FINA2204 Tutorial 3: Hedging with futures

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Agenda

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Recap: hedging

- A definition from the US Commodity Futures Trading Commission (CFTC, 2006).
- Two scenarios - current position vs. anticipated transaction
- *“Taking a position in a futures market opposite to a position held in the cash market (aka, spot market or physical market) to minimize the risk of financial loss from an adverse price change;”*
- *“Or a purchase or sale of futures as a temporary substitute for a cash transaction that will occur later.”*

Recap: hedging

Scenario type	Spot market position	Hedge action	Example
Current position – long cash	**Already** own the asset	Short hedges (want protection against price **decreases**)	Grain trader currently holds 1,000 tonnes of wheat in inventory.
Current position – short cash	**Already** obligated to deliver asset but don't own it yet	Long hedges (want protection against price **increases**)	Copper exporter signed a contract to deliver 500 tonnes in 1 month.
Anticipated transaction – long cash	**Will** own the asset in the future	Short hedges (want protection against price **decreases**)	Famer expects to harvest 1,000 tonnes of corn in 3 months.
Anticipated transaction – short cash	**Will** need to obtain the asset in the future	Long hedges (want protection against price **increases**)	Airline will buy 1 million gallons of jet fuel in 4 months

Recap: hedging

1. Long hedgers: **consumers** of commodities

- Airlines – lock in jet fuel for future consumption
- Food manufactures – secure raw material prices
- Importer – hedge foreign currency purchases against depreciation of domestic currency

2. Short hedgers: **producers** of commodities

- Farmer - lock in crop prices before harvest
- Mining companies – secure prices for metals before sales
- Exporter – hedge foreign currency receipts against depreciation

Recap: hedging

Example 3.1 A short hedge

It is May 15. An oil producer has negotiated a contract to sell 1 million barrels of crude oil. The price in the sales contract is the spot price on August 15. Quotes:

Spot price of crude oil: \$60 per barrel
August oil futures price: \$59 per barrel

The oil producer can hedge with the following transactions:

May 15: Short 1,000 August futures contracts on crude oil
August 15: Close out futures position

After gains or losses on the futures are taken into account, the price received by the company is close to \$59 per barrel.

- What will be the outcome of this hedge if the spot price of crude oil on August 15 is (i) \$55 per barrel, (ii) \$65 per barrel?

Recap: hedging

August spot price = \$55/bbl (so F_T = \$55)

Date	Spot	Step 1 Futures
May 15	N/A	Sell 1,000 August futures @ \$59/bbl SV = 1,000 x 1,000 x \$59 = \$59 million
Aug 15	Step 2 Sell 1m bbls @ \$55 Revenue = \$55 million	Buy 1,000 August futures @ \$55/bbl SV = 1,000 x 1,000 x \$55 = \$55 million Step 3 Futures profit = \$4 million
	Effective revenue = \$55 million + \$4 million = \$59 million (= \$59/bbl)	

Short hedge locked in revenue of \$59/bbl.

Recap: hedging

August spot price = \$65/bbl (so F_T = \$65)

Date	Spot	Futures
May 15		Sell 1,000 August futures @ \$59/bbl SV = 1,000 x 1,000 x \$59 = \$59 million
Aug 15	Sell 1m bbls @ \$65 Revenue = \$65 million	Buy 1,000 August futures @ \$65/bbl SV = 1,000 x 1,000 x \$65 = \$65 million Futures profit = -\$6 million
	Effective revenue = \$65 million - \$6 million = \$59 million (= \$59/bbl)	

Short hedge locked in revenue of \$59/bbl.

Recap: hedging

Implication

- The hedger did not lock in the May 15 spot price of $60.
- Instead, it locked in the May 15 **futures price** of $59, which reflects the market's forward valuation and cost-of-carry, not the current spot.
- Futures and spot prices may differ at the start due to interest rates, storage, convenience yield, etc $(\boldsymbol{F_0 = S_0 \times e^{rT}})$.
- At maturity, futures converge to spot, but the hedge outcome is based on the initial futures price, **not** the initial spot price.

Problem 3.9

- *Does a perfect hedge always succeed in locking in the current spot price of an asset for a future transaction? Explain your answer.*

Answer 3.9

- No. Consider, for example, the use of a forward (futures) contract to hedge a known cash inflow in a foreign currency. The forward (futures) contract locks in the forward exchange (futures) rate, which is in general different from the spot exchange rate.

