

**1 Wind veer and speed in turbulent Ekman flow part I:
2 scaling analysis and universal profile model**

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6 Abstract

7 The profiles of wind speed and direction in turbulent Ekman flow are formulated
8 based on asymptotic theory and data from direct numerical simulation. The profile
9 of the streamwise component follows the classical viscous, logarithmic and wake
10 scaling. In the outer layer, the velocity component profiles can be described by an
11 Ekman-spiral with adapted boundary conditions that result in a reduction of the
12 spiral-like rotation. The span-wise component poses a conceptual challenge to the
13 channel-flow analogy in the context of asymptotic matching; it exhibits a mixed
14 scaling in the surface layer, but follows outer scaling for most of the outer layer.
15 Viscous stress scales universally across the boundary layer in inner units while
16 the total stress becomes universal as a function of outer height. This implies a
17 mixed scaling for the turbulent stress and eddy viscosity across the inner layer
18 and convergence to a universal scaling as function of the outer height across the
19 outer layer for increasing scale separation vide Reynolds numbers.

20 **Keywords** Direct Numerical Simulation · Scale Separation · Ekman Layer · Surface
21 Layer · Hodograph

22 **1 Introduction**

23 The Coriolis force bends the apparent path of motion on a rotating sphere and
24 establishes geostrophic equilibrium when in balance with a pressure gradient force.
25 Wind veer away from the wind direction in geostrophic equilibrium is (i) due to
26 direct frictional effects in the very vicinity of the surface and (ii) due to turbulence
27 which exerts indirect frictional effects; these effects cause a slow-down of the mean
28 wind reducing the Coriolis force thus turning the wind in favor of the pressure
29 gradient force. Not only does the veering set the frame of reference for surface

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layer theory, it also has effects at small and large scales from large-scale dispersion via plume spreading to cyclone spin-down (Svensson and Holtslag 2009) and on the capabilities of data assimilation and accuracy of surface flux estimates (Brown et al. 2005). From a large-scale perspective, the veering of wind across the planetary boundary layer determines the amount of cross-isobaric mass-flux, commonly referred to as 'Ekman pumping' (Ekman 1905), and it is thus a key factor in the life-cycle of large-scale synoptic systems. Within the atmospheric boundary layer (ABL), directional shear of the wind in the upper part of the surface layer may cause a systematic yaw for tall wind power generation devices where blades reach into the Ekman layer, i.e. that part of the boundary layer where the wind starts to turn; an exact estimate of such effects is critical in the site assessments for wind farms (Calaf et al. 2010; Mirocha et al. 2018).

In the planetary boundary layer, wind veer is characterized by the surface veering angle α defined as the angle between the negative surface shear stress τ_{sfc} and the geostrophic wind. Surface veering α and geostrophic drag $Z \equiv u_*/G$, where the friction velocity $u_* \equiv \sqrt{|\tau_{\text{sfc}}|/\rho}$, uniquely determine the surface drag τ_{sfc} in a turbulent Ekman flow. In any quantitative description of the surface layer, the friction velocity u_* is the dynamic scale and α defines the horizontal alignment of the frame of reference, i.e. the rotation of the surface friction with respect to the outer wind direction. Knowledge about u_* and α is thus a prerequisite for any quantitative theory of the surface layer, and Rossby and Montgomery (1935) constrained the two parameters based on integral relations in the ABL. Asymptotic similarity theory was later used by Tennekes (1973); Blackadar and Tennekes (1968), and–based on his seminal direct numerical simulations (DNS) of Ekman flow–, Spalart (1989) suggested a modification to take into account effects of low to intermediate Reynolds numbers. Later on, constants were re-evaluated with a focus on the ABL based on observations (Högström 1988, 1996) and numerical modelling (Spalart et al. 2008, 2009; Ansorge and Mellado 2014; Ansorge 2019).

Attempts were also undertaken to obtain profiles of the wind speed: One approach is to match the inner and outer layer at a reference height as suggested by Etling (2002); Emeis (2018) (Sec. 21.10; Eq. 21.48); they choose the Prandtl-layer height z_{Prandtl} to match the wind speed profiles, which, however, requires external prescription of $\alpha(z_{\text{prandtl}})$, the veering at that height. A one-dimensional profile with constant veering is given by Emeis et al. (2007, Sec. 3; Eq. 3.1-3.19).

Gryning et al. (2007) present an extension of the wind-speed profile beyond the surface layer using a neutral reference profile and a stability correction; Kelly and Gryning (2010), based on a probabilistic representation of stratification, develop a model for the long-term mean wind speed in the ABL and compare this with observation at different sites; Kelly and Troen (2016) demonstrate the effect of such improved model for wind-energy applications. In consideration of the large scale separation in geophysical flow, the rotation of the wind in the surface layer is often assumed negligible, and above investigations merely focus on the wind speed; that means, the veering of the wind with height is not described and there is little knowledge on the profile of the span-wise velocity component and the precise shape of the hodograph in the limit of a truly neutral Ekman boundary layer. A climatology of wind turning in the ABL is given by Lindvall and Svensson (2019) Klein et al. (2021) use a statistical turbulence modelling approach that yields a

78 two-component velocity profile, but they also find that the exact representation of
79 turning is challenging.

80 Ekman-layer models are roughly based on Ekman's seminal 1905 paper in
81 combinations with additional assumptions. One option is a prescribed profile shape
82 for eddy viscosity (Ellison (1955)), another are two-layer models of the ABL that
83 take into account rotational effects at higher altitudes, for instance when the wind
84 speed needs to be evaluated at heights on the order of 100 – 200 m, a particular
85 concern when it comes to wind-power forecasting (Etling 2002; Emeis 2018; Optis
86 et al. 2014). Despite rotational effects being considered, the formulation of these
87 models for the outer layer and analysis of their performance primarily focuses
88 on wind speed. Still, in 2018, Jiang et al. recognized that the outer part of the
89 Ekman boundary layer receives less attention in comparison with the surface layer
90 and study the neutral problem by Large-Eddy simulation (LES). They focus on
91 the wind speed and find an extended logarithmic layer when considering the wind
92 speed instead of the shear-aligned component, and they eventually demonstrate
93 by means of an analytical model that this vertical extension of the logarithmic
94 layer may be explained by a transfer of stress to the span-wise velocity component
95 where it is assumed that the shear vector $\tau(z)$ and stress vectors $(\partial_z U, \partial_z V)$ are
96 aligned.

97 More recently, Ghannam and Bou-Zeid (2021) treated the horizontally av-
98 eraged momentum budget to show that departures from shear-alignment in the
99 vicinity of the surface result in an integral of the wind veer (α_M in their notation)
100 over the height to very high accuracy ($\int_{z_0}^H \sin \alpha_M$ in their notation; their Eq. (16)).
101 Classic surface-layer similarity is recovered when the angle α_M does not depend
102 on height, i.e., the wind veer is constant across the surface layer. If, however, the
103 wind veer depends on height, the profiles of stress and mean velocities depart from
104 the scalings implied by classic surface-layer similarity.

105 Turbulent Ekman flow is considered here as a conceptual model of the homo-
106 geneous, stationary ABL over a flat surface under neutral stratification. Universal
107 profiles of the wind vector for turbulent Ekman flow not only are a well-described
108 limit for theoretical exploration or higher-order approaches taking into account
109 possible effects of stratification, roughness or other physical complications encoun-
110 tered in the real geophysical system. While, on first sight, the study of such a
111 strongly idealized case appears as an academic problem, it contains the essence
112 of surface similarity as it is used in most atmospheric models, be it conceptual
113 or numeric ones. More complex accounts generally refer to the homogeneous sta-
114 tionary problem as a base state: (i) Roughness is commonly incorporated by a
115 linear transformation of vertical scale involving the roughness parameter z_0 and
116 for larger roughness also a displacement height (Monin and Yaglom 1975; Jacobs
117 and Van Boxel 1988; Högström 1988); (ii) Stability can be accounted for by a
118 linearization around the neutrally stratified profile (Monin 1970; Monin and Ya-
119 glom 1975; Högström 1988, 1996; Sakagami et al. 2020); (iii) Non-stationarity in
120 the pressure-gradient forcing can be accounted for by a linear damped-oscillator
121 approach around the base state (Momen and Bou-Zeid 2016); (iv) Barotropic and
122 baroclinic effects on the velocity profile require to consider the height-dependence
123 of the veer and stress misalignment (Momen et al. 2018; Ghannam and Bou-Zeid
124 2021). Furthermore, such a solution can serve as better initial condition for numer-
125 ical simulation of the flow, to minimize the length of initial transient periods, or

as benchmark for turbulence closures that can be tuned to reproduce the neutral limit case.

Despite the strong simplifications implied by our choice of set-up, there is no straightforward approach to solving this well-defined problem. Large-Eddy simulation not only needs to be tuned for the surface shear stress and veering angle, but it also relies on sub-grid closures that commonly assume alignment of the turbulent stress with gradients. This pre-requisite is not fulfilled when the wind rotates with height. Esau (2004) investigated the representation of the Ekman boundary layer by dynamical subgrid closures and Zikanov et al. (2003) proposed a closure for the wind profile using a linearized representation of the eddy viscosity. Despite advances in analysis of this simplified set-up (Jiang et al. 2018), there is yet insufficient understanding for a quantitative generalization of the results to arbitrary external forcing (manifest in variation of the Reynolds number) – and indeed the fundamental questions pertaining to such relatively simple dynamics of turbulence are not reflected in the research on LES for the ABL over the past 50 years (Stoll et al. 2020).

At the same time, an increasing amount of high-quality and high-resolution data from turbulence-resolving approaches is emerging due to recent advances in high-performance computing and its application to geophysical problem sets; the geophysical range of scale separation, however, is—and it will remain so for the foreseeable future—out of reach for such simulation (Dimotakis 2005). Here, the routinely employed concept of Reynolds-number similarity can help. It postulates the existence of *fully developed turbulence* believed to occur for a sufficiently large but finite Reynolds number (Barenblatt and Goldenfeld 1995). (Already in 1998, this in fact lead Moin and Mahesh to the question *how high a Re is high enough?*) Certain statistics of fully developed turbulence, such as dissipation (Dimotakis 2005) or profiles of mean velocity (Barenblatt 1993), become independent of the Reynolds number when appropriately scaled; other statistics, such as the near-wall maximum in velocity fluctuation depend on Re (Baars and Marusic 2020) and externality of the flow may exert an impact on near-wall scaling (da Silva et al. 2014). It appears that for certain statistics in Ekman flow, fully-developed turbulence is reached with the Reynolds numbers that became possible due to an increase of computing capabilities over the past decades.

