Towards a Universal Veering Profile for Turbulent Ekman Flow at arbitrary Reynolds number - Part 2

LES and DNS of Turbulent Ekman Flow

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Abstract

~~The~~ turbulent Ekman ~~layer~~ flow is the simplest representation of the atmospheric boundary layer that exhibits two key properties: the logarithmic increase of the wind speed near the surface and a turning of the wind vector aloft. In part I of this work, we derived a ~~theoretic~~ formulation of the mean wind velocity profiles of the stationary turbulent Ekman flow based on direct numerical simulation~~s~~ (DNS). Here, we explore~~d~~ the feasibility of extrapolating these ~~theoretical~~ profiles to atmospheric Reynolds numbers using large-eddy simulation~~s~~ (LES). We compare~~d~~ ~~the~~ theoretical wind speed profiles of a ~~very~~ low - Reynolds - number turbulent Ekman flow to ~~simulations by~~ results from DNS and LES. This necessitate~~d~~ - s the inclusion of viscous effects, ~~that are~~ usually neglected in LES ~~codes~~, along with modifications to the standard bottom boundary condition. Analysis of the grid resolution dependency showed a convergence of LES results towards the theoretical profiles for low and high Reynolds numbers. LES results confirmed that within the logarithmic layer (the lowest 10 % of the boundary layer), the turning of the wind vector contributes approximately one-third to the total rotation. The notable agreement of the theoretical formulation and LES results raised our confidence in the reliability of the theoretical profiles for high Reynolds numbers. This reinforces their utility as a true reference for intermediate and a quasi-reference for higher Reynolds numbers simulations.

Keywords — Large-Eddy Simulation · Scale separation · Ekman layer · Prandtl layer · Hodograph

1 Introduction

Large Eddy simulation~~s~~ (LES) is ~~are~~ widely used to model turbulent flow~~s~~ in the atmospheric boundary layer (ABL) [Stoll et al., 2020]. This includes various applications, such as simulating wind turbines, wind farms, turbine wakes, and interactions within wind farm clusters [Porté-Agel et al., 2011, Mehta et al., 2014, Breton et al., 2017]. Furthermore, LES is applied ~~over~~ ir complex environments like mountainous or urban areas [Stoll et al., 2020, García-Sánchez et al., 2018]. These applications typically focus on the lower segment of the ABL on the rotating earth, known as the Prandtl or surface layer. In this layer, the vertical wind speed profile commonly exhibits a logarithmic increase, followed by prominent a change of wind direction within the Ekman layer above. Accurate characterization of how wind speed and direction vary with height is of great importance for wind-power forecasting and projection [Optis et al., 2014].

The simplest representation of the ABL taking into account rotation *and* the quasi non-turbulent, free atmosphere aloft is the turbulent Ekman flow. It is a horizontally homogeneous, statistically stationary boundary-layer flow over a rotating flat surface. Here we consider the problem under neutral stratification, where the potential temperature is constant across the whole domain. Turbulent Ekman flow shows the key characteristics shaping the real ABL: a logarithmic layer and the Ekman spiral. Near the surface, viscous forces dominate the flow. The stationary ABL-height is then defined by the interplay between turbulent growth due to the shear instability of the configuration [Lilly, 1966] and rotational suppression of turbulence due to the Coriolis effect. The system's statistical equilibrium is defined by one single parameter, the Reynolds number **Re**, signifying the scale separation between the largest and smallest scale within the problem. The largest scales are given by shear-induced eddies constrained by the boundary layer~~'s~~ height, while the smallest scales reside within the dissipative range, where strong viscous forces suppress ~~all~~ turbulence. The dimensional solution of the turbulent Ekman layer is influenced by three parameters: the geostrophic wind G, the Coriolis parameter f, and the kinematic viscosity ν. These three parameters ~~collectively~~ define the Reynolds number $Re_D = G/\sqrt{\frac{1}{2}\nu f}$ ($Re_D = GD/\nu$ and $D = \sqrt{2\nu/f}$). als Gleichung mit Nummer freistellen

~~In~~ part I of this publication~~, we~~ presents a theoretical framework to p wind direction and speed across the turbulent Ekman flow, drawing from the works of Csanady [1967], Tennekes [1973], and Spalart [1989]. While earlier descriptions of the turbulent Ekman layer were limited to specific parts of the boundary layer, we give a comprehensive theoretical description of the mean velocity profiles that covers the complete ABL and quantifies the Reynolds-number dependency. ~~dary layer. This means that our theoretical framework predicts the wind speed and wind direction for every height in the boundary layer for arbitrary Renolds numbers of turbulent Ekman layers as defined above.~~ [do we have to be more specific and more clearly define *the* turbulent Ekman flow (TEF)?] We have shown that the theoretical velocity profiles are in excellent agreement for low and intermediate Reynolds number~~s~~. NO PARAGRAPH BREAK HERE

However, it remains uncertain, if the extrapolation of the theory towards atmospheric **Re** is valid. The substantial separation of scales in the ABL~~s~~ renders direct numerical simulation (DNS) of the entire turbulent flow at scale impossible. Within the framework of LES, only the larger eddies are resolved, that contain the main share of kinetic energy, while turbulent mixing below the subfilter-scale is parameterized. This technique allows for the simulation of the ABL~~s with~~ at very high (atmospheric) Re~~ynolds numbers~~.

Here, ~~In our study~~, we examine three different Reynolds numbers Re_D: 1 600, 150 000, and 1 000 000. While the low-Re case ($Re_D = 1600$) is also investigated by DNS (cf. part 1 of this study), $Re_D = 1.5 \cdot 10^5$ and 10^6 correspond to the scale separation found in typical atmospheric boundary layers. This approach raises our confidence in the LES results to a new level for the direct comparison with a resolving simulation. from DNS Furthermore, it allows us to extend the insight to atmospheric scale where a direct approach is clearly infeasible for computational constraints.

An assumption in LES usage presumes accurate reproduction of first-order velocity profiles [Fedorovich et al., 2004]. ~~Our investig~~ Here, we compare LES outcomes to theoretical solutions to determine the validity of this assumption under different conditions. As LES inherently introduces the grid size as an additional (artificial) parameter, we conduct a thorough analysis of its impact on simulation results. Esau [2004] and Jiang et al. [2018] have executed similar LES analyses of neutral Ekman layers, however, they introduce an additional parameter by selecting an arbitrary roughness length z_0, whereas we derive the parameter z_0 corresponding to a fluid with a kinematic viscosity of $\nu = 1.5 \cdot 10^{-5}$ over flat surface.

The mean velocity profiles for all **Re** exhibit a dual scaling nature: in proximity to the lower boundary, the velocity profiles collapse when scaled using viscous (inner) units ($z^+ = zu_\star/\nu$, $U^+ = U/u_\star$). From the top perspective, the profiles converge when scaled using outer units ($z^- = z/\delta$, $U^- = U/G$, with $\delta = u_\star/f$ representing the boundary layer height). The outer and inner velocities G and $u_\star$, respectively, are linked by a semi-empirical law Spalart [1989]. At the lower boundary, the x-axis of the inner units aligns with the shear stress, while the x-axis of the outer units aligns with the geostrophic wind. The angle between both axes is denoted ~~as~~ by $\alpha_\star$.

The content is structured as follows. Section 2 provides a complete mathematical description of the turbulent Ekman layer's velocity profiles. A description of the simulated cases and the numerical set-up is given in section 3, followed by the presentation of results and its comparison to the theoretical profiles in section 4. We conclude in section 5.

2 A Universal Velocity Profile for the Turbulent Ekman Layer

A universal velocity profile for the turbulent Ekman layer is developed in part I of this paper. In this chapter, we give a brief mathematical description.

