区间估计:

置信度 Confidence level $P(\theta\in[\hat\theta_1,\hat\theta_2])$

置信系数 Confidence coefficient $\inf P_\theta(\theta\in[\hat\theta_1,\hat\theta_2])$

精确度: $E_\theta(\hat\theta_2-\hat\theta_1)$

$\hat\theta_U(X)$: 置信上限 $\hat\theta_L(X)$: 置信下限.

枢轴变量法:

① 单正态参数

(1) σ 已知 求 μ

$$T=\sqrt{n}\frac{\bar X-\mu}{\sigma}\sim N(0,1)\quad \left[\bar X-\frac{\sigma}{\sqrt n}u_{\frac{\alpha}{2}},\ \bar X+\frac{\sigma}{\sqrt n}u_{\frac{\alpha}{2}}\right]$$

(2) σ 未知 求 μ

$$T=\sqrt{n}\frac{\bar X-\mu}{S}\sim t_{n-1}\quad \left[\bar X-\frac{\sigma}{\sqrt n}t_{n-1}(\tfrac{\alpha}{2}),\ \bar X+\frac{\sigma}{\sqrt n}t_{n-1}(\tfrac{\alpha}{2})\right]$$

(1) μ 已知 求 σ^2

$$T=\frac{nS_\mu^2}{\sigma^2}\sim\chi_n^2\quad\left[\frac{nS_\mu^2}{\chi_n^2(\frac{\alpha}{2})},\ \frac{nS_\mu^2}{\chi_n^2(1-\frac{\alpha}{2})}\right]$$

(4) μ 未知 求 σ^2

$$T=\frac{(n-1)S^2}{\sigma^2}\sim\chi_{n-1}^2\quad\left[\frac{(n-1)S^2}{\chi_{n-1}^2(\frac{\alpha}{2})},\ \frac{(n-1)S^2}{\chi_{n-1}^2(1-\frac{\alpha}{2})}\right]$$

② 双正态参数.

$X_1\cdots X_m\sim N(\mu_1,\sigma_1^2)$ $Y_1\cdots Y_n\sim N(\mu_2,\sigma_2^2)$

* $m=n$ 时 $X-Y\sim N(\mu_1-\mu_2,\sigma_1^2+\sigma_2^2)$ 转化成单正态.

* $m\neq n$ 时

(1) σ_1^2,σ_2^2 已知 求 $\mu_2-\mu_1$

$$U=\frac{(\bar Y-\bar X)-(\mu_2-\mu_1)}{\sqrt{\sigma_1^2/m+\sigma_2^2/n}}\sim N(0,1)\quad \bar Y-\bar X\pm u_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}$$

(2) σ_1^2,σ_2^2 未知但 $\sigma_1^2=\sigma_2^2=\sigma^2$ 求 $\mu_1-\mu_2$

$$[(\bar X-\bar Y)-(\mu_1-\mu_2)]/[\sigma\sqrt{\tfrac1m+\tfrac1n}]\sim N(0,1)$$

$$[(m-1)S_X^2+(n-1)S_Y^2]/\sigma^2\sim\chi^2_{m+n-2}$$

$$\frac{(\bar X-\bar Y)-(\mu_1-\mu_2)}{\sqrt{\frac{(m-1)S_X^2+(n-1)S_Y^2}{m+n-2}\left(\frac1m+\frac1n\right)}}\sim t_{m+n-2}\quad \bar X-\bar Y\pm t_{m+n-2,\frac{\alpha}{2}}\sqrt{\frac{(m-1)S_X^2+(n-1)S_Y^2}{m+n-2}\left(\frac1m+\frac1n\right)}$$

(4) μ_1,μ_2 未知 求 $\frac{\sigma_1}{\sigma_2}$

$$\frac{S_Y^2/\sigma_2^2}{S_X^2/\sigma_1^2}\sim F_{n-1,m-1}\quad\left[\frac{S_X^2}{S_Y^2}F_{n-1,m-1,1-\frac{\alpha}{2}},\ \frac{S_X^2}{S_Y^2}F_{n-1,m-1,\frac{\alpha}{2}}\right]$$

(3) σ_1^2,σ_2^2 未知且 $\sigma_1^2\neq\sigma_2^2$

用 S_1,S_2 近似 σ_1,σ_2 (大样本条件下) 转化成(1)求解.

