Homework 7

Exercise 7.6

Let $X \sim$ Beta(α, β). Find $Var[X]$.
From a) we know that $E[X] = \dfrac{\alpha}{\alpha+\beta}$, and we have two hints available: $\mathrm{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ and $\mathrm{B}(x+1,y) = \mathrm{B}(x,y)\frac{x}{x+y}$.

$$Var[X] = E[X^2] - E[X]^2 = \frac{\alpha(\alpha+1)(\alpha+\beta) - \alpha^2(\alpha+\beta+1)}{(\alpha+\beta+1)(\alpha+\beta)^2} = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

$$\begin{aligned}
E[X^2] &= \int_0^1 x^2 \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}dx = \frac{1}{\mathrm{B}(\alpha,\beta)}\int_0^1 x^{\alpha+1}(1-x)^{\beta-1}dx \\
&= \frac{1}{\mathrm{B}(\alpha,\beta)}\mathrm{B}(\alpha+2,\beta) = \frac{1}{\mathrm{B}(\alpha,\beta)}\mathrm{B}(\alpha+1,\beta)\frac{\alpha+1}{\alpha+1+\beta} \\
&= \frac{1}{\mathrm{B}(\alpha,\beta)}\mathrm{B}(\alpha,\beta)\frac{\alpha+1}{\alpha+1+\beta}\frac{\alpha}{\alpha+\beta} = \frac{\alpha+1}{\alpha+\beta+1}\frac{\alpha}{\alpha+\beta}
\end{aligned}$$

Exercise 7.9

Let X follow a normal distribution with mean μ and variance σ^2.

a) Find $E[2X+4]$. $E[2X+4] = 2E[X] + 4 = 2\mu + 4$, because E is linear.

b) Find $E[X^2]$.
$Var[X] = E[X^2] - E[X]^2$, so $E[X^2] = Var[X] + E[X]^2 = \sigma^2 + \mu^2$.

d) Check your results numerically for $\mu = 0.4$ and $\sigma^2 = 0.25$ and plot the densities of all four distributions.
In R script HW7-ex7_9.R.

Exercise 7.10

R: Take $n = 5, 10, 100, 1000$ samples from the $N(2,6)$ distribution 10000 times. Plot the theoretical density and the densities of $\bar{X}$ statistic for each n. Intuitively, are the results in correspondence with your calculations? Check them numerically.
In R script HW7-ex7_10.R.