2.1 Drag-Law

The geostrophic drag $Z \equiv u_\star/G$ and the angle between the shear stress and the geostrophic wind $\alpha_\star$ are two key parameters of the Ekman flow. They can be estimated using a semi-empirical drag-law based on Spalart [1989], which describes them as functions of only the Reynolds number:

$$\frac{G}{u_\star}\cos\phi^\star = \frac{1}{\kappa}\log Re_\tau + C - A_r, \tag{1a}$$

$$\sin\phi^\star = A_i\frac{u_\star}{G}, \tag{1b}$$

$$\alpha_\star = \phi^\star - \frac{C_5}{Re_\tau}, \tag{1c}$$

use separate numbered equation for Re_\tau

with $Re_\tau = \frac{Re_D^2}{2}\frac{u_\star^2}{G^2}$, $\kappa = 0.416$, $A_r = 4.80$, $A_i = -5.57$, $C = 5.4605$, $C_5 = -57.8$. This law is in excellent agreement with DNS in the range $400 \leq Re_D \leq 1\,600$ as demonstrated by Ansorge and Mellado [2014].

2.2 Total profile

As frame of reference we use the coordinate system Oxyz with Ox in the direction of the surface stress, Oz normal to the surface, and Oy in the span-wise direction normal to Oxz. The profile of the stream-wise component of the velocity is seperated into three layers, which are the viscous layer U_{visc}, the logarithmic layer U_{log}, and the Ekman layer U_{EK}. U_{visc} and U_{log} are matched by their formulation and combined to U_{inner}. The span-wise component of the velocity is separated into two layers, namely the inner layer V_{inner}, and the Ekman layer V_{EK}. The smooth transition between consecutive layers is achieved using a transfer function:

$$w_* = \frac{1}{2}\left(\operatorname{erf}\left[\sigma_T \log\left(\frac{z}{z_T}\right)\right] + 1\right), \tag{2}$$

where σ_T is a transition scale that defines the width of the transition and z_T is the height of the transition, where the upper and the lower layer equally contribute to the velocity ($w_*(z_T) = 0.5$).

$$U = (1 - w_{outer})U_{inner} + w_{outer}U_{EK}, \tag{3a}$$

$$V = (1 - w_{outer})V_{inner} + w_{outer}V_{EK}. \tag{3b}$$

2.3 Profiles in the Outer Layer

the

In the outer layer, the vertical change of the Coriolis force causes a pronounce

~~Above the boundary layer, in the outer layer, the~~ ~~the coriolis force and the wind speed equals the geostrophic wind G. For the stationary case, the horizontal equations of motion are solved by~~

$$U_{EK} = G + Ae^{-\tilde{z}}\cos\tilde{z}, \tag{4a}$$

$$V_{EK} = -Ae^{-\tilde{z}}\sin\tilde{z}, \tag{4b}$$

where the x-axis is aligned with the geostrophic wind and $A = 8.4u_\star$, $\tilde{z} = (z - z_r)/D_E$, $z_r = 0.12\delta$, and $D_E = 3\delta/4\pi \approx 0.24\delta$. ~~The parameters are deduced from DNS.~~ The transition from the logarithmic layer to the Ekman layer is located at $z_T^- = 0.3 - 120/Re_D$ [in script at 0.28-2.25*np.sqrt(1./re)] with a transition scale of $\sigma_T = 2$ for the stream-wise velocity.

2.4 Shear-Aligned Velocity

In the viscous sublayer, the span-wise velocity is close to zero and the shear-aligned velocity is described by the law of the wall:

$$U^{\alpha_\star +} = z^+, \tag{5}$$

(the index $\alpha_\star$ indicates the alignment with the direction of the shear stress). Around $z^+ = 5$, the velocity is beginning to deviate from its linear profile and the buffer layer forms the transition between viscous layer and logarithmic layer. From the surface up to the buffer layer, the stream-wise velocity is described by

$$U^{\alpha_\star +}_{visc} = \frac{z^+}{1 + c_1 (z^+)^2} + (c_2 z^+ - a_{match}) \frac{1 + \tanh[0.2(z^+ - 22)]}{2} + c_3 e^{-c_4 (z^+ - 22)^2}. \tag{6}$$

With $c_1 = 0.00185$, $c_2 = 0.195$ [adapt c_2?], $c_3 = 0.4$, $c_4 = 0.35$. The coefficient $a_{match} = 3.5727$ is chosen to match the u-profile in the logarithmic layer above at $z^+ = 40$.

The logarithmic region of the stream-wise velocity is

$$U^{\alpha_\star +}_{\log} = \frac{1}{\kappa} \log z^+ + C, \tag{7}$$

with the von-Kármán constant $\kappa = 0.416$, and $C = 5.4605$. The lower part of $U^{\alpha_\star +}$ is described by

$$U^{\alpha_\star +}_{inner} = \begin{cases} U^{\alpha_\star +}_{visc}, & z^+ < 40. \\ U^{\alpha_\star +}_{log}, & z^+ > 40. \end{cases} \tag{8}$$

The inner profile is then blended to the Ekman profile using eq. 3 with $\sigma_T = 2$ and $z_T^- = 0.28 - 2.25\sqrt{1/Re_D}$.

2.5 Span-wise Velocity

In the viscous layer, the shear-aligned span-wise velocity is described as

$$V^{\alpha_\star}_{visc} = \frac{G}{\delta^+} v_{ref} (w_v z^+ - 1 + exp[-w_v z^+]). \tag{9}$$

The choice of $v_{ref} = 18.85$ and $w_v = 0.2353$ leads to excellent agreement with the DNS data.

Above the viscous layer a log-like transition to the Ekman layer is modeled by

$$V^{\alpha_\star}_{log} = \frac{G}{\delta^+} (a_{log} + b_{log} \log(z^+) + c_{log} z^+). \tag{10}$$

A smooth transition to V_{visc} at $z^+ = 10$ and the condition $V_{log}(z^- = 0.3) = V_{EK}(z^- = 0.3) =: v_{03}$ lead to

$$c_{log} = \frac{v_{03} - v_{10} - d_{10} z_{10} \log(z_{03}/z_{10})}{z_{03} - z_{10}}, \tag{11}$$

$$a_{log} = v_{10} - d_{10} z_{10} \log z_{10} + c_{log} z_{10} (1 - \log z_{10}), \tag{12}$$

$$b_{log} = (d_{10} - c_{log}) z_{10}. \tag{13}$$

Here, $d_{10} = 4.01$, $v_{10} = 27.3$. Because of the **Re** dependence of V_{Ek}, v_{03} also depends on **Re**. $V^{\alpha_\star}_{log}$ is blended into V_{EK} using eq. 3 with $\sigma_T = 2$ and $z_T^- = 0.28 - 2.25\sqrt{1/Re_D}$.

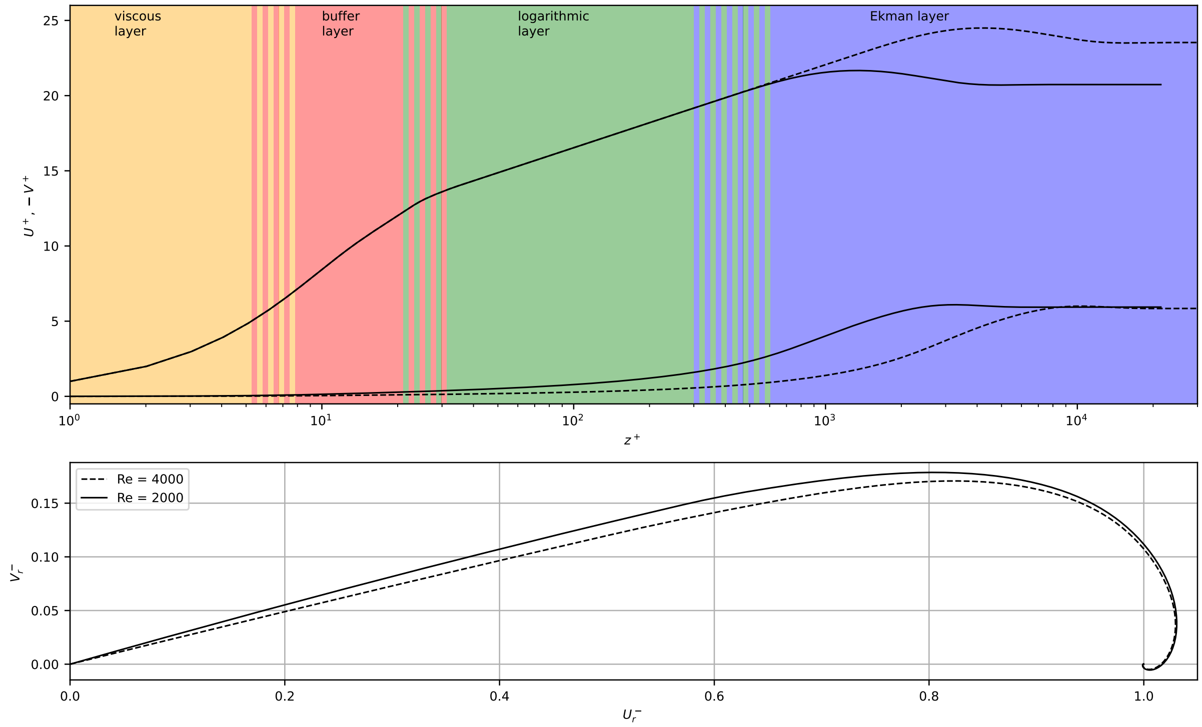

Figure 1: a) Theoretical shear-aligned velocity profile (U^+,$-V^+$) of the turbulent Ekman flow for $Re_D = 2000$ (——) and $Re_D = 4000$ (- - - -) and the different layers (colors). b) Hodograph of the geostrophy-aligned velocity components
.

