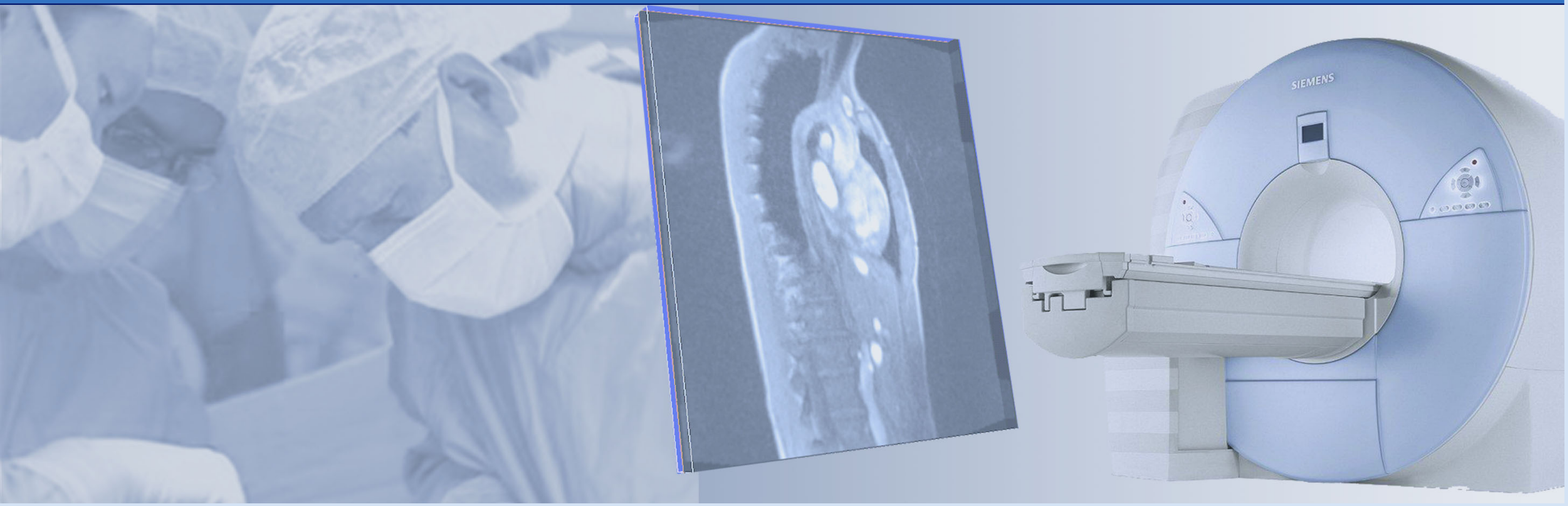
Tutorial computer- and robot-assisted surgery



NATIONALES CENTRUM
FÜR TUMORERKRANKUNGEN
PARTNERSTANDORT DRESDEN
UNIVERSITÄTS KREBSCENTRUM UCC

getragen von:
Deutsches Krebsforschungszentrum
Universitätsklinikum Carl Gustav Carus Dresden
Medizinische Fakultät Carl Gustav Carus, TU Dresden
Helmholtz-Zentrum Dresden-Rossendorf

Sebastian Bodenstedt
Translational Surgical Oncology

Any questions from the lecture?

Review Fourier Transform

Fourier series

- Any periodic function can be expressed through (infinite) sine and cosine functions

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos(n \cdot x) + \sum_{n=1}^{\infty} b_n \sin(n \cdot x)$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(n \cdot x) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(n \cdot x) dx$$