非正态变量—大数定律近似求解.

① 二项分布 $X_i\sim B(1,\theta)$

$$\frac{\sqrt n(\bar X_n-\theta)}{\sqrt{\theta(1-\theta)}}\sim N(0,1)\quad \frac{n}{n+Z_{\frac{\alpha}{2}}^2}\left[\bar X_n+\frac{Z_{\frac{\alpha}{2}}^2}{2n}\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\bar X_n(1-\bar X_n)}{n}+\frac{Z_{\frac{\alpha}{2}}^2}{4n^2}}\right]$$

$$\theta^2(n+Z_{\frac{\alpha}{2}}^2)-\theta(2n\bar X_n+Z_{\frac{\alpha}{2}}^2)+n\bar X_n^2\le 0$$

或直接用 $\bar X$ 代替 θ, 可得 $\bar X\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\bar X(1-\bar X)}{n}}$

② 泊松分布

$$\frac{\sqrt n(\bar X-\theta)}{\sqrt\theta}\sim N(0,1)\ \text{or}\ \frac{\sqrt n(\bar X-\theta)}{\sqrt{\bar X}}\sim N(0,1)\quad \bar X\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\bar X}{n}}$$

置信域: 置信度定为 $(1-\alpha)^{\frac1k}$ 后转化为单变量运算.

例: σ 未知 $X_i\sim N(\mu_1,\sigma_1^2)$ $Y_i\sim(\mu_2,\sigma_2^2)$ X Y 独立

$r=(1-\sqrt{1-\alpha})/2$ $\boxed{\bar X\pm t_{m-1,r}\frac{S_X}{\sqrt m}}\times\boxed{\bar Y\pm t_{n-1,r}\frac{S_Y}{\sqrt n}}$

X, Y 不独立时, 假设 $m=n$ 时

$r=\frac{\alpha}{4}$ 则 $P\left(\left|\frac{\sqrt m(\bar X-\mu_1)}{S_X}\right|>t_{m-1,r}\ \text{or}\ \left|\frac{\sqrt n(\bar Y-\mu_2)}{S_Y}\right|>t_{n-1,r}\right)\le\frac{\alpha}{2}+\frac{\alpha}{2}=\alpha$

假设检验部分.

基本概念:

简单假设: 单点 ($H_0:\theta=1\Leftrightarrow H_1:\theta\neq1$) 注意其它参数也要确定

拒绝域: D 接受域: D^c

检验函数: $\varphi(x)=\begin{cases}1 & T(x)>C\\ r & T(x)=C\\ 0 & T(x)<C\end{cases}$

Neyman-Pearson Principle: 在一类错误控制在 α 以内时, 二类错误尽量得小.

功效函数 power function: $\pi(\theta)=P_\theta(X\in D)=\begin{cases}\alpha(\theta) & \theta\in\Theta_0\\ 1-\beta(\theta) & \theta\in\Theta_1\end{cases}$

P-value: 度量证据的强弱.

$p=P(|T|>|T_0|\mid H_0)$ 注意单边和双边的p也要分单向和双向.