2.6 Profile

The velocity profiles are given for two Reynolds numbers in Fig. 1. In the viscous sublayer, the shear-aligned component U^+ increases linearly with height up to the buffer layer between $z^+ \approx 5$ and $z^+ \approx 30$, where it transitions into the logarithmic layer. Then, U^+ increases logarithmically up to the Ekman layer and reaches its supergeostrophic maximum. Above, it decreases to its geostrophic value. The length of the logarithmic layer increases with **Re**. The spanwise component V^+ remains close to zero up to the middle of the logarithmic layer, where the transition to the Ekman layer takes place ($z^- \approx 0.28 - 2.25\sqrt{1/Re_D}$). The profiles of V^+ of all Reynolds numbers have a similar shape but are shifted in z^+. The similar V^+ and the growth of U^+ leads to a smaller angle $\alpha_\star$ between surface shear stress and geostrophic wind for higher **Re**, which is visible in the hodograph.

3 Case description and numerical set-up

3.1 Settings

An incompressible, turbulent Ekman flow over a hydrodynamically smooth surface is simulated for three different Reynolds numbers $Re_D = 10^3; 1.5 \times 10^5; 10^6$, hereafter $Re1$, $Re2$, and $Re3$, respectively. The input parameters are given in table 1.

The domain is rotating around the z-axis with an angular velocity corresponding to the Coriolis parameter $f = 10^{-4}$ 1/s, ~~a value representative of mid-latitude synoptic systems~~. The stratification of the flow is truly neutral, i.e., the potential temperature $\Theta = const.$ for the whole domain. Nevertheless, the boundary layer height is not growing infinitely but results from a balance between shear production and rotational suppression of turbulence. At the sufficiently far aloft of the boundary-layer height upper boundary, a no-penetration boundary condition is used and the horizontal components of

Table 1: ~~Key~~ parameters of the simulated cases. f, ν, and G are input parameters, while $u_\star$, $\alpha_\star$, and $\delta = u_\star/f$ are resulting properties of the flow ~~according to Spalart [1989]~~

Name	Re_D	Re_τ	f [s^{-1}]	ν [m^{2}s^{-1}]	G [ms^{-1}]	$u_\star$ [ms^{-1}]	$\alpha_\star$ [°]	δ [m]
Re1	1.6×10^3	3.0×10^3			0.0438	0.00211	16.8	21.1
Re2	1.5×10^5	7.3×10^6	10^{-4}	1.5×10^{-5}	4.108	0.1048	8.5	1048
Re3	1×10^6	2.2×10^8			27.39	0.5785	7.0	5785

Table 2: Simulations and grid parameters: ReX stands for one of the Reynolds numbers Re1, Re2, and Re3. Δ is the grid cell size, $\delta = u_\star/f$ is the boundary layer height, n_i is the number of grid cells in the direction Oi, L_x and L_z are the domain sizes in the horizontal and vertical direction, respectively

Name	Δ^-	n_x	n_y	n_z	L_x/δ	L_z/δ
ReX_50	1/50	192	192	128	3.84	5.0
ReX_100	1/100	384	384	216	3.84	4.5
ReX_150	1/150	576	576	288	3.84	3.7
ReX_200	1/200	768	768	384	3.84	4.1
ReX_dyn	1/200	768	768	384	3.84	4.1

the wind are ~~forced to equal the~~ geostrophic ~~wind through~~ a Dirichlet-type boundary condition. At the bottom, a constant flux layer is assumed and Monin-Obukhov similarity theory (MOST) is used to calculate the surface momentum fluxes. The Navier-Stokes equations are integrated using a 3rd-order low-storage Runge-Kutta method. For scalar advection a 5th-order Wicker-Skamarock scheme is employed. A comprehensive description of the LES model is given by Maronga et al. [2020a].

To study the effect of resolution on the simulations, four different grid resolutions are chosen for each Reynolds number case. The grid cell size Δ is around $\delta/50$, $\delta/100$, $\delta/150$, and $\delta/200$ one coarse, two medium, and one fine resolution, respectively (see table 2).

The grid spacing inside the boundary layer is isotropic up to ~~the height~~ $z = 1.3\delta$. Aloft, the grid spacing along Oz is stretched by 3% per grid point until a maximum spacing of $(\Delta z)_{\max} = 6\Delta x$. The number of vertical grid points is chosen such that $L_z \geq 3\delta$. Different domain heights are caused by numerical requirements of the FFT-solver. In the upper third of the domain, Rayleigh damping is active to avoid reflection~~s~~ from the ~~upper~~ boundary.

The flow is initialized by a ~~1d~~-model with a Reynolds-average based turbulence parametrization. At the beginning of the simulation, random perturbations are imposed on the velocity field to trigger turbule~~[illegible]ility~~. The resulting imbalance between pressure force and Coriolis force results in an inertial oscillation of the period $T_{io} = 2\pi/f$, where a part of the flow's mean kinetic energy [illegible] between U- and V-component. The oscillation ~~slowly~~ decays over time and eventually vanishe~~s~~. In order to obtain ~~reliable~~ profiles, we use a spin-up time of 1.5 T_{io} and perform a horizontal domain average over 2 T_{io}.

In part I of this publication, the DNS of $Re_D \leq 1\,600$ is carried out for a horizontal domain size of $(0.54\Lambda_{Ro})^2$ - $(1.08\Lambda_{Ro})^2$, where $\Lambda_{Ro} = G/f$ is the Rossby radius. For these Reynolds numbers, $u_\star/G$ is around 0.05 so $L_x \approx 10\delta$ However, $u_\star/G$ decreases with increasing Reynolds number and is around 0.02 for $Re_D = 10^6$. A domain size of half the Rossby radius would then extend to $L_x \approx 25\delta$. Such a large domain would imply immense computational costs. Spalart et al. [2008] used a horizontal domain of $L_x = 2\delta$, arguing that this length include the largest outer-layer eddies according to Csanady [1967]. During a sensitivity test of the domain size we observed that simulations with domain sizes $L \geq 4\delta$ often tend to accumulate turbulence kinetic energy in the upper half of the the boundary layer. This TKE increases over several inertial oscillations with energy mostly on the scale of the domain size. Such a development was not observed in the DNS. We could successfully avoid such a behaviour by using a domain of size $L = 3\delta$ in combination with a shifted periodic boundary condition in y-direction, as described by Munters et al. [2016]. Although for the Ekman flow the direction of the mean flow is only aligned with the x-direction of the grid near the surface, a shift of the boundary condition by $L/3$ significantly helped to suppress the accumulation of TKE in the upper half of the boundary layer.

In LES, the turbulent transport on the subgrid scale (SGS) needs to be modeled by a dedicated model, the SGS model. The model ~~code~~ of PALM [Maronga et al., 2020a] offers two different SGS models: a 1.5-order closure after Deardorff [1980] and a dynamic closure after Heinz [2008]. For most of the simulations, the 1.5-order closure is used, but the simulations with $\Delta^- = 200^{-1}$ are repeated using the dynamic closure.