Fourier Transform

- Generalization of the complex Fourier series

$$f(x)=\int_{-\infty}^{\infty} F(u)\cdot e^{2\pi i u x}\,du$$

where

$$e^{\pm i\theta}=\cos(\theta)\pm i\cdot\sin(\theta)$$

- F(u) holds the amplitude A and phase θ of sine functions for each u

$$A\cdot\sin(u\cdot x+\theta)$$

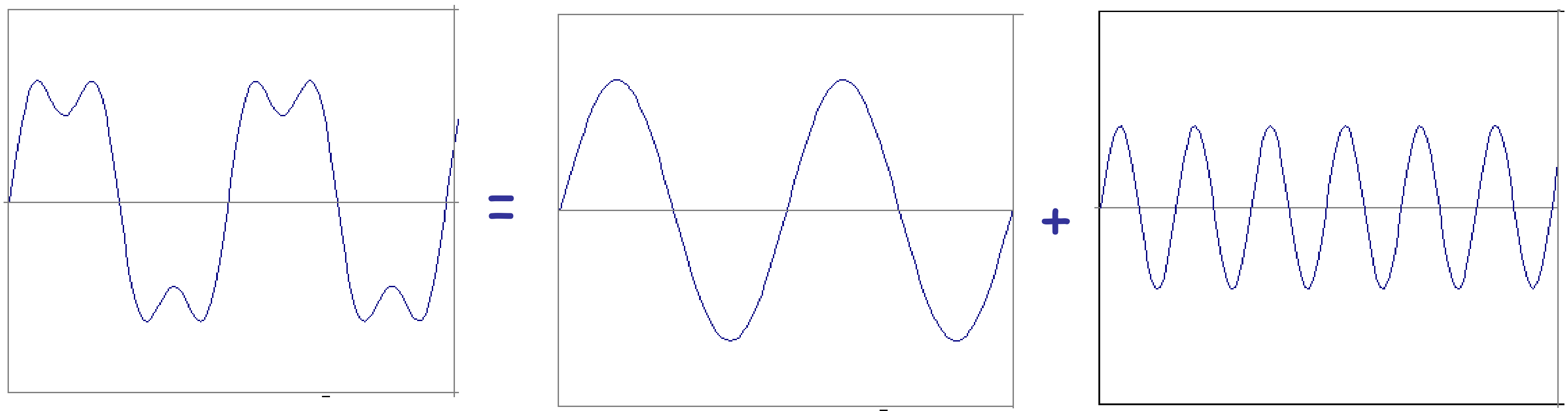
Fourier Transform

- Fourier Transform

$$F(u) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i u x} \, dx$$

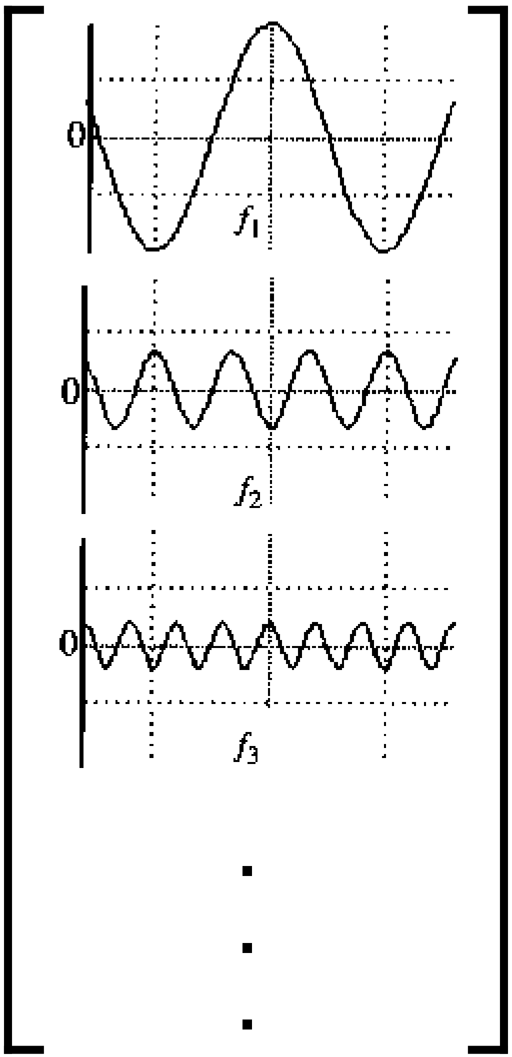
- Inverse Fourier Transform

$$f(x) = \int_{-\infty}^{\infty} F(u) \cdot e^{2\pi i u x} \, du$$



Fourier Transform – Change in basis

$$M * f(x) = F(\upsilon)$$



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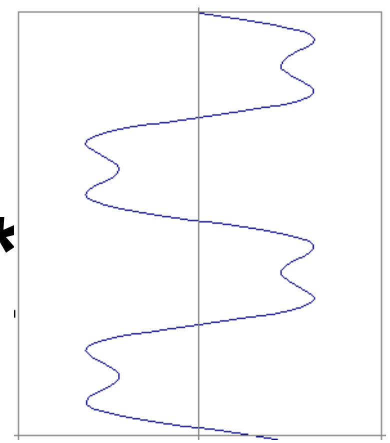

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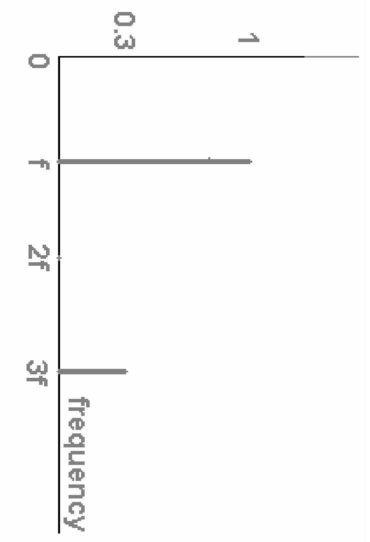
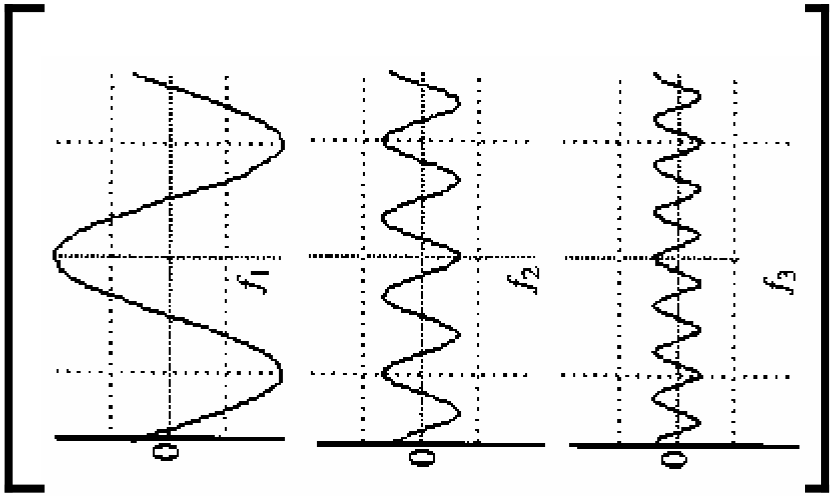


Fourier Transform – Change in basis

$$M^{-1} * F(\upsilon) = f(x)$$



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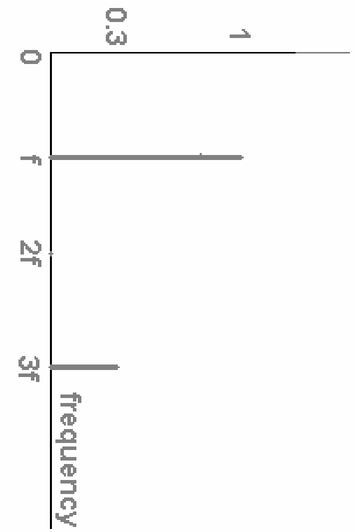

