

By choice of the grid, it is for the first grid point (z_1)

$$z_1 = c_\delta \delta \Rightarrow z_1^- = c_\delta.$$

The height argument of the log-law is, however, an inner height, and we need to use z^+ . Using the relation $z^- = z^+/\mathbf{Re}_\tau$, we get

$$z_1^+ = c_\delta \mathbf{Re}_\tau$$

for the height of the first grid node in inner units. And a similar relation holds for the rough equivalent:

$$\frac{z_1}{z_0} = \frac{c_\delta \delta}{z_0} = c_\delta \frac{u_\star}{f} \frac{u_\star}{c_\nu \nu} = \frac{c_\delta}{c_\nu} \mathbf{Re}_\tau.$$

This underlines the role of c_ν in linking the classic inner ('+') scale and the rough inner unit (z_0), it is namely

$$\frac{z^+}{z/z_0} = c_\nu$$

. First, this shows that apart from any variation in $c_n u$ the scaling with z_0 is the classic inner ('+') scaling. Second, the small variation of c_ν (estimated/tuned against the DNS results[!!!]) across the Reynolds number (less than the already weak variation in u_\star and orders of magnitudes less than the variation in Re_τ) shows that the analogy between aerodynamically rough and smooth flows holds quite nice, but not exact. It remains to be seen whether these small variations are due to technical details of the LES formulation under changing scale separation and different action of the SGS scheme, or whether these are an actual physical property of the flow.

With regards to the velocity boundary condition, we now see that

$$U_1 = U(z_1) = \frac{u_\star}{\kappa} \ln \frac{z_1}{z_0} = \frac{u_\star}{\delta} \ln \mathbf{Re}_\tau - \ln \frac{c_\delta}{c_\nu}$$

where the first term ($\ln \mathbf{Re}_\tau$) captures the dependency of the grid on the Reynolds number and the second term reflects the tuning parameter used to get a correct u_\star in the LES.