This paper exploits the robust features of mean velocity profiles from direct numerical simulation across a range of Reynolds numbers to formulate both the streamwise and span-wise components of the mean velocity vector as a function of the Reynolds number.

2 Problem formulation and numerical approach

We consider here incompressible, turbulent Ekman flow, that is, the turbulent flow over a flat rotating plate, as a physical model for the truly neutral ABL. The f-plane approximation is applied such that rotation only acts on horizontal velocity components; we thus neglect rotational effects on the horizontal components of velocity and dynamical effects due to latitudinal variation of the rate of rotation.

169 2.1 Notation and governing equations

170 The dimensional velocity vector of the numerical simulations is $\underline{U} = (U_1, U_2, U_3) =$
 171 (U, V, W) over the coordinate system $Oxyz$, where an approximate alignment (plus/
 172 minus few degrees) of the direction Ox with the surface shear stress is achieved. We
 173 consider velocity profiles only, i.e. all velocities are averaged over horizontal planes
 174 and in time, that is, they correspond to an Ensemble average. The coordinate Oz
 175 points away from the wall, and Oy points in the span-wise direction normal to
 176 Oxz . For analysis of the results, we use two coordinate systems that are (i) exactly
 177 aligned with the surface shear stress

$$\tau_{\text{sfc}} = \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = -\nu \left(\frac{\partial U}{\partial z} \hat{e}_x + \frac{\partial V}{\partial z} \hat{e}_y \right) \quad (1a)$$

178 and labelled by an upper index α as in \underline{U}^α for the velocity vector, and (ii) the
 179 coordinate system aligned with the free-atmosphere geostrophic wind labelled by
 180 an upper index G as in \underline{U}^G . We denote the square root of the modulus of surface
 181 shear, the surface friction, by

$$u_* = \sqrt{\|\tau_{\text{sfc}}\|} \quad (1b)$$

182 and let $Z_* = G/u_*$; the surface veering angle α_* is the angle between τ and the
 183 geostrophic wind

$$\alpha_* = \sphericalangle(G, \tau_{\text{sfc}}). \quad (1c)$$

184 Analogously, we denote the height-local veering of the wind $\alpha(z) = \sphericalangle(G, \underline{U}(z))$,
 185 where $\underline{G} = (G_1, G_2, 0)$ is the geostrophic wind vector.

186 We consider the incompressible Navier–Stokes equations for the three velocity
 187 components on the f-plane in a framework that is governed by (i) geostrophic wind
 188 magnitude $G = \sqrt{G_1^2 + G_2^2}$, (ii) Coriolis parameter f (representing the angular
 189 rotation), and (iii) kinematic viscosity ν . In absence of external variability, this
 190 system converges to a statistically steady state in the sense that flow statistics
 191 do not depend on time; and this state is defined by a Reynolds number, the only
 192 non-dimensional parameter that governs the system. We use the geostrophic wind
 193 as velocity and the Coriolis parameter f as time scale for the non-dimensional
 194 framework. This implies the Rossby radius $A_{\text{Ro}} = G/f$ as length scale, such that
 195 one Reynolds number governing the problem reads as

$$\text{Re}_A = \frac{GA_{\text{Ro}}}{\nu}. \quad (2)$$

196 The scales used in defining Re_A are of limited relevance for description of the
 197 turbulent flow state. The turbulence scale separation in a wall-bounded flow is
 198 commonly characterized by the friction Reynolds number (Jiménez 2012):

$$\text{Re}_\tau = \frac{u_* \delta}{\nu} = \delta^+ = \frac{\text{Re}_A}{Z_*^2}, \quad (3)$$

199 where $\delta = u_*/f$ and we use a superscript '+' to denote normalization by inner
 200 turbulence scales (u_*, ν) . Another common measure of scale separation is the
 201 Reynolds number

$$\text{Re}_D = \frac{GD}{\nu} \quad (4)$$

202 defined by the laminar Ekman layer thickness $D = \sqrt{2\nu/f}$.

203 The governing equations non-dimensionalized by G , f , and A_{Ro} read as

$$\frac{\partial u_i}{\partial t} = \frac{\partial \pi}{\partial x_i} - u_j \frac{\partial u_i}{\partial x_j} + \epsilon_{i2j}(u_j - g_j) + \frac{1}{Re_A} \frac{\partial^2 u_i}{\partial x_j^2} \quad (5a)$$

$$\frac{\partial u_j}{\partial x_j} = 0, \quad (5b)$$

204 where $u_i = U_i/G$ are the non-dimensional components velocity, π is non-dimen-
 205 sional pressure, $g_j = G_j/G$ are non-dimensionalized components geostrophic wind
 206 (with $g_1^2 + g_2^2 = 1$ by construction), and ϵ is the Levi-Civita tensor. These equations
 207 are solved inside a bounded cube of size $L_x \times L_y \times L_z$ with periodic boundary condi-
 208 tions in the lateral (streamwise and spanwise) directions, a no-slip–no-penetration
 209 boundary at $z = 0$, and a no-penetration, free-slip boundary at $z = L_z$.

210 2.2 Numerical simulations

211 The problem is solved numerically by tLab¹, an open-source tool-suite to simulate
 212 and analyze turbulent flows. We use here a fourth-order–five-step Runge–Kutta
 213 integration and sixth-order compact schemes for spatial derivatives in all direc-
 214 tions. The incompressibility constraint is enforced by a fractional step approach
 215 where the Poisson equation for the pressure field is solved to machine accuracy
 216 using a combined spectral/compact approach as described in Mellado and Ansorge
 217 (2012).

218 Simulations used here are shown in Tab. 1. We extend an existing set of simula-
 219 tions for $Re_A \in \{125\,000; 281\,250; 500\,000\}$ (gray shading; cf. Ansorge and Mellado
 220 2014, 2016) by new simulations at higher Reynolds numbers up to $Re_A = 1.28 \times 10^6$
 221 with a horizontal domain extent up to 3.3×10^4 viscous units. In total, this yields
 222 one order of magnitude variation in terms of the scale separation in the boundary
 223 layer.

224 3 Scaling behavior of the flow for Re_τ up to 3000

225 The generalization of profiles to arbitrary Reynolds numbers requires sufficient
 226 scale separation in the simulations, not only to quantify the effect of the Reynolds
 227 number on low-order flow statistics, but also to assess the corresponding rate-of-
 228 change to eventually allow for an extrapolation of the findings. While the simula-
 229 tions previously available (gray shading in Tab. 1) give confidence in a first-order
 230 representation of the turbulent problem, the estimation of higher-order effects such
 231 as the dependency of the Reynolds number requires a broader scale separation that
 232 is made available by the two new simulations at increased Reynolds number (cf.
 233 Tab. 1). Data at such scale separation has been obtained previously (cf. Spalart
 234 et al. 2008, 2009), but we also need high confidence with respect to the convergence
 235 of simulation data for slow oscillations and with respect to sampling convergence,
 236 which translates to two further requirements on the data: First, data should be free

¹ <https://github.com/turbulencia/tlab>

Table 1 Direct numerical simulation data sets used in this work. Re_A and Re_D refer to the Reynolds number defined in terms of the Rossby radius Λ and Ekman-layer thickness D respectively. L_{xy} is the domain size in the stream- and span-wise direction. The grid is given by the number of grid points in the stream-wise (N_x), span-wise (N_y) and vertical (N_z) directions respectively. The resolution in the span-wise and stream-wise directions are given as Δx^+ and Δy^+ . The grid in the vertical is stretched, and resolution at the wall is given by Δz^+ .

Re_A	Re_D	L_{xy}/Λ	$N_x \times N_y \times N_z$	Δx^+	Δy^+	$\Delta z^+ _{z=0}$
125 000	500	1.08	$2048 \times 2048 \times 192$	4.1	4.1	1.05
281 250	750	1.08	$3072 \times 3072 \times 384$	5.6	5.6	1.60
500 000	1 000	1.08	$3072 \times 6144 \times 512$	9.3	4.7	1.14
845 000	1 300	0.54	$2560 \times 5120 \times 640$	8.9	4.5	0.99
1 280 000	1 600	0.54	$3860 \times 7680 \times 960$	8.6	4.3	1.00

Table 2 DOIs and reference to the openly accessible data set at refubium repository

Re_D	DOI	Reference
500	10.17169/refubium-42505	Ansorge (2024a)
1000	10.17169/refubium-42507	Ansorge (2024b)
1300	10.17169/refubium-42508	Ansorge (2024c)
1600	10.17169/refubium-42509	Ansorge (2024d)

237 of artifacts from long-term oscillations across the vertical extent of the domain—
238 primarily, simulations should be free of effects originating from the inertial oscil-
239 lation; this is achieved here by replacing the mean value of the three-dimensional
240 velocity fields by the time mean over a whole inertial oscillation. Second, high
241 accuracy is also needed in terms of the statistical convergence of averages, bulk
242 measures and large-scale structures; this requires a domain size $L_x > \mathcal{O}(\delta_{95})$. We
243 use here $L_x = L_y = 1.08\Lambda$ for cases with $Re \leq 1000$ and $L_x = L_y = 0.54\Lambda$ for
244 $Re \geq 1300$ which corresponds to $L_x/\delta_{95} \approx 25$ for $Re_D = 500$ and $L_x/\delta_{95} \approx 18$ for
245 $Re_D = 1300$.