3.2 Viscosity and roughness length

In LES, one postulates that a sufficient part of the largest eddies is resolved so as to represent dominant the non-linear effects of turbulent mixing [Pope, 2004]. Below these resolved scales, turbulence is modeled as a more or less isotropically acting diffusive agent by a closure model (dynamic, Deardorff, see above). Thus, molecular friction is not considered directly, but only by virtue of a turbulence model linking the resolved and dissipative scales. In their seminal works on the spectral energy transfer in homogeneous isotropic turbulence, Kolmogorov and Obukhov (Kolmogorov [1941],Obukhov [1941]) showed that the energy transfer rate across the spectrum is in fact a constant. This implies that the transfer rate across the cut-off scale in LES does not depend on the magnitude of the viscous scale, presupposed that (i) the cut-off scale of the LES is well within the inertial range and (ii) the LES turbulence is approximately isotropic and homogeneous at the smallest resolved scales. Consequently, SGS-models of LES do not necessarily require explicit information about the actual viscosity of the fluid or other viscous parameters. In LES ~~of flows with relatively~~ at low **Re** or very high resolution ~~leads to results where~~ the ~~model yields~~ eddy viscosities may fall below the molecular viscosity of air ~~$\nu = 1.5 \times 10^{-5} m^2/s$~~. ~~The eddy viscosity~~ in the context of PALM's Deardorff clo where $c_0 = 0.1$ [Deardorff, 1980], Δ is the grid size and e is the SGS-TKE, calculated by a prognostic equation. Hence, very low e as well as fine resolution can lead to $K_m < \nu$. When this is the case, ν cannot be ignored anymore, and we let $K_m = c_0 \Delta \sqrt{e} + \nu$. The governing equation of the velocity components in PALM reads

$$\frac{\partial u_i}{\partial t} = -\frac{\partial u_i u_j}{\partial x_j} - \epsilon_{ijk} f_j u_k + \epsilon_{i3j} f_3 u_{g,j} - \frac{1}{\rho_0}\frac{\partial \pi^*}{\partial x_i} + \frac{\partial}{\partial x_j}\left(K_m\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right) \tag{14}$$

for a neutrally stratified flow. In the limit of either very well resolved simulations or very low **Re** (or both), $K_m \to \nu$, so the last term on the right hand side of eq. (14) becomes $\nu \frac{\partial^2 u_i}{\partial x_j^2}$, which complies with the Navier-Stokes equations for an incompressible fluid.

In contrast to the interior closure of the LES where the direct effect of ν is of small relevance compared to the eddy viscosity–at least for high **Re**–, this is not true for the viscid effects at the bottom boundary. Here, a constant flux layer is usually assumed below the first grid point and the Monin–Obukhov similarity theory is used to compute the friction velocity and the stresses at the first half grid point (where the horizontal velocities are located on the Arakawa-C staggered grid):

$$u_\star = \frac{\kappa (U^2 + V^2)^{0.5}}{\ln(z/z_0)}, \tag{15}$$

$$-\overline{u''w''}_0 = \frac{\kappa U u_\star}{\ln(z/z_0)}. \tag{16}$$

In these expressions, viscosity enters indirectly by virtue of the roughness length z_0 when considering the law of the wall for a smooth surface:

$$u^+ = \frac{1}{\kappa}\ln(z^+) + C^+ = \frac{1}{\kappa}\ln\left(\frac{z^+}{z_0^+}\right). \tag{17}$$

This leads to the expression

$$z_0^+ = z_0\frac{u_\star}{\nu} = \exp\left(-\kappa C^+\right), \tag{18}$$

and for an aerodynamically smooth flow, $z_0^+ \cong 0.1$ (using the same values as in eq. 7). This is known to be the minimal roughness length of a turbulent boundary layer (see e.g. Kraus [2008]). The roughness length in SI-units

$$z_0 = z_0^+\frac{\nu}{u_\star} \approx 0.1\frac{\nu}{u_\star} \tag{19}$$

hence depends on the viscosity of the fluid, which means that—given fixed surface properties—the choice of z_0 implies a particular value for the viscosity ν.

When using MOST for the surface fluxes it is assumed that the height of the first grid point lies inside of the logarithmic layer. Again, the limit of low **Re** and high resolution requires adaptations to this boundary condition. In the case of very fine resolution, the first grid point might fall into the buffer layer or even the viscous layer of the flow, so the equations of MOST are no longer adequate to calculate the local stress. To avoid that, we follow the recommendation of Kawai and Larsson [2012] to use the horizontal velocity from a higher layer z_{sl} to compute the mean stress in the constant flux layer. Furthermore, we adopt the boundary condition suggested by Maronga et al. [2020b] and use the domain averaged velocities:

$$u_{*,mean} = \frac{\kappa < u_h(z_{sl}) >}{\ln(z_{sl}/z_0)}, \tag{20}$$

where $u_h = \sqrt{u^2+v^2}$ and angle brackets refer to the horizontal average over the entire domain. The mean stress is then used as a boundary condition at the first grid point ($z = z_1$). It is distributed locally to the stresses in x- and y-direction via

$$\overline{u'w'}_0(x,y,z_1) = -u_{*,mean}^2\frac{u(x,y,z_1)}{\sqrt{< u >^2(z_1) + < v >^2(z_1)}}, \tag{21}$$

and accordingly for $\overline{v'w'}_0$. This way, the domain average of the stress components yield the total stress of eq. 20 ($u_{*,mean}^2 = \sqrt{< \overline{u'w'}_0 >^2 + < \overline{v'w'}_0 >^2}$). As reference height we use $z_{sl}^- \approx 0.1$. By using this higher reference height for the boundary condition we solve two problems: First, the reference height is inside of the logarithmic layer, second, we use a velocity from a region where the flow is much better resolved than close to the surface.

4 Results and Discussion

In this chapter, we compare the results from the LES to the theoretical bulk parameters (4.1) and velocity profiles (4.2) and discuss the dynamics of the turbulent flow in LES.

4.1 Bulk parameters

A key parameter governing the turbulent state of the flow is the geostrophic drag: it puts the surface friction in relation to the geostrophic wind and thus illustrates the conversion of mean-flow kinetic energy to turbulence. The ratio of $u_\star$ to the theoretical value $u_{\star,th}$ resulting from

Table 3: Simulation results

Name	Δ^+	$u_\star/u_{\star,th}$	$\alpha_{\star,th}-\alpha_\star$	κ_{LES}	$z_{0,LES}/z_0$	δ_{95}/δ
Re1_50	59	1.021	1.5°	-	-	0.71
Re1_100	30	1.008	1.1°	0.52	0.14	0.66
Re1_150	20	1.004	1.0°	0.48	0.33	0.64
Re1_200	15	1.003	0.7°	0.46	0.47	0.62
Re1_dyn	15	1.002	0.9°	0.43	0.78	0.58
Re2_50	1.5×10^5	1.008	1.1°	-	-	0.77
Re2_100	7.3×10^4	1.001	0.5°	0.53	0.02	0.66
Re2_150	4.9×10^4	1.000	0.2°	0.47	0.15	0.60
Re2_200	3.7×10^4	1.000	0.1°	0.44	0.49	0.59
Re2_dyn	3.7×10^4	1.000	0.0°	0.42	1.01	0.55
Re3_50	4.5×10^6	1.007	0.7°	-	-	0.76
Re3_100	2.2×10^6	1.001	0.5°	0.53	0.01	0.65
Re3_150	1.5×10^6	1.000	0.3°	0.47	0.07	0.60
Re3_200	1.1×10^6	1.001	0.1°	0.44	0.33	0.59
Re3_dyn	1.1×10^6	1.000	0.2°	0.43	0.62	0.56

the simulations are shown in Tbl. 3. All values are close to one, while the strongest deviations are observed for the coarsest simulations. A slight dependence on the resolution can be observed for Re1, as $u_\star$ steadily approaches $u_{\star,th}$ with increasing resolution. For Re2 and Re3, all but the coarsest resolution nearly exactly match the theoretical value. The choice of z_0 and the geostrophic wind G determine the magnitude of $u_\star$ in a non-trivial way. From a top perspective, the horizontal velocity increases from its geostrophic value to the supergeostrophic maximum and then decreases with height. The horizontal mean velocity at $z^- \approx 0.1$ is used to calculate $u_\star$ according to eq. 20. It is remarkable that the choice of $z_0^+ = 0.1031$ leads to a value of $u_\star$ very close to the prediction by semi-empirical considerations (cf. Spalart [1989]).

