FINA2204 Tutorial 7: Mechanics of Options

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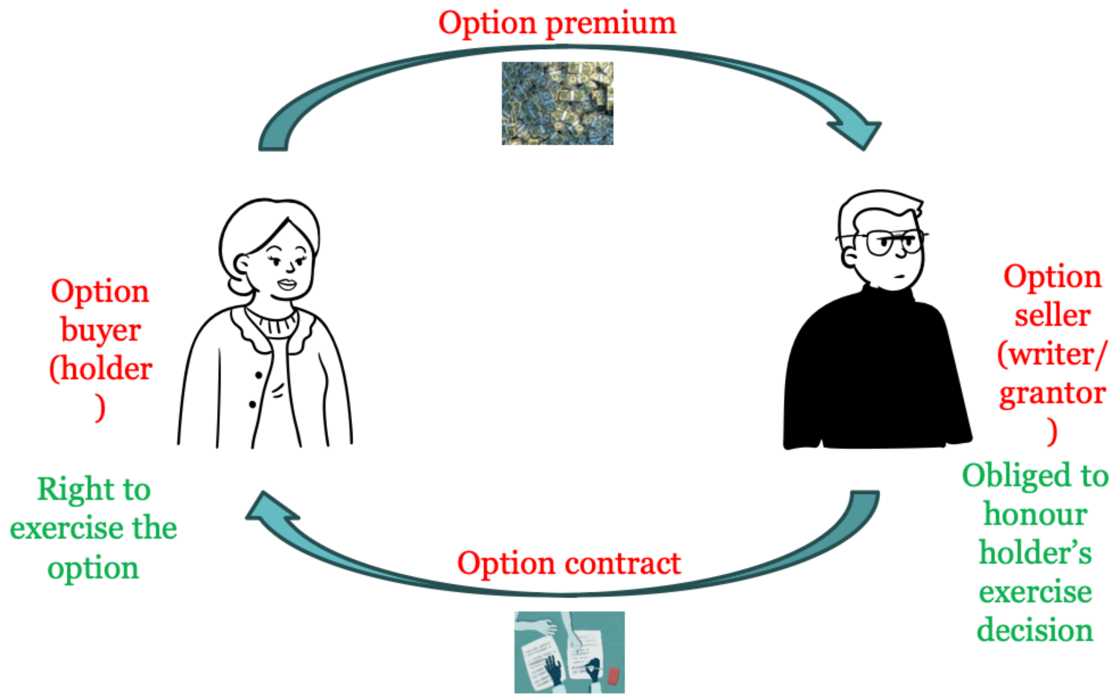
Agenda

- Problem 9.9
- Problem 9.10
- Problem 9.12
- Problem 9.13
- Problem 9.14
- Problem 9.15
- Problem 9.16
- Problem 9.17

Recap: definition

- An option is a contract that gives the holder the **right**, but not the obligation, to buy (call option) or to sell (put option) an underlying asset (such as a stock, bond, or commodity) at a predetermined price (strike price, K) on or before a specified date (expiration date).



Recap: features

1. Asymmetric risk / return: the buyer’s loss is capped at the premium (C), while the seller’s downside can be very large, unlimited for calls, substantial for puts (*the non-linear payoffs*).
2. Asymmetric obligations: the buyer hold a right, while the seller must fulfill the contract if exercised.
3. Zero-sum nature: one side’s gain from the option is exactly the other side’s loss.

Recap: options vs. forwards

- Similarities: both are financial derivatives whose value comes from un underlying asset, commonly used for hedging, speculation, or arbitrage.
- Differences

options	forwards
Right	Obligation
Protect from adverse price movement, and meanwhile benefit from favorable price movement.	Lock in fixed price but the outcome can end up significantly worse than with no hedging.
Upfront payment	No premium cost upfront
OTC / exchange	OTC

Recap: options vs. forwards

- **Example**: A U.S. company will need €1,000,000 in 3 months
- Current spot rate: \$1.10/€; forward 3-month rate: \$1.10/€; option 3-month call on euro: strike = \$1.10/€; premium =\$0.02/€
- *Using a forward*
- Company locks in \$1.10/€
- In 3 months, no matter the market rate, cost is fixed at:

$$1{,}000{,}000 \times 1.10 = \$1{,}100{,}000$$

If spot in 3 months = \$1.00/€, Hedge costs \$100,000 more than if the company had remained unhedged.

Recap: options vs. forwards

- *Using an option*
- Company buys a call option on € with strike $1.10, pays premium = $0.02/€.
- Premium cost today = $20,000.
- In 3 months: If spot = $1.00/€ → **do not exercise**, buy at market $1,000,000 + $20,000 premium = $1,020,000.
- In this case, choosing a forward instead of an option would cost you an additional $80,000.

Problem 9.16

- *The treasurer of a corporation is trying to choose between options and forward contracts to hedge the corporation's foreign exchange risk. Discuss the advantages and disadvantages of each.*

Answer 9.16

- Forward contracts **lock in** the exchange rate that will apply to a particular transaction in the future.
- Options provide **insurance** that the exchange rate will not be worse than some level.
- The advantage of a forward contract is that uncertainty is eliminated as far as possible. The disadvantage is that the outcome with hedging can be ***significantly worse*** than the outcome with no hedging.
- This disadvantage is not as marked with options. However, unlike forward contracts, options involve an **up-front cost**.

Recap: American vs. European

- European option (vanilla, standard): an option can **only** be exercised on the expiration date.
- American option: an option can be exercised on **any date** before the expiation date.
- The American option gives you all the rights of the European option plus **extra flexibility** (you can exercise earlier).

Recap: American vs. European

- More flexibility can not make it less valuable, at worst, you just do not use the extra flexibility (American = European).
- This flexibility only has value when early exercise is more advantageous than waiting until maturity (American > European).
- So American option must be *at least as much as* a European option **(American ≥ European)** because it gives the holder all the same rights plus the extra flexibility to exercise at any time before expiry.

Recap: American vs. European

- Example: Call on dividend-paying stock (**American > European)**
- Stock price today: \$100; Strick price: \$90; Dividend tomorrow: \$5
- If you do not exercise early (European):
- Tomorrow stock goes ex-dividend → price drops to \$95; Option payoff: \$5 (95-90)
- If you exercise early (American, today):
- Buy stock at \$90, get stock worth \$100; Tomorrow collect \$5 dividend → total = \$10 gain (stock drop to 95 + 5 dividend)
- So in this case, the American call should be about \$5 above the European call today.

Recap: American vs. European

- **Arbitrage opportunity**
- Suppose the American option were cheaper than the European option.
- Strategy: An arbitrageur could: (1) sell the European option to collect more money upfront; (2) buy the American option by paying less money upfront.
- The arbitrageur is now guaranteed to be at least as well off, since both options behave the same at expiration, but the American option has extra rights. That's a **"free lunch".**

Problem 9.13

- *Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.*

Answer 9.13

- The holder of an American option has all the same rights as the holder of a European option **and more**. It must therefore be worth at least as much. If it were not, an arbitrageur could short the European option and take a long position in the American option.

Recap: American vs. European

Reflect the possibility the option will gain more value before expiry.
To calculate it properly, we need a price model (Binomial tree; BSM)

- (European) option price = intrinsic value + time value

5 July 2024 (11.30am, AEST)

Price = Intrinsic value (IV) + Time value (TV)
2.070 = Max(44.63 – 43.50,0) + TV
2.070 = 1.13 + 0.94

Underlying stock price (S) = $44.63

Code	Type	Expiry date	Exercise price	Option price*	Implied Volatility**
BHPET7	Call (A)	15 Aug 2024	$43.50	$\boxed{\$2.070}$	19.14%

Recap: American vs. European

- American option price = intrinsic value + time value + (early-exercise flexibility premium)
- For calls on non-dividend-paying stock → this premium = 0; for calls on dividend-paying stock → this premium > 0.
- Details: an American option can be exercised anytime, therefore its value must be *at least as high as* its intrinsic value.
- Because if the market price of the option ever dropped below its intrinsic value, an arbitrageur could buy the option and immediately exercise it for a guaranteed profit.

