Let u, v and w (and the corresponding Reynolds stress tensor) be defined on a grid $Ox \times Oy \times Oz$ where the unit vector $\hat{e}_x$ is turned by θ with respect to the [negative] direction of surface shear stress [1] We require the velocities on the grid $Ox_\star \times Oy \times Oz_\star$ that is aligned with the surface shear stress, i.e.

$$\left(\begin{array}{c} \hat{e}_{x_\star} \\ \hat{e}_y \\ \hat{e}_{z_\star} \end{array} \right) = \left(\begin{array}{ccc} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{array} \right) \left(\begin{array}{c} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{array} \right). \tag{1}$$

This implies

$$\begin{aligned} u_\theta &= u\cos\theta + w\sin\theta \\ w_\theta &= -u\sin\theta + w\cos\theta. \end{aligned}$$

Let $c_\theta \equiv \cos\theta$, $s_\theta \equiv \sin_\theta$ and $m_\theta = \sin\theta\cos\theta$. The corresponding Reynolds stress tensor is

$$\left(\begin{array}{ccc} u_\theta u_\theta & u_\theta v_\theta & u_\theta w_\theta \\ u_\theta v_\theta & v_\theta v_\theta & v_\theta w_\theta \\ u_\theta w_\theta & v_\theta w_\theta & w_\theta w_\theta \end{array} \right) = \left(\begin{array}{ccc} (uc_\theta + ws_\theta)^2 & uvc_\theta + wvs_\theta & (uc_\theta + ws_\theta)(-us_\theta + wc_\theta) \\ & vv & (-us_\theta + wc_\theta)v \\ & & (-us_\theta + wc_\theta)^2 \end{array} \right) \tag{2}$$

$$= \left(\begin{array}{ccc} u^2c_\theta^2 + w^2s_\theta^2 + 2uwm_\theta & uvc_\theta + wvs_\theta & m_\theta(w^2 - u^2) + uw(c_\theta^2 - s_\theta^2) \\ & v^2 & -uvs_\theta + wvc_\theta \\ & & u^2s_\theta^2 + w^2c_\theta^2 - 2uwm_\theta \end{array} \right) \tag{3}$$

$$= \left(\begin{array}{c} u^2c_\theta + w^2s_\theta \end{array} \right) \tag{4}$$

[1] Here, negative means that we consider the grid aligned with the positive vector of the velocity gradient at the surface