Recap: arguments for and against hedging

- Arguments in favour of hedging

1. Hedging reduces exposure to market risks (e.g., interest rates, commodity prices).

- Arguments against hedging

1. Shareholders can diversify or hedge themselves;
2. If competitors don't hedge, hedging can hurt relative performance.
3. Hedge losses can be hard to explain when the business is profitable.

Recap: basis risk

- What is the 'basis' here?

$$Basis = Spot\ price\ (S) - Futures\ price\ (F)$$

Consider a <u>short hedge</u> initiated at time t_1 and liquidated at time t_2.

F_1 : Futures price at time t_1
F_2 : Futures price at time t_2
S_2 : Spot price at time t_2
b_2 : Basis at time t_2 (= $S_2 - F_2$)

t_1 ------------ ---------------------- t_2

Sell futures @ F_1

Buy futures @ F_2
Sell spot @ S_2

Price received for asset	S_2
Gain on futures	$F_1 - F_2$
Net amount received	$S_2 + (F_1 - F_2) = F_1 + b_2$

F1 is fixed and the only uncertain part is b2

Recap: basis risk

Consider a long hedge initiated at time t_1 and liquidated at time t_2.

F_1 : Futures price at time t_1
F_2 : Futures price at time t_2
S_2 : Spot price at time t_2
b_2 : Basis at time t_2 (= $S_2 - F_2$)

t_1 ------------ ----------------------- t_2

Buy futures @ F_1

Sell futures @ F_2
Buy spot @ S_2

Price paid for asset	S_2
Gain on futures	$F_2 - F_1$
Net amount paid	$S_2 - (F_2 - F_1) = F_1 + b_2$

Recap: basis risk

$$Change\ in\ basis = \Delta(S - F)$$

- Positive → strengthening basis
- Negative → weakening basis
- Considering a short hedge, basis strengthens means S falls less **relative** to F, so the futures buy-back price (F) is lower than expected and the spot price (S) is higher than expected, both improving the net outcome.

Problem 3.10

- *Explain why a short hedger's position improves when the basis strengthens unexpectedly and worsens when the basis weakens unexpectedly.*

Answer 3.10

- The basis is the amount by which the spot price exceeds the futures price. A short hedger is long the asset and short futures contracts. The value of his or her position therefore improves as the basis increases. Similarly it worsens as the basis decreases.

Problem 2.23

- *Sixty futures contracts are used to hedge an exposure to the price of silver. Each futures contract is on 5,000 ounces of silver. At the time the hedge is closed out, the basis is $0.20 per ounce. What is the effect of the basis on the hedger's financial position if (a) the trader is hedging the purchase of silver and (b) the trader is hedging the sale of silver?*

Answer 3.23

- The excess of the spot over the futures at the time the hedge is closed out is $0.20 per ounce.
- If the trader is hedging the purchase of silver, the price **paid** is the futures price plus the basis. The trader therefore loses $60 \times 5{,}000 \times \$0.20 = \$60{,}000$.
- If the trader is hedging the sales of silver, the price **received** is the futures price plus the basis. The trader therefore gains $60,000.

Recap: basis risk

- In practice, even if you hedge, price movements in the futures and the underlying may not fully offset (too good to be true).
- Three reasons:

1. Asset being hedged may not be identical to asset underlying the futures contract (i.e. cross hedging);
2. Hedger is uncertain at to exact date when asset will be bought or sold (e.g., timing uncertainty);
3. Hedge may require futures contract to be closed out prior to expiration date (e.g., margin call).

Recap: basis risk (cross hedging)

