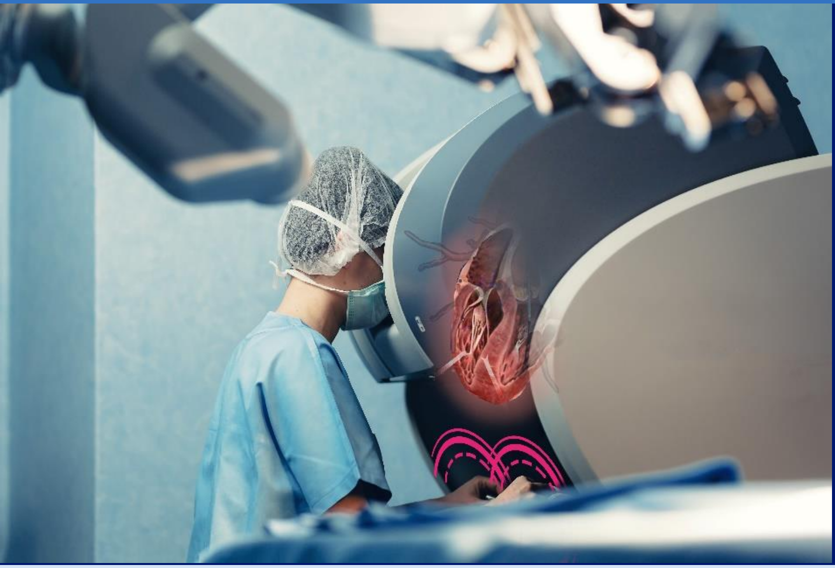
Computer- and robot-assisted Surgery



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NATIONALES CENTRUM
FÜR TUMORERKRANKUNGEN
PARTNERSTANDORT DRESDEN
UNIVERSITÄTS KREBSCENTRUM UCC

getragen von:
Deutsches Krebsforschungszentrum
Universitätsklinikum Carl Gustav Carus Dresden
Medizinische Fakultät Carl Gustav Carus, TU Dresden
Helmholtz-Zentrum Dresden-Rossendorf

Lecture 2

CT Imaging

Imaging - Summary

X-Ray

Ultrasound

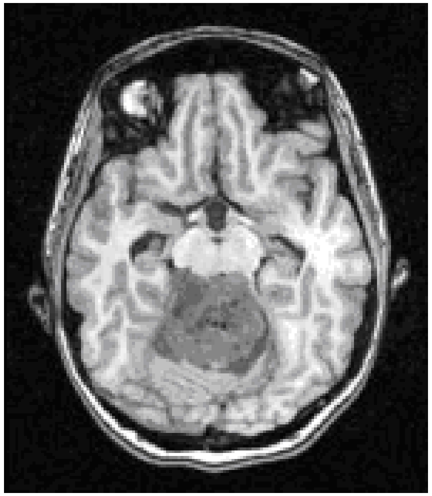
NCT

Tomographic Imaging

Computed tomography

.CTisus.

Magnetic resonance imaging



Tomographic Imaging provides cross-sectional images or slices of the human body

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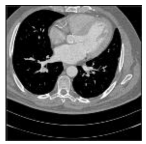
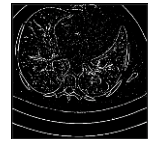
Workflow

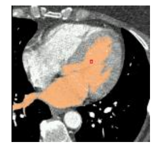
Image Acquisition



Preprocessing

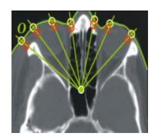


Segmentation

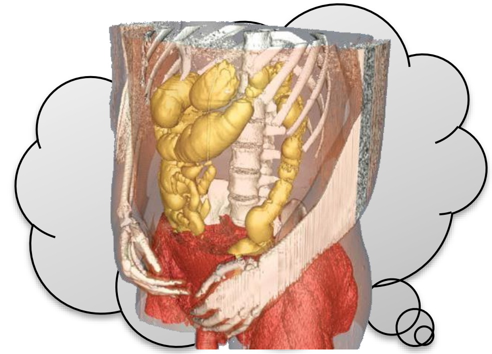


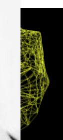
Modeling

Registration



Visualisation





Content

How does compute tomography work?

What is measured, how it is measured?

How are the cross-sectional images reconstructed?

What methods exist?

