We identify the total shear stress τ as

$$\tau = \underbrace{\mu\frac{\partial \overline{u}}{\partial y}}_{\text{viscous stress}} - \underbrace{\rho\overline{u'v'}}_{\text{Reynolds' stress}}, \tag{1}$$

where y is distance from the wall, μ dynamic viscosity ($\nu = \mu/\rho$) and ρ is density. Next, observe that the amplitude of motion due to oscillation of an infinite plate decays as $\exp\left[-y/A\right]$. Hence, we use for the damping of fluid oscillation due to a fixed wall, the model

$$1 - e^{-\tilde{y}}, \tag{2}$$

where $\tilde{y} = y/A$. Now, we express the stress, according to Prandtl

$$\begin{aligned}\frac{\tau}{\rho} &= \nu\frac{\partial \overline{u}}{\partial y} + r\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}} && \text{(3a)}\\ &= \nu\frac{\partial \overline{u}}{\partial y} + r l_1 l_2\left(\frac{\partial \overline{u}}{\partial y}\right)^2 && \text{(3b)}\\ &= \nu\frac{\partial \overline{u}}{\partial y} + \kappa^2 y^2\left(\frac{\partial \overline{u}}{\partial y}\right)^2, && \text{(3c)}\end{aligned}$$

where κ is the von-Kármán constant and l Prandtl's mixing length. This model is well-known to hold appropriately in fully developed turbulent flow.

Near a wall, however, the turbulence is not fully developed, but damped by the presence of that very wall such that the prefactor $1 - e^{-\tilde{y}}$ can be taken into account in Reynolds' stres term:

$$\frac{\tau}{\rho} = \nu\frac{\partial \overline{u}}{\partial y} + \kappa^2 l^2\left(1 - e^{-\tilde{y}}\right)^2\left(\frac{\partial \overline{u}}{\partial y}\right)^2 \tag{4}$$

Non-dimensionalize Eq. (4) using $u_\star = \sqrt{\tau_{\text{wall}}/\rho}$ and the wall unit $y_+ = \nu/\sqrt{\tau_{\text{wall}}/\rho}$:

$$\tau^+\left(= \frac{\tau}{\tau_{\text{wall}}}\right) = \frac{\partial u^+}{\partial y^+} + \kappa^2 y^{+2}\left(1 - e^{-\widetilde{y^+}}\right)^2\left(\frac{\partial u^+}{\partial y^+}\right)^2 \tag{5}$$

In the constant-flux layer, it is $\tau = \tau_{\text{Wall}}$, such that

$$\begin{aligned}0 &= \frac{\partial u^+}{\partial y^+} + \kappa^2 y^{+2}\left(1 - e^{-\widetilde{y^+}}\right)^2\left(\frac{\partial u^+}{\partial y^+}\right)^2 - 1 && \text{(6)}\\ 0 &= \left(\frac{\partial u^+}{\partial y}\right)^2 + \frac{1}{\kappa^2 y^{+2}\left(1 - e^{-\widetilde{y^+}}\right)^2}\left(\frac{\partial u^+}{\partial y^+} - 1\right) && \text{(7)}\end{aligned}$$

and we solve for $\partial_{y^+} u^+$ to obtain

$$\frac{\partial u^+}{\partial y^+} = \frac{2}{1 + \sqrt{1 + 4\kappa^2 y^{+2} \left(1 - e^{-\widetilde{y^+}}\right)}} \tag{8}$$