正态分布假设检验:

① 对均值

	H_0	H_1	检验统计量及其分布	否定域
σ 已知	$\mu=\mu_0$ $\mu\le\mu_0$ $\mu\ge\mu_0$	$\mu\neq\mu_0$ $\mu>\mu_0$ $\mu<\mu_0$	$U=\sqrt n(\bar X-\mu_0)/\sigma$ $U\vert_{\mu=\mu_0}\sim N(0,1)$	$\vert U\vert>u_{\frac{\alpha}{2}}$ $U>u_\alpha$ $U<-u_\alpha$
σ 未知	$\mu=\mu_0$ $\mu\le\mu_0$ $\mu\ge\mu_0$	$\mu\neq\mu_0$ $\mu>\mu_0$ $\mu<\mu_0$	$T=\sqrt n(\bar X-\mu_0)/S$ $T\vert_{\mu=\mu_0}\sim t_{n-1}$	$\vert T\vert>t_{n-1}(\frac{\alpha}{2})$ $T>t_{n-1}(\alpha)$ $T<-t_{n-1}(\alpha)$

② 对方差

	H_0	H_1		
μ 已知	$\sigma^2=\sigma_0^2$ $\sigma^2\le\sigma_0^2$ $\sigma^2\ge\sigma_0^2$	$\sigma^2\neq\sigma_0^2$ $\sigma^2>\sigma_0^2$ $\sigma^2<\sigma_0^2$	$\chi_\mu^2=nS_\mu^2/\sigma_0^2$ $\chi_\mu^2\vert_{\sigma_0^2}\sim\chi_n^2$	$nS_\mu^2/\sigma_0^2<\chi_n^2(1-\frac{\alpha}{2})$ 或 $>\chi_n^2(\frac{\alpha}{2})$ $nS_\mu^2/\sigma_0^2>\chi_n^2(\alpha)$ $nS_\mu^2/\sigma_0^2<\chi_n^2(1-\alpha)$
μ 未知	$\sigma^2=\sigma_0^2$ $\sigma^2\le\sigma_0^2$ $\sigma^2\ge\sigma_0^2$	$\sigma^2\neq\sigma_0^2$ $\sigma^2>\sigma_0^2$ $\sigma^2<\sigma_0^2$	$\chi^2=(n-1)S^2/\sigma_0^2$ $\chi^2\vert_{\sigma_0^2}\sim\chi_{n-1}^2$	$(n-1)S^2/\sigma_0^2<\chi_{n-1}^2(1-\frac{\alpha}{2})$ 或 $>\chi_{n-1}^2(\frac{\alpha}{2})$ $(n-1)S^2/\sigma_0^2>\chi_{n-1}^2(\alpha)$ $(n-1)S^2/\sigma_0^2<\chi_{n-1}^2(1-\alpha)$

③ 两个正态的均值

	H_0	H_1		
$\sigma_1^2\sigma_2^2$ 已知	$\mu_2-\mu_1=\mu_0$ $\mu_2-\mu_1\le\mu_0$ $\mu_2-\mu_1\ge\mu_0$	$\mu_2-\mu_1\neq\mu_0$ $\mu_2-\mu_1>\mu_0$ $\mu_2-\mu_1<\mu_0$	$U=\frac{(\bar Y-\bar X)-\mu_0}{\sqrt{\sigma_1^2/m+\sigma_2^2/n}}$ $U\vert_{\mu_0}\sim N(0,1)$	$\vert U\vert>u_{\frac{\alpha}{2}}$ $U>u_\alpha$ $U<-u_\alpha$
$\sigma_1^2\sigma_2^2$ 未知	$\mu_2-\mu_1=\mu_0$ $\mu_2-\mu_1\le\mu_0$ $\mu_2-\mu_1\ge\mu_0$	$\mu_2-\mu_1\neq\mu_0$ $\mu_2-\mu_1>\mu_0$ $\mu_2-\mu_1<\mu_0$	$T_w=\frac{\bar Y-\bar X-\mu_0}{S_w}\sqrt{\frac{mn}{m+n}}$ $S_w^2=\frac{1}{m+n-2}[(m-1)S_1^2+(n-1)S_2^2]$ $T_w\vert_{\mu_0}\sim t_{n+m-2}$	$\vert T_w\vert>t_{n+m-2}(\frac{\alpha}{2})$ $T_w>t_{n+m-2}(\alpha)$ $T_w<-t_{n+m-2}(\alpha)$