Recap: American vs. European

- Example: American call option
- Setup: stock price = \$100, strike price = \$90.
- Intrinsic value = max (100-90, 0) = \$10
- Suppose the market price of the call = \$8.
- Stock price (\$8) < intrinsic value (\$10) → **arbitrage opportunity**
- Strategy: Trader buys the call for \$8; immediately exercises;
- Gains = \$10 from exercise - \$8 cost of option = \$2 risk-free profit

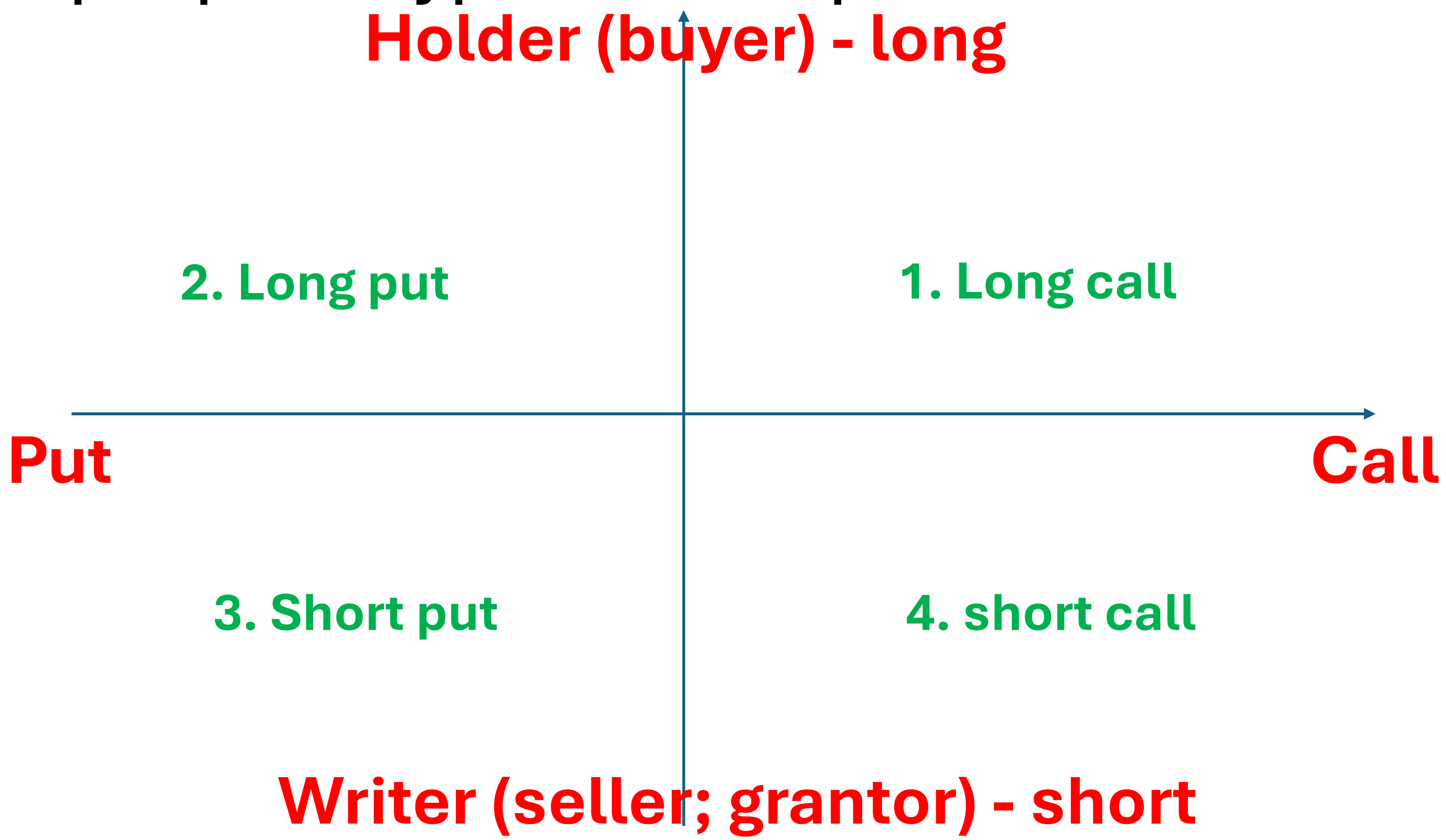
Problem 9.14

- *Explain why an American option is always worth at least as much as its intrinsic value.*

Answer 9.14

- The holder of an American option has the right to exercise it immediately. The American option must therefore be worth at least as much as its intrinsic value. If it were not an arbitrageur could lock in a sure profit by buying the option and exercising it **immediately**.

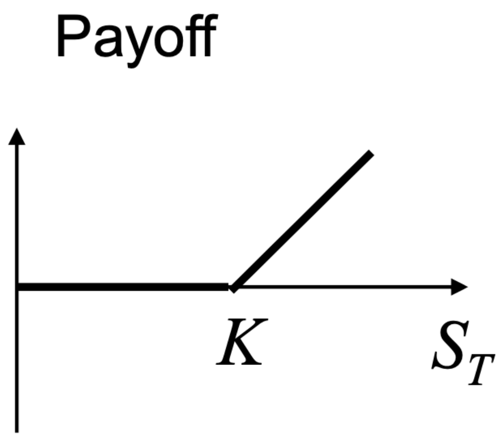
Recap: option types – four quadrants



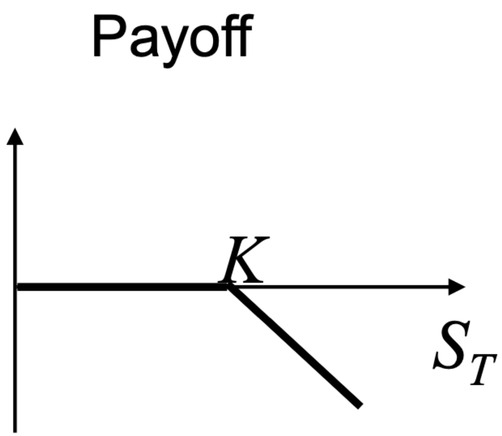
Recap: option types

Non-linear payoffs
Holder's curve bends upward (convex) → limited downside, unlimited or large upside.
Writer's curve bends downward (concave) → limited upside, potentially very large downside.