246 Bulk parameters of the simulations are given in Tab. 3. The surface stress is
247 characterized by u_* and α_* in relation to the geostrophic wind vector and dis-
248 cussed in more detail as the drag law below in Sec. 4.1 (we find the expected slight
249 decrease of u_*/G and α_* with increasing Re). The boundary-layer height estimated
250 from the 95% stress reduction, δ_{95} , is around 0.6δ to 0.66δ . Interestingly, the in-
251 tegrated TKE $\int_0^\delta edz$ stays constant when normalized by the friction velocity u_*
252 while the integrated dissipation $\int_0^\delta edz$ exhibits inviscid scaling when normalized
253 by the magnitude G of the geostrophic wind. (TKE and dissipation normalized
254 as $fG^{-3} \int edz$ and $u_*^{-3} \int edz$, exhibit substantial dependence on Re for the varia-
255 tion of u_* .) This indicates that the bulk dissipation is governed by the forcing
256 G -irrespective of Re . Changes in Re , however, affect the level and organization of
257 turbulence, and the parameter representing this dependency is the friction veloc-
258 ity u_* which describes the turbulence production processes in the surface layer, in
259 particular in the buffer layer.

260 Velocity profiles in inner units ($U^{\alpha+}(z^+)$, Fig. 1a) and outer units ($U^{\alpha-}(z^-)$,
261 Fig. 1b) are in accordance with previous work (Coleman et al. 1992; Spalart et al.
262 2008, 2009; Ansorge and Mellado 2014; Ansorge 2019): The profiles of the shear-
263 aligned streamwise velocity component are well-collapsed for $Re_D > 500$ below
264 $z^- \approx 0.15$ (circles in Fig. 1a); the case with $Re_D = 500$ is only transitionally

Table 3 Bulk characterization of the simulations for different Reynolds numbers Viscous Reynolds number Re , friction Reynolds number Re_τ , friction velocity u_* , surface veering angle α_* , normalized boundary layer depth δ_{95}/δ , inner normalization of vertically integrated TKE, outer normalization of vertically integrated dissipation.

Re	$\delta^+ = \text{Re}_\tau$	u_*/G	α_*	δ_{95}/δ	$f u_*^{-3} \int_0^\delta \epsilon dz$	$G^{-3} \int_0^\delta \epsilon dz$
500	479	0.0619	25.5	0.66	0.88	1.31
750	886	0.0561	21.0	0.65	0.90	1.34
1000	1403	0.0530	18.8	0.62	0.92	1.30
1300	2122	0.0501	17.9	0.59	0.85	1.24
1600	2978	0.0482	17.2	0.61	0.91	1.21

turbulent and there is no distinct inner-outer scale separation. The logarithmic law is appropriate for $50 < z^+ < 0.15 \text{Re}_\tau$, where $z^+ = \text{Re}_\tau z^-$. While the profiles $U^{\alpha*+}(z^+)$ diverge between different Re beyond $z^- = 0.15$, the corresponding profiles of the velocity deficit $(U^{\alpha*}(z^-) - G_1^\alpha)/u_*$ agree closely, irrespective of Re . This illustrates the inner–outer scale-duality in this external flow with inner scaling being appropriate in the inner layer and outer scaling in the outer layer. Also in the outer layer of the flow, u_* (and not the magnitude of the geostrophic wind G) governs the inviscid normalization, i.e. a scaling independent of the Reynolds-number.

No collapse is found for the profiles of spanwise velocity when considered in inner units, $V^{\alpha*+}(z^+)$. When normalized in outer units, the deficit profiles of spanwise velocity $(V^{\alpha*}(z^-) - G_2^\alpha)/u_*$ agree well beyond $z^- \approx 0.3$. This is a much higher level in comparison with the streamwise component that collapses also within the overlap layer, i.e. much closer to the surface (circles in Fig. 1b). The value of $V^{\alpha*+}(z^-) - G_2^\alpha$ is sensitive to the wind veering for $z \rightarrow 0$ as—for use of the shear-aligned component—it has to approach the value $-G_2^\alpha = |G| \sin \alpha \neq 0$ in view of the no-slip boundary condition. While low-Re effects appear to be present for $\mathcal{O}(\text{Re}) < 10^3$, the spanwise component converges to an Re -independent limit within the range of scale separation considered here, i.e.

$$G_2^\alpha/u_* = Z_* \sin \alpha \rightarrow \text{const. for } \text{Re} \rightarrow \infty, \quad (6)$$

which has indeed already been found by Spalart (1989), who estimates the constant from an integral relation.

The viscous stress

$$S_{\text{visc}} = \nu \sqrt{\left(\frac{\partial U}{\partial z} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2} \quad (7a)$$

exhibits universal scaling when considered as $S_{\text{visc}}^+(z^+)$ (Fig. 2a); this normalization is also appropriate in the outer layer where the viscous stress is, however, small. Small deviations from the universal profile are observed for the smallest Reynolds number $\text{Re} = 500$; we attribute these to low-Re effects in the only transitionally turbulent flow ($\text{Re}_\tau = 479$). In contrast to the viscous stress, the total stress follows outer normalization, i.e. $S^+(z^-)$ is universal; a discrepancy in the inner layer does not occur as the total stress is approximately constant in the viscous and buffer layer, and a rescaling of the height would have no effect there;

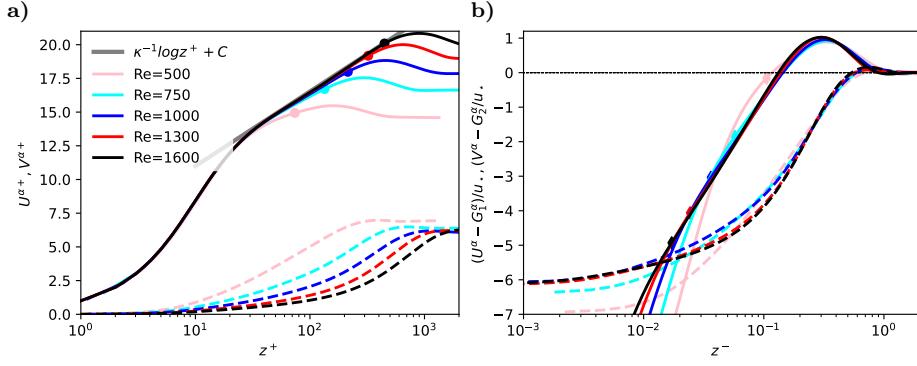


Fig. 1 a) Shear-aligned velocity profiles in inner units; circles mark the height $z^- = 0.15$; b) Shear-aligned velocity deficit in outer units; diamonds mark the grid point closest to the height $z^+ = 50$

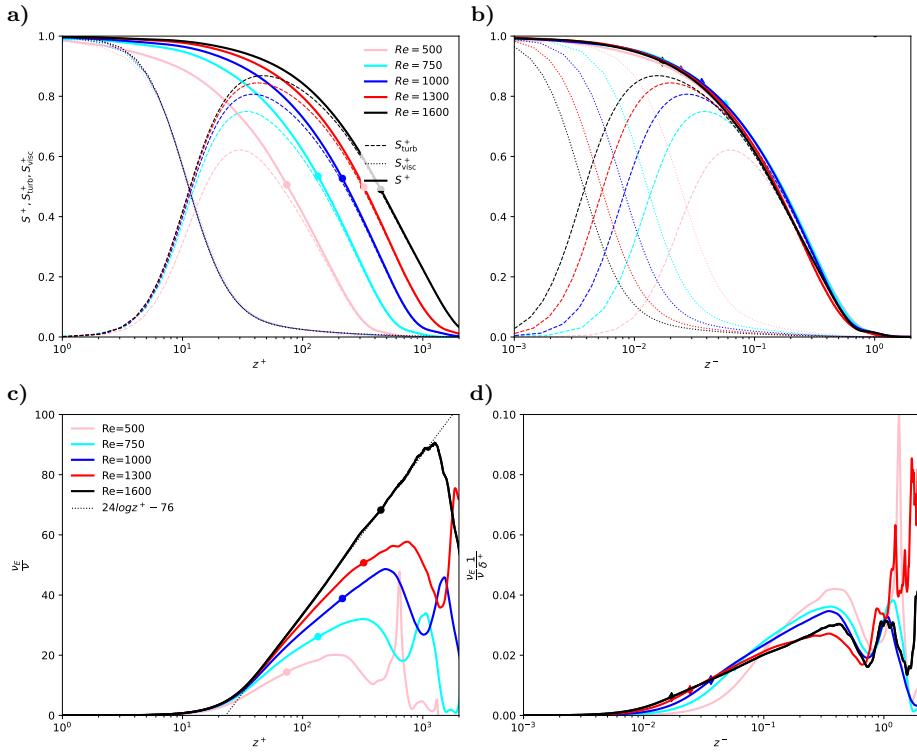


Fig. 2 a-b) Profiles of the turbulent stress S_{turb}^+ (dashed), the viscous stress S_{visc}^+ (dotted), and the total stress, $S^+ = S_{\text{visc}}^+ + S_{\text{turb}}^+$ (solid) as a function of inner height (a) and outer height (b). c-d) Normalized eddy viscosity ν_E (solid) plotted versus inner height (c) and outer height (d). (c) uses inner normalization; (d) uses normalization by $\nu \delta^+$ which approximately collapses data in the outer layer. Different colors are for different Reynolds numbers (cf. Tab. 1). Circles in (a) and (c) denote the height $z^- = 0.15$, diamonds in (b) and (d) are for $z^+ = 50$ as in Fig. 1

above, outer scaling is appropriate for the well-established dynamics in the overlap region of inner and outer layer. This, however, implies a mixed scaling for the turbulent stress,

$$S_{\text{turb}} = \sqrt{\overline{u'w'}^2 + \overline{v'w'}^2}, \quad (7b)$$

where dashed quantities u' , v' , w' indicate deviations from the mean and the overbar denotes horizontal and time averaging. Indeed, S_{turb} only follows inner normalization below $z^+ \lesssim 20$ (where the turbulent contribution is negligible). In the outer layer, where $S_{\text{visc}} \rightarrow 0$, S_{turb}^+ follows outer normalization for $z^- \gtrsim 0.15$ —with increasing accuracy for larger Re and larger distance from the surface. In the overlap region, i.e. for $z^+ > 20$ and $z^- < 0.15$, the mixed scaling for the turbulent stress can be expressed as

$$S_{\text{turb}}^+(z^+, \text{Re}_\tau) = S^+(z^-) - S_{\text{visc}}^+(z^+), \quad (7c)$$

where $z^- = z^+/\text{Re}_\tau$.