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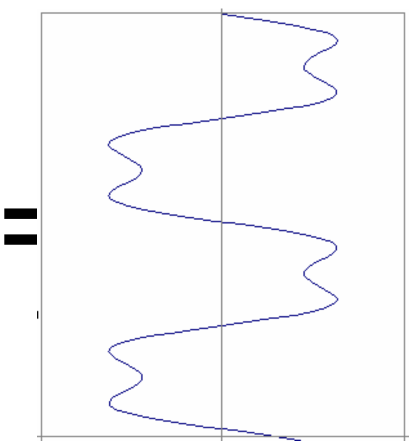


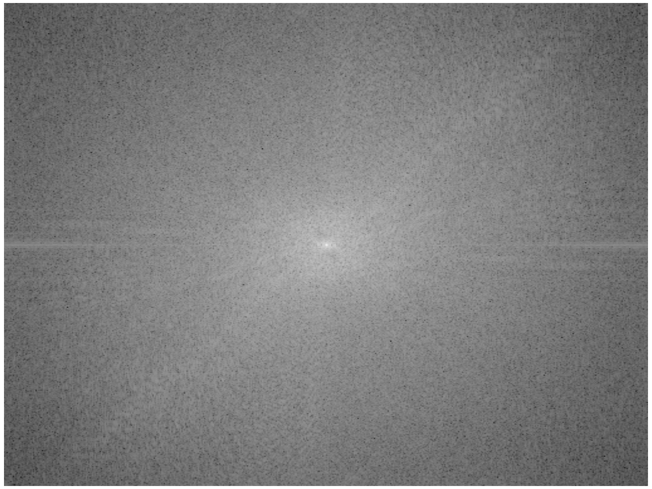
2D Fourier Transform

- Fourier Transform

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot e^{-2\pi i(ux+vy)}\,dxdy$$

- Inverse Fourier Transform

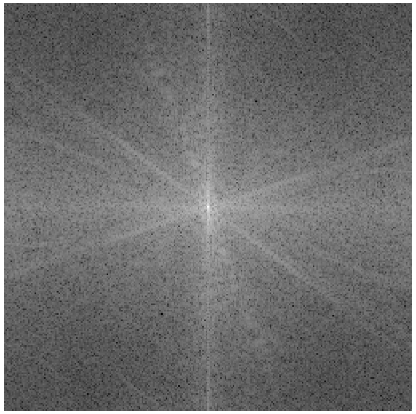
$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\cdot e^{2\pi i(ux+vy)}\,dudv$$



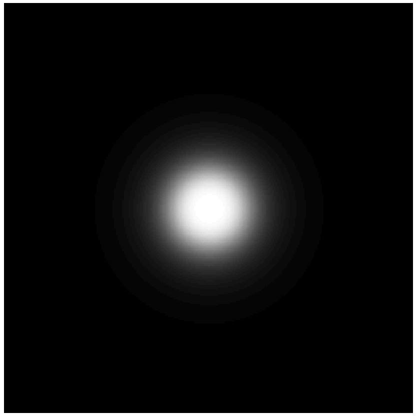
Low-pass filter

Original image

FFT of original image



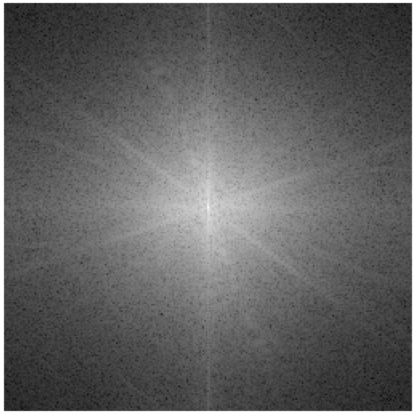
Low−pass filter



Let the low frequencies pass and eliminating the high frequencies.

Low−pass image

FFT of low−pass image

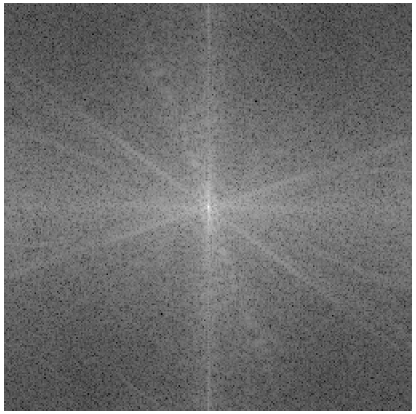


Generates image with overall shading, but not much detail

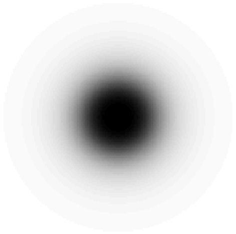
High-pass filter

Original image

FFT of original image



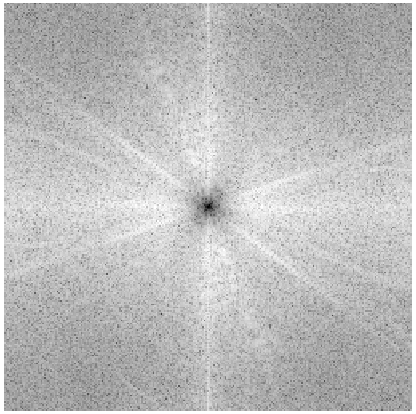
High−pass filter



Lets through the high frequencies (the detail), but eliminates the low frequencies (the overall shape). It acts like an edge enhancer.