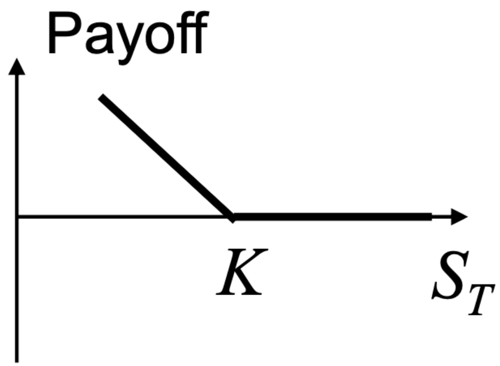
Long Call



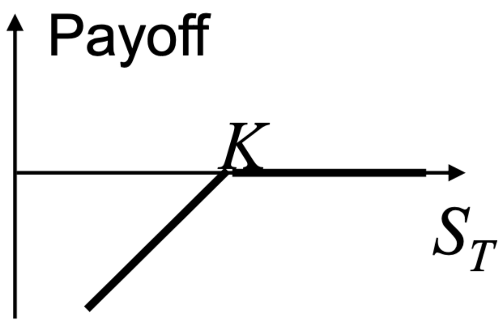
Short call



Long put



Short put



Recap: option types

- **How to identify option type from payoff diagram**

1. Look for payoff > 0

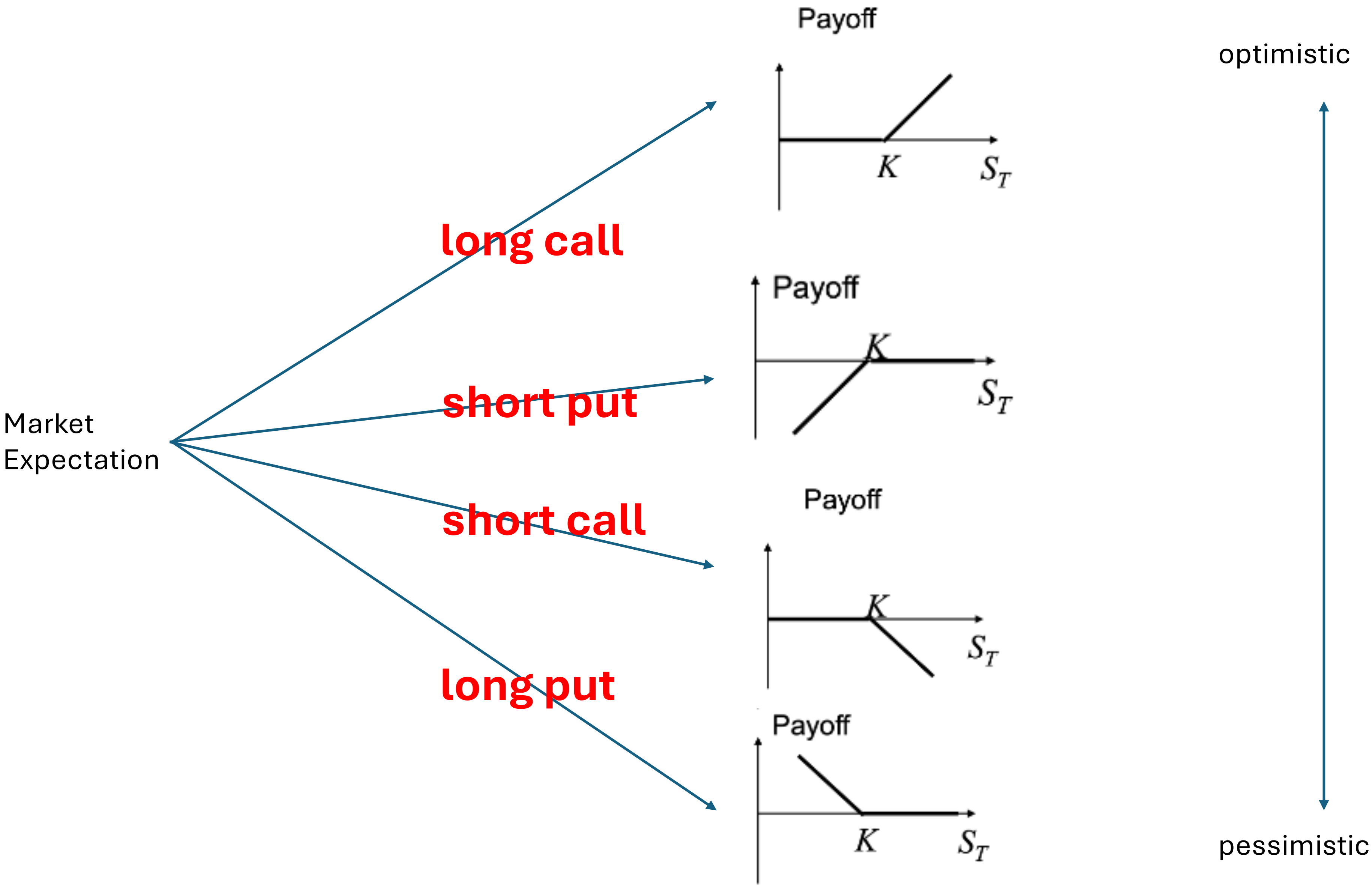
- If profit is on the right of K → long call (benefits from rising stock)
- If profit is on the left of K → long put (benefits from falling stock)

2. If no profit region (payoff ≤ 0)

- If the flat (0) region is on the left of K → short call (loses if stock rises)
- If the flat (0) region is on the right of K → short put (loses if stock falls)

Recap: option types

- **Market expectations**
- Each option reflects a different belief about future stock price movement.
- These beliefs form four distinct levels of optimism vs. pessimism.
- Most Optimistic → Long Call (expects strong rise).
- Moderately Optimistic → Short Put (expects stable or rise).
- Moderately Pessimistic → Short Call (expects stable or slight fall).
- Most Pessimistic → Long Put (expects strong fall).


Market Expectation
long call
short put
short call
long put
Payoff
K
S_T
Payoff
K
S_T
Payoff
K
S_T
Payoff
K
S_T
optimistic
pessimistic

Market Summary > NVIDIA Corp

177.82 USD

+177.78 (444,450.00%) ↑ all time

Closed: 15 Sept, 12:17 am GMT-4 • Disclaimer

After hours 177.53 −0.29 (0.16%)

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max

Open	177.77	Mkt cap	4.32T	52-wk high	184.48
High	178.60	P/E ratio	50.61	52-wk low	86.63
Low	176.45	Div yield	0.022%	Qtrly div amt	0.010

Long call: *"I expect NVIDIA to rise to $250, driven by AI demand."*

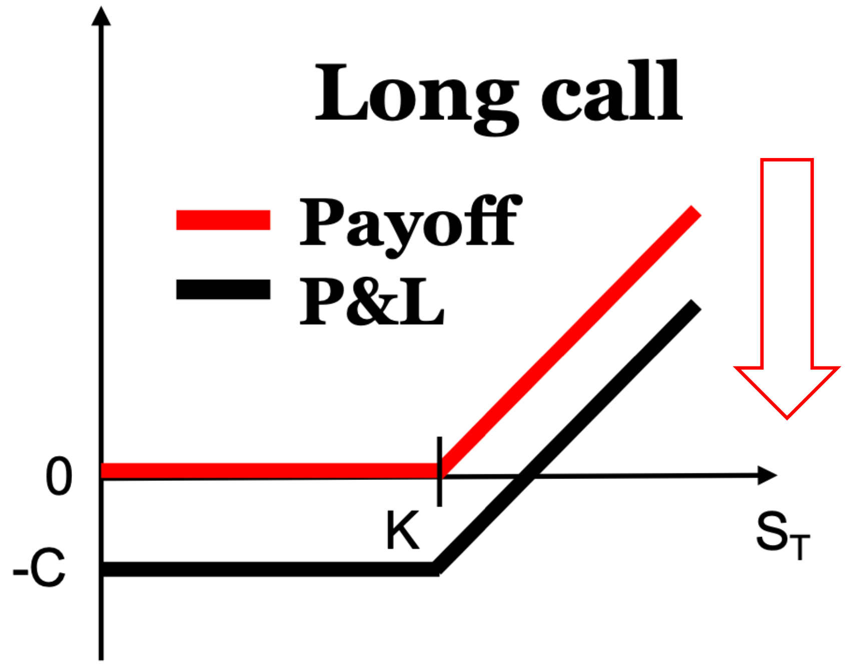
Short put: *"I expect NVIDIA to stay above $178, supported by earnings strength."*

Short call: *"I expect NVIDIA to dip slightly, since its valuation looks stretched."*

Long put: *"I see NVIDIA dropping toward $120 if AI hype fades."*

Recap: option payoff vs. P&L

- Payoff shows the theorical gain from the option at expiry, ignoring cost.
- To find the actual profit, we use P&L
- At expiration date (T):
- Long call payoff = max (S-K, 0)
- Short call payoff = -max (S-K, 0)
- Or min (K-S, 0)
- Long call P&L = -C + max (S-K, 0)
- Short call P&L = C - max (S-K, 0)