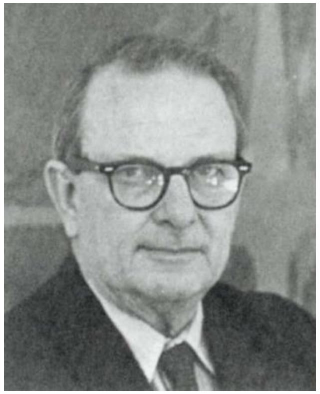
- Relation X-ray – CT
- Iterative Reconstruction
- Radon-Transformation
  - Fourier-Reconstruction
  - Filtered Backprojection
- Representation of slices

COMPUTED TOMOGRAPHY (CT)

CT- History

- 1895 Röntgen discovers X-ray beams
- 1917 Radon develops Radon transformation
- 1963 Cormack publishes method for calculation of absorption distribution (*Journal of Applied Physics*)
- 1971 Hounsfield develops CT, first human examination
- 1979 Cormack and Hounsfield receive Nobel prize

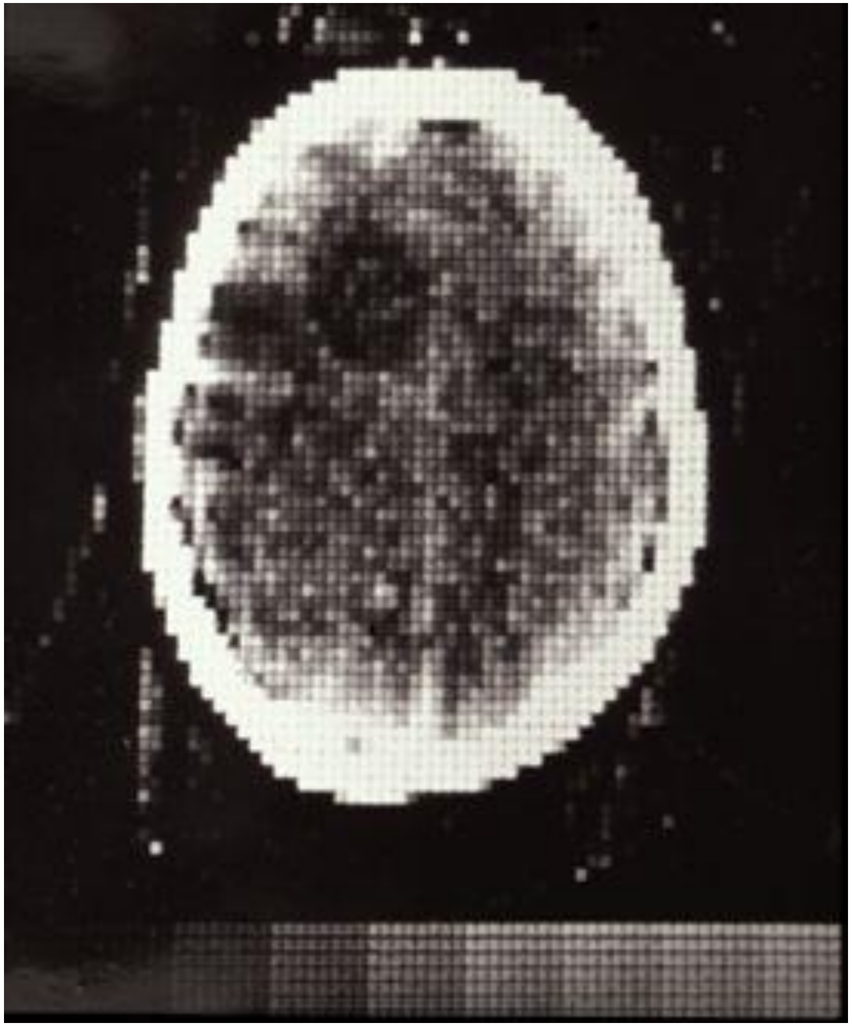
Röntgen



Cormack

Hounsfield

CT – Past / Present



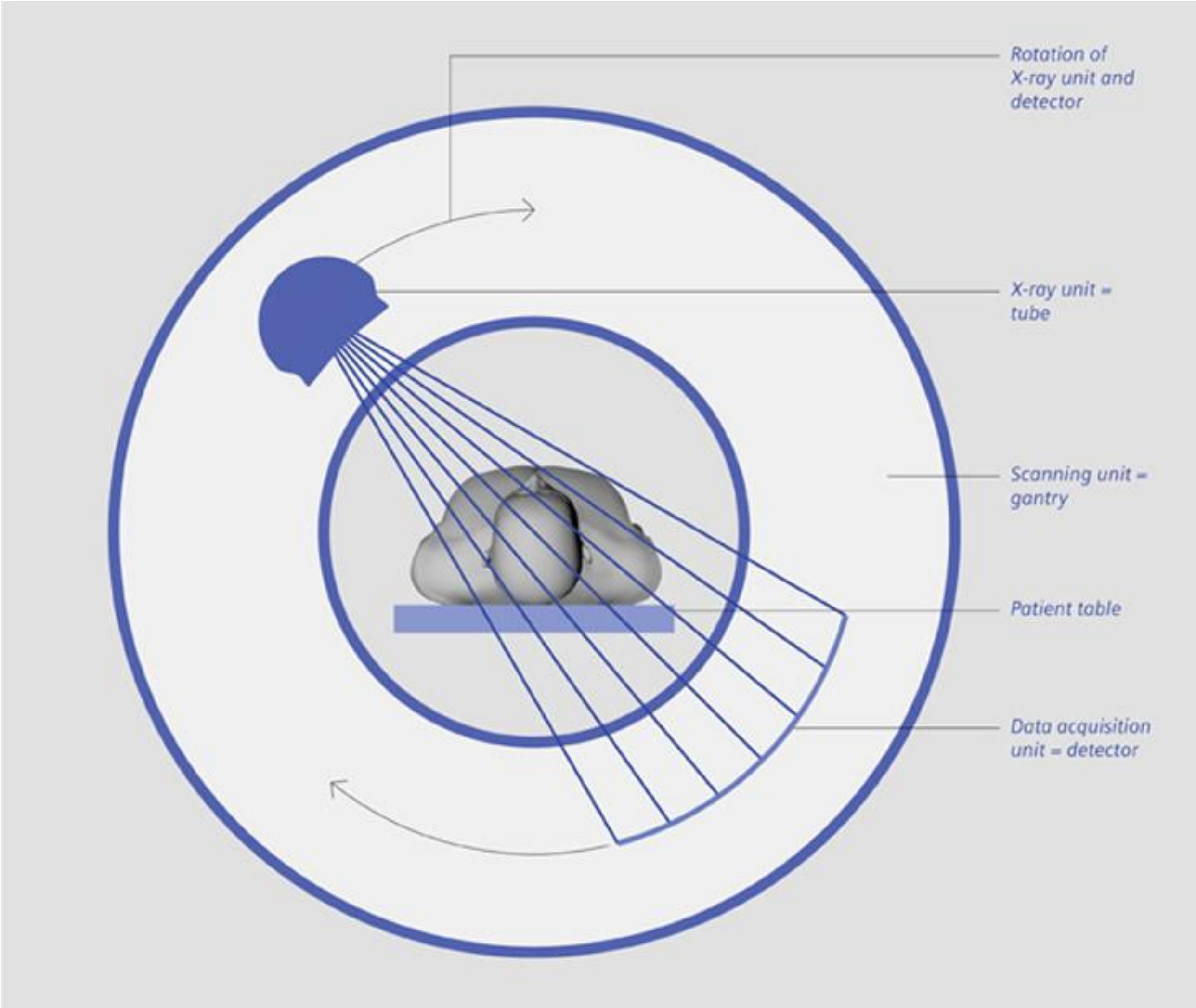
Atkinson Morley's Hospital, October 1971

Functional principle

- X-ray measurements are used for Imaging
- Many X-ray measurements taken from different angles produce cross-sectional (tomographic) images
- A tomographic image consists of several voxels (volumetric pixel), that posess a certain gray value depending on the type of tissue