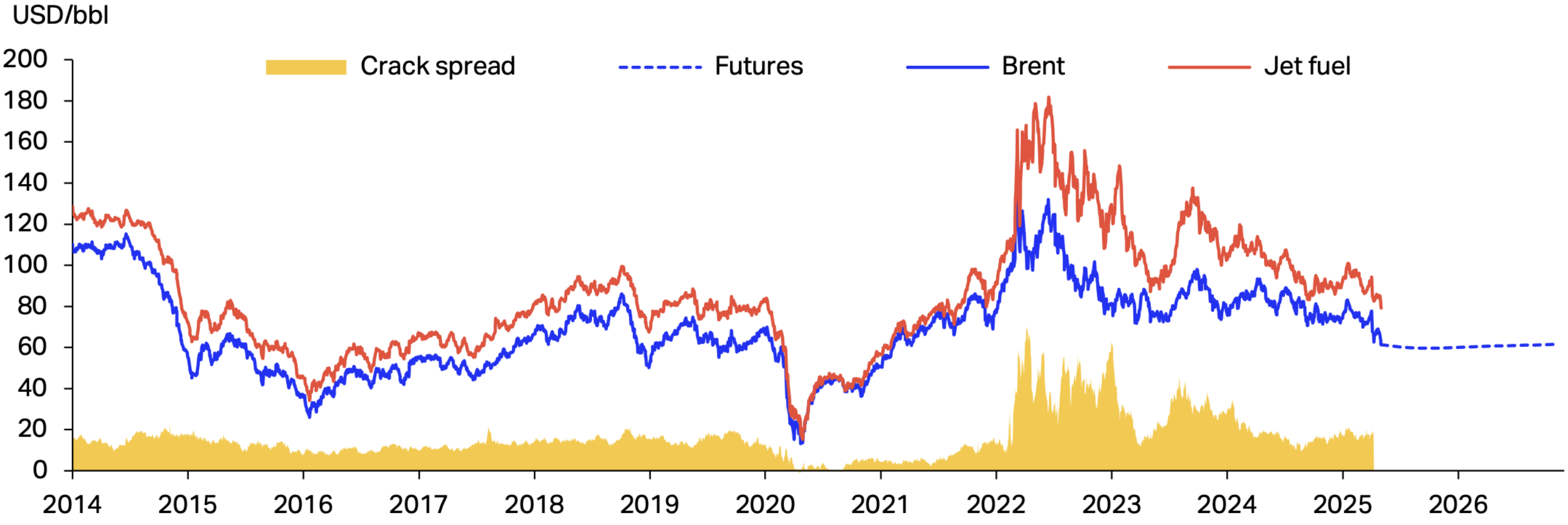
Table 3.2 Data to calculate minimum variance hedge ratio when heating oil futures contract is used to hedge purchase of jet fuel

Month i	*Change in heating oil futures price per gallon* (ΔF)	*Change in jet fuel price per gallon* (ΔS)
1	0.021	0.029
2	0.035	0.020
3	–0.046	–0.044
4	0.001	0.008
5	0.044	0.026
6	–0.029	–0.019
7	–0.026	–0.010
8	–0.029	–0.007
9	0.048	0.043
10	–0.006	0.011
11	–0.036	–0.036
12	–0.011	–0.018
13	0.019	0.009
14	–0.027	–0.032
15	0.029	0.023

Source: textbook, p.80

Recap: basis risk (cross hedging)

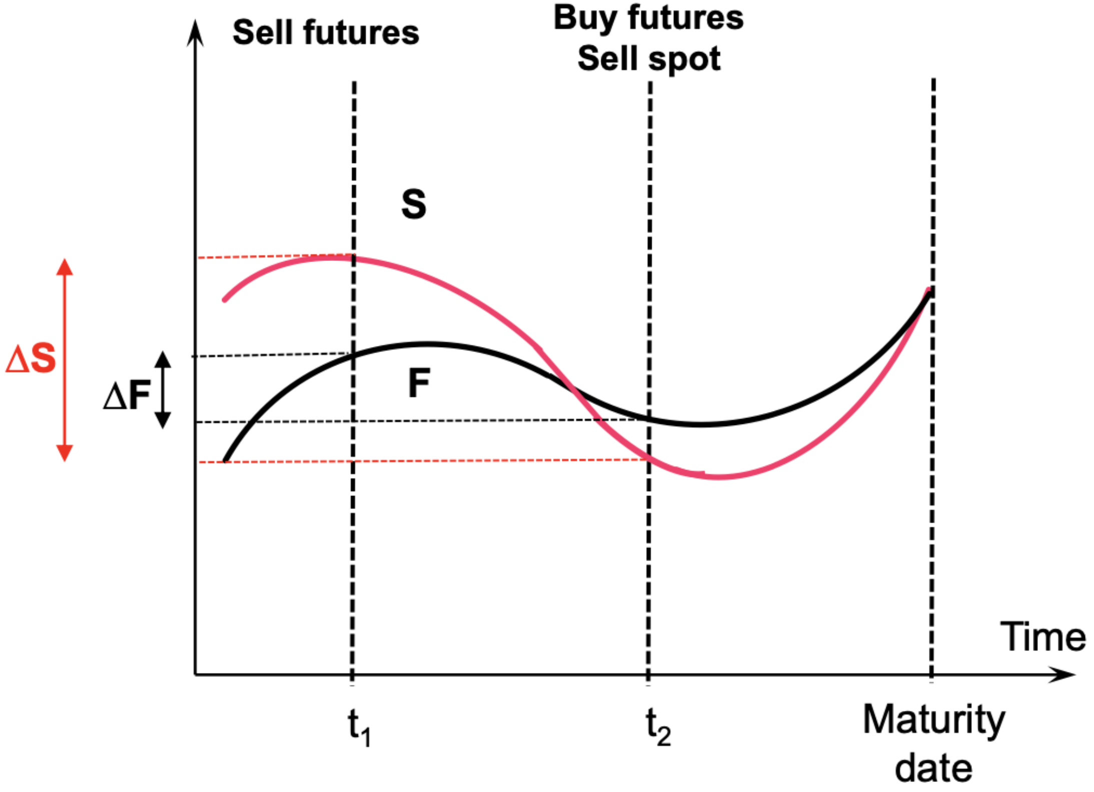
Brent crude oil price with futures curve, jet fuel price, and jet crack spread, USD per barrel