④ 两个正态方差

	H_0	H_1		
$\mu_1\mu_2$ 已知	$\sigma_2^2=\sigma_1^2$ $\sigma_2^2\le\sigma_1^2$ $\sigma_2^2\ge\sigma_1^2$	$\sigma_2^2\neq\sigma_1^2$ $\sigma_2^2>\sigma_1^2$ $\sigma_2^2<\sigma_1^2$	$F_*=S_{2*}^2/S_{1*}^2$ $F_*\vert_{\sigma_2^2=\sigma_1^2}\sim F_{n,m}$ $S_{1*}^2=\frac1m\sum(X_i-\mu_1)^2$ $S_{2*}^2=\frac1n\sum(Y_i-\mu_2)^2$	$F_*<F_{n,m}(1-\frac{\alpha}{2})$ 或 $>F_{n,m}(\frac{\alpha}{2})$ $F_*>F_{n,m}(\alpha)$ $F_*<F_{n,m}(1-\alpha)$
$\mu_1\mu_2$ 未知	$\sigma_2^2=\sigma_1^2$ $\sigma_2^2\le\sigma_1^2$ $\sigma_2^2\ge\sigma_1^2$	$\sigma_2^2\neq\sigma_1^2$ $\sigma_2^2>\sigma_1^2$ $\sigma_2^2<\sigma_1^2$	$F=S_2^2/S_1^2$ $F\vert_{\sigma_2^2=\sigma_1^2}\sim F_{n-1,m-1}$ $S_1^2=\frac{1}{m-1}\sum(X_i-\bar X)^2$ $S_2^2=\frac{1}{n-1}\sum(Y_i-\bar Y)^2$	$F<F_{n-1,m-1}(1-\frac{\alpha}{2})$ 或 $>F_{n-1,m-1}(\frac{\alpha}{2})$ $F>F_{n-1,m-1}(\alpha)$ $F<F_{n-1,m-1}(\alpha)$

似然比检验.

基本概念: $H_0:\theta\in\Theta_0\Leftrightarrow H_1:\theta\in\Theta_1$, $\lambda(x)=\frac{L(\hat\theta_{MLE,0})}{L(\hat\theta_{MLE})}=\frac{\sup_{\theta\in\Theta_0}f(x;\theta)}{\sup_{\theta\in\Theta}f(x;\theta)}$

检验函数

$$\varphi(x)=\begin{cases}1 & \lambda<\lambda_0\ (\text{or } -2\log\lambda>C)\\ r & \lambda=\lambda_0\ (\text{or } -2\log\lambda=C)\\ 0 & \lambda>\lambda_0\ (\text{or } -2\log\lambda<C)\end{cases}$$

显著程度 $E_\theta(\varphi(x))\le\alpha\ \forall\theta\in\Theta_0$, 则检验具有显著性 α.

正态分布的 LRT:

例: $H_0:\mu=\mu_0\Leftrightarrow H_1:\mu\neq\mu_0$, σ 未知 T检验.

$$L(\hat\omega)=(2\pi\hat\sigma_\omega^2)^{-\frac n2}\exp\left[-\frac{1}{2\hat\sigma_\omega^2}\sum_{i=1}^n(X_i-\mu_0)^2\right]=(2\pi\hat\sigma_\omega^2)^{-\frac n2}\exp(-\tfrac n2)$$

$$L(\hat\Omega)=(2\pi\hat\sigma_\Omega^2)^{-\frac n2}\exp\left[-\frac{1}{2\hat\sigma_\Omega^2}\sum_{i=1}^n(X_i-\bar X)^2\right]=(2\pi\hat\sigma_\Omega^2)^{-\frac n2}\exp(-\tfrac n2)$$

$$\hat\sigma_\omega^2=\frac1n\sum_{i=1}^n(X_i-\mu_0)^2\quad \hat\sigma_\Omega^2=\frac1n\sum_{i=1}^n(X_i-\bar X)^2$$