The Eddy viscosity plays a crucial part when modelling profiles and the vertical transport in turbulent flow. In analogy to the Fick-law for molecular diffusion, the eddy diffusivity is the effective diffusivity that yields the turbulent transport S_{turb} based on the strain rate. For the symmetries in the flow (horizontal homogeneity, and $W = 0$), it is

$$\nu_E = \frac{S_{\text{turb}}}{\sqrt{\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}} = \nu \frac{S_{\text{turb}}}{S_{\text{visc}}}. \quad (8a)$$

The inner normalization of ν_E is obtained when dividing by the molecular viscosity:

$$\nu_E^+ = \nu_E/\nu = S_{\text{turb}}/S_{\text{visc}}. \quad (8b)$$

Under this normalization, the profiles of eddy viscosity collapse below $z^+ \approx 20$ with a tendency towards better collapse at higher z^+ for higher Reynolds number (up to $z^+ \approx 50$ for $\text{Re} = 1600$; Fig. 2c). In the outer layer, the eddy viscosity is characterized by a distinct minimum at $z^- \approx 0.6 - 0.8$, and we find that the following mixed normalization of ν_E by the geostrophic wind and friction velocity collapses the value of ν_E at this minimum (cf. Fig. 2d):

$$\nu_E^- = \nu_E^+ \frac{1}{\delta^+} = \nu_E \frac{1}{\nu} \frac{\nu}{u_* \delta} = \nu_E \frac{f}{u_*^2}. \quad (8c)$$

Substantial variation of the profiles is, however observed below and above this minimum for different Re which illustrates that this normalization is probably not generally appropriate across the outer layer.

The organization of the flow with $\text{Re}_\tau = 2978$ is depicted in terms of the turbulence kinetic energy in Fig. 3. In vicinity of the wall, at $y^+ \approx 10$, (Fig. 3a), elongated streaks aligned with the surface shear stress dominate. Moving away from the wall, to $y^+ \approx 150$ (well within the logarithmic region), the structures are larger and more isotropic, but they are still largely aligned with the surface shear stress. In the upper part of the outer layer, around $y^+ \approx 1000$, no clear signature of the surface veering direction is found, and intense TKE structures (bright yellow) are organized on a large spatial scale with weaker eddies (greenish structures) and quiescent regions in between.

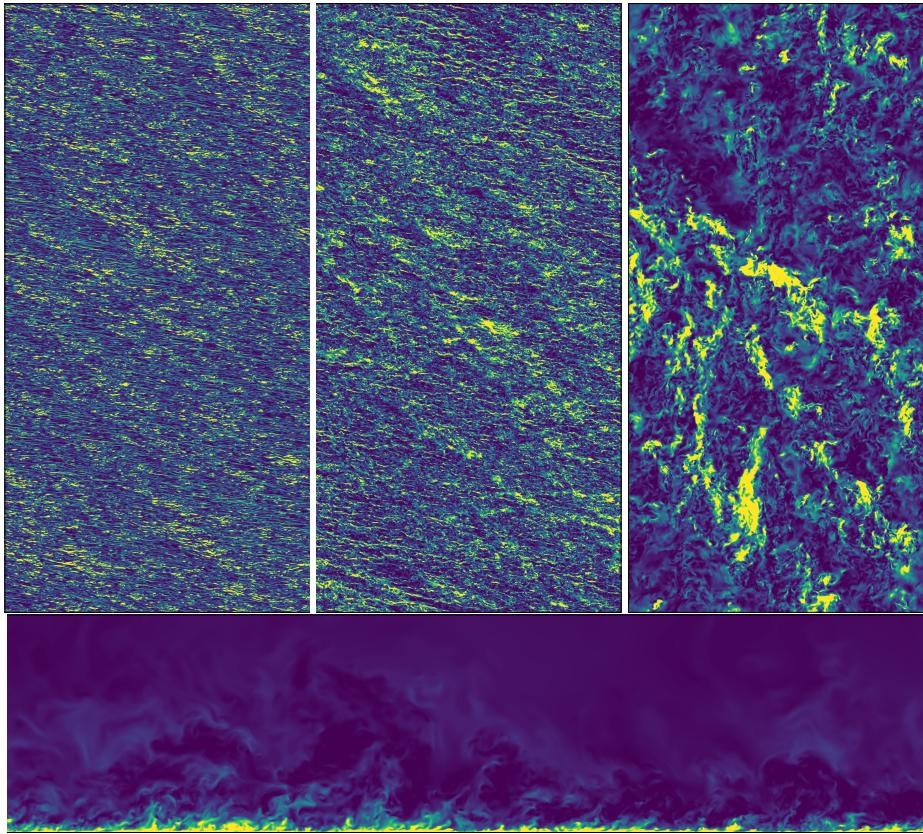


Fig. 3 Horizontal slices of turbulence kinetic energy in the Buffer layer ($i=10$), logarithmic Layer ($i=100$), and outer layer ($i=400$) of the case with $Re_\tau = 2978$; coloring between percentiles 4 and 96 of the respective image. Lower panel: streamwise–vertical intersect through the domain

330 4 A universal velocity profile for the turbulent Ekman layer

331 We now turn to the formulation of a general velocity profile that is fully determined
 332 by the only parameter of the idealized problem, namely a Reynolds number repre-
 333 senting the scale separation or geometric size of the problem. This precludes, first,
 334 a drag law wherewith we begin this section (4.1). Based on the scaling arguments
 335 put forward in Sec. 3, we then develop, second, a formulation of the wind vector
 336 in the Ekman layer (Sec 4.2). Finally, we come up with a separate formulation of
 337 the, third, stream-wise and, fourth, span-wise velocity components in the overlap
 338 and inner regions of the flow.

339 4.1 Drag-law

340 A drag-law for Ekman flow determines—as a function of Reynolds number alone—
 341 the surface drag. This can be formulated by the normalized surface friction, u_*

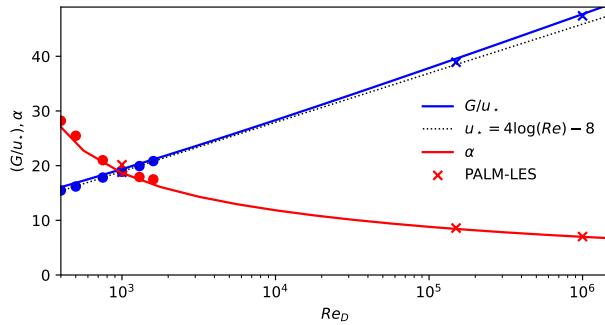


Fig. 4 Variation of geostrophic drag, Z_* , and surface veering, α_* , with Reynolds number according to the theory by Spalart et al. (1989) and as estimated from DNS data

(Eq. (1b), also termed geostrophic drag), and the direction of surface shear stress, α_* (Eq. (1c), also termed wind veer). A non-zero veering of the wind is a rather special case in comparison with most turbulent flows considered in an engineering context, and it confronts us with a situation where the most appropriate coordinate system for analysis (namely that aligned with the surface shear stress) is a priori unknown. We compare our DNS data against a semi-empirical drag-law based on integral consideration (Spalart 1989) and find, as demonstrated in previous work (Ansorge and Mellado 2014), excellent agreement in the range $400 < \text{Re} < 1600$, representing a factor of 16 in variation of viscosity.

We also find that the solution of the transient equation involved in estimation of u_* for a given Reynolds number Re_D is approximated reasonably by the formulation

$$Z_* = 4 \log(\text{Re}_D) - 8 \quad (9)$$

which quantifies the 'weak' dependence of u_* on the Reynolds number as an approximately logarithmic one, at least for problems with a scale separation on the order that is relevant to geophysical problems ($Re_D < 10^8$).

4.2 Profile in the Ekman layer

Formulations for the outer layer that take into account the rotation (and thus deviation from the channel-flow analogy) need to be matched to the framework of surface similarity. A smooth transition from the inner layer to the Ekman layer, where the wind is characterized by a turning of its mean direction, is not easily achieved. Optis et al. (2014), for instance, define an "*effective geostrophic wind vector that has the same magnitude of the observed surface geostrophic wind and is rotated by the angle α* [their nomenclature]" to overcome the unsteady transition when approaching the Ekman layer from below. Such rotation of the wind vector is *a posteriori* justified by the observational data that the model outcomes are compared to. This need for a connection of the two reference frames is a manifestation of a mismatch in the theoretical treatment of the inner and outer layer in this rotating flow configuration.

The text-book solution for Ekman flow makes use of the physical boundary conditions (BCs) for the ABL (no-slip at the bottom and geostrophic wind in the

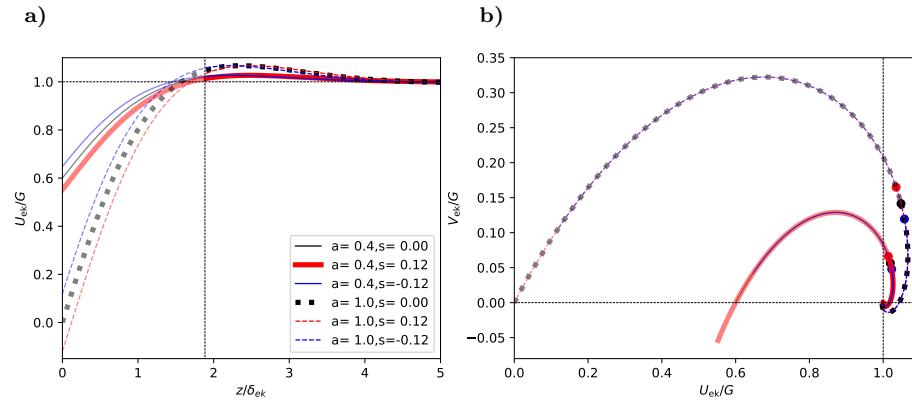


Fig. 5 a) Generalized Ekman-profile of the geostrophic-aligned component U_{ek} . b) Hodograph for the geostrophic-aligned and pressure-gradient aligned components U_{ek} and V_{ek} . Thick, black dashed line shows the classic solution. The height corresponding to $z^- = 0.30$ is marked by the dashed line in panel (a) and by filled circles in panel (b). The hodograph and profiles above this reference height are shown as solid lines, below as opaque line.