Source: IATA Sustainability and Economics, Platts, ICE, Updated: 06/2025 Next Update: 12/2025

Recap: basis risk (timing uncertainty)

A short hedge by a wheat farmer:



Outcome: Futures profit much less than fall in wheat revenue.

Problem 3.8

- *In the CME Group’s corn futures contract, the following delivery months are available: March, May, July, September, and December. State the contract that should be used for hedging when the expiration of the hedge is in (a) June, (b) July, and (c) January.*

Answer 3.8

- A good rule of thumb is to choose a futures contract that has a delivery month **as close as possible to, but later than**, the month containing the expiration of the hedge. The contracts that should be used are therefore:
- (a) July
- (b) September
- (c) March

Recap: basis risk (margin call)

- Case: Metallgesellschaft hedged long-term fixed-price oil contracts with short-term futures. Falling oil prices triggered heavy margin calls, draining cash and leading to a $1.33 billion boss.
- Takeaway: hedging fails if daily settlement cash needs can’t be met.

Business Snapshot 3.2 Metallgesellschaft: Hedging gone awry

Sometimes rolling hedges forward can lead to cash flow pressures. This problem was illustrated dramatically by the activities of a German company, Metallgesellschaft (MG), in the early 1990s.

MG sold a huge volume of 5- to 10-year heating oil and gasoline fixed-price supply contracts to its customers at 6 to 8 cents above market prices. It hedged its exposure with long positions in short-dated futures contracts that were rolled forward. As it turned out, the price of oil fell and there were margin calls on the futures positions. Considerable short-term cash flow pressures were placed on MG. The members of MG who devised the hedging strategy argued that these short-term cash outflows were offset by positive cash flows that would ultimately be realized on the long-term fixed-price contracts. However, the company’s senior management and its bankers became concerned about the huge cash drain. As a result, the company closed out all the hedge positions and agreed with its customers that the fixed-price contracts would be abandoned. The outcome was a loss to MG of $1.33 billion.

Source: textbook, p.89

Problem 3.20

- *A futures contract is used for hedging. Explain why the daily settlement of the contract can give rise to cash flow problems.*

Answer 3.20

- Suppose that you enter into a short futures contract to hedge the sale of an asset in six months. If the price of the asset rises sharply during the six months, the futures price will also rise and you may get margin calls. The margin calls will lead to cash outflows. Eventually the cash outflows will be offset by the extra amount you get when you sell the asset, but there is a **mismatch in the timing** of the cash outflows and inflows. Your cash outflows occur earlier than your cash inflows. A similar situation could arise if you used a long position in a futures contract to hedge the purchase of an asset and the asset's price fell sharply. An extreme example of what we are talking about here is provided by Metallgesellschaft (see Business Snapshot 3.2).

Recap: how many contracts?