则 $\lambda=\left(\frac{\hat\sigma_\Omega^2}{\hat\sigma_\omega^2}\right)^{\frac n2}$ or $\lambda^{\frac2n}=\frac{\sum(X_i-\bar X)^2}{\sum(X_i-\mu_0)^2}$

$$\lambda^{\frac2n}=\frac{\sum(X_i-\bar X)^2}{\sum(X_i-\bar X)^2+n(\bar X-\mu_0)^2}=\frac{1}{1+\frac{t^2}{n-1}}\quad t(x)=\frac{\sqrt n(\bar X-\mu_0)}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n(X_i-\bar X)^2}}$$

$\varphi(x)=\begin{cases}1 & t<-t_{n-1,\frac{\alpha}{2}}\quad t>t_{n-1,\frac{\alpha}{2}}\\ 0 & \text{otherwise}\end{cases}$ 这与前结论一致.

例: $H_0:\mu=\mu_0\Leftrightarrow H_1:\mu\neq\mu_0$ σ 已知

$$-2\log\lambda=\frac{1}{\sigma^2}\sum_{i=1}^n[(X_i-\mu_0)^2-(X_i-\bar X)^2]=\left(\frac{\sqrt n(\bar X-\mu_0)}{\sigma}\right)^2>C^2$$

这样也回到了上结论.

而 $\mu\le\mu_0\Leftrightarrow H_1:\mu>\mu_0$ σ 未知.

$$L(\theta)=(2\pi\sigma^2)^{-\frac n2}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^n(X_i-\bar X)^2-\frac{n(\bar X-\mu)^2}{2\sigma^2}\right)$$

$\bar X>\mu_0$ 时 $\hat\sigma_{\omega,1}^2=\frac1n\sum_{i=1}^n(X_i-\mu_0)^2$ $\hat\mu=\mu_0$

$\bar X\le\mu_0$ 时 $\hat\sigma_{\omega,2}^2=\hat\sigma_\Omega^2=\frac1n\sum_{i=1}^n(X_i-\bar X)^2$ $\hat\mu=\bar X$

$$L(\hat\omega)=\begin{cases}L(\hat\Omega) & \bar X\le\mu_0\\ (2\pi\hat\sigma_{\omega,1}^2)^{-\frac n2}\exp(-\frac n2) & \bar X>\mu_0\end{cases}$$

$$\lambda=\begin{cases}1 & \bar X\le\mu_0\\ \left(\frac{\sum(X_i-\bar X)^2}{\sum(X_i-\mu_0)^2}\right)^{\frac n2}=\left(1+\frac{t^2}{n-1}\right)^{-\frac n2} & \bar X>\mu_0\end{cases}\quad D:\{\vec X: t(\vec X)>c\}$$

例: $\sigma=\sigma_0$, μ 已知. 卡方检验.

$$L(\hat\omega)=(2\pi\sigma_0^2)^{-\frac n2}\exp\left[-\frac{1}{2\sigma_0^2}\sum_{i=1}^n(X_i-\mu)^2\right]$$

$$L(\hat\Omega)=(2\pi\hat\sigma_\Omega^2)^{-\frac n2}\exp\left(-\frac n2\right)$$

$$\lambda=\left(\frac{\hat\sigma_\Omega^2}{\sigma_0^2}\right)^{\frac n2}\exp\left[\frac n2-\frac{1}{2\sigma_0^2}\sum_{i=1}^n(X_i-\mu)^2\right]=e^{\frac n2}U^{\frac n2}\exp\left(-\frac{nU}{2}\right)$$

$U=\frac1n\sum_{i=1}^n\left(\frac{X_i-\mu}{\sigma_0}\right)^2$ $D:\{U<C_1\ \text{or}\ U>C_2\}$

例: $\sigma=\sigma_0$, μ 未知.

$$\lambda=\left(\frac{\hat\sigma_\Omega^2}{\sigma_0^2}\right)^{\frac n2}e\left[\frac n2-\frac{1}{2\sigma_0^2}\sum(X_i-\bar X)^2\right]\quad U=\frac1n\sum_{i=1}^n\left(\frac{X_i-\bar X}{\sigma_0}\right)^2$$

UMP检验:

概念:在α显著性下第二类错误最小.