372 free atmosphere) and a constant eddy viscosity. Specifying the boundary conditions
 373 at top and bottom eliminates one mode of the analytical solution, and it determines
 374 the magnitude of the spiral. In doing so, one has to assume that the solution
 375 is appropriate across the entire ABL, which is not the case: The dynamics put
 376 forth by Ekman in 1905 are not appropriate in the surface layer of the ABL;
 377 better approximations exist for the logarithmic, buffer, and viscous sublayers. In
 378 view of this situation, we use an adapted Ekman spiral that does not enforce the
 379 boundary conditions at the surface but at a different height. This is achieved by
 380 considering the Ekman spiral only in the Ekman layer, thus giving way for the
 381 well-established logarithmic and viscous-layer profiles in the lower surface layer.
 382 Based on the derivation in App. A.1, this profile is given by

$$\frac{1}{G} \begin{pmatrix} U_{\text{ek}} \\ V_{\text{ek}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-z_{\text{ek}}} \left[a_{\text{ek}} \begin{pmatrix} -\cos z_{\text{ek}} \\ \sin z_{\text{ek}} \end{pmatrix} + b_{\text{ek}} \begin{pmatrix} \sin z_{\text{ek}} \\ \cos z_{\text{ek}} \end{pmatrix} \right]. \quad (10a)$$

383 with $z_{\text{ek}} = \delta_{\text{ek}}(z^- - s_{\text{ek}})$. The right-hand-side consists of two modes with magnitude
 384 a_{ek} and b_{ek} shifted by $\pi/2$ with respect to each other. In the classic case, the second
 385 mode governed by b_{ek} is incompatible with the surface boundary condition. While
 386 this is not the case here for the general form of the profile, the phase shifts can also
 387 be captured by the parameter s_{ek} , and we stick with a single-modal approach,
 388 i.e., we let $b_{\text{ek}} = 0$.

389 This single-modal solution is characterized by three parameters, (i) an Ekman-
 390 layer depth scale δ_{ek} , (ii) the magnitude parameter of the spiral a_{ek} , and (iii) a
 391 zero-crossing point for the velocity s_{ek} . The effects of varying these parameters
 392 are illustrated in Fig. 5 where the classic Ekman solution is recovered by setting
 393 $a_{\text{ek}} = 1$, $s_{\text{ek}} = 0$ and $\delta_{\text{ek}} = \sqrt{2\nu/f}$. These parameters are *a priori* unknown as
 394 they need to conform to the turbulent state of the boundary layer; we use our
 395 DNS data to arrive at best estimates for them.

396 **The Ekman-layer depth scale** δ_{ek} is fundamentally defined by the eddy vis-
 397 cosity. However, we have seen in Section 3 that a characteristic value for the

398 eddy diffusivity is not easily obtained for its strong dependence on the Reynolds
 399 number and distance from the surface. We therefore resort to the physical man-
 400 ifestation of the eddy diffusivity in an Ekman layer, and use the boundary layer
 401 depth $\delta_{\text{ek}} = 0.66\delta \times 2\pi$. For the relation $\delta_{\text{ek}} = \sqrt{2\nu_{\text{ek}}/f}$, this yields $\nu_{\text{ek}} \propto u_*^2/f$ in
 402 accordance with the observations in Sec. 3 (Eq. 8c).

403 **The magnitude parameter of the Ekman spiral**, a_{ek} , defines the super-
 404 geostrophic maximum of the wind profile aloft the logarithmic layer. Our simu-
 405 lations suggest this maximum of the velocity deficit remains constant when nor-
 406 malized by u_* as shown in Fig. 6. The numerical value of a_{ek} is estimated from
 407 visual comparison, and we find $a_{\text{ek}} = 8.4u_*$; while this appears rather large, it is
 408 pre-multiplied by $e^{-z_{\text{ek}}}$ which has already decreased to $\mathcal{O}(0.1)$ at the height of this
 409 maximum. This choice ascertains that the velocity deficits $U/u_* - Z_*$ and $V/u_* - Z_*$
 410 do not depend on the velocity scale u_* , but only on G as

$$U_{\text{ek}}/u_* - Z_* \propto a_{\text{ek}}Z_* = 8.4G. \quad (11)$$

411 **The offset parameter** s_{ek} defines the zero-crossing height of the profile (in
 412 contrast to δ_{ek} , which determines the thickness across which the wind veering
 413 takes place). Physically, this offset can be understood as the height at which the
 414 surface was located assuming a perfect Ekman flow down to the surface. As this
 415 is not the case, and gradients are steeper in the highly turbulent boundary layer
 416 flow encountered when approaching the surface, the offset is smaller than zero (the
 417 fully turbulent boundary layer is actually thinner than an Ekman layer would be).
 418 From our DNS data, we estimate $s_{\text{ek}} = -0.12$.

419 In summary, the outer layer of Ekman flow is characterized by a turning of
 420 the wind velocity and the super-geostrophic maximum that is sustained by mo-
 421 mentum convergence at the inflection point of the velocity profile. The super-
 422 geostrophic maximum of streamwise velocity and a secondary minimum aloft the
 423 bulk-turbulent part of the boundary layer are well-described by a classic Ekman
 424 spiral with adapted boundary conditions and a shift in reference height. Corre-
 425 sponding profiles are shown in comparison with data from three DNS runs in
 426 Fig. 6. The idealized profiles capture the secondary minimum and convergence to
 427 the geostrophic equilibrium in the non-turbulent flow very well.

428 4.3 Streamwise velocity component

429 Well-established theories exist for the streamwise velocity profile, which in non-
 430 rotating flows is aligned with the surface shear stress due to the geometry. These
 431 theories cover various regimes based on their distance from the wall and the relative
 432 influence of viscosity, turbulence, and interaction with the outer flow region, with
 433 the logarithmic law for the mean velocity serving as a central reference point.

434 In immediate vicinity to the surface, local turbulent mixing cannot occur for
 435 the no-slip/no-penetration boundary condition, and the mean velocity is described
 436 by a viscous profile of the form

$$U^{\alpha_*+} = z^+ \quad (12a)$$

437 where the direction of the velocity points into the exact opposite direction of the
 438 wall shear stress τ . In absence of roughness elements and for small roughness

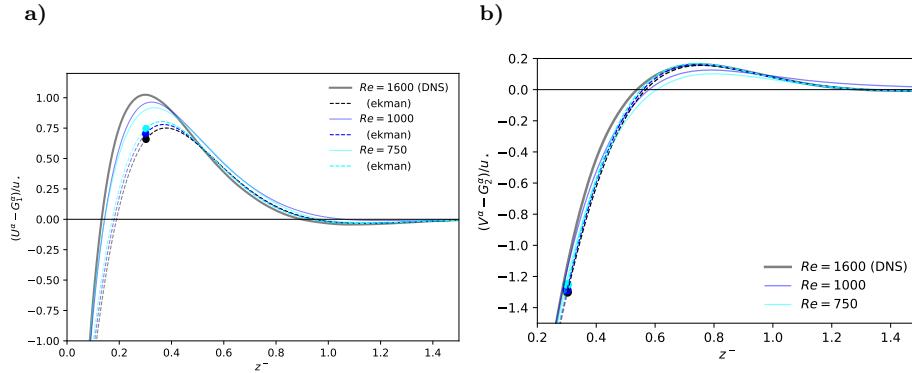


Fig. 6 Shear-aligned velocity deficit for the streamwise (panel (a)) and spanwise (panel (b)) components of the mean velocity U^{α} and V^{α} . Solid lines show DNS data, dashed lines the Ekman profiles U_{ek} and V_{ek} as defined in Eq. 10. Variations in U_{ek} and V_{ek} are a consequence of the normalization and related to changes in u_{*} and α_{*} among the different Re_D .

439 ($z_0^+ < 5$), this linear regime is known as viscous sub-layer Foken (2002); Foken
 440 et al. (1978). In fact, this law of the wall has no degree of freedom given the
 441 drag, i.e. once u_{*} and α_{*} are defined. However, theoretical foundation is lacking
 442 for the exact shape of the velocity profile in the buffer layer; though crucial for
 443 turbulence production, it is commonly understood as a transition region between
 444 the linear profile at the surface and the logarithmic profile aloft. A pure blending
 445 from the linear velocity profile into the logarithmic one is, however, not reasonable
 446 as both the linear and logarithmic profile overestimate the velocity in the buffer
 447 layer. We therefore introduce a two-step correction procedure, accounting for the
 448 smaller-than linear growth beyond $z^+ \approx 5$, and assuring smooth matching with
 449 the logarithmic law at $z^+ = 40$:

$$U_{\text{inner}}^{\alpha_{*}+} = \frac{z^+}{1 + c_1(y^+)^2} + (c_2 z^+ - a_{\text{match}}) \frac{1 + \tanh[0.2(z^+ - 22)]}{2} + c_3 e^{-c_4(z^+ - 22)^2}. \quad (12b)$$

450 We use here

$$c_1 = 0.00185; \quad c_2 = 0.195; \quad c_3 = 0.4; \quad c_4 = 0.35.$$

451 The second and third terms on the right hand side vanish for $z^+ \ll 22$, and
 452 $c_1 = 0.00185$ implies an approximately 5% correction at $z^+ = 5$ and an 18.5%
 453 correction at $z^+ = 10$. The second and third term on the R.H.S. of eq. (12b) are
 454 an empirical fit to the velocity profiles observed in the buffer layer and appear
 455 independent of the Reynolds number for the range observed here. The coefficient
 456 a_{match} , which has no effect in the viscous sublayer, is then used to match this
 457 formulation to the logarithmic law employed above.

458 In the logarithmic region, we use the profile

$$U_{\log}^{\alpha_{*}+} = \frac{1}{\kappa} \log z^+ + C \quad (12c)$$

459 with the von-Kármán constant $\kappa = 0.416$ and the boundary condition $C = 5.4605$.
 460 For this logarithmic law, $a_{\text{match}} = 3.569861$ for a matching at $z^+ = 40$.