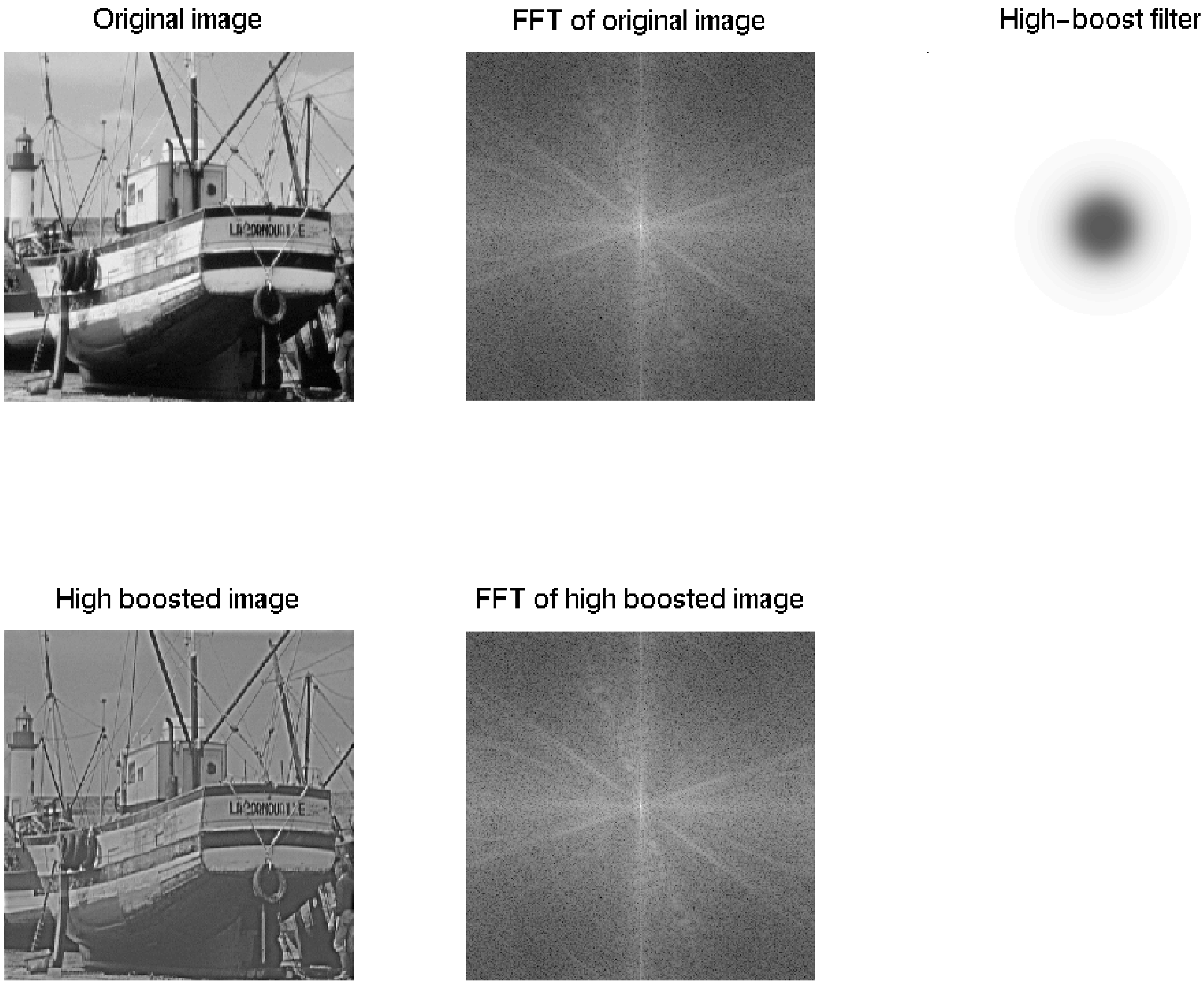
High−pass image

FFT of high−pass image



NCT

Boosting high frequencies



Discrete Fourier Transform

- Fourier Transform

1D $$F(u)=\frac{1}{N}\sum_{x=0}^{N-1} f(x)\cdot e^{\frac{-2\pi i u x}{N}}$$

2D $$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cdot e^{-2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

