Homework 4

Exercise 4.9

b) Find the quantile function of the uniform random variable.

$$x = F_x(F_x^{-1}(x)) = \frac{F_x^{-1}(x) - a}{b - a} \iff F_x^{-1}(x) = x(b - a) + a$$

d) Let $X \sim$ Uniform$(-1, 3)$. Find such z that $P(X < z + \mu_x) = \frac{1}{5}$.

$$F_x^{-1}(\frac{1}{5}) = \frac{1}{5}(3 - (-1)) + (-1) = -\frac{1}{5} = z + \mu_x \iff z = -\frac{6}{5}$$

We know that μ_x is the mean of uniform RV X, so $\mu_x = 1$.

e) R: Check your result from d) using simulation.

In R script HW4-ex4_9.R.

Exercise 4.16

a) Let $X \sim \mathbf{N}(\mu, \sigma)$. Find the PDF of $Y : \log(Y) = X$.

We will use formula for transformation of a RV: $f_y(y) = f_x(g^{-1}(y))|\frac{d}{dy}g^{-1}(y)|$. Since $\log(Y) = X$, we already know $g^{-1}(y) = \log(y)$.

$$f_y(y) = f_x(\log(y))|\frac{d}{dy}\log(y)| = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(\log y - \mu)}{2\sigma^2}} \cdot |y^{-1}|$$

b) R: Sample from the lognormal distribution with parameters $\mu = 5$ and $\sigma = 2$. Plot a histogram of the samples. Then log-transform the samples and plot a histogram along with the theoretical normal PDF.

In R script HW4-ex4_16.R.