We assume, the spanwise velocity follows a profile of the following shape:

$$w = w_1 + \alpha \log\left(\frac{z}{z_1}\right) + \beta(z - z_1) \quad (1)$$

$$\Rightarrow \frac{\partial w}{\partial z} = \frac{\alpha}{z} + \beta \quad (2)$$

We now determine α and β such that:

$$\begin{array}{rcl} \frac{\partial w}{\partial z}\Big|_{z_1} & = & C_1 \\ w(z_2) & = & C_2 \end{array} \quad (3)$$

The lower boundary yields

$$C_1 = \alpha/z_1 + \beta \Leftrightarrow \left\{ \begin{array}{rcll} \beta & = & C_1 - \alpha/z_1 & \text{viz. (i)} \\ \alpha & = & z_1(C_1 - \beta) & \text{viz. (ii)} \end{array} \right. \quad (4)$$

We can now substitute β or α in Eq. (1) to yield:
(i) when substituting β

$$\begin{aligned} w &= w_1 + \alpha \log\left(\frac{z}{z_1}\right) + \left(C_1 - \frac{\alpha}{z_1}\right)(z - z_1) && (5) \\ w - w_1 - C_1(z - z_1) &= \alpha\left[\log\left(\frac{z}{z_1}\right) - \frac{z}{z_1}\right] && (6) \\ \Rightarrow \alpha &= \frac{C_2 - w_1 - C_1(z_2 - z_1)}{\log(z_2/z_1) - z_2/z_1} && (7) \end{aligned}$$

(ii) when substituting α

$$\begin{aligned} w &= w_1 + z_1(C_1 - \beta)\log\left(\frac{z}{z_1}\right) + \beta(z - z_1) && (8) \\ w - w_1 - C_1 z_1 \log(z/z_1) &= \beta\left[z - z_1\left(1 + \log\left(\frac{z}{z_1}\right)\right)\right] && (9) \\ \Rightarrow \beta &= \frac{C_2 - w_1 - C_1 z_1 \log(z_2/z_1)}{z_2 - z_1(1 + \log(z_2/z_1))} && (10) \end{aligned}$$