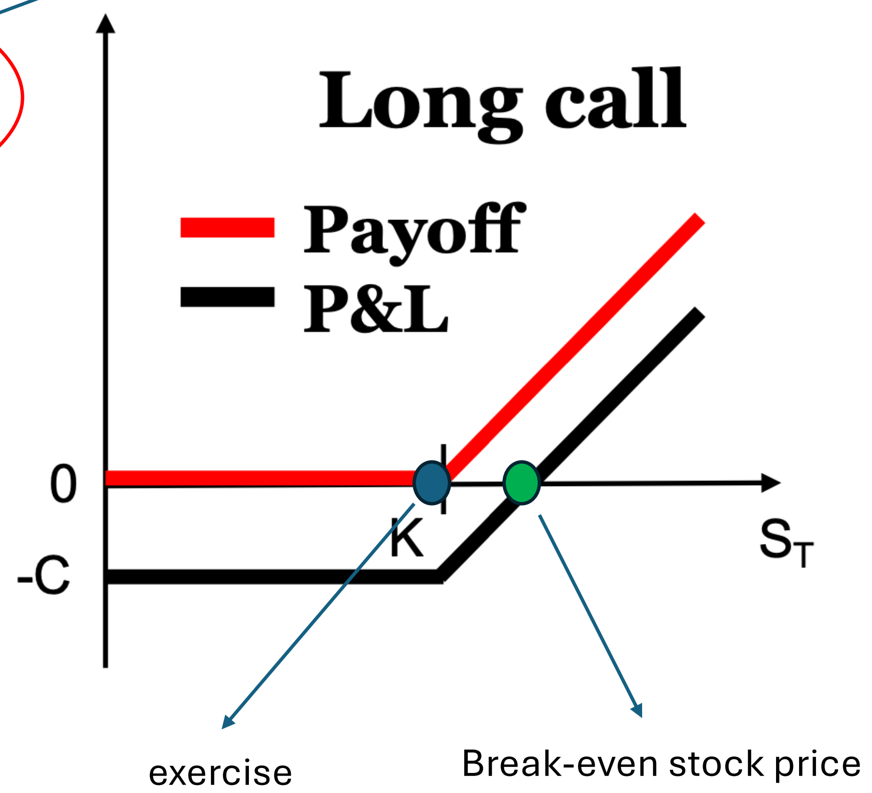


Equivalate expressions

Recap: option payoff vs. P&L

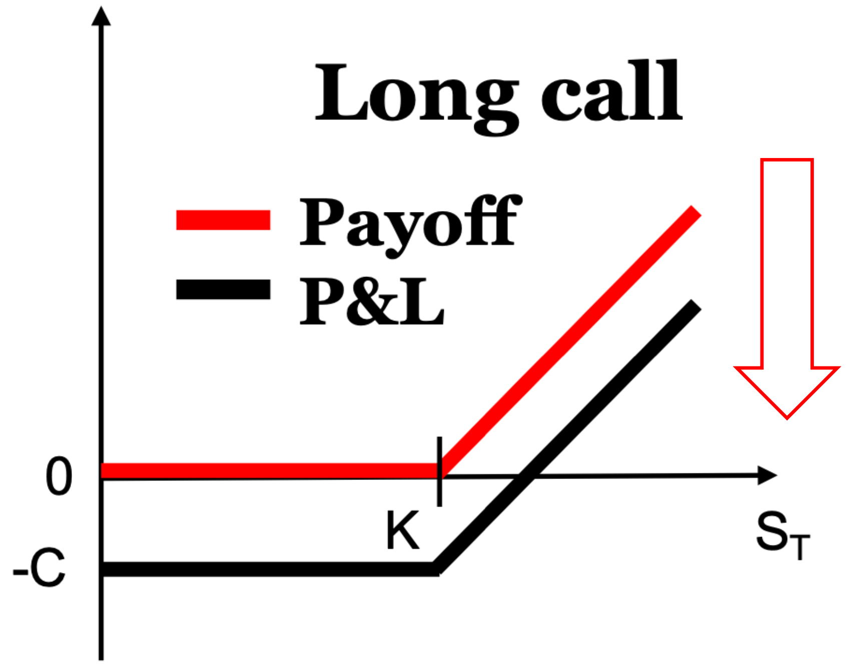
Sellers make a profit

- If S = K, call option is "at-the-money", holder is indifferent
- If S < K, call option is "out-of-the-money", holder lets option lapse
- **If S > K , call option is "in-the-money", holder exercises (blue dot).**
- **Only If S > K + C, the holder earns a net profit (green dot, *break-even point*).**



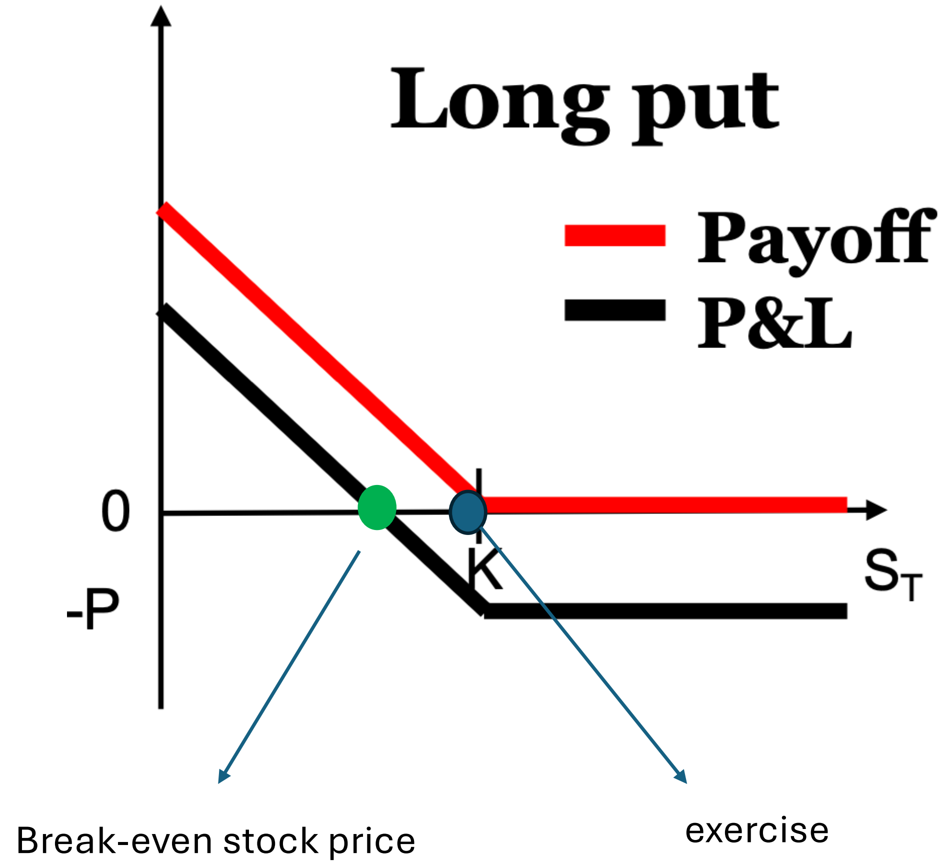
Recap: option payoff vs. P&L

- **Math derivation**
- Long call = -C + max (S-K, 0)
- Holder's goal: ensure P&L > 0
- Condition to exercise:
- S – K > 0 → S > K
- Condition to profit after paying premium
- –C + (S-K) > 0 → S > K + C