Another key parameter of the Ekman flow is the surface veering angle angle $\alpha_\star$ between the negative surface shear stress and the geostrophic wind. For the LES, the direction of the negative stress is represented by the first derivatives of the velocity components calculated by 3rd order forward finite differences: $\alpha_\star = \tan^{-1}(\langle\partial v/\partial z\rangle_0/\langle\partial u/\partial z\rangle_0) - \tan^{-1}(V_G/U_G)$. Table 3 shows that all LES yield an $\alpha_\star$ smaller than the theoretical value $\alpha_{\star,th}$. In general, we observe an increase of $\alpha_\star$ with resolution. The finest resolution of $Re2$ and $Re3$ even reach 99% of the theoretical value while $Re1\_200$ only reaches 95%. ~~However,~~ Fig. 2 illustrates ~~shows~~ that this is not caused by poor quality of the simulation since all of the simulations of $Re1$ follow the theoretical hodograph quite closely and even overestimate the angle at the respective height by a little. In contrast to the higher Re, the veering of the wind vector still continues in the lower parts of the boundary layer. Hence, ~~So~~ for the low Re case, the general trend of a higher $\alpha_\star$ with finer resolution is caused by the approach of the final $\alpha_\star$ with decreasing height of the first grid point.

4.2 Logarithmic layer stream-wise velocity

Figure 3 shows the velocity profiles from the LES and the theoretical profiles for ~~all~~ each of the three **Re**. In general, the velocity profiles of the LES agree well with the theoretical profiles: The course of the theoretical Ekman layer is matched and the simulations exhibit a logarithmic layer for the U-component. For Re1, the lowest grid point should fall into the buffer layer, which is visible as the curved course of the U-component below the logarithmic layer in the theoretical profile. The best resolved simulation with Deardorff-closure even seems to follow the course of the upper part of the buffer layer, but with a resolution of $\Delta^+ = 15$ this is a coincidence caused by a well-known S-shape of velocity profiles close to rigid walls. The lowest points of the LES of

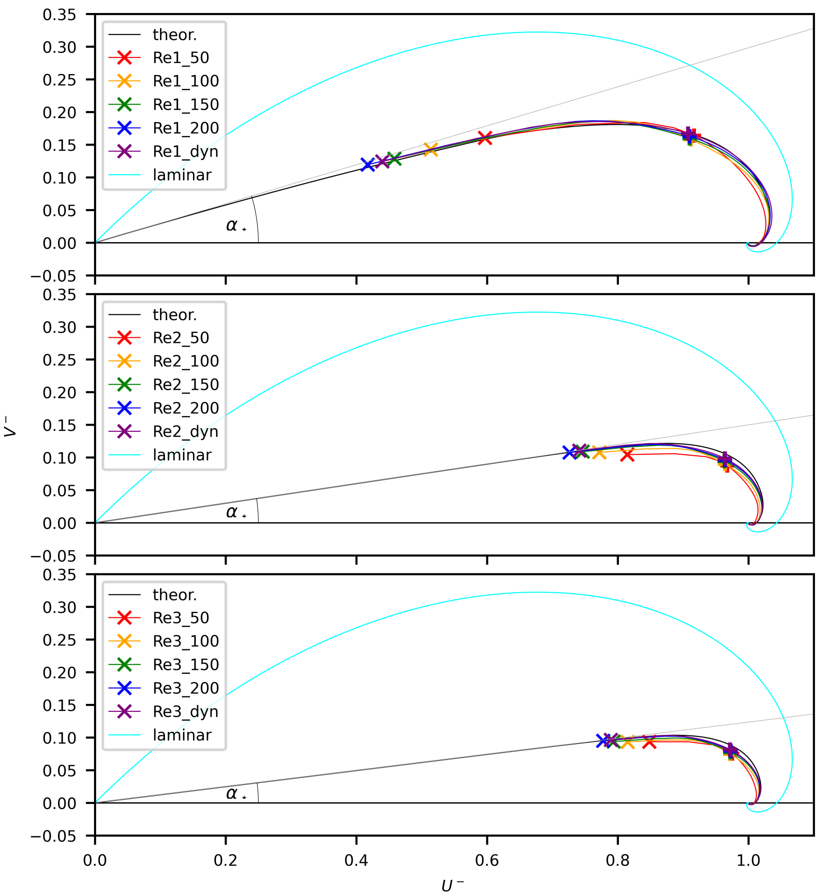

Figure 2: Geostrophy aligned hodographs of the LES in comparison to the theoretical and laminar hodographs. A $\times$ indicates the first grid point of the LES. At the points marked with $+$, the velocity is used to determine $u_\star$

the higher **Re** fall into the logarithmic layer. This log-layer mismatch arises from a competition between the scales $u_\star$ and z and other velocity and length scales introduced by the discretization of the dynamical system [Mason and Thomson, 1992, Brasseur and Wei, 2010]. In other words, at the lower boundary, the relevant eddies are too small to be resolved by the grid and their contribution~~s~~ to the flow ha~~ve~~s to be modeled. Also , The vertical component is massively restricted by the non-permeability of the wall, known as blocking effect. On the contrary, the SGS-closure assumes isotropic turbulence, which is not the case on the grid scale near the wall.

In the logarithmic region, the profile of the u-component should follow the logarithmic law of eq. 7 with the Kármán measure $\kappa = (z^+\partial U^+/\partial z^+)^{-1}$ =const. ~~stant within an ideal logarithmic layer~~. In the low-Reynolds-number case, the viscous sublayer represents about 0.5% of the boundary layer, while this layer is not visible for the high-Reynolds-number cases, where the share drops to $10^{-5} - 10^{-7}$. Above the viscous sublayer, the theoretical profile shows a near-constant value for κ up to $z^- \approx 0.1$ for the case Re1 and up to $z^- \approx 0.12$ for the cases Re2 and Re3. In order to estimate κ_{LES} from the simulations, we perform a linear regression ~~between~~ of the mean velocity in x-direction and the logarithm of ~~the~~ height between the seventh grid point and the grid point corresponding to the height $z^- = 0.1$ for $Re1\_X$ and $z^- = 0.12$ for $Re2\_X$ and $Re3\_X$. We consider only points above the 7th grid point following arguments of Maronga et al. [2020b], that in PALM, the mean velocity profiles follow MOST above the seventh grid point. The number of values for each regression is 6, 12, and 18 for ReX_100, ReX_150 and ReX_200, respectively. We do not compute κ for the coarsest simulations since there are only six grid points below $z^- \approx 0.12$. The resulting κ_{LES} for the other simulations are shown in

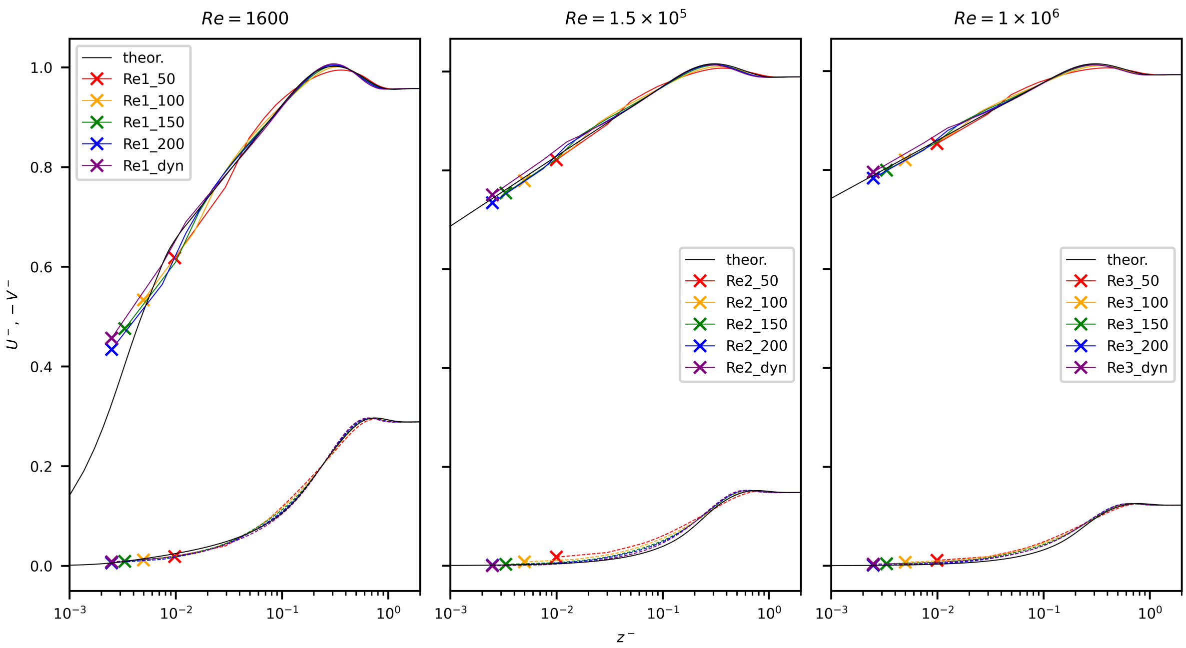

Figure 3: Shear-aligned velocity profiles in outer scaling. Solid lines: u-component, dashed lines: v-component. The lowest grid point is marked by x