- Inverse Fourier Transform

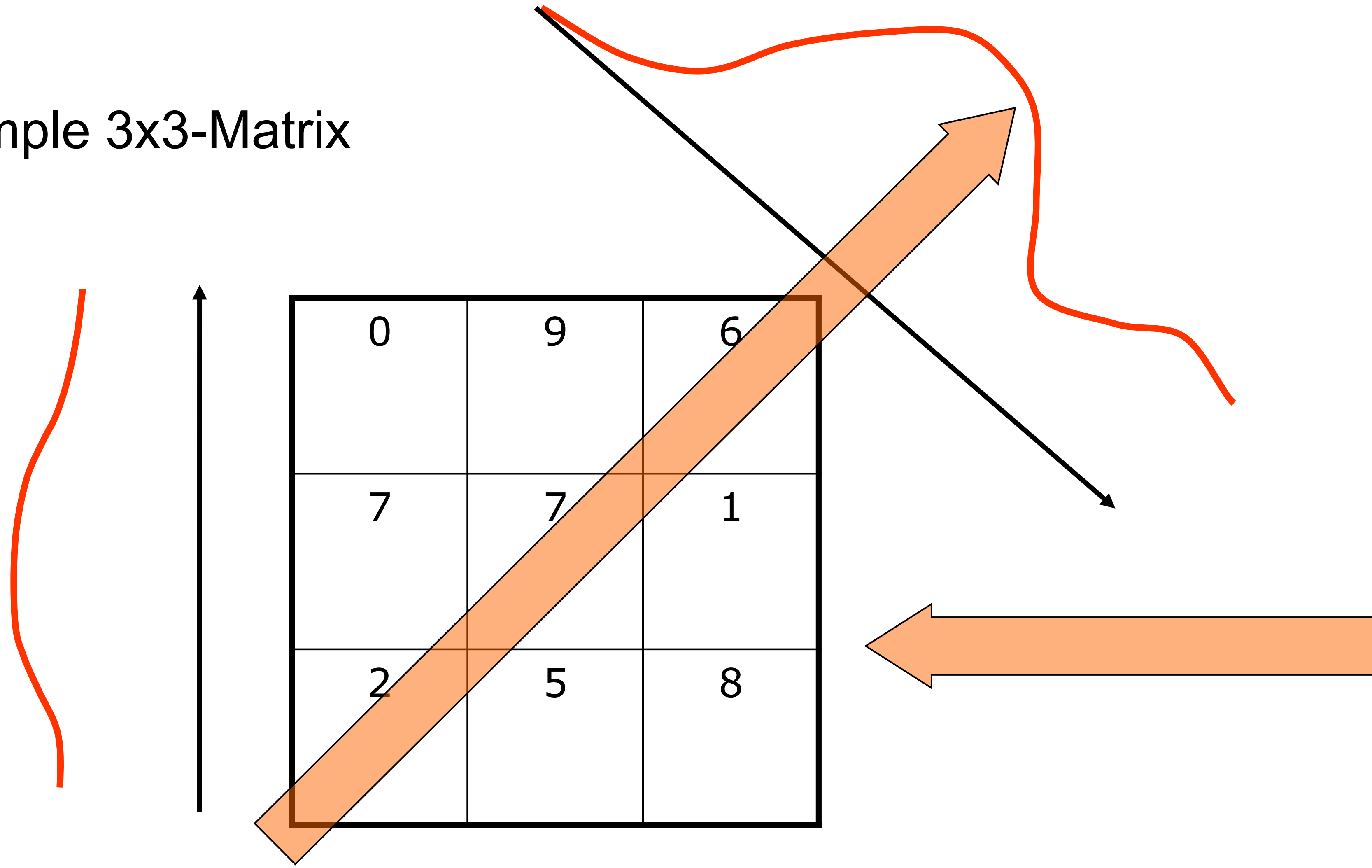
1D $$f(x)=\frac{1}{N}\sum_{u=0}^{N-1} F(u)\cdot e^{\frac{2\pi i u x}{N}}$$

2D $$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\cdot e^{2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Reminder Iterative Reconstruction