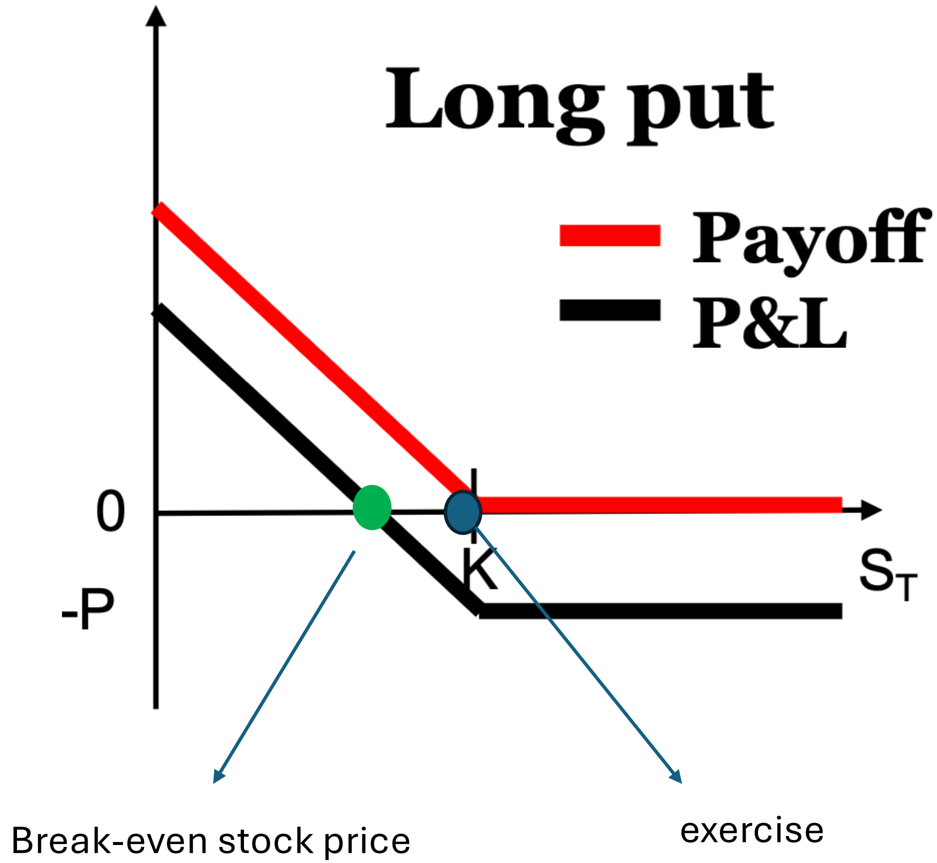
Recap: option payoff vs. P&L

- At expiration date (T):
- Long put payoff = max (K-S, 0)
- Short put payoff = -max (K-S, 0)
- Or min (S-K, 0)
- Long put P&L = -C + max (K-S, 0)
- Short put P&L = C - max (K-S, 0)

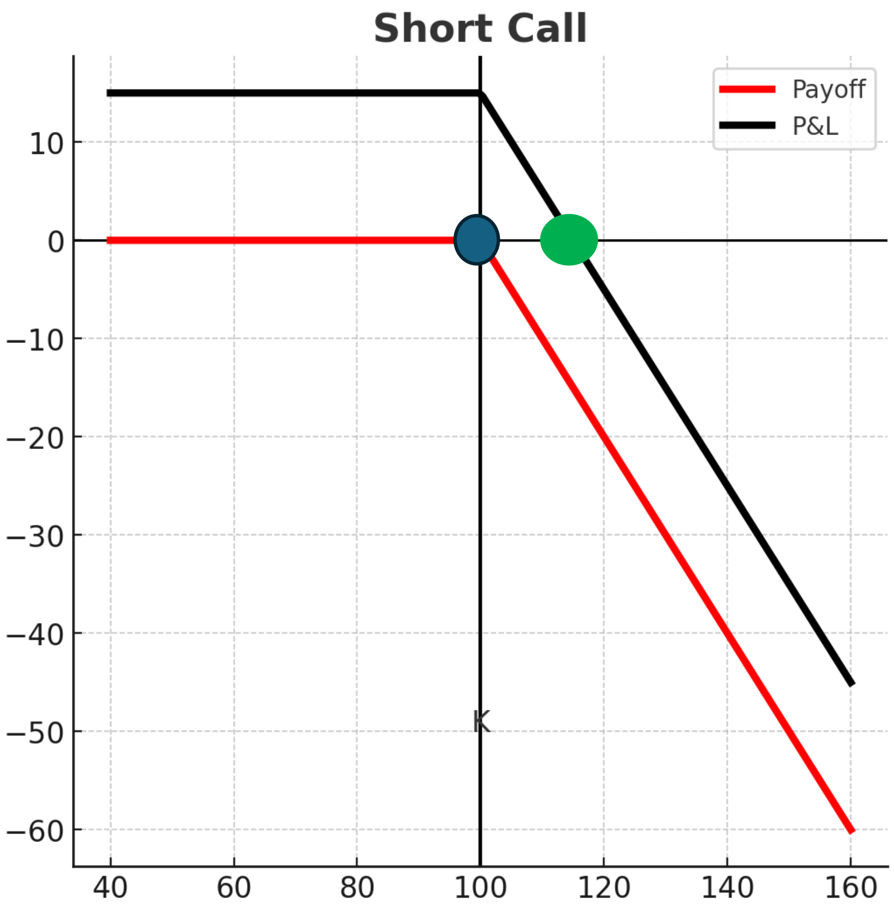


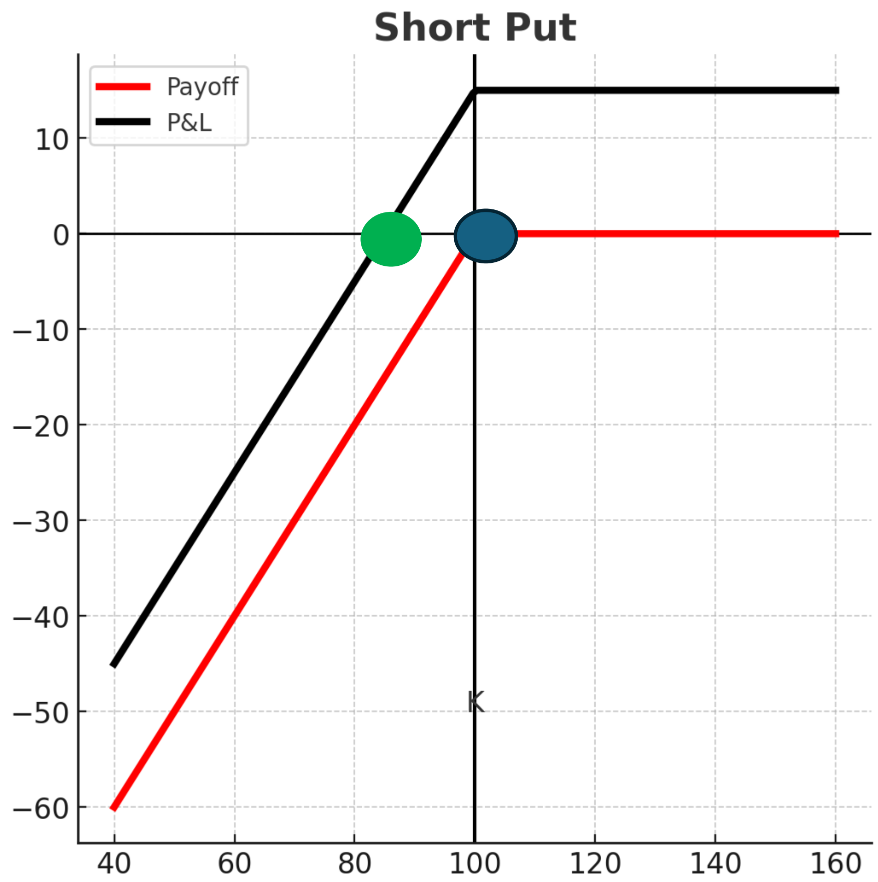
Recap: option payoff vs. P&L

- If S = K, call option is "at-the-money", holder is indifferent
- If S > K, call option is "out-of-the-money", holder lets option lapse
- **If S < K , call option is "in-the-money", holder exercises (blue dot).**
- **Only If S < K - C, the holder earns a net profit (green dot, *break-even point*).**