- Goal: choose the right number of futures contracts to avoid under-hedging or over-hedging.
- Step 1 – calculate the **minimum variance hedge ratio** h*:

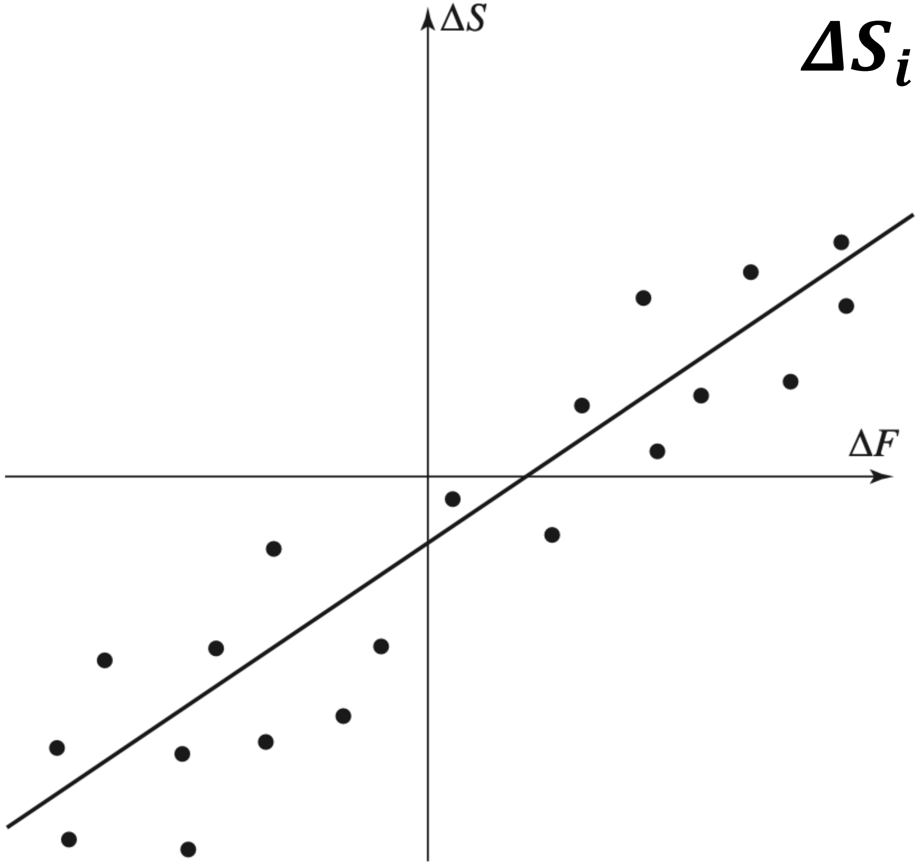
$$h^* = \rho \times \frac{\sigma_S}{\sigma_F}$$

where:

- ρ = correlation between % changes in spot (S) and futures (F) prices
- σ_S = standard deviation of % changes in spot price
- σ_F = standard deviation of % changes in futures price

Recap: how many contracts?

$$\Delta S_i = \alpha + \beta \Delta F_i + \varepsilon_i,$$



Slope (β) = h*

Figure 3.2 Regression of change in spot price against change in futures price

Source: textbook, p.78

Recap: how many contracts?

- We regress spot changes on futures changes:

$$\overbrace{\boxed{\Delta S_i}}^{\text{Actual data}} = \overbrace{\boxed{\alpha + \beta \Delta F_i}}^{\text{Prediction}} + \varepsilon_{i,}\ i = 1, \dots, n.$$

- OLS chooses α, β to minimize the sum of squared residuals

$$SSR(\alpha, \beta) = \varepsilon_i^2 = \sum_{i=1}^{n} (\Delta S_i - \alpha - \beta \Delta F_i)^2$$

- Differentiate SSR and set to zero:

$$\frac{\partial SSR}{\partial \alpha} = -2 \sum_i (\Delta S_i - \alpha - \beta \Delta F_i) = 0$$

$$\sum_i \Delta S_i = n\alpha + \beta \sum_i \Delta F_i$$

$$\alpha = \overline{\Delta S} - \beta \overline{\Delta F}$$

Recap: how many contracts?

$$\frac{\partial SSR}{\partial \beta} = -2\sum_i \Delta F_i(\,\Delta S_i - \alpha - \beta\Delta F_i) = 0$$

$$\boldsymbol{\alpha}\sum_i \Delta F_i + \beta\sum_i (\Delta F_i)^2 = \sum_i \Delta F_i \Delta S_i$$

$$(\overline{\boldsymbol{\Delta S}} - \boldsymbol{\beta}\overline{\boldsymbol{\Delta F}})\sum_i \Delta F_i + \beta\sum_i (\Delta F_i)^2 = \sum_i \Delta F_i \Delta S_i$$

$$n\overline{\Delta S}\,\overline{\Delta F} - \beta \mathrm{n}\overline{\Delta F}^2 + \beta\sum_i (\Delta F_i)^2 = \sum_i \Delta F_i \Delta S_i$$

$$\beta\left[\sum_i (\Delta F_i)^2 - \mathrm{n}\overline{\Delta F}^2\right] = \sum_i \Delta F_i \Delta S_i - n\overline{\Delta S}\,\overline{\Delta F}$$

Recap: how many contracts?

