

Homework 5

Exercise 5.3

Let $p(x, y) = 6(x - y)^2$ be the PDF of a bivariate random variable (X, Y) where both variables range from zero to one.

a) Find CDF.

$$\begin{aligned} F(x, y) &= \int_0^y \int_0^x 6(t - s)^2 dt ds = \int_0^y \int_0^x 6t^2 - 12ts + 6s^2 dt ds = \\ &= \int_0^y 2t^3 - 6t^2 s + 6ts^2 \Big|_0^x ds = \int_0^y 2x^3 - 6x^2 s + 6xs^2 ds = 2x^3 y - 3x^2 y^2 + 2xy^3 \end{aligned}$$

b) Find marginal distributions.

$$\begin{aligned} p_X(x) &= \int_0^1 6(x - y)^2 dy = \int_0^1 6x^2 - 12xy + 6y^2 dy = 6x^2 - 6x + 2 \\ p_Y(y) &= \int_0^1 6(x - y)^2 dx = \int_0^1 6x^2 - 12xy + 6y^2 dx = 6y^2 - 6y + 2 \end{aligned}$$

c) Find conditional distributions.

$$\begin{aligned} p_{X|Y}(x|y) &= \frac{p_{X,Y}(x, y)}{p_Y(y)} = \frac{6(x - y)^2}{6y^2 - 6y + 2} \\ p_{Y|X}(y|x) &= \frac{p_{X,Y}(x, y)}{p_X(x)} = \frac{6(x - y)^2}{6x^2 - 6x + 2} \end{aligned}$$

Exercises 4.17c and 5.1

In R scripts HW5-ex4_17.R and HW5-ex5_1.R.