Neyman-Pearson定理:

$H_0:\theta=\theta_0 \leftrightarrow H_1:\theta=\theta_1$ 简单假设.

$$\varphi(x)=\begin{cases}1 & f(x;\theta_1)>Cf(x;\theta_0)\\ r & f(x;\theta_1)=Cf(x;\theta_0)\\ 0 & f(x;\theta_1)<Cf(x;\theta_0)\end{cases} \quad E_{\theta_0}\varphi(X)=\alpha$$

则φ是UMP且具有显著性α.

一般方法:(从单点拓展至复合)

① 找到θ_0 in Θ_0最接近Θ_1

② 在Θ_1中任取θ_1

③ 根据NP lemma证明$H_0:\theta=\theta_0 \leftrightarrow H_1:\theta=\theta_1$是UMP

④ 证明φ_{θ_1}与θ_1无关 $\varphi_{\theta_1}=\varphi$

⑤ 则φ为$H_0:\theta=\theta_0 \leftrightarrow H_1:\theta<\theta_1$的UMP

⑥ 如果$E_\theta\varphi(X)\le\alpha,\ \theta\in\Theta_0$,则$\varphi$是$H_0:\theta\in\Theta_0 \leftrightarrow H_1:\theta\in\Theta_1$显著性$\alpha$的检验.

前提:指数分布族. R或其子域.

非参数检验:

① sign test

$$S_i=\begin{cases}1 & Z_i>0\\ -1 & Z_i<0\\ 0 & Z_i=0\end{cases} \quad n_+\sim B(n_0,\theta) \quad n_0=n_++n_-$$

$H_0:\theta=0.5 \Leftrightarrow H_1:\theta\neq0.5$

$$P=\sum_{i=0}^{x_0}C_{n_0}^{i}0.5^{n_0}+\sum_{i=n_0-x_0}^{n_0}C_{n_0}^{i}0.5^{n_0} \quad x_0=\min\{n_+,n_-\}$$

大样本下的sign test:

$$U=\frac{X-EX}{\sqrt{\mathrm{Var}X}}=\frac{X-0.5n_0}{\sqrt{\frac{n_0}{4}}}=\frac{2X-n_0}{\sqrt{n_0}}\longrightarrow N(0,1)$$

$$p=1-\Phi(|u|)+\Phi(-|u|)$$

② Wilcoxon 符号秩和检验.

将Z_i中的0去掉,按绝对值从小到大排序后将其次序赋为其秩. 令$P(W^+\le d|H_0)=P(W^+\ge c|H_0)=\frac{\alpha}{2}$

$$W^+=\sum_{i=1}^{n}U_iR_i \quad R_i\text{为秩} \quad U_i=\begin{cases}1 & Z_i>0\\ 0 & Z_i<0\end{cases}$$

大样本方法:

$$E(W^+)=\frac{n(n+1)}{4} \quad D(W^+)=\frac{1}{24}n(n+1)(2n+1)$$

否定域$\{|W^{+*}|>u_{\frac{\alpha}{2}}\}$ $\quad W^{+*}=\frac{W^+-E(W^+)}{\sqrt{D(W^+)}}$

③ Wilcoxon两样本秩和检验.

$X_1\cdots X_m, Y_1\cdots Y_n$ 由小到大排列并赋秩.

$Y_i=X_i+\theta$ 观察Y的秩.

$W=R_1+\cdots+R_n \quad N=n+m$

大样本方法: $E(W)=\frac{n(N+1)}{2}$

$D(W)=\frac{mn(N+1)}{12}$

$$W^*=\frac{W-E(W)}{D(W)}\to N(0,1)$$

$H_0:\theta=0$	$\lvert W^*\rvert\ge u_{\frac{\alpha}{2}}$
$\theta\le0$	$W^*\ge u_\alpha$
$\theta\ge0$	$W^*\le -u_\alpha$

④ 拟合优度检验.