Quelle: Siemens

Functional principle

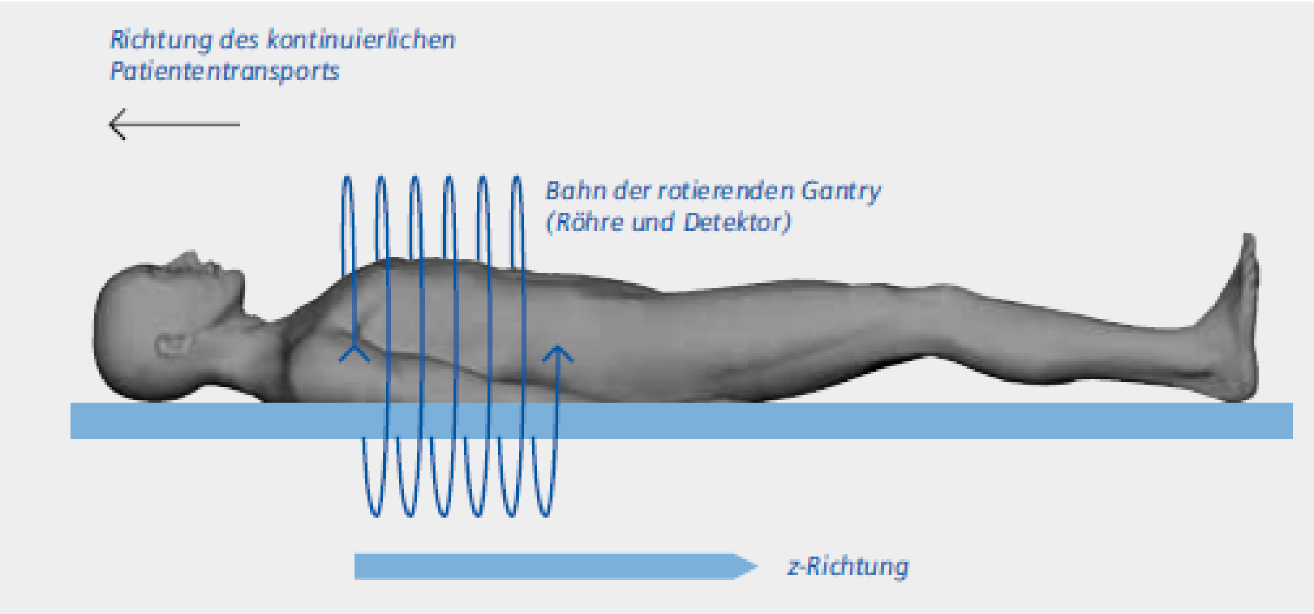


Quelle: Siemens

Functional principle

Technical setup

- Rotating source, rotating detector
- In addition: feed in z-direction, spiral recording

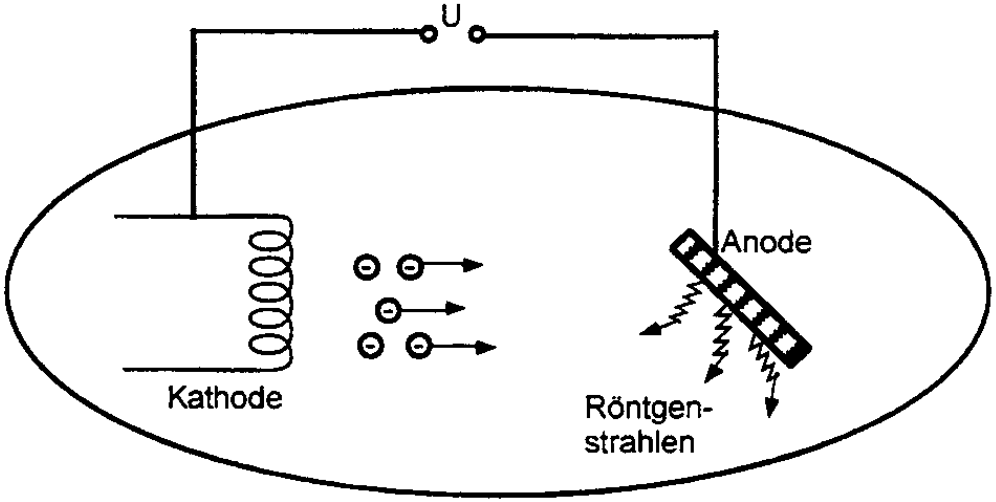


Quelle: Siemens

X-ray - Repetition

X-ray

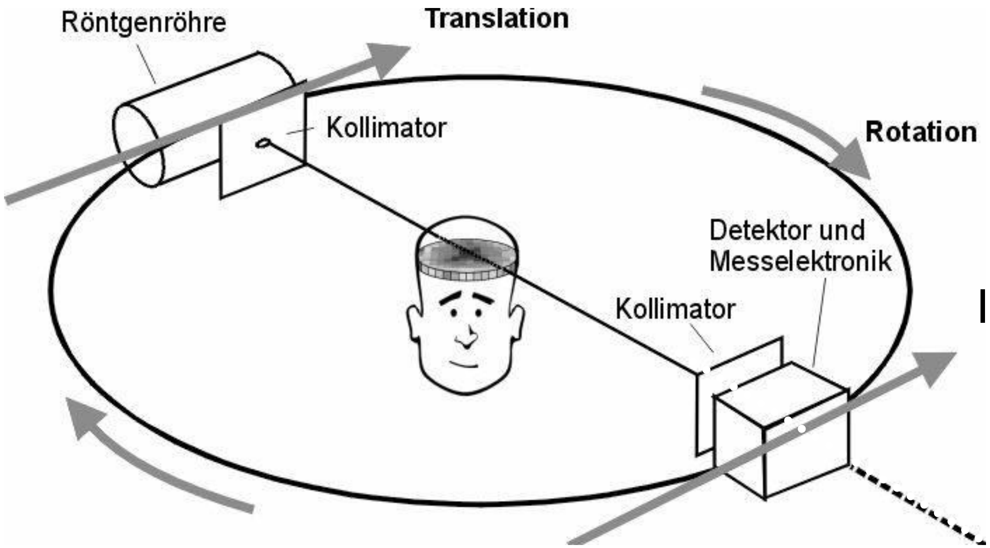
- Transmission of an X-ray beam with intensity N_0
- The intensity N of the beam that crossed the tissue is measured
- The attenuation of the beam is characteristic of the traversed tissue
- The different attenuation is used for imaging



$$N = N_0 e^{-\mu l}$$

X-ray -> CT: What and how it is measured?