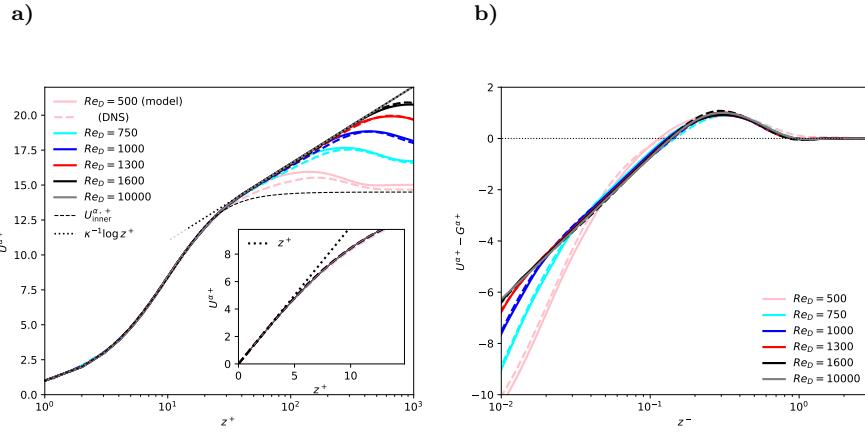


Fig. 7 Shear-aligned profiles of velocity components $U^{\alpha*+}$ in inner (left) and outer (right) units.

461 4.4 Spanwise velocity component

462 The background rotation and associated veering of the surface wind implies a
 463 non-zero profile for the span-wise velocity which challenges the conventional as-
 464 sumptions related to the channel-flow analogy: While the analogy with channel
 465 flow in vicinity of the wall implies that the streamwise component be zero or at
 466 least small, the veering requires a value of $V_{top} = U_G \sin \alpha_*$ in the free stream (and
 467 thus also at the top of the boundary layer if we assume that any substantial veloc-
 468 ity gradient is confined to the turbulent part of the flow). This continuous rotation
 469 of the wind vector is conveniently visualized by velocity hodographs aligned with
 470 the outer, geostrophic flow (cf. Fig. 5b) and normalized by the geostrophic wind.
 471 The geometry of the flow and its drag imply the following for any hodograph: (i)
 472 the boundary conditions at the surface, (ii) the boundary condition at the top,
 473 and (iii) the inclination of the hodograph at the origin by the surface veering:

$$V^{\alpha*}(z = 0) = 0, \quad (13a)$$

$$\lim_{z \rightarrow \infty} V^{\alpha*} = G \sin \alpha_* \quad (13b)$$

$$\partial_{z+} V^{\alpha*+} \Big|_{z=0} = 0. \quad (13c)$$

474 Outer scaling of the velocity profile further implies that the velocity deficit
 475 of $(V^{\alpha*} - G^{\alpha*})/u_*$ be a universal function of the outer height z^- . In the outer
 476 region of the flow (for $z^- \mapsto 1$), $f_V(z^-)$, should govern the spanwise velocity
 477 profile, as is supported by our DNS data (Fig. 1b); above $z^- \approx 0.3$, this profile is
 478 very well approximated by the Ekman-turning derived above (Eq. (10); Fig. 6b).
 479 While this deficit is a signature of outer rotation, it is inappropriate to extend this
 480 general relation to the surface where inner scales matter: On the one hand, the
 481 variation of the spanwise velocity deficit across the boundary layer (i.e. between
 482 $0 < z^- < 1$) must match the difference implied by the drag law (u_*, α_*) and the
 483 constant value of $V^{\alpha*}$ around $z^- = 0.3$. On the other hand, provided the outer
 484 velocity deficit is Re independent—the Re -dependence of α_* and u_* implies that
 485 this difference cannot be constant as a function of Re . We hence ask, how does the

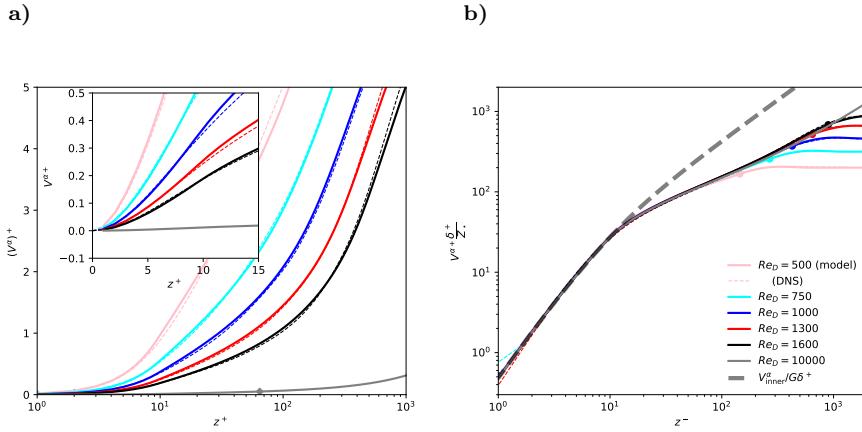


Fig. 8 Profiles of shear-aligned span-wise velocity $(V^\alpha)^+$ versus inner height. Dashed lines show DNS data, thick, opaque lines are from the semi-empirical theory developed above. Panel (a) shows standard inner normalization, panel (b) the inviscid normalization yielding a universal profile for the spanwise component of velocity in the inner layer.

span-wise component scale when the surface is approached? Clearly, the spanwise contribution is small in comparison with the streamwise component throughout much of the layer below $z^- \approx 0.3$. However, we cannot assume $V = 0$ if a smooth matching between the inner and outer layers shall be achieved. In this context, we first realize that the velocity deficit $(V^{\alpha*} - G^{\alpha*})/u_*$ approaches a Re-independent constant around $C_{V0} = Z_* \sin \alpha = 6.1$ at the surface; deviations from this constant are only found for the lowest Reynolds numbers which is in accordance with the low-Re correction suggested by Spalart (1989). This constrains the wind veer, and it quantitatively shows that the decreasing wall friction manifest in an increase of Z_* exactly compensates the decrease of wind turning measured by $\sin \alpha_*$.

If the difference across the boundary layer is constant (C_{V0}) vs. Re, the averaged gradient $\partial_{z^+} V^{\alpha*+}$ of the spanwise velocity component must decrease as $1/\delta^+$ with increasing Re_τ . Hence, it should—at a fixed height—be $V^{\alpha*} \propto (\delta^+)^{-1}$. A profile that agrees with the constraints of the profile at the surface and exploits the dependence of $V^{\alpha*}$ on δ^+ is

$$V^{\alpha*} \frac{\delta^+}{G} = f_{V,visc}(z^+) = v_{ref} (\omega_v z^+ - 1 + \exp[-\omega_v z^+]), \quad (14)$$

where v_{ref} controls the magnitude of the profile and ω_v sets the height at which the profile transitions into an approximately linear one. We find excellent agreement with the DNS data for $500 \leq Re_D \leq 1600$ below $z^+ \approx 15$ with

$$v_{ref} = 18.85; \quad \omega_v = 0.2353$$

(cf. Fig. 8b).

For the adjacent surface layer, we find a log-like transition from the quasi-linear profile inner profile around $z^+ = 10$ to a linear profile with increasing Re (Fig. 8b). We model this transition by

$$f_{V,\log}(z^+) = \frac{V_{log}(z^+)}{G} \delta^+ = a_{\log} + b_{\log} \log z^+ + c_{\log} z^+. \quad (15)$$

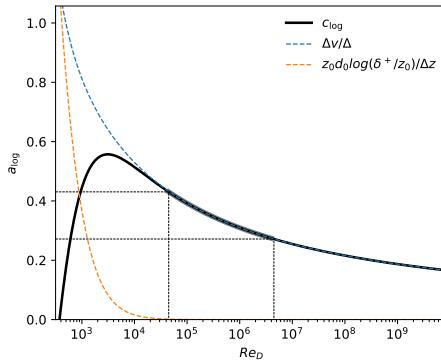


Fig. 9 Coefficients a_{\log} , b_{\log} and c_{\log} (cf. Eq. 15) as a function of the viscous Reynolds number Re_D . The approximate range of scale separation relevant for atmospheric application is found in between the dotted lines, where $a_{\log} \simeq 0$ and $c_{\log} \simeq 0.4$

508 This surface-layer profile matches the inner (viscous) scaling in vicinity of the
 509 surface to the outer (Ekman) scaling above $z^- = 0.3$ when constrained by the
 510 viscous profile at the bottom and the Ekman profile at the top:

$$f_{V,\log}(z^+ = 10) = f_{V,\text{visc}}(z^+ = 10) =: v_{10} \simeq 27.3 \quad (16a)$$

$$\frac{\partial}{\partial z^+} [f_{V,\log}]_{z^+ = 10} = \frac{\partial}{\partial z^+} [f_{V,\text{visc}}]_{z^+ = 10} =: d_{10} \simeq 4.01 \quad (16b)$$

$$f_{V,\log}(z^+ = 0.3\delta^+) = V_{\text{ek}}^{\alpha*}(z^- = 0.3)\delta^+ =: v_{03} \quad (16c)$$

511 where v_{03} is determined by $V_{\text{ek}}(0.3)$ and $U_{\text{ek}}(0.3)$ and depends on Re . Given the
 512 Ekman formulation of the velocity profile introduced in Sec. 4.2, one may express
 513 v_{03} using the Ekman profile introduced in Sec. 4.2 together with the approximation
 514 for $u_*(\text{Re})$ found in Eq. (9). While the Re -dependency of a_{\log} , b_{\log} , c_{\log} is small,
 515 it shows up in Fig. 1 where the normalized profiles of spanwise velocity become
 516 more convex with increasing Re . We can now quantify this effect by means of the
 517 change of c_{\log} versus Re which is shown in Fig. 9 (cf. Appendix A.2; a_{\log} and b_{\log}
 518 are then determined by the universal values of v_{10} and d_{10}).