Recap: option payoff vs. P&L





Problem 9.15

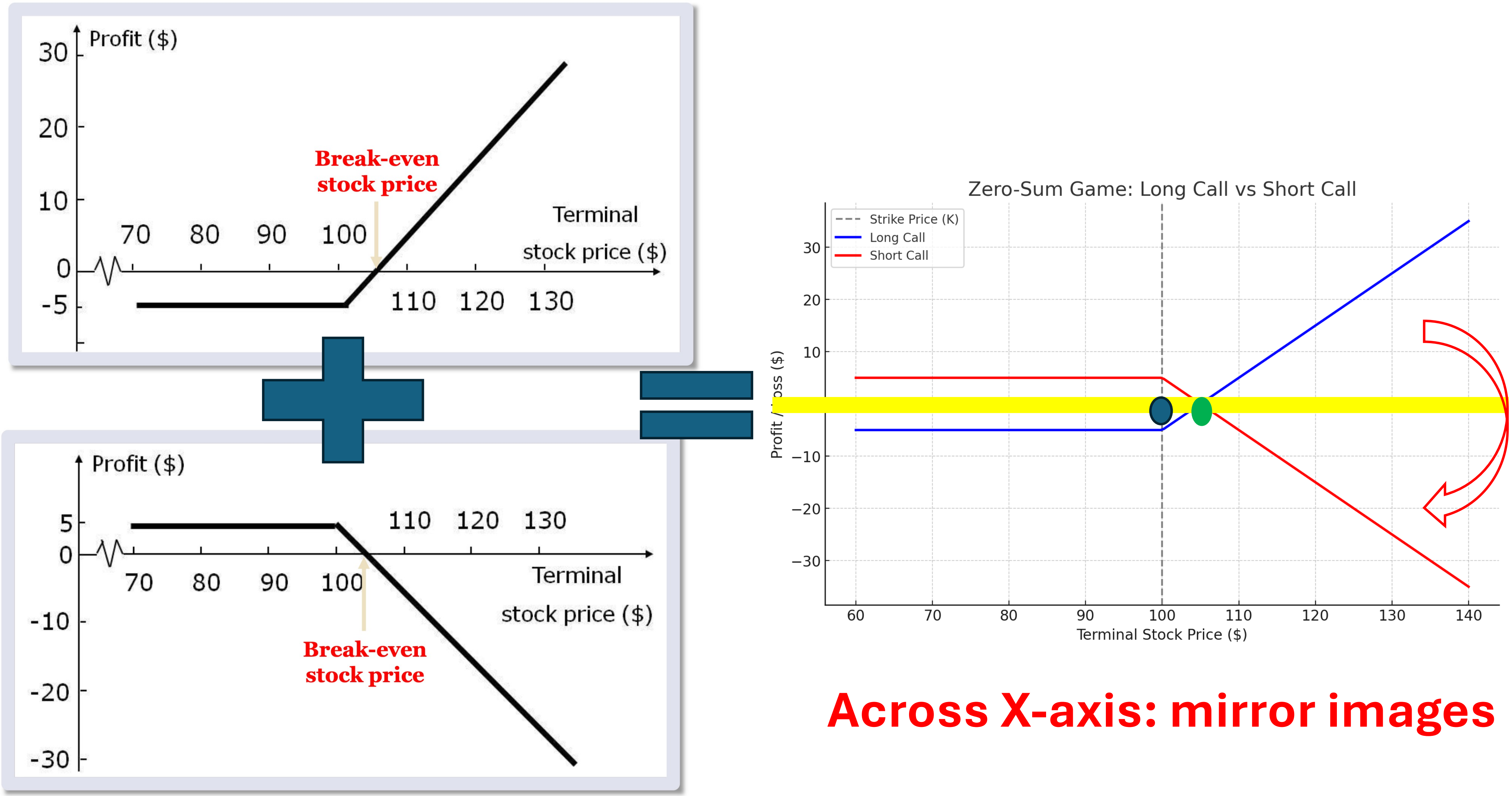
- *Explain carefully the difference between writing a put option and buying a call option.*

Answer 9.15

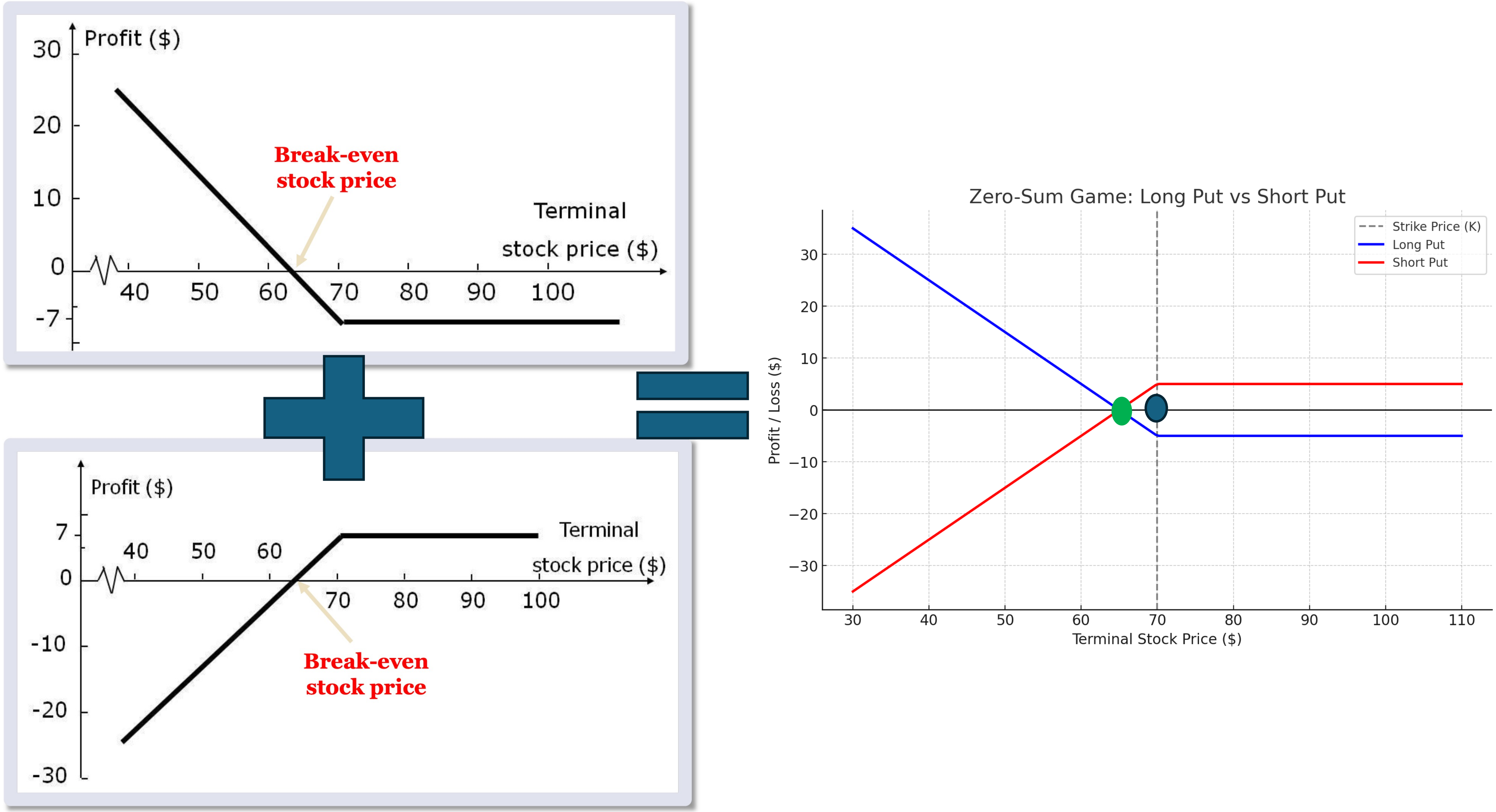
- Writing a put gives a payoff of $min(S - K, 0)$. Buying a call gives a payoff of $max(S - K, 0)$. In both cases the potential payoff is $S - K$.
- The difference is that for a written put, the counterparty chooses whether you get the payoff (and will allow you to get it only when it is negative to you).
- For a long call, you decide whether you get the payoff (and you choose to get it when it is positive to you.)