CT: Calculation of the attenuation coefficient μ(x,y) for every voxel P(x,y)



Quelle: W. Kalender

Intensity profile

Attenuation profile
(Projection)

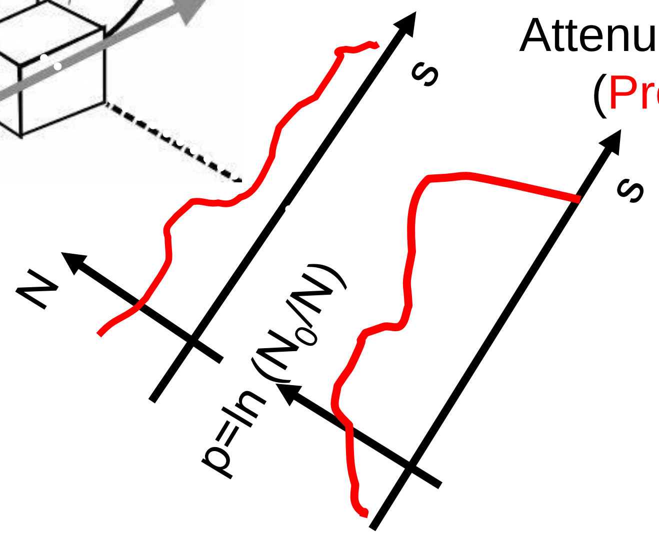


Image reconstruction (Image of the attenuation coefficients)

X-ray → CT: What is measured?

CT: Calculation of the attenuation coefficient μ(x,y) for every voxel P(x,y) with width l in one slice

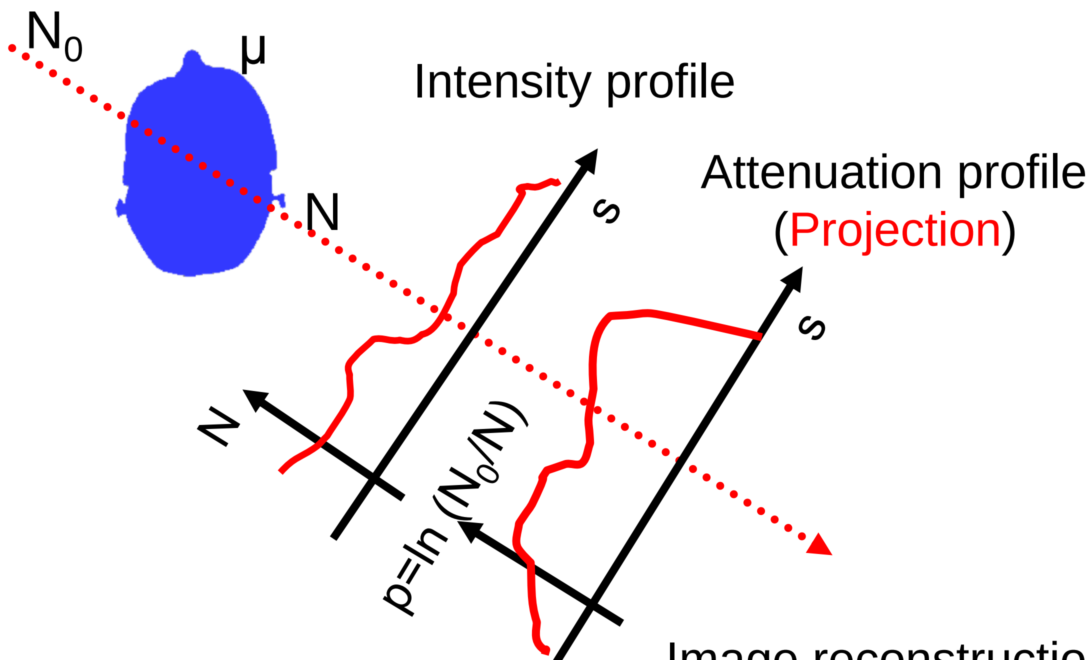


Image reconstruction (Image of the attenuation coefficients)