- Centered cross-product

$$\sum_i (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_i X_i Y_i - n\,\bar{X}\bar{Y},$$

- Centered sum of squares

$$\sum_i (X_i - \bar{X})^2 = \sum_i X_i^2 - n\bar{X}^2\,,$$

- Substitute both into the equation:

$$\beta = \frac{\sum(\Delta F_i - \overline{\Delta F})(\Delta S_i - \overline{\Delta S})}{\sum(\Delta F_i - \overline{\Delta F})^2}, \qquad this\ is\ exacly, \beta = \mathrm{h}^* = \frac{Cov(\Delta S, \Delta F)}{Var(\Delta F)}$$

Recap: how many contracts?

- By definition of correlation:

$$\rho = \frac{Cov(\Delta S, \Delta F)}{\sigma_S \sigma_F}$$

$$Cov(\Delta S, \Delta F) = \rho \sigma_S \sigma_F$$

- Substitute into h*

$$h^* = \frac{Cov(\Delta S, \Delta F)}{Var(\Delta F)} = \frac{\rho \sigma_S \sigma_F}{\sigma_F^2} = \rho \frac{\sigma_S}{\sigma_F}$$

- ρ = the direction of co-movement
- $\frac{\sigma_S}{\sigma_F}$ = the relative volatility

Recap: how many contracts?

- Let Q_A be size of spot position being hedged, Q_F be size of one futures contract.
- **Optimal number of futures contracts** (N^*) to use in a hedge:

$$N^* = \frac{h^* Q_A}{Q_F}$$

- N^* is simply how many futures you need to match your hedge ratio to the size of your real-life position.

Problem 3.16

- *The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?*

Answer 3.16

- The optimal **long** hedge ratio is

$$h^* = 0.7 \times \frac{1.2}{1.4} = 0.6$$

- The beef producer requires a long position in 200000 × 0.6 = 120,000 lbs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

$$N^* = 0.6 \times \frac{200,000}{40,000} = 3$$

Problem 3.18

- *On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3. What strategy should the investor follow?*

Answer 3.18

- A **short** position in

$$N^* = 1.3 \times \frac{50{,}000 \times 30}{50 \times 1{,}500} = 26$$

- contracts is required. It will be profitable if the stock outperforms the market in the sense that its return is greater than that predicted by the capital asset pricing model.

Problem 3.25

- *A company wishes to hedge its exposure to a new fuel whose price changes have a 0.6 correlation with gasoline futures price changes. The company will lose \$1 million for each 1 cent increase in the price per gallon of the new fuel over the next three months. The new fuel's price change has a standard deviation that is 50% greater than price changes in gasoline futures prices. If gasoline futures are used to hedge the exposure what should the hedge ratio be? What is the company’s exposure measured in gallons of the new fuel? What position measured in gallons should the company take in gasoline futures? How many gasoline futures contracts should be traded? Each futures contract is on 42,000 gallons.*

Answer 3.25

- The hedge ratio (h^*) should be 0.6 (correlation)× 1.5 (relative volatility) = 0.9.
- The company has an exposure to the price of **100 million gallons (see the next slide for details)** of the new fuel. It should therefore take a position of 90 million gallons in gasoline futures. Each futures contract is on 42,000 gallons. The number of contracts required is therefore

$$N^* = \frac{0.9 \times 100{,}000{,}000}{42{,}000} = 2142.9$$

- or, rounding to the nearest whole number, 2143.

Answer 3.25

- Calculation Details for 100 million gallons
- Since it states that ‘the company will lose $1 million for each 1 cent increase’
- If Q = exposure in gallons,
- And $\Delta\mathrm{P} = 0.01$ (dollars per gallon),
- Then
- $Loss = Q \times \Delta\mathrm{P}$
- Solving for Q:

$$Q = \frac{Loss}{\Delta P} = \frac{\$1\, million}{0.01\, dollars\ pergallon} = 100\, million\ gallons$$