Iterative Reconstruction technique

Example 3x3-Matrix



Iterative Reconstruction technique

Goal of the reconstruction

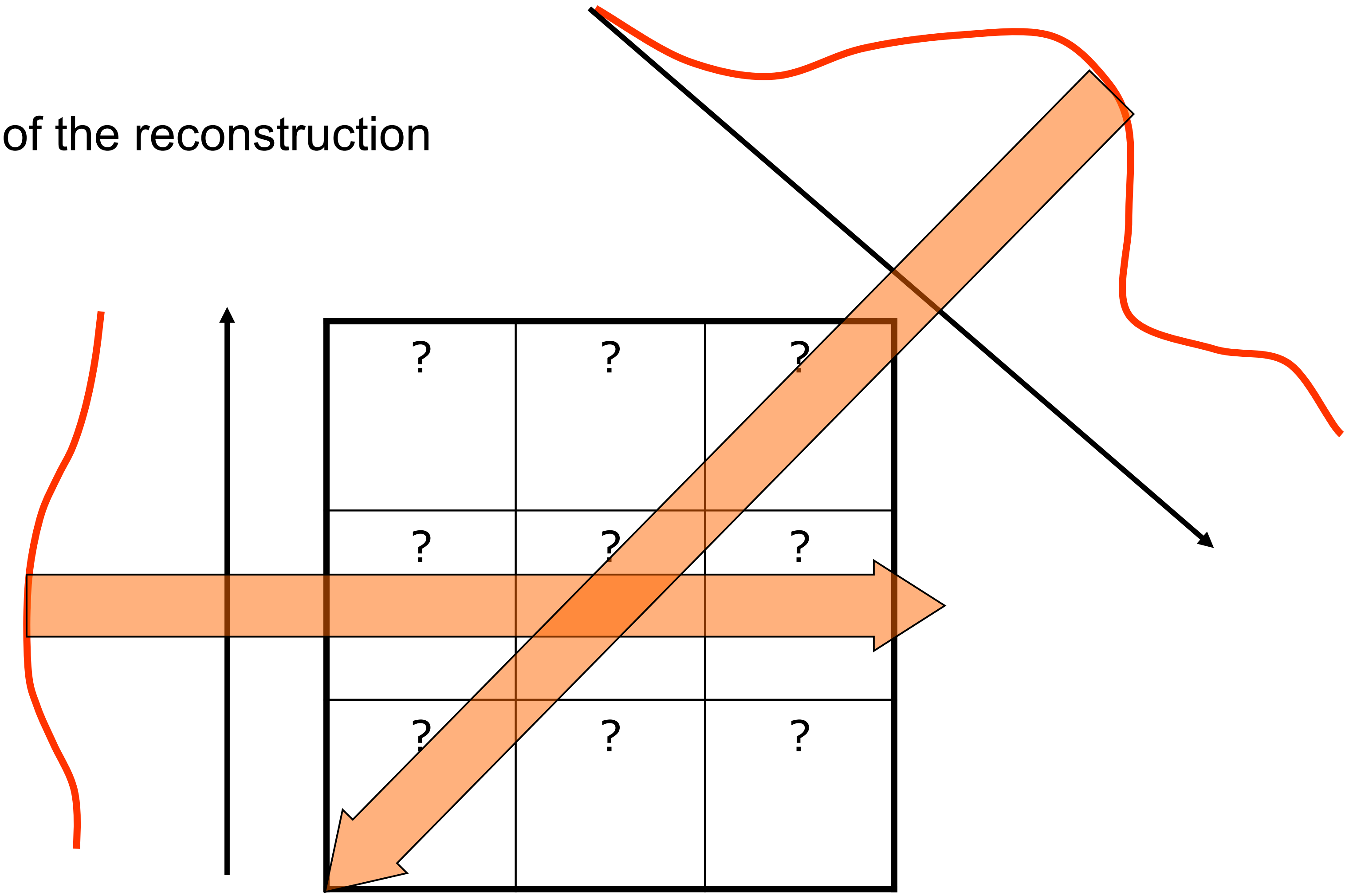


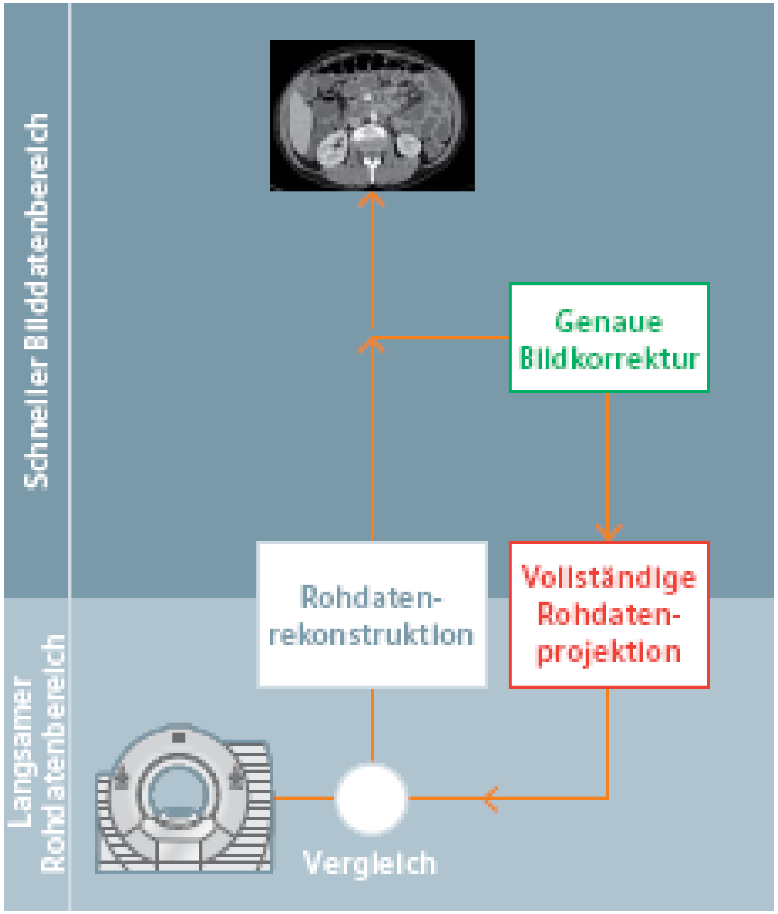
Image reconstruction - iterative

How can we calculate an image from the measurements?

Approximation of the image:

1. Estimate
2. Correction
3. Iteration

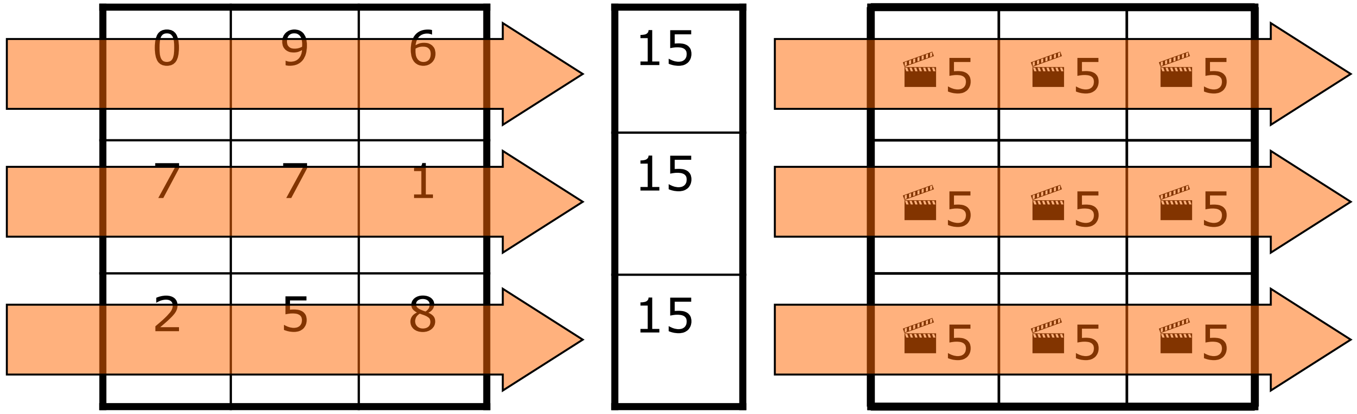
- First approximation derived from direction 1
- Creation of correction profiles
- Stop criteria: min. error or max. number of iteration