Intensity equation

$$N = N_0 \mathrm{e}^{-\int \mu \, \mathrm{dl}}$$

Attenuation equation

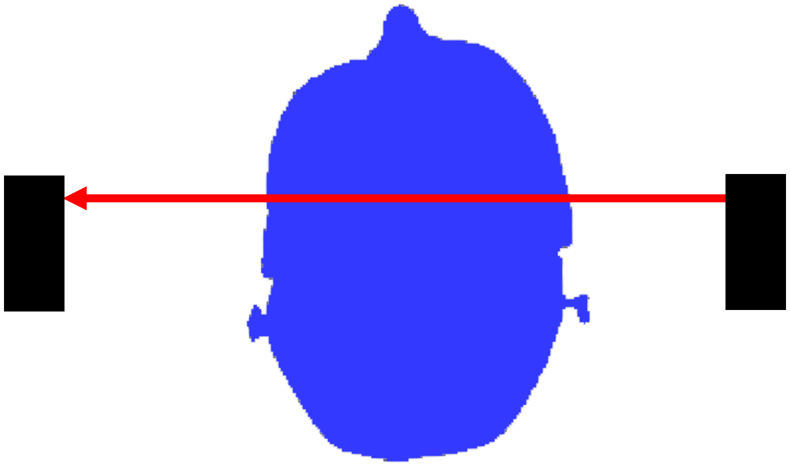
$$\ln \frac{N_0}{N} = \int \mu dl$$

$$\mu(x, y) = ???$$

Slice recording

For every 2D slice recordings from several perspectives

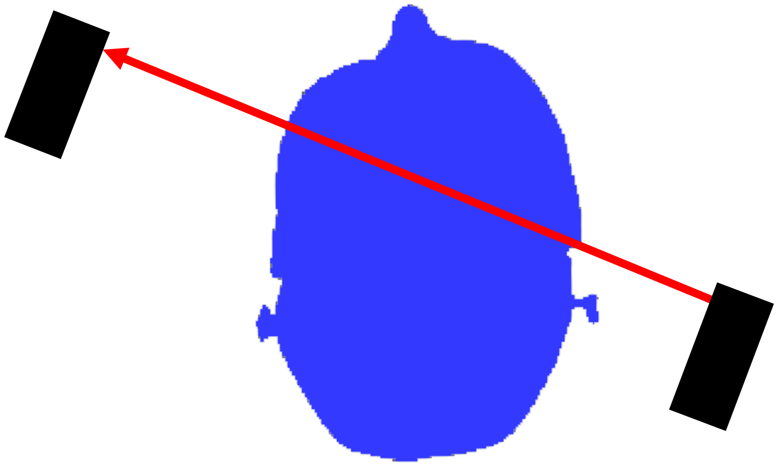
Example: Pencil beam CT (1. Generation)



Slice recording

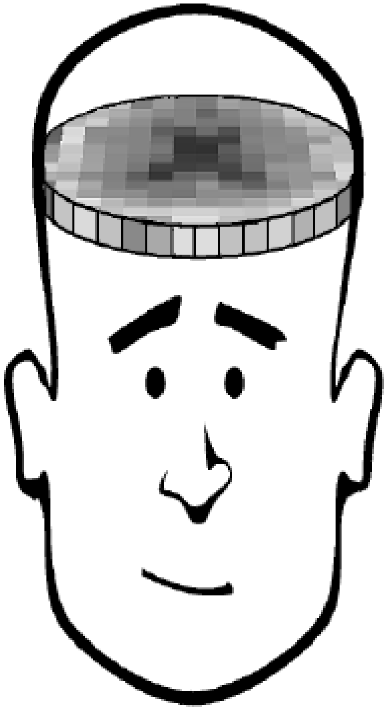
For every 2D slice recordings from several perspectives

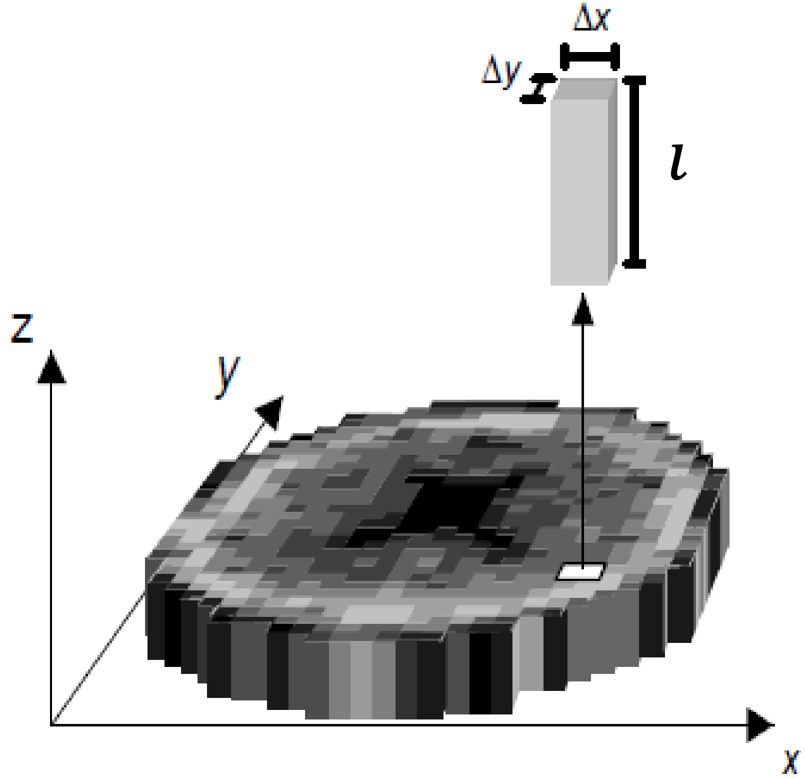
Example: Pencil beam CT (1. Generation)



