

Homework 3

Exercise 3.7

Describe:

- **Real-world example of two events \mathcal{A} and \mathcal{B} that are dependent but become conditionally independent if conditioned on a third event \mathcal{C} .**

Event \mathcal{A} is that Karen knows a lot about certain subject and event \mathcal{B} is that Karen talks a lot about the subject. This two events are dependent, but if conditioned to \mathcal{C} , the event that says the information is shared over Facebook, the knowledge about discussed topic becomes irrelevant.

- **Real-world example of two events \mathcal{A} and \mathcal{B} that are independent, but become dependent if conditioned on some third event \mathcal{C} .**

Event \mathcal{A} is that person 1 has COVID-19 and \mathcal{B} is that person 2 has it. They are independent, but if conditioned to \mathcal{C} , the event that says they live in the same household, they become dependent.

Exercise 4.3

a) **Let X and Y be independent discrete variables. Find the PMF of $Z = X + Y$. Hint: Use the law of total probability.**

$$P(Z = z) = P(X + Y = z) = \sum_{y=-\infty}^{\infty} P(X = z - y | Y = y) \cdot P(Y = y) = \sum_{y=-\infty}^{\infty} P(X = z - y) \cdot P(Y = y).$$

In the last step we used the information that X and Y are independent. In the second step we used law of total probability.