理论分布:

$$F:\begin{pmatrix}a_1 & a_2 & \cdots & a_r\\ p_1 & p_2 & \cdots & p_r\end{pmatrix}$$

$H_0: P(X=a_i)=p_i \quad i=1,\cdots r$

ν_i: a_i的频数 ν_i/n: a_i的频率.

$$K_n=K_n(X_1\cdots X_n;F)=\sum_{i=1}^{r}\frac{(\nu_i-np_i)^2}{np_i}\to\chi^2_{r-1}$$

否定域为$D:\{\vec{x}:K_n>\chi^2_{r-1}(\alpha)\}$

⑤ 列联表独立性检验.

	1 ⋯	j ⋯	s	Σ
1	n_{11} ⋯	n_{1j} ⋯	n_{1s}	$n_{1\cdot}$
⋮		⋮	⋮	⋮
i	n_{i1} ⋯	n_{ij} ⋯	n_{is}	$n_{i\cdot}$
⋮	⋮	⋮	⋮	⋮
r	n_{r1} ⋯	n_{rj} ⋯	n_{rs}	$n_{r\cdot}$
Σ	$n_{\cdot1}$ ⋯	$n_{\cdot j}$ ⋯	$n_{\cdot s}$	n

$$K_n=n\left(\sum_{i=1}^{r}\sum_{j=1}^{s}\frac{n_{ij}^2}{n_{i\cdot}n_{\cdot j}}-1\right)$$

H_0:二者独立,则$K_n\to\chi^2_{(r-1)(s-1)}$

Reject H_0 when $p(k_0)<\alpha$.

ps:

似然比双样本检验.

$$L(\Omega)=(2\pi)^{-\frac{m+n}{2}}(\hat{\sigma}^2_{1,\Omega})^{-\frac{m}{2}}(\hat{\sigma}^2_{2,\Omega})^{-\frac{n}{2}}\exp\left(-\frac{m+n}{2}\right)$$

$$L(w)=(2\pi)^{-\frac{m+n}{2}}(\hat{\sigma}^2_w)^{-\frac{m+n}{2}}\exp\left(-\frac{m+n}{2}\right)$$

$$\hat{\sigma}^2_{1,\Omega}=\frac{1}{m}\sum_{i=1}^{m}(X_i-\bar{X})^2$$

$$\hat{\sigma}^2_{2,\Omega}=\frac{1}{n}\sum_{i=1}^{n}(Y_i-\bar{Y})^2$$

$$\hat{\sigma}^2_w=\frac{1}{m+n}\left[\sum_{i=1}^{m}(X_i-\bar{X})^2+\sum_{j=1}^{n}(Y_j-\bar{Y})^2\right]$$

$$\lambda=\frac{(\sigma^2_{\Omega,1})^{\frac{m}{2}}(\sigma^2_{\Omega,2})^{\frac{n}{2}}}{(\sigma^2_w)^{\frac{m+n}{2}}}$$

常见分布及其矩母函数及其期望.

泊松分布 $P(\lambda)$	$p(k)=\frac{\lambda^k}{k!}e^{-\lambda}$ $k=0,1,\cdots$	λ	λ	
几何分布 $G(p)$	$p_k=(1-p)^{k-1}p$ $k=1,2\cdots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	
均匀分布 $U(a,b)$	$p(x)=\frac{1}{b-a}$ $a<x<b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
指数分布 $Exp(\lambda)$	$p_x=\lambda e^{-\lambda x}$ $x\ge0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$M(s)=\frac{\lambda}{\lambda-s}$
Gamma分布 $G(\alpha,\lambda)$	$p_x=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$ $x\ge0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$M(s)=(1-\frac{s}{\lambda})^{-\alpha}$
χ^2_n分布	$p(x)=\frac{x^{n/2-1}e^{-x/2}}{\Gamma(n/2)2^{n/2}}$ $x\ge0$	n	$2n$	$M(s)=(1-2s)^{-\frac{n}{2}}$
正态分布	$p_x=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$M(s)=e^{\frac{s^2}{2}}$