What is visualized in the CT image?

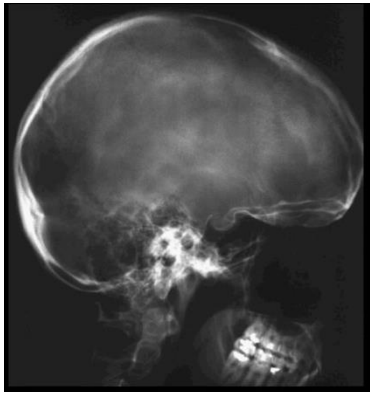
Linear attenuation coefficient averaged for each volume element





Quelle: W. Kalender

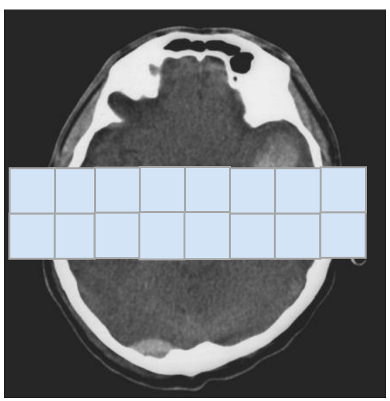
Why do sectional images have higher contrast?



X-ray Image

Skull Imaging is insufficient (X-ray)

$$Kontrast(I_1, I_2) = \frac{I_1 - I_2}{I_1 + I_2}$$



CT Image

700	40	70	50	40	50	670
680	30	20	50	30	60	740

1620
1600

In the x-ray image bone structures are dominant which results in lower contrast

Interaction und Feedback

- https://pingo.coactum.de/ -> 5766

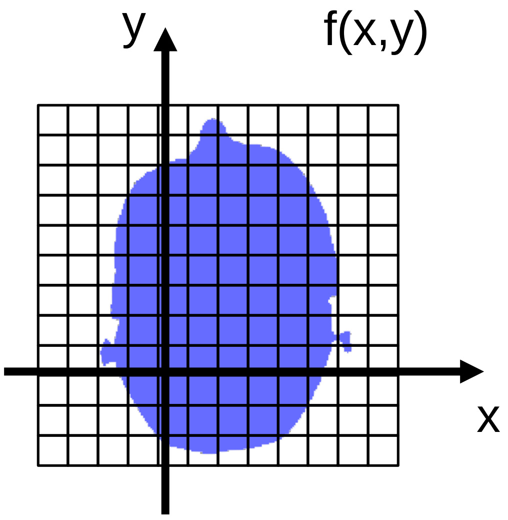
Reconstruction CT Images

How can the CT slices be reconstructed ?

A slice is defined as a matrix *f(x,y)* = *μ(x,y)*

Three possible methods:

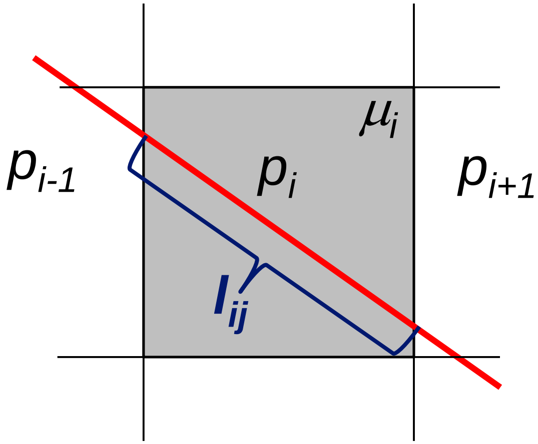
- Iterative reconstruction
- Fourier reconstruction
- Filtered backprojektion