Recap: zero-sum nature

- Horizontal-axis as symmetry line
- Long and short positions are **mirror images** across the x-axis.
- Call options
- Long call: profit if stock price ↑(right side of K).
- Short call: mirror image → loss if stock price ↑.
- Put options
- Long put: profit if stock price ↓ ; Short put: loss if price ↓
- takeaway: **zero-sum game**



Across X-axis: mirror images



Profit ($)
30
20
10
0
-7
40
50
60
70
80
90
100
Break-even
stock price
Terminal
stock price ($)
Profit ($)
7
0
-10
-20
-30
40
50
60
70
80
90
100
Terminal
stock price ($)
Break-even
stock price
Zero-Sum Game: Long Put vs Short Put
Strike Price (K)
Long Put
Short Put
Profit / Loss ($)
30
20
10
0
−10
−20
−30
30
40
50
60
70
80
90
100
110
Terminal Stock Price ($)

Problem 9.9

- *Suppose that a European call option to buy a share for \$100.00 costs \$5.00 and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.*

Answer 9.9

- *Ignoring the time value of money*, the holder of the option will make a profit if the stock price (S) at maturity of the option is greater than \$105.
- This is because the payoff to the holder of the option is, in these circumstances, greater than the \$5 paid for the option.
- The option will be exercised if the stock price at maturity is greater than \$100.
- Note that if the stock price is between \$100 and \$105 the option is exercised, but the holder of the option takes a loss overall. The profit from a long position is as shown in Figure S9.1.

Answer 9.9

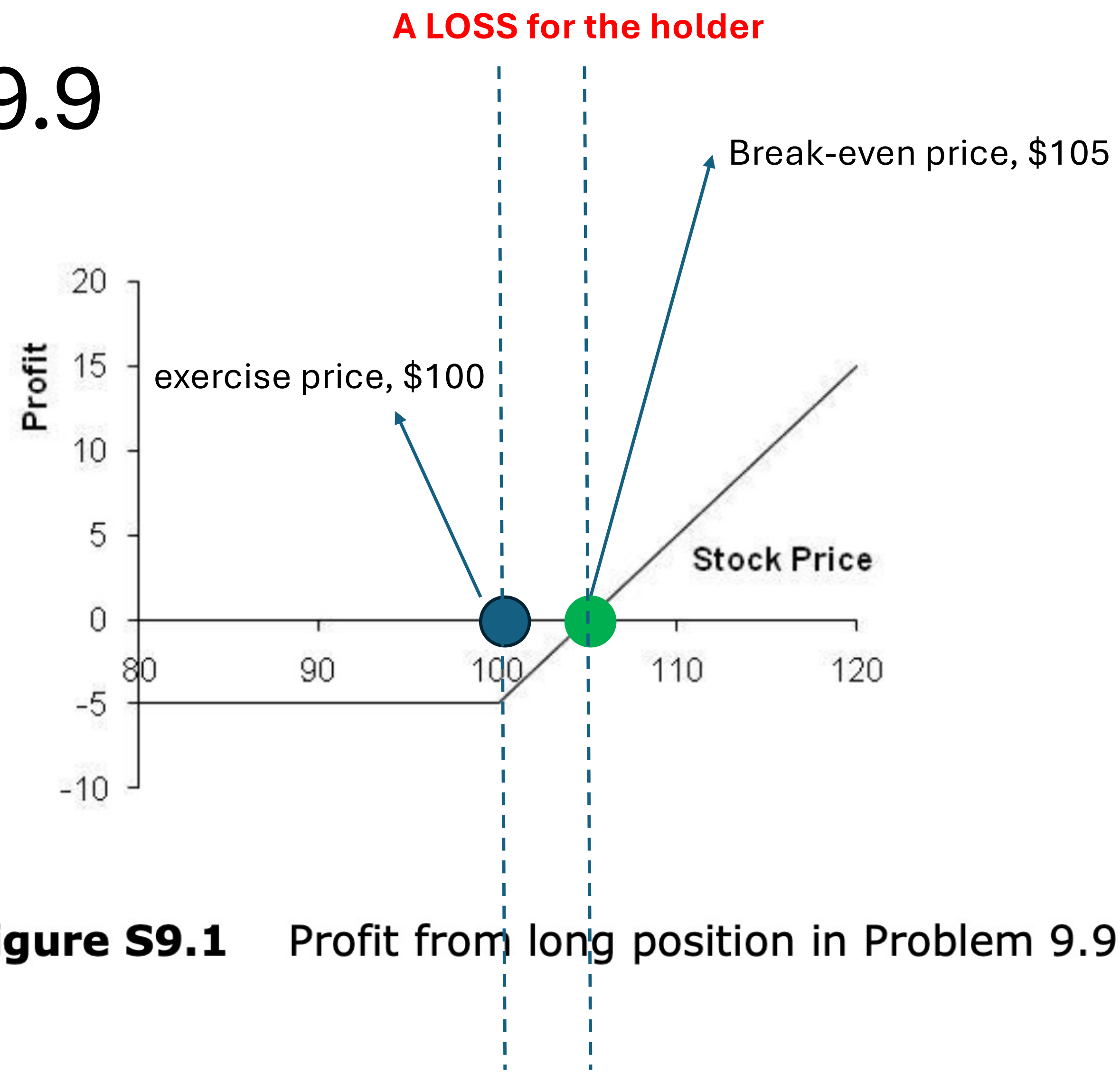


Figure S9.1 Profit from long position in Problem 9.9

Problem 9.10

- *Suppose that a European put option to sell a share for \$60 costs \$8 and is held until maturity. Under what circumstances will the seller of the option (the party with the short position) make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.*

Answer 9.10

- *Ignoring the time value of money*, the seller of the option will make a profit if the stock price (S) at maturity is greater than \$52.00.
- This is because the cost to the seller of the option is in these circumstances less than the price received for the option.
- The option will be exercised if the stock price at maturity is less than \$60.00.
- Note that if the stock price is between \$52.00 and \$60.00 the seller of the option makes a profit even though the option is exercised. The profit from the short position is as shown in Figure S9.2.