Quelle: Siemens

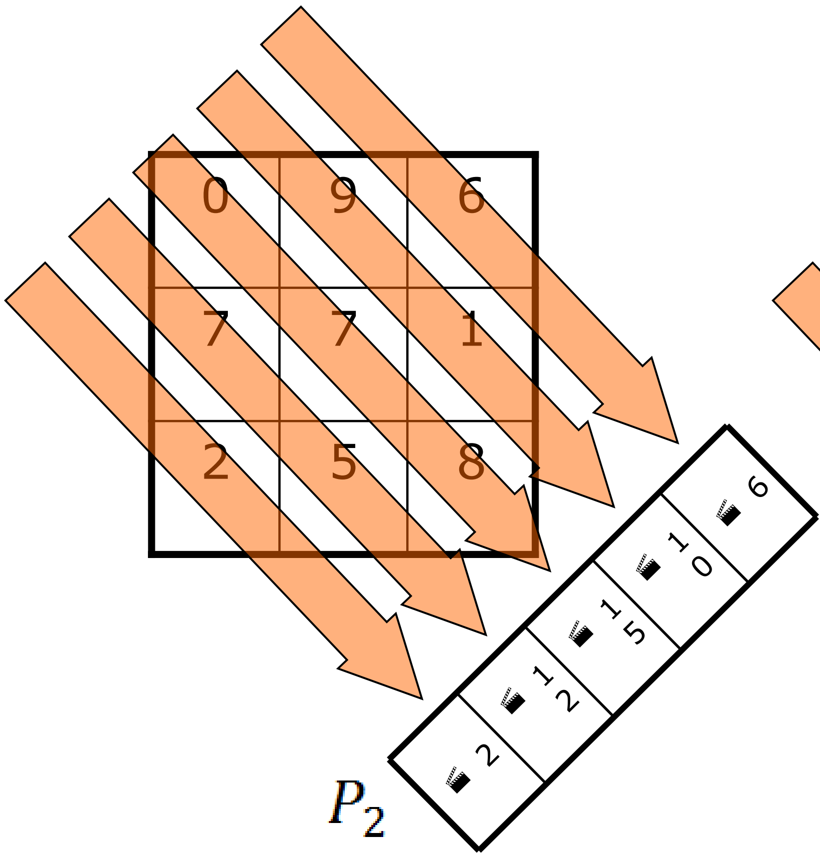
Estimate

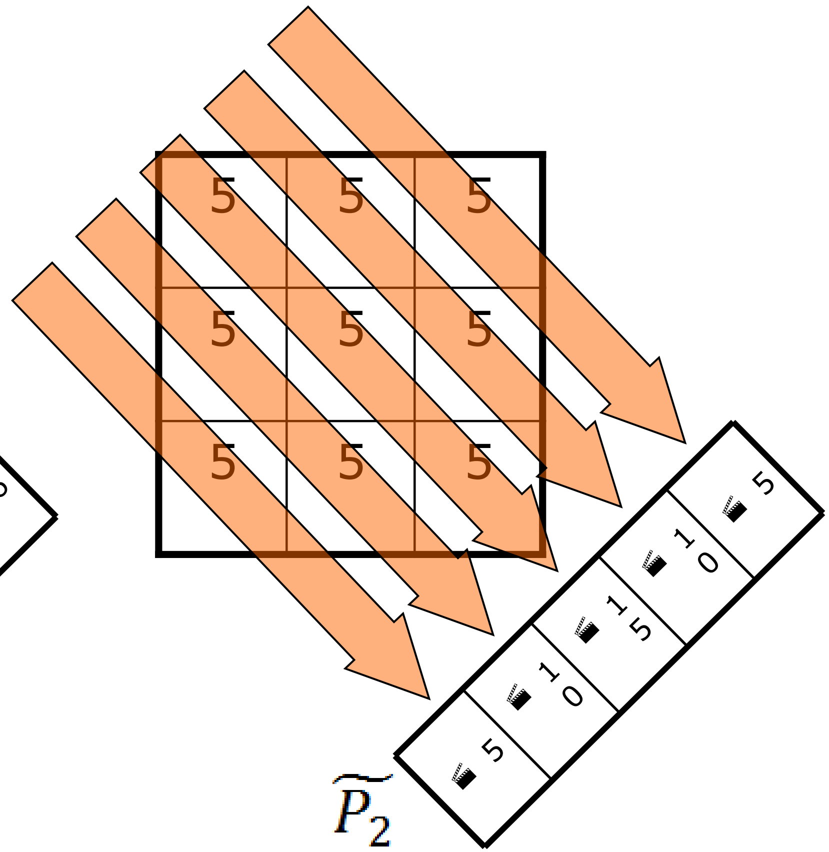
1. projection direction: 1. Estimate of the matrix



