Homework 1

Exercise 1.12

Show that the standard measurable space on $\Omega = \{0, 1, ..., \infty\}$

a) equipped with geometric measure is a discrete probability space:
For $k \in \Omega : P(X = k) = p(1-p)^k$
We need to show that $P(\Omega) = 1$. Since Ω is discrete,

$$P(\Omega) = \sum_{w \in \Omega} P(w) = \sum_{k=0}^{\infty} P(k) = \sum_{k=0}^{\infty} p(1-p)^k = p\sum_{k=0}^{\infty}(1-p)^k = \frac{p}{1-(1-p)} = 1.$$

We saw that $p(1-p)^k$ is a geometric series and used formula for its infinite sum: $\frac{a_1}{1-q}$, with first term of the series $a_1 = p$ and ratio $q = (1-p)$.

b) equipped with Poisson measure is a discrete probability space:
For $k \in \Omega : P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$
We need to show that $P(\Omega) = 1$. Since Ω is discrete,

$$P(\Omega) = \sum_{w \in \Omega} P(w) = \sum_{k=0}^{\infty} P(k) = \sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda}\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{-\lambda}e^{\lambda} = 1.$$

We used that $\sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$ is a Taylor series that converges to e^{λ}.

c) Define another probability measure on this measurable space.
We could set $P(w) = \frac{1}{n}$ for any n elements (let's say for $w < n$), and $P(w) = 0$ for all others (for $w \geq n$).

d)R: Draw 1000 samples from the Poisson distribution $\lambda = 10$ (rpois) and compare relative frequencies with theoretical probability measure.
In R script HW1-ex1_12.R.