Answer 9.10

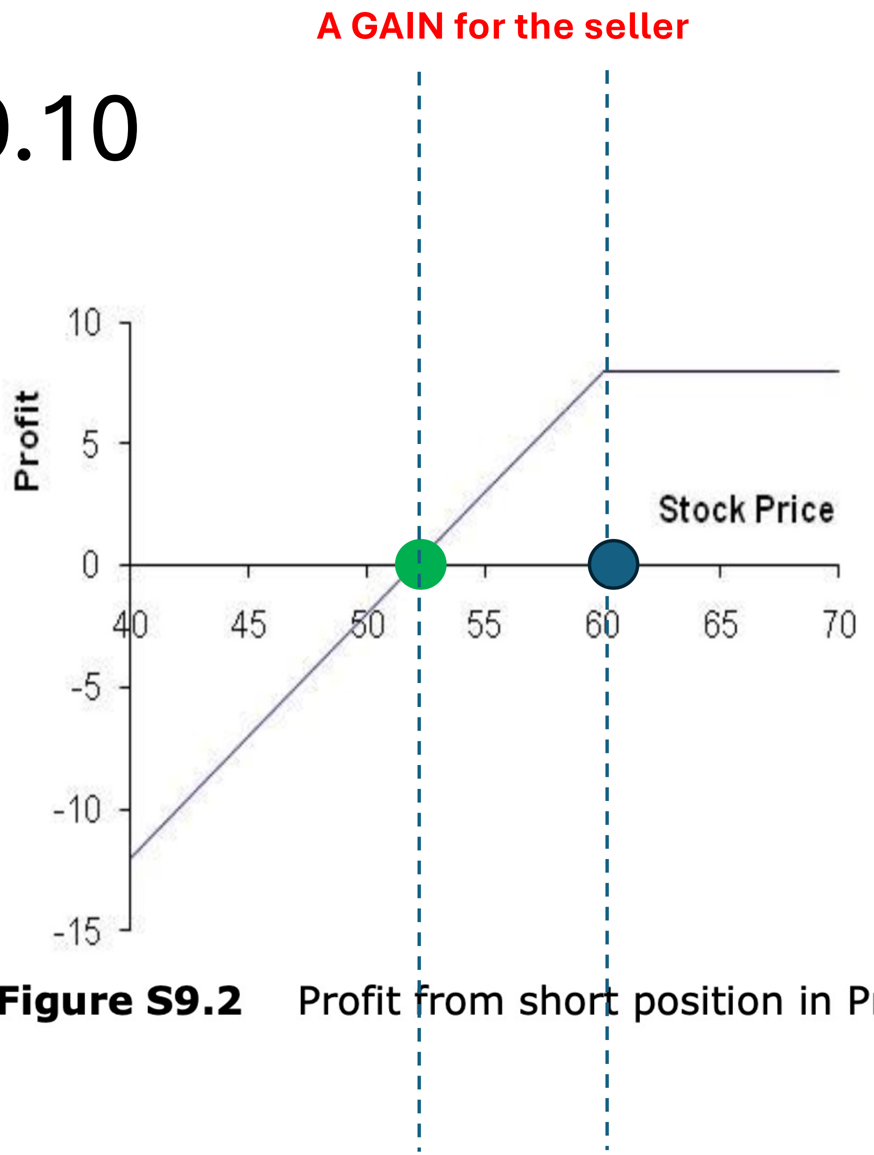


Figure S9.2 Profit from short position in Problem 9.10

Problem 9.12

- *A trader buys a call option with a strike price of \$45 and a put option with a strike price of \$40. Both options have the same maturity. The call costs \$3 and the put costs \$4. Draw a diagram showing the variation of the trader's profit with the asset price.*

Answer 9.12

- Figure S9.4 shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:

a) When the asset price less than \$40, the put option provides a payoff of $40 - S$ and the call option provides no payoff. The options cost \$7 and so the total profit is $33 - S$.

b) When the asset price is between \$40 and \$45, neither option provides a payoff. There is a net loss of \$7.

Answer 9.12

c) When the asset price greater than \$45, the call option provides a payoff of $S - 45$ and the put option provides no payoff. Taking into account the \$7 cost of the options, the total profit is $S - 52$.

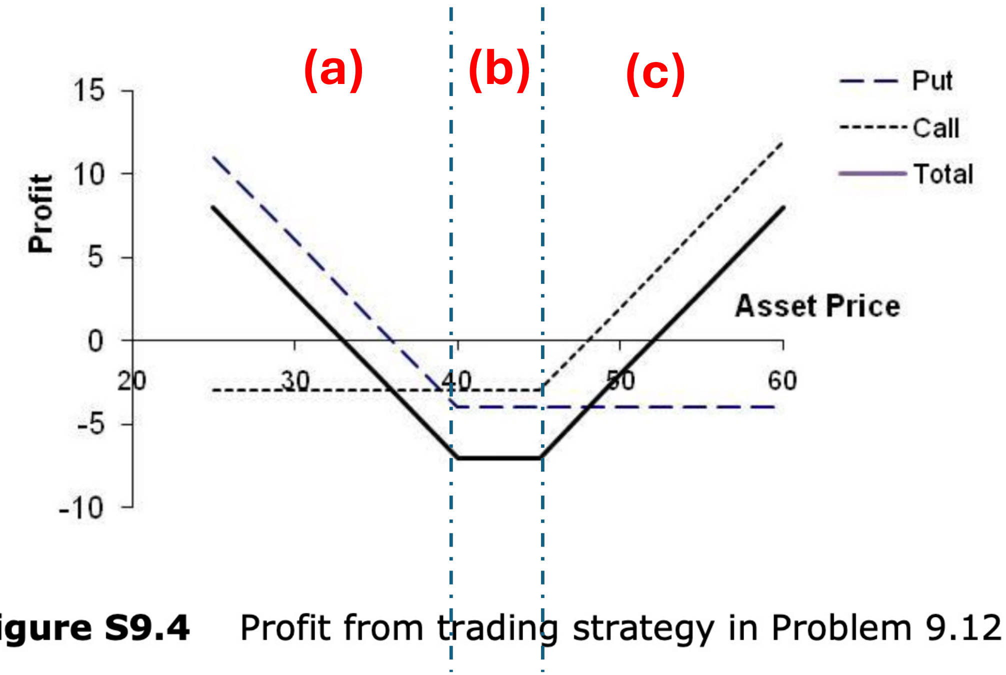


Figure S9.4 Profit from trading strategy in Problem 9.12

Recap: exchange-traded option

- The ET option is listed on an **exchange**, which provides standardization and liquidity.
- Example:

NasdaqGS - Delayed Quote • USD

Apple Inc. (AAPL) ☆ Follow + Add holdings

234.07 +4.04 +(1.76%) **233.97** -0.10 (-0.04%)

At close: 12 September at 4:00:01 pm GMT-4 After hours: 12 September at 7:59:54 pm GMT-4

19 Sep 2025 | All strike prices | List | All options

Calls In the money

Contract name	Last trade date (GMT-4)	Strike	Last price	Bid	Ask	Change	% change	Volume	Open interest	Implied volatility
AAPL250919C00005000	12/9/2025 2:39 pm	5	228.97	228.70	229.50	+3.57	+1.58%	76	510	1,075.00%
AAPL250919C00010000	12/9/2025 9:38 am	10	219.50	223.70	224.70	+0.93	+0.43%	5	27	1,010.94%
AAPL250919C00015000	22/8/2025 3:50 pm	15	213.35	218.70	219.70	0.00	0.00%	2	2	870.31%
AAPL250919C00020000	31/7/2025 9:40 am	20	188.90	213.40	214.70	0.00	0.00%	10	0	962.89%

Source: Yahoo Finance

Recap: exchange-traded option

- ET options **are not** adjusted for cash dividends.
- ET options are adjusted for stock splits.
- An n-for-m stock split should cause:

1. The stock price ($K \rightarrow K^*$)

$$K^* = K \times \frac{m}{n}$$

2. The number of shares ($N \rightarrow N^*$)

$$N^* = N \times \frac{n}{m}$$

- Even though the share count and price per share change, the total value you hold stays **the same** ($K^* \times N^* = K \times N$).

Recap: exchange-traded option

- Example: consider a call option to buy 1,000 shares for $30 per share. How should terms be adjusted for a (a) 2-for-1 stock split, and (b) a 25% stock dividend.

(1) 2-for-1 stock split, where n=2 and m=1,

k*=30*1/2=$15; n*=1,000*2/1=2,000

(2) 25% stock dividend means each shareholder receives **25% more shares.** If you had 4 shares, you get 1 extra share, and now 5 shares.

It can be seen as 5-for-4 stock split, where n=5 and m=4.

Problem 9.17

- *Consider an exchange-traded call option contract to buy 500 shares with a strike price of $40 and maturity in four months. Explain how the terms of the option contract change when there is*

a) *A 10% stock dividend*

b) *A 10% cash dividend*

c) *A 4-for-1 stock split*

Answer 9.17

a) 10% stock dividend = a 11-for-10 stock split

The option contract becomes one to buy 500 × 11/10 (n/m) = 550 shares with an exercise price of 40 × 10/11 (m/n) = 36.36.

b) There is no effect. The terms of an options contract are not normally adjusted for cash dividends.

c) A 4-for-1 stock split, where n=4 and m=1

The option contract becomes one to buy 500 × 4 = 2,000 shares with an exercise price of 40/4 = 10.