Iterative Reconstruction

General reconstruction equations

In every Voxel p_i, through which a beam j with length l_{ij} passes, the radiation intensity is weakend by the attenuation coefficient μ_i, weighted by l_{ij}. This type of attenuation applies to all rays. You get a series of equations describing the whole system:



$$P_j = \sum_{i=0}^{N-1} l_{ij} \cdot \mu_i$$

Mit P_j – Attenuation along of the j-th projection beam,

μ_i – *Attenuation coefficient* for the i-th image element,

l_{ij} – Lenght of the j-th projection beam inside the i-th voxel

N – Number of voxel

Image reconstruction – direct methods

Modern CT-Scans:

- Number of projections for every slice: 800-1500
- Number of measurement for every projection: 600-1200

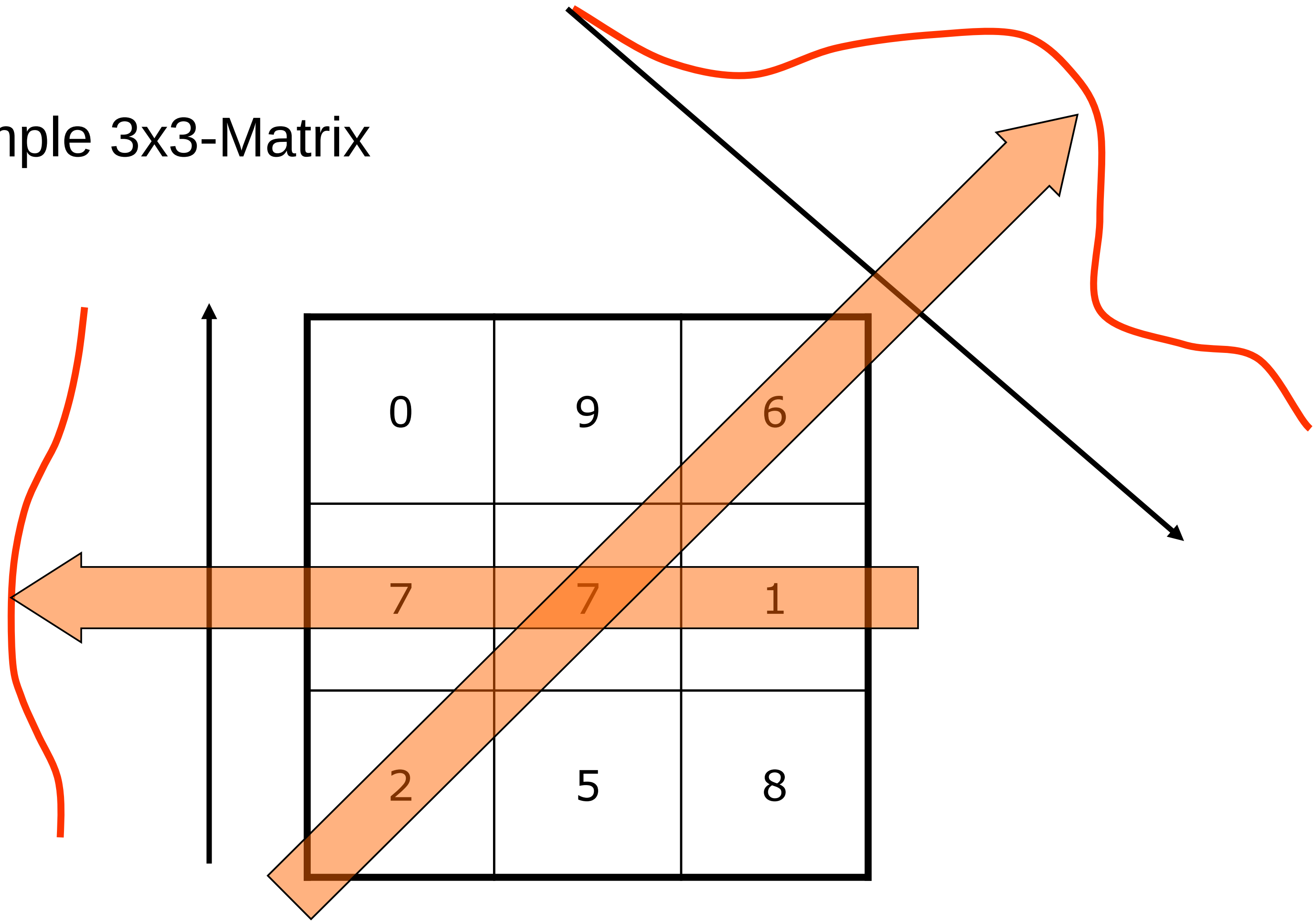
→ 1500*1200 = 1.800.000 measurement for every slice

512x512-Bildern: 262.144 unknown

→ Iterative Reconstruction

Iterative Reconstruction

Example 3x3-Matrix



Iterative Reconstruction

