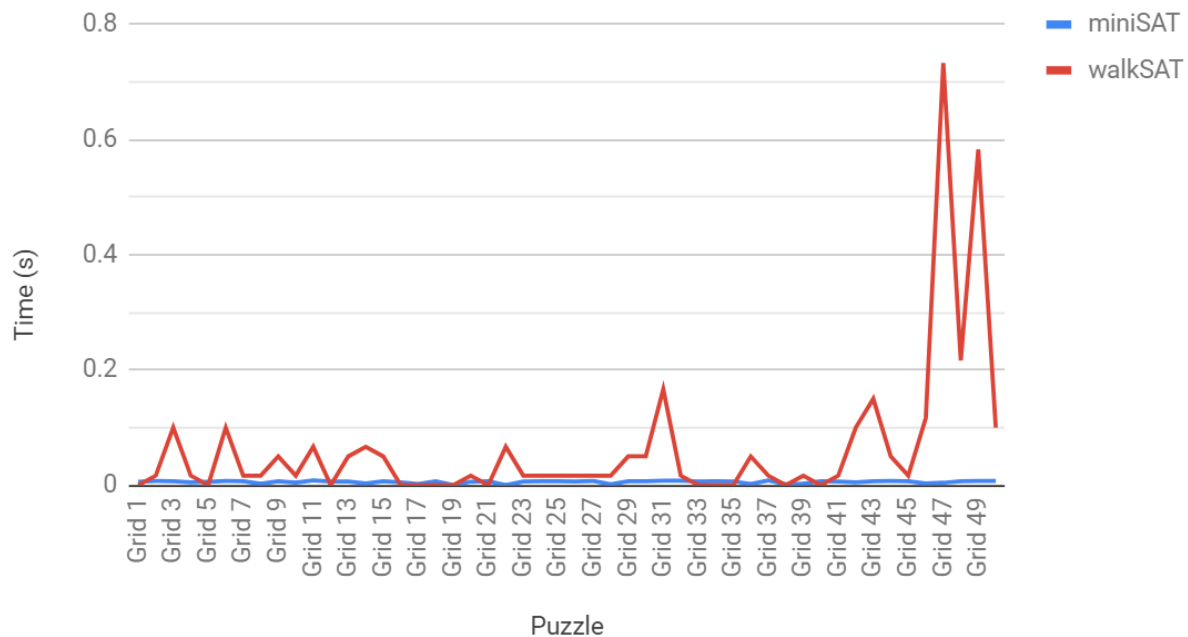
Dependent on how the input of the Sudoku puzzles were encoded, the computational time needed varies with the size of the input. In the case of multiple encodings, we modified the *sud2sat.py* script to accept a wide variety of text file sudoku encodings. The additional parsing slows down the computation, but the added functionality makes it more versatile to different inputs. Below is a list of the accepted encodings for *sud2sat2.py*:

- Separated by '.'
- Separated by '|'
- Separated by '+'
- Separated by '-'
- Separated by ''

3 Use of other SAT solvers - walkSAT

WalkSAT is another SAT solver. Walksat attempts to find a satisfying model of a generalized cnf formula just as miniSAT does. WalkSAT proved to be much slower than the given miniSAT solver. Using the command “./walksat -numsol 1”, walkSAT will stop after finding the first satisfiable solution but is capable of finding many. MiniSAT out performed walkSAT with an average runtime of 0.005626 as compared to walksAT’s average runtime of 0.0640000.

Run-time Comparison - miniSAT vs. walkSAT



Runtime Comparion - miniSAT vs. walkSAT		
Puzzle	miniSAT	walkSAT
Grid 1	0.005682	0.000000
Grid 2	0.007173	0.016667
Grid 3	0.007033	0.100000
Grid 4	0.004792	0.016667
Grid 5	0.005828	0.000000
Grid 6	0.007300	0.100000
Grid 7	0.006783	0.016667

Grid 8	0.002326	0.016667
Grid 9	0.006963	0.050000
Grid 10	0.004443	0.016667
Grid 11	0.008062	0.066667
Grid 12	0.006566	0.000000
Grid 13	0.006312	0.050000
Grid 14	0.003213	0.066667
Grid 15	0.006758	0.050000
Grid 16	0.004687	0.000000
Grid 17	0.002127	0.000000
Grid 18	0.006610	0.000000
Grid 19	0.000000	0.000000
Grid 20	0.006105	0.016667
Grid 21	0.006931	0.000000
Grid 22	0.000000	0.066667
Grid 23	0.006251	0.016667
Grid 24	0.006606	0.016667
Grid 25	0.006741	0.016667
Grid 26	0.006550	0.016667
Grid 27	0.007483	0.016667
Grid 28	0.001601	0.016667
Grid 29	0.006717	0.050000
Grid 30	0.006825	0.050000
Grid 31	0.007770	0.166667
Grid 32	0.007621	0.016667
Grid 33	0.006297	0.000000
Grid 34	0.006940	0.000000
Grid 35	0.006591	0.000000
Grid 36	0.002071	0.050000
Grid 37	0.008056	0.016667

Grid 38	0.001686	0.000000
Grid 39	0.002369	0.016667
Grid 40	0.006751	0.000000
Grid 41	0.006467	0.016667
Grid 42	0.004782	0.100000
Grid 43	0.006736	0.150000
Grid 44	0.007224	0.050000
Grid 45	0.006706	0.016667
Grid 46	0.002884	0.116667
Grid 47	0.004185	0.733333
Grid 48	0.006865	0.216667
Grid 49	0.007483	0.583333
Grid 50	0.007365	0.100000
Sum	0.281317	3.200007
Average	0.005626	0.064000