Tbl. 3. The coefficient of determination is above 0.99 for all fits. κ_{LES} is decreasing with finer resolution for all **Re**. While the values for ReX_100 are rather high with a value around 0.53, finer resolution leads to more realistic values around 0.46 for $Re1\_200$ and around 0.44 for $Re2\_200$ and $Re3\_200$. The dynamic subgrid closure yields lower values for κ_{LES}: in $Re1\_dyn$ we even see $\kappa_{LES} = 0.39$, while $Re2\_dyn$ and $Re3\_dyn$ yield 0.42 and 0.43, respectively. Figure 4 shows the Kármán measure in the logarithmic layer. An increase in resolution leads to a profile of the Kármán measure closer to the theoretical curve for all cases. Above the first grid points, where the Kármán measure jumps between the values, the curve smoothens and approaches the expected value of κ (at least for the finer resolutions). However the first grid points, the Kármán measure is heavily influenced by the grid cell size Δ. In accordance with the observations of Maronga [2014] and Maronga and Reuder [2017], the kinks in the Kármán measure diminish around the seventh grid point for all simulations. Above the seventh grid point, the profile is no longer influenced by this artefact and becomes relatively constant. Exceptions are the simulations ReX_50, where no constant κ can be observed.

As expected for an LES with isotropic grid spacing, the resolution near the surface is a critical part of the simulation. We can support our above observations by taking a look ~~on~~ at the two-point correlation in the critical layers of the simulations. The two point correlation is defined as

$$B_{xx}(x^*) = \langle u(x - x^*)\, u(x) \rangle / \sigma_u^2. \tag{22}$$

As suggested by Wurps et al. [2020], in an isotropic grid with periodic horizontal boundary conditions the turbulent structures of the velocity's w-component in y-direction can serve as an indicator how well the flow is resolved. Figure 5 shows the two-point correlation $B_{ww}(y^*)$ at ~~in~~ a height of $z^- \approx 0.08$, ~~which is~~ right inside of the logarithmic layer. We define the average size of a structure in the flow σ as the distance where the two-point correlation drops to 0.3 (horizontal line in figure). The number of cells by which the average structure is resolved is equal to σ/Δ. All **Re** show a similar behaviour: the average size of the structure decreases with finer resolution and seems to approach a limit. The structure size of ReX_150 and ReX_200 are very close together, which indicates beginning convergence. Furthermore, the number of

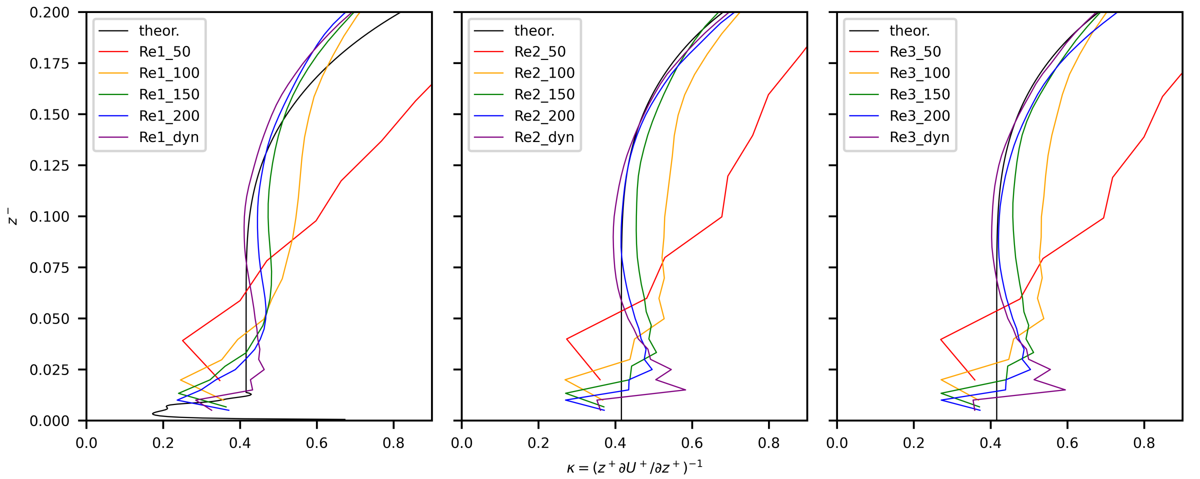

Figure 4: Kármán measure κ in the logarithmic region and above for different Reynolds numbers and resolutions

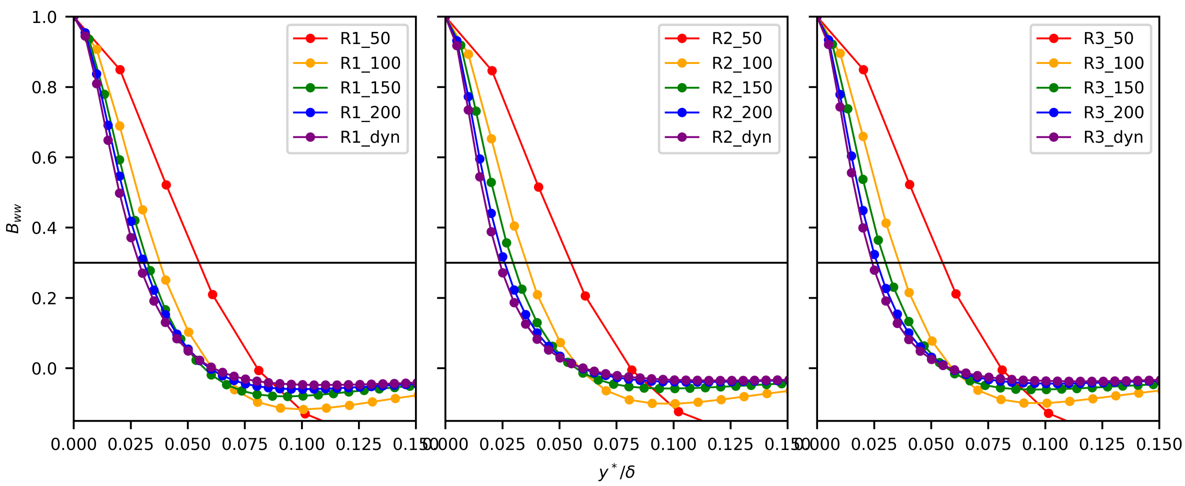

Figure 5: Two-point correlation of w-component in y-direction at $z^- \approx 0.08$. The horizontal line is at $B_{ww} = 0.3$

cells by which the structures are resolved exceed 4 from the resolution of ReX_150 on. This corresponds to the resolution from which on a logarithmic layer with a constant κ can be seen. It also fits to the rule of thumb given in Wurps et al. [2020], that in a sufficiently resolved part of an LES the average structures of the w-component in y-direction should be resolved by at least 4 cells. The dynamic closure shows structures that are slightly smaller than the structures of ReX_200, which is caused by a tendency to lower eddy-viscosities and, hence, a weaker coupling of neighboring grid cells.

