1. Fall $Re_D = 10^6$

$Re_\tau = \delta^+ = 2.28 \times 10^8$

$\mathcal{O}(k_m/\nu) \cong 10^4$

$\Rightarrow Re_{\tau,LES} \cong 2\times 10^4$

$\tilde{Re}_{\tau,LES} \cong 2\times 10^4 \quad (\nu \ll k_m)$

$\sqrt{Re_\tau / Re_{\tau,LES}} \cong 100$

$\Rightarrow$ verschiebt Plateau in $\frac{z^+}{\sqrt{Re_\tau}}$ um ca. Faktor 100 ~~nach oben~~

oder nur in $\sqrt{\frac{Re_\tau}{Re_{\tau,LES}}} : \frac{1}{140}$

2. Fall $Re_D = 10^4$

$Re_\tau = \delta^+ = 6.4\times 10^4$

$\mathcal{O}(k_m/\nu) \cong 10^0$

$\Rightarrow Re_{\tau,LES} \cong 6.4\times 10^4$

$\tilde{Re}_{\tau,LES} \cong 3\times 10^4$

$\sqrt{Re_\tau / Re_{\tau,LES}} \cong \sqrt{0.5}$

verschiebt Plateau um ≈ 1

≈ 1

3. Fall $Re_D = 1.6\times 10^3$

$Re_\tau = 3\times 10^3$

$\mathcal{O}(k_m/\nu) \cong 5\times 10^{-2}$

$\Rightarrow Re_{\tau,LES} \cong 6\times 10^4 \cong 2\, Re_\tau$

$\tilde{Re}_{\tau,LES} \cong \frac{20}{21} Re_\tau \cong Re_\tau$

$\left(\frac{\nu}{k_m} = 20;\ \frac{1}{1+\frac{\nu}{k_m}} = \frac{1}{21}\right)$

um ≈ 0.2

≈ 1

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effektive Friction-Re in LES

a) $Re_{\tau,LES} = \frac{u_* \delta}{k_m} \Rightarrow Re_{\tau,LES} = Re_\tau \frac{\nu}{k_m}$

b) $\tilde{Re}_{\tau,LES} = \frac{u_* \delta}{k_m + \nu} \Rightarrow \tilde{Re}_{\tau,LES} = Re_\tau \frac{\nu}{k_m+\nu} = \frac{\nu}{k_m\left(1+\frac{\nu}{k_m}\right)} = \frac{\nu}{k_m}\left(\frac{1}{1+\frac{\nu}{k_m}}\right)$

analog $z^{LES} = \frac{z u_*}{k_m} = \frac{z u^*}{\nu}\frac{\nu}{k_m} = z^+ \frac{\nu}{k_m}$

Townsend-Scaling ist höheninvariant unter der Vertikalkoordinate

$$\frac{z^+}{\sqrt{Re_\tau}} = \frac{z u^*}{k_m}\frac{k_m}{\nu}\frac{1}{\sqrt{\frac{u_*\delta}{k_m}\frac{k_m}{\nu}}} = \frac{z^{LES}}{\sqrt{Re_{\tau,LES}}}\sqrt{\frac{k_m}{\nu}}$$