Original　　Projection P_1　　1^{st} Estimate

Iterative Reconstruction technique

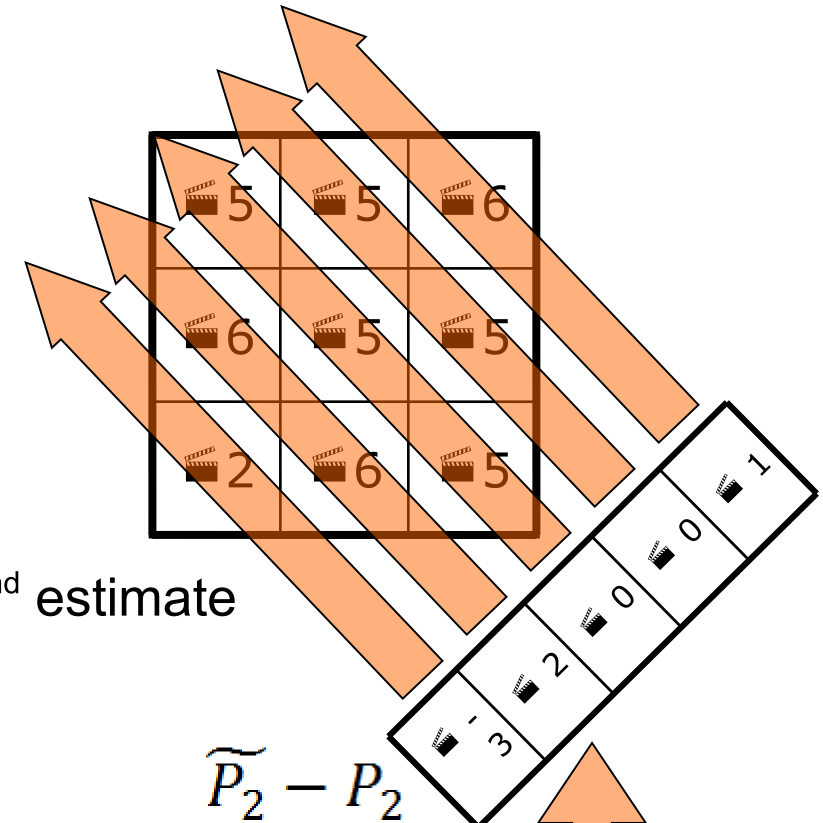




2	12	15	10	6
5	10	15	10	5
-3	2	0	0	1

Iterative Reconstruction technique

0	9	6
7	7	1
2	5	8



2^{nd} estimate

Lengths normalized to 1

$\widetilde{P_2} - P_2$

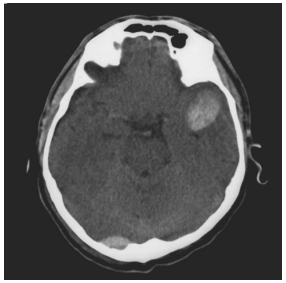
2	12	15	10	6
5	10	15	10	5
-3	2	0	0	1

Exercise: Iterative Reconstruction

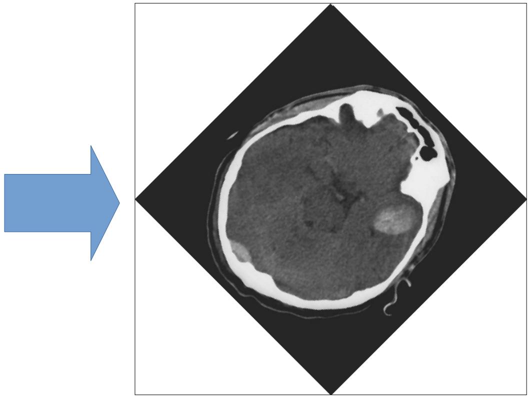
- Exercise: Implement method for iterative reconstruction
  - Reconstruct 2 images

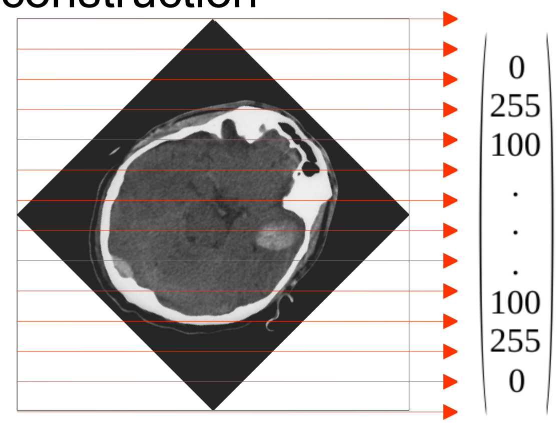
- **Solution**

- Estimate rays



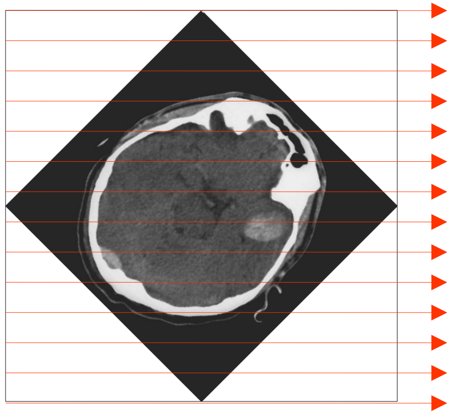
Current reconstruction





Exercise: Iterative Reconstruction

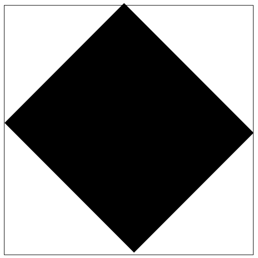
- Estimate



$$\vec{r}_e = \begin{pmatrix} 0 \\ 255 \\ 100 \\ \cdot \\ \cdot \\ \cdot \\ 100 \\ 255 \\ 0 \end{pmatrix}$$

- Form delta to measurement: $\delta \vec{r} = \vec{r}_m - \vec{r}_e$

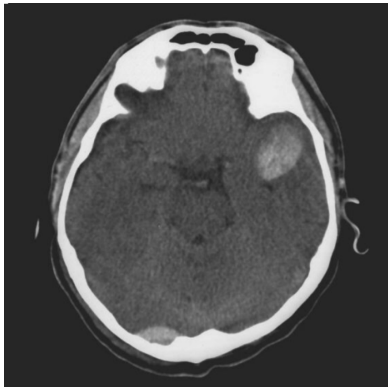
- $\delta \vec{r}$ Scale and subtract from reconstruction
  - Scaling requires number of traversed pixels
  - Idea: Rotate mask and count number of relevant pixels



Exercise: Iterative Reconstruction

- Fill matrix with $\delta\vec{r}$:

- Rotate back and update estimate:



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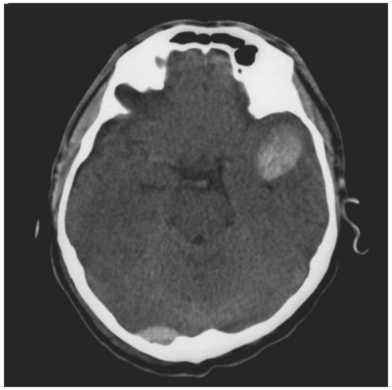

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