Figure 6 shows $B_{ww}(y^*)$ at the 8th grid point above the bottom. Hence, the corresponding heights differ between the simulations, that is $z^- = 0.16, 0.08, 0.53, 0.04$ for ReX_50, ReX_100, ReX_150, ReX_200, respectively. For $Re1$, the curves almost collapse perfectly while for $Re2$ and $Re3$ we see slightly larger structures for ReX_50 and slightly smaller structures for ReX_dyn. This means that even 8 points above the lower boundary the smallest size of the turbulent structures rather depends on the grid cell size than on the actual height above the ground. According to the above findings and Maronga [2014], the flow should start to be well resolved from here on (above the seventh grid point). And indeed, the number of resolving cells is very close to 4 for $Re1$ and between 3 and 4 for $Re2$ and $Re3$, which is close to the recommended 4

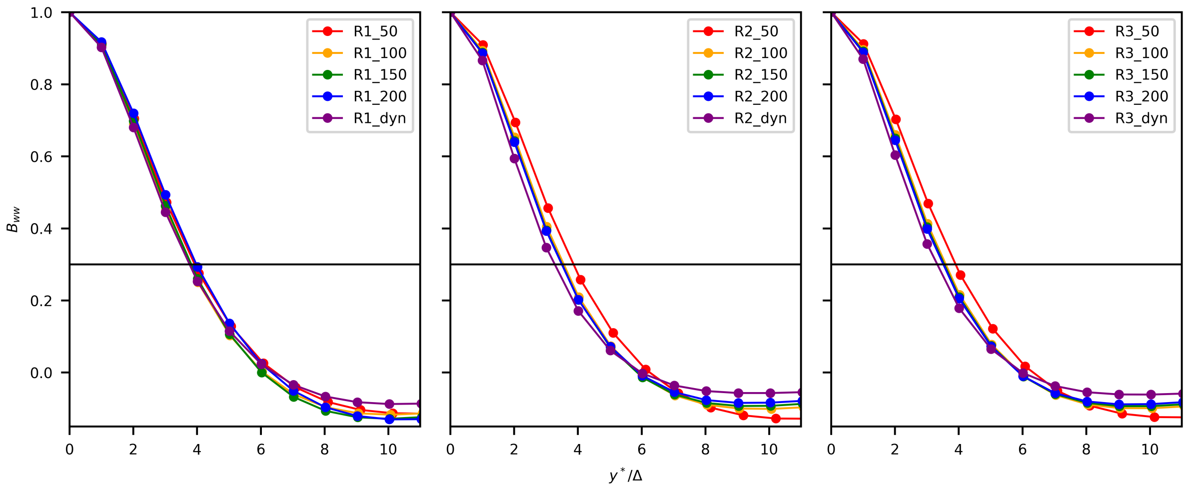

Figure 6: Two-point correlation of w-component in y-direction at the 8th grid point above the surface

cells.

4.3 Logarithmic layer span-wise velocity

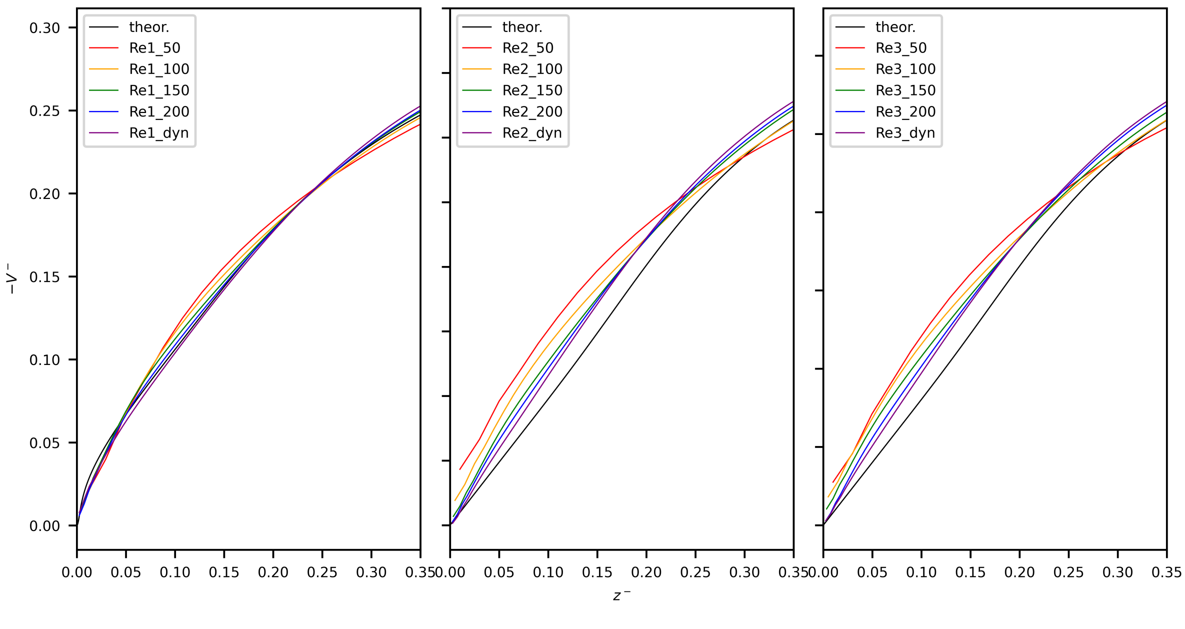

Figure 7: Span-wise velocity component in outer scaling

Figure 7 shows the span-wise velocity component in the lower part of the boundary layer. In the lowest part of $Re1$ ($z^- \lessapprox 0.05$), a part of the viscous layer is visible, where the velocity is slightly underestimated by the the LES. Above, the simulations $Re1\_200$ and $Re1\_dyn$ are very close to the theoretical curve. Lower resolution leads to an overestimation of the velocity in the logarithmic layer.(*Around $z^- = 0.25$, while blending into the Ekman layer, the curves cross each other and the coarse simulations underestimate the velocity. For the higher Re all simulations overestimate the velocity in the logarithmic layer, while coarser resolution leads to a higher velocity and finer resolution leads to a lower velocity and better agreement with the theoretical curve. Again, we see a crossing of the curves, between $z^- = 0.2$ and $z^- = 0.25$. A

(*), which is consistent with a lack of large eddies that translates in a lack of mixing and thus a reduced turbulen possible explanation of the fast incr and the steeper course of the coarser simulations is that the layers are coupled via less cells.

4.4 Ekman layer

Above the logarithmic layer, the Ekman layer follows, characterized by a change of wind direction. The course of the wind velocity vector is visualized by hodographs, as plotted in Fig. 2. The hodograph of Re1_X is followed quite closely by all resolutions. Hence, all simulations–even Re1_50–are resolved sufficiently to closely capture the course of the wind vector in the Ekman layer. The higher **Re** behave differently in the sense that the hodographs lie inside of the theoretical hodograph. An increased resolution ensures that at least the lowest grid point approaches the hodograph while the course of the hodograph's upper right part still does not reach the theoretical curve.

In Fig. 2, the cross indicates the lowest grid point and the plus indicates the height where the velocity for the boundary condition is taken from. To avoid taking a velocity from the first layer of the simulation, where the turbulent flow is poorly resolved, we took the horizontal velocity near $z^- = 0.1$, where the mean velocity already veered away from the direction of the surface stress by around one third of α. However, the veering does not seem to influence the resulting bulk stress $u_\star$ at the bottom: all but the coarsest resolutions yield a $u_\star$ very close to the theoretic reference.

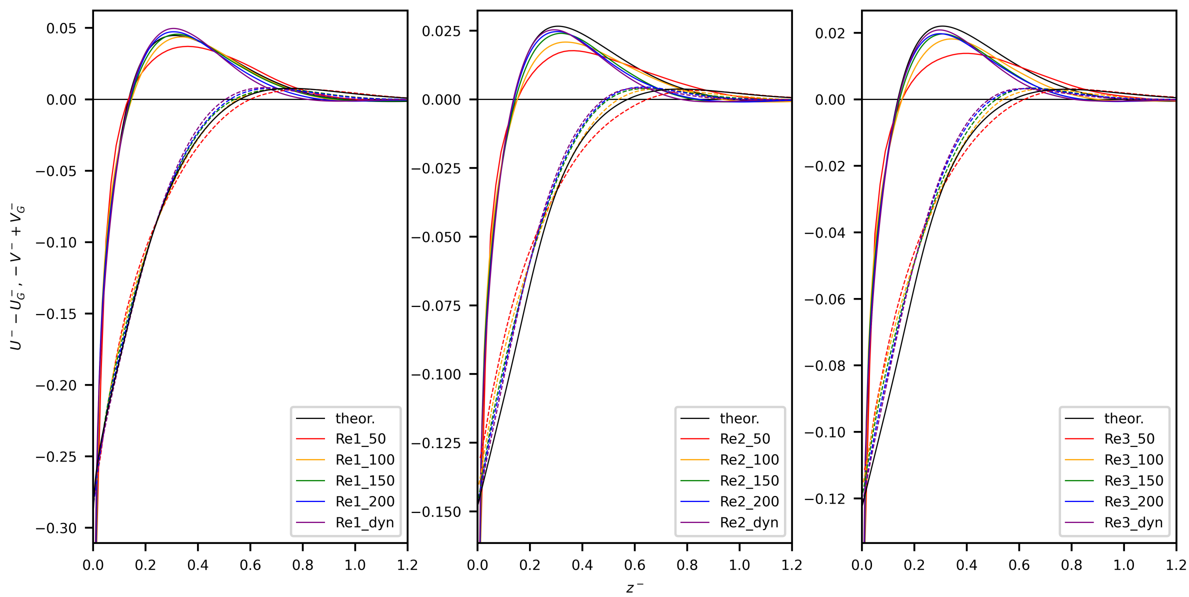

Figure 8: Shear-aligned velocity deficit in outer scaling. Solid lines: U-component, dashed lines: V-component

Figure 8 shows the shear-aligned velocity deficit of the whole boundary layer. The strongest deviations from the theoretical profiles can be seen for the coarsest LES near the super-geostrophic maximum of the U-component. Finer resolution leads to a very good agreement with the extent of the maximum. However, for $Re2$ and $Re3$, the maximum velocity in x-direction is still not reached by the LES and the approach of the geostrophic wind in the upper part of the boundary layer takes place at lower heights than in theory. This is even more obvious for the V-component, where the maximum value is at a significantly lower height than in the theoretical curve. In general, a finer resolution leads to a lower location of the maximum. The maximum at lower heights might also explain the too high velocities in the logarithmic layer for the higher Re.

(*) For the first time, an LES of the Ekman layer is \emph{a posteriori} valida

5 Conclusions

We study ~~have analyzed LES of a~~ truly neutral Ekman boundary layers ~~—~~absent of capping inversions, non-stationarity, and surface heterogeneity ~~— within the context of a semi-empirical drag-law and wind-profile model (Part 1)~~. (*) Despite the common assumption that viscosity as a parameter drops out in LES as a consequence of the turbulence closure, we considered a range of scale separations (Reynolds numbers) and found that for an exact representation of the surface turning and wall friction as well as the wind-turning profile, viscous effects need consideration. The LES suffer from the dilemma, that in Ekman flow, some aspects of crucial importance happen on relatively small scales, such as the rotation-surface interaction which is confined to the inner layer.

With the wind-profile formulation developed in part 1 of this work, we have ~~a means at hands that serves as~~ a ~~true~~ reference based on first principl for intermediate-Re simulations, and ~~is considered~~ a quasi-reference for the higher-Re simulations. While we acknowledge that both the LES and the wind-profile model suffer from assumptions for high Reynolds number, their quantitative agreement across a wide range of scale separations hints towards the consistency. In particular, we understand that the grid convergence of LES towards the theoretical profiles underpins the inviscid scaling hypotheses in the development of the theory underlying part 1 of this paper.

We simulated three different Reynolds numbers while having DNS results for the lowest **Re**. For the low **Re**, viscous forces have a significant contribution to the balance of forces on the grid scale of the LES, hence we adapted the LES code to consider the fluid's viscosity in addition to the modeled eddy-viscosity. We did not introduce a roughness length z_0 at the lower boundary as additional parameter but deduced z_0 according to the law of the wall and the shear velocity expected according to the semi-empirical law by Spalart [1989]. The interplay of geostrophic wind, shear velocity, and roughness length in the simulation showed remarkable consistency, which supports the value of the adapted boundary condition at the bottom Maronga and Reuder [2017]. fix brackets... The dependence of the LES solution on the grid cell size was investigated through a comparison of four different resolutions. The setups of all **Re** use similar grid sizes in terms of the outer scale ($\Delta^- = \Delta/\delta = const.$) but different grid sizes in terms of the inner scale ($\Delta^+ = \delta^+ \cdot const. = Re_\tau \cdot const.$). This means that from an inner scale perspective the high Reynolds numbers were much less good resolved.

The convergence towards the theoretical profile expressed itself in different major aspects of the flow. To reach the expected total rotation $\alpha_\star$, a sufficient resolution was necessary. Furthermore, a sufficient vertical resolution was needed to simulate a logarithmic layer with a constant Kármán measure in some vertical extent.

A resolution of 150 grid levels below δ resolves well the boundary layer across all **Re**, in agreement with the findings of Wurps et al. [2020]. Their study demonstrated successful resolution of the neutral simulation with more than 100 grid levels within the boundary layer δ_{95}. The ratio δ_{95}/δ is approximately 2/3 (gradually decreasing with Reynolds number). Therefore, having 150 grid levels within δ roughly corresponds to around 100 grid levels within δ_{95}.

In summary, we synthesize some technical recommendations for the correct simulation of Ekman layer dynamics on a process level:

- (3 There should exist a logarithmic layer, or explicit consideration of viscous interaction with the surface; otherwise the assumptions of the surface closure (MOST / dynamic wall model) will fail.
- (2 $V|_{z=\Delta} < u_\star$ for an accurate matching of the hodograph and proper quantification of $\alpha_\star$. [where is this in the results?] this can be seen from Fig. 7, but we might make it somewhat more explicit wh
- (1 The roughness parameter (z_0) essentially defines a Reynolds number of the LES Problem
- (5 For very high resolution or relatively small Reynolds number, the viscous friction needs to be taken into account as the modeled eddy viscosity may locally drop to zero.

(4 • The parameters $\alpha_\star$ and $u_\star$ characterizing the bulk Ekman dynamics (across the vertical extent of the boundary layer) are matched by the LES if—in a three-dimensionally isotropic grid—more than 150 to 200 grid points are used in the vertical direction.

When these considerations are taken into account, LES becomes possible at uncommonly low Reynolds number and resolution, which allows a quantitative comparison to state-of-the art DNS. ~~direct numerical simulation results~~.

Some resolution and best-practice constraints developed in this work are strong, in some cases even prohibitive. They result from the externality of the flow, i.e. the presence of non-turbulent fluid aloft which leads to a duality of scales (cf. Introduction): both the inner and outer dynamics need to be visible to the resolved LES scales, at least to some extent. Despite these relatively strong constraints, we appreciate the capability of LES ~~Large-Eddy simulation~~ which, for example in case Re3, can appropriately match both $u_\star$, $\alpha_\star$ and the hodograph with the first grid point located at about 10^6 wall units. This means, there is a gap in resolution in comparison to a true DNS of five to six orders of magnitude per direction; illustrating that a DNS at this scale would be prohibitive and will remain so for a foreseeable time. From this inner, or small-scale perspective, a requirement of few hundred grid points across the boundary layers is not a lot.

We did not see a second logarithmic layer as Jiang et al. [2018] neither in the theoretical formulation nor in our simulations. A reason might be their introduction of the additional parameter z_0.

The theoretical formulation of mean velocity profiles within the turbulent Ekman layer can serve as comprehensive benchmark for model frameworks such as LES. These reference profiles offer a deeper analysis beyond the conventional assessment of the logarithmic increase in wind speed within the Prandtl layer. Comparing simulation results to the expected bulk parameters $u_\star$ and $\alpha_\star$ and to the hodograph and conducting a detailed evaluation of the wind speed profiles can yield valuable insights into the correct interplay among the model's grid resolution, turbulence closure, and boundary conditions. In the future, expanding the theoretical profiles to include additional aspects, such as temperature stratification, could potentially provide an even more valuable reference.

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