519 4.5 Matching of the inner and outer layer profiles

520 The formulations introduced above are continuous across the transition from the
 521 inner to the outer layer. However, the requirement of smooth derivatives would
 522 over-constrain the velocity profiles and is hence not applied. This can, in particular
 523 for low or extremely high Re , cause discontinuity in the derivatives of the velocity
 524 profiles around the transition from the inner to the outer layer. To avoid such
 525 artificial discontinuity, the profiles are blended by an error-function transition
 526 using a blending height $z_{\text{blend}}^- = 0.28 - 2.25\sqrt{1/\text{Re}_T}$ and a transition thickness
 527 $z_{\text{trans}} = 2$, such that the weighting function $\omega(z^-)$ becomes

$$\omega(z^-) = \frac{1}{2} \left[\text{erf} \left(z_{\text{trans}} \log \frac{z^-}{z_{\text{blend}}} \right) + 1 \right], \quad (17)$$

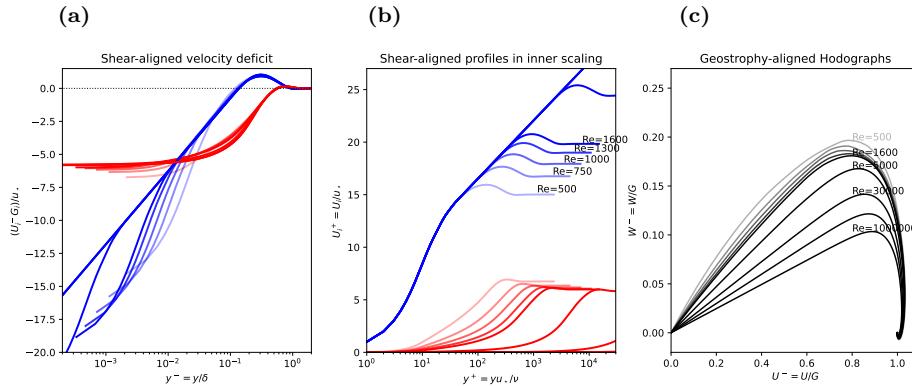


Fig. 10 (a) Velocity deficit, (b) velocity profile in shear-aligned hodographs and (c) hodograph in geostrophy-aligned coordinates. In panels (a) and (b), blue lines correspond to the streamwise component and red to the spanwise.

528 hence $u_{\text{total}} = (1 - \omega) \times u_{\text{inner}} + \omega \times u_{\text{outer}}$ and similar for v .

529 The resulting velocity profiles across the entire boundary layer are shown in
 530 inner and outer scaling as well as in hodograph- view in Fig. 10. The shear-aligned
 531 velocity deficit is shown in outer scaling highlighting the universality in the outer
 532 layer. The logarithmic scaling of the streamwise component is encountered as
 533 straight blue lines in panels (a) and (b) where the extent of the logarithmic range
 534 increases with Re towards lower values of z^- and higher values of z^+ depending
 535 on the scaling. Importantly, the logarithmic region is widening for increasing
 536 Reynolds number – irrespective of the scaling. In this simple inner scaling, the
 537 spanwise velocity (which follows a mixed scaling) does not collapse but seems to
 538 depend on Re (the collapse is seen in Fig. 8). However, the velocity deficit in outer
 539 units becomes approximately universal, also across the inner layer; this reflects
 540 the compensation of reduced turning (α) by increased drag (u_*), and is consistent
 541 with the theoretical considerations discussed in 4.1.

542 The spanwise velocity at a fixed height scales approximately as Re_τ^{-1} (Sec. 4.4).
 543 However, the fraction of turning that is encountered within the inner layer of the
 544 flow amounts to about 1/3 of the total wind veer (Fig. 12a). This is because, in
 545 inner units, the inner layer grows as Re_τ which exactly compensates the reduced
 546 gradient of spanwise velocity. The hodographs show the well-known Ekman shape
 547 with the laminar profile as an outer limit and 'flatter' hodographs, corresponding
 548 to less turning, for increasing Reynolds number.

549 5 Discussion

550 5.1 Implications for surface-layer scaling

551 Eq. (14) establishes a universal mixed scaling for the spanwise velocity in the vis-
 552 ous layer: While it requires the vertical coordinate to be expressed in inner units,
 553 the velocity itself is normalized by the geostrophic wind, and becomes inversely
 554 proportional to the friction Reynolds number $Re_\tau = \delta^+ / \nu$ when considered at a fixed
 555 height. In vicinity of the surface, such mixed scaling has already been identified

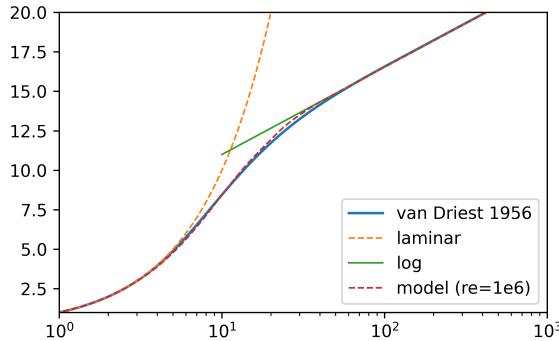


Fig. 11 Near-wall velocity profile according to the van-Driest scaling (blue, solid) in comparison with the present model (red, dashed), the viscous law of the wall (orange dashed), and the logarithmic law (green, solid)

556 for higher-order statistics in convective flows (Mellado et al. 2016; Li et al. 2018),
 557 where large scales leave their signatures in vicinity of the surface. It is important
 558 to note here that, while V is a first-order statistic from a statistical perspective,
 559 the spanwise velocity is a higher-order correction term from the perspective of sim-
 560 ilarity theory and from the viewpoint of the channel-flow analogy that is routinely
 561 employed in the surface layer. Further, this is consistent with the scaling for the
 562 velocity hodograph found in Eq. (11) where the friction velocity also drops out.

563 In the surface layer, there is not only a mixed scaling—as we had already iden-
 564 tified in the viscous layer—, but we cannot find a universal function onto which the
 565 profiles of spanwise velocity collapse. This additional degree of freedom reflects
 566 the inner–outer matching problem for the spanwise velocity. Rather than giving
 567 a universal profile for this region, we resort here to a parametric description of
 568 the problem in terms of the function $f_{V,\log}$ determined by the parameters a_{\log} ,
 569 b_{\log} , c_{\log} which can be estimated based on the above scaling considerations for
 570 any Reynolds number. We note that, once the parameter a_{\log} is known, the pa-
 571 rameters b_{\log} and c_{\log} can be estimated solely based on $f_{V,\text{visc}}$, i.e. using the value
 572 v_{10} and d_{10} found for the viscous region of the flow. For the range of Reynolds
 573 number relevant to geophysical problems ($10^4 \lesssim \text{Re}_D \lesssim 10^6$), the variation of c_{\log}
 574 is, however, rather small.

575 5.2 Comparison with other theories

576 An alternative approach that considers viscous effects close to the surface is the
 577 van-Driest scaling (Van Driest 1956), where an exponential damping of Prandtl's
 578 mixing length is considered near the wall to yield

$$\frac{\partial u^+}{\partial z^+} = \frac{2}{1 + \sqrt{1 + (2\kappa z^+)^2 (1 - \exp[-z^+])}}; \quad (18)$$

579 the spanwise component is zero as no rotational effects are considered. Compar-
 580 ing our proposed formulation for the stream-wise velocity in the inner layer to

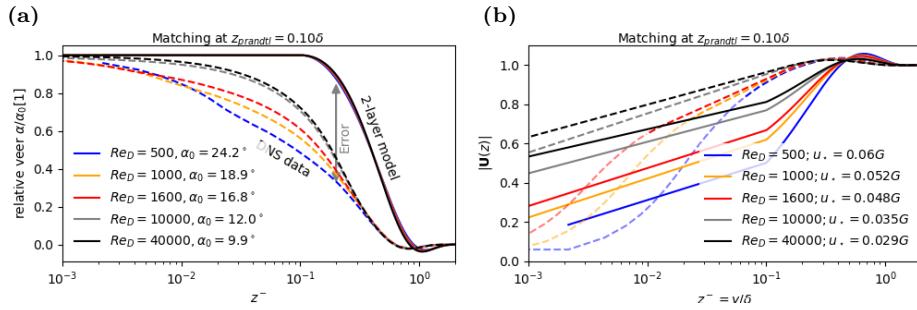


Fig. 12 Comparison of the DNS data (dashed lines) with the two-layer model proposed by Etling (solid lines) for different Reynolds number. Left panel shows the relative wind veer, right panel shows the velocity magnitude. The Etling model is calibrated by the surface veer from the DNS and the roughness parameter is chosen according to the correction factor $\exp \kappa A$ such that the total veer and velocity magnitude agree. Profiles in the buffer layer, defined here as $z^+ < 30$ are shown as opaque dashed lines as the two-layer model does not consider viscous effects.

Van Driest's formulation yields later convergence of the velocity onto the logarithmic profile while, over all, it serves as an excellent model of the streamwise velocity component (Fig. 11): Notable deviations (on the order of few percent) only occur in the region $10 < z^+ < 30$, where the velocity transitions from the linear to the logarithmic profile.

For the higher layers of the ABL, the Ekman spiral is the simplest model available. When employed across the entirety of the ABL, the spiral of a turbulent Ekman layer is flattened with respect to Ekman's laminar solution, which corresponds to a reduction of the veering angle both at the surface and throughout the ABL. We, however, find that a modified version of the Ekman spiral explicitly taking into account the surface boundary condition, is a consistent model and yields excellent agreement with the velocity profiles from DNS (Sec. 4.2).

A two-layer model consolidating both the logarithmic and Ekman layer can be obtained following the arguments by Etling (2008), cf. Emeis (2018). Given a surface veering and a matching height (extent of the logarithmic layer), a formulation for the velocity profile across both the logarithmic and the outer layer is obtained. A comparison using the surface veering based on our model and a matching height of $z_{\text{prandtl}} = 0.05\delta$, which gives better results than the matching height of 0.1δ suggested by Etling (2008), is shown in Fig. 12. The overall shape of velocity magnitude is matched apart from the viscous and buffer layer (cf. Fig. 12b) that is neglected by the two-layer model. However, quantitative departures on the order of 10% occur across the inner layer: it turns out that the non-rotating profile, of the two-layer model in the logarithmic region yields too low overall velocity as the spanwise component contributes to the velocity across the inner layer. Deviations also occur with respect to wind direction; despite the rather low matching height, a substantial fraction of the rotation occurs within the lower part of the ABL and about 20% of the wind veer is not captured by the two-layer model. As the overall veer is given, the non-captured veer close to the surface is then compensated across the logarithmic layer. In the upper part of the boundary layer, both profiles match well.

