# AURMC Estimator

```r
library(AURMC)
```

## Estimator

Each patient is observed until their time of censoring $C$ or death $T$. Let $X = \min(T, C)$ denote the observation time, and $\delta = \mathbb{I}(T \le C)$ the status indicator. While under observation, measurements are recorded for a process $\{D(t), 0 < t < \tau\}$. For example, $D(t)$ may take the value 1 while a patient is in the hospital, and 0 otherwise. Define the target estimand:

$$\mu = \int_0^\tau \mathbb{E}\{D(t)\}dt,$$

which is the area under the curve (AUC) up to the truncation time $\tau$. A consistent estimator for $\mu$ is given by:

$$\hat{\mu} = \int_0^\tau \hat{S}(t)\frac{\bar{D}(t)}{\bar{Y}(t)}dt,$$

$$\bar{D}(t) = \frac{1}{n}\sum_{i=1}^n D_i(t)\mathbb{I}(X_i \ge t),$$

$$\bar{Y}(t) = \frac{1}{n}\sum_{i=1}^n \mathbb{I}(X_i \ge t).$$

Here $\hat{S}(t)$ is the Kaplan-Meier estimate of the survival probability at time $t$, $\bar{D}(t)$ is the mean value of the measurement among subjects remaining at risk, and $\bar{Y}(t)$ is the proportion of the sample remaining at risk.

### Method of integration

Let $\hat{E}(t) = \hat{S}(t)\bar{D}(t)/\bar{Y}(t)$ denote the estimated value of $\mathbb{E}\{D(t)\}$ at time $t$. Partition the interval $[0, \tau)$ as:

$$0 = t_0 \le t_1 < \cdots < u_K \le u_{K+1} = \tau,$$

where $\{t_k : 1 \le k \le K\}$ are the times at which the value of $\hat{E}(t)$ changes. $\hat{\mu}$ may be calculated in several ways, all of which provide consistent estimation. The left sum is:

$$\hat{\mu}_{\text{left}} = \sum_{k=1}^{K+1} \hat{E}(t_{k-1}) \cdot (t_k - t_{k-1}).$$

The right sum is:

$$\hat{\mu}_{\text{right}} = \sum_{k=1}^{K+1} \hat{E}(t_k) \cdot (t_k - t_{k-1}).$$

Lastly, the trapezoidal sum is:

$$\hat{\mu}_{\text{trapezoid}} = \sum_{k=1}^{K+1} \left\{\frac{\hat{E}(t_{k-1}) + \hat{E}(t_k)}{2}\right\} \cdot (t_k - t_{k-1}).$$

**Example**

Consider the following data for a single subject:

```r
data <- data.frame(
  idx = c(1, 1, 1, 1, 1, 1),
  time = c(0, 1, 2, 3, 4, 5),
  status = c(1, 1, 1, 1, 1, 0),
  value = c(5, 4, 3, 2, 1, 0)
)
show(data)
#>   idx time status value
#> 1   1    0      1     5
#> 2   1    1      1     4
#> 3   1    2      1     3
#> 4   1    3      1     2
#> 5   1    4      1     1
#> 6   1    5      0     0
```

The value of the repeated measure curve can be tabulated as:

```r
tab <- AURMC::TabRMC(data)
show(tab)
#>   time nar y haz surv d exp
#> 1    0   1 1   0    1 5   5
#> 2    1   1 1   0    1 4   4
#> 3    2   1 1   0    1 3   3
#> 4    3   1 1   0    1 2   2
#> 5    4   1 1   0    1 1   1
#> 6    5   1 1   0    1 0   0
```

Here `nar` is the number at risk at the beginning of the interval, `y` is $\bar{Y}(t)$, `haz` is the hazard of death, `surv` is the survival probability, `d` is $\bar{D}(t)$, and `exp` is $\hat{E}(t) = \hat{S}(t)\bar{D}(t)/\bar{Y}(t)$.

The AUC can be calculated as:

```r
# Left sum.
left <- AURMC::AURMC(data, int_method = "left")
cat("Left sum:\n")
#> Left sum:
show(left)
#>       method tau auc se lower upper p
#> 1 asymptotic   5  15  0    15    15 0

# Right sum.
right <- AURMC::AURMC(data, int_method = "right")
cat("Right sum:\n")
#> Right sum:
show(right)
#>       method tau auc se lower upper p
#> 1 asymptotic   5  10  0    10    10 0

# Trapezoidal sum.
right <- AURMC::AURMC(data, int_method = "trapezoid")
cat("Trapezoidal sum:\n")
#> Trapezoidal sum:
show(right)
#>       method tau  auc se lower upper p
#> 1 asymptotic   5 12.5  0  12.5  12.5 0
```