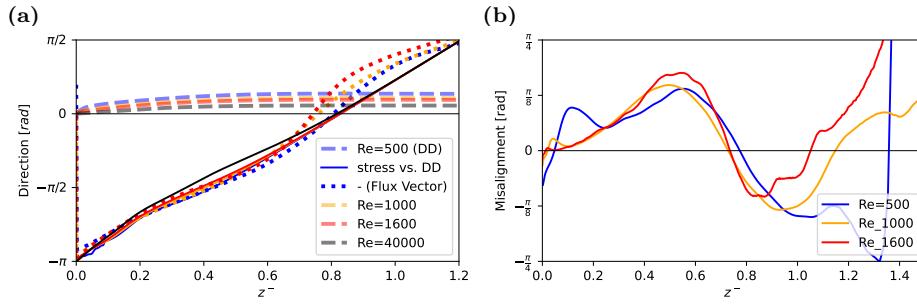


Fig. 13 Panel (a) shows the wind direction (DD) as thick dashed lines and the direction of stress relative to DD as thin solid lines, both according to our profile model. The direction of the negative flux vector ($u'w'$, $v'w'$) based on DNS data is shown as a dotted line. In panel (b), we plot the misalignment, i.e. the angle, between the flux vector and the shear vector, where the shear vector is defined accordingly as $(\partial_z U, \partial_z V)$.

The interpretation of flux and gradient profiles in terms of the K-theory (cf. Sec. 3, Fig. 2) suggests a certain universality in the inner layer, while a global collapse, i.e. across the boundary layer, is not obtained. While the K-Profiles concern the total stress, a consistent formulation of the turning would also require its partitioning to the individual components, i.e. the orientation of the stress vector ($u'w'$, $v'w'$) in the horizontal plane. If K-theory shall be used, this stress vector needs to be anti-parallel to the corresponding stress vector $(\partial_z U, \partial_z V)$. Fig. 13 shows the direction of the velocity, gradient and stress vectors across the boundary layer. It turns out that the negative stress vector with respect to the wind direction and the flux vectors (absolute) have an approximately similar direction. It appears that both rotate by about 270° ($3\pi/2$) across the boundary layer. Apart from the lower part of the Ekman layer ($0.2 < z^- < 0.7$), where there is a slight dependence on Re , the direction of stress appears to be universal, which is a consequence of the Ekman profile introduced in Sec. 4.2. However, this implies a misalignment between the flux and the stress on the order of the wind turning, and indeed Fig. 13b shows a misalignment up to $\pi/8$. This is in accordance with the expectation by (Townsend 1976, Chap. 7.18) and prevents the transfer of energy from the mean flow to turbulence at these heights, thus preventing a boundary-layer growth. While the behavior in the inner layer seems to depend on Re , there emerges universality in the misalignment across the outer layer, suggesting that a consideration of misalignment in the context of K-theory is possible when developing formulations for higher-order quantities.

6 Conclusions

We investigate the wind veer and fundamental scaling properties of the velocity profiles in Ekman flow. Based on scaling considerations and direct numerical simulation spanning a decade in external separation from $Re_A = 1.25 \times 10^5$ to $Re_A = 1.28 \times 10^6$, we derive a universal formulation for the both horizontal components of the velocity profile in a stationary, smooth Ekman layer. This formulation is consistent with the DNS data and also yields reasonable results at geophysical scale separation; the logarithmic law of the mean velocity is recovered with the

well-known limits and deviations towards the surface and Ekman layer. The classic formulation of the Ekman layer, employing the surface boundary condition, is replaced by a modified solution that can be obtained by Ekman's system of governing equations, but using different boundary conditions that are more appropriate of the actual situation encountered in the planetary boundary layer. The three parameters that characterize this boundary condition are estimated based on DNS data.

To quantify the spanwise velocity component consistently across the boundary layer, we derive a universal scaling of the spanwise velocity component in a shear-aligned reference frame. For the *inner layer*, we find the mixed scaling

$$V^{\alpha*}/G = \frac{1}{\delta^+} f(z^+) \quad (19)$$

that is, the spanwise velocity normalized by the outer velocity scale is a universal function of the inner height and Friction Reynolds number $Re_r = \delta^+$. This scaling is derived here based on scaling considerations, and it is in excellent agreement with the DNS data available. In the outer layer, the spanwise velocity follows outer scaling, consistently with the Ekman model discussed above (Sec. 4.2).

Our results suggest that there is no lower limit of the turning. Hence—despite its very large scale separation / huge Reynolds number—the ABL is not in the 'limit' of high Reynolds number from the perspective of wind veer, but always in a regime where changes in Re impact on the vertical partitioning of rotation.

660 A Appendices

661 A.1 Laminar Ekman solution with consideration of inner layer

662 The following Ekman system is obtained by averaging the Navier–Stokes equations horizontally
663 and over time and neglecting the turbulent transport terms (turbulence can then be considered
664 via the eddy-viscosity concept through variations in the viscosity ν):

$$\begin{pmatrix} \partial_t U \\ \partial_t V \end{pmatrix} = \begin{pmatrix} fV & +\nu\partial_z^2 U \\ -f(U-G) & +\nu\partial_z^2 V \end{pmatrix} \quad (20a)$$

$$\Rightarrow \partial_t(U+iV) = f(V-i(U-G)) + \nu\partial_z^2(U+iV) \quad (20b)$$

665 In stationary conditions, this system is solved by

$$\hat{u}(z) = U_\infty + e^{-\gamma z} [A \cos \gamma z + B \sin \gamma z] \quad (20c)$$

$$\hat{v}(z) = V_\infty + e^{-\gamma z} [-A \sin \gamma z + B \cos \gamma z] \quad (20d)$$

666 where the constants U_∞ , V_∞ set the top boundary condition and A and B set the bottom
667 boundary condition. The most common boundary condition for a surface Ekman layer is $A =$
668 $U_\infty = G$, $B = 0$, and $V_\infty = 0$. The lower boundary condition, however, neglects the existence
669 of the surface layer, and it appears reasonable to define $A = cG$ where $c < 1$ is a constant that
670 incorporates the increased shear in the surface layer. Given a 'matching height' z_{match} and
671 normalized matching height $\xi = \gamma z_{match}$ in the upper part of the inner layer, we can match
672 the Ekman profile to the inner layer by letting

$$\begin{aligned} u(z_{match}) &\equiv u_{match} = U_\infty + e^{-\xi} [A \cos \xi + B \sin \xi] \\ v(z_{match}) &\equiv v_{match} = V_\infty + e^{-\xi} [-A \sin \xi + B \cos \xi] \end{aligned} \quad (21a)$$

$$\Rightarrow \begin{pmatrix} u_{match} - U_\infty \\ v_{match} - V_\infty \end{pmatrix} = e^{-\xi} \begin{pmatrix} A \\ B \end{pmatrix} \begin{pmatrix} \cos \xi & +\sin \xi \\ -\sin \xi & +\cos \xi \end{pmatrix} \quad (21b)$$

$$(21c)$$

674 Matching the profile at $\xi = 0$, one obtains $A = \Delta u_{\text{match}}$ and $B = -\Delta w_{\text{match}}$; and when the
 675 direction Ox is aligned with the geostrophic wind, we obtain the textbook-case $A = |\mathbf{G}|$ and
 676 $B = 0$.

677 Otherwise, choosing $B \neq 0$ allows to introduce a phase shift of the Ekman rotation with
 678 respect to the decay of the wind spiral. However, in our context, the thickness and position of
 679 the spiral can already be controlled by the eddy viscosity and an offset in ξ , here we let $B = 0$.

680 A.2 Matching the spanwise velocity profiles in the inner layer

681 The spanwise profile in vicinity of the surface is given by $V/G = f_{V,\text{visc}}\delta^+$ with

$$f_{V,\text{visc}} = v_{\text{ref}} (\omega_v z^+ - 1 + e^{-\omega_v z^+}) \quad (22a)$$

$$f_{V,\log} = a_{\log} + b_{\log} \log z^+ + c_{\log} z^+ \quad (22b)$$

682 Matching the profiles and gradient $z_0 = 10^+$ and the value at $z_1 = 0.3\delta^+$ yields

$$v_{\text{ref}} (\omega_v z_0 + e^{-\omega_v z_0}) = v_0 = a_{\log} + b_{\log} \log z_0 + c_{\log} z_0 \quad (23a)$$

$$v_1 = a_{\log} + b_{\log} \log z_1 + c_{\log} z_1 \quad (23b)$$

$$v_{\text{ref}} \omega_z (1 - e^{-\omega_z z_0}) = d_0 = \frac{b_{\log}}{z_{10}} + c_{\log} \quad (23c)$$

683 The gradient condition implies $b_{\log} = (d_0 - c_{\log})z_0$, and yields

$$v_0 - z_0 d_0 \log z_0 = a_{\log} + c_{\log} (z_0 - z_0 \log z_0) \quad (24a)$$

$$v_1 - z_0 d_0 \log z_1 = a_{\log} + c_{\log} (z_1 - z_0 \log z_0) \quad (24b)$$

$$\Rightarrow c_{\log} = \frac{\Delta v - z_0 d_0 \log z_1 / z_0}{\Delta z} \quad (24c)$$

684 with $\Delta z = z_1 - z_0$ and $\Delta v = v_1 - v_0$. Then, the coefficient a_{\log} is estimated as

$$a_{\log} = v_0 - z_0 d_0 \log z_0 - \frac{\Delta v - z_0 d_0 \log z_1 / z_0}{\Delta z} [z_0 - z_0 \log z_0]. \quad (24d)$$

685 We note that $\log(z_1/z_0)/(z_1 - z_0) \rightarrow 0$ for large z_1 , and as $z_1 = 0.3\delta^+$, this implies that the
 686 second term in c_{\log} only plays a role at low and intermediate Re. Then, a_{\log} can be estimated
 687 as

$$a_{\log} \simeq v_0 - z_0 \left[d_0 \log z_0 - \frac{\Delta v}{\Delta z} (1 - \log z_0) \right] \quad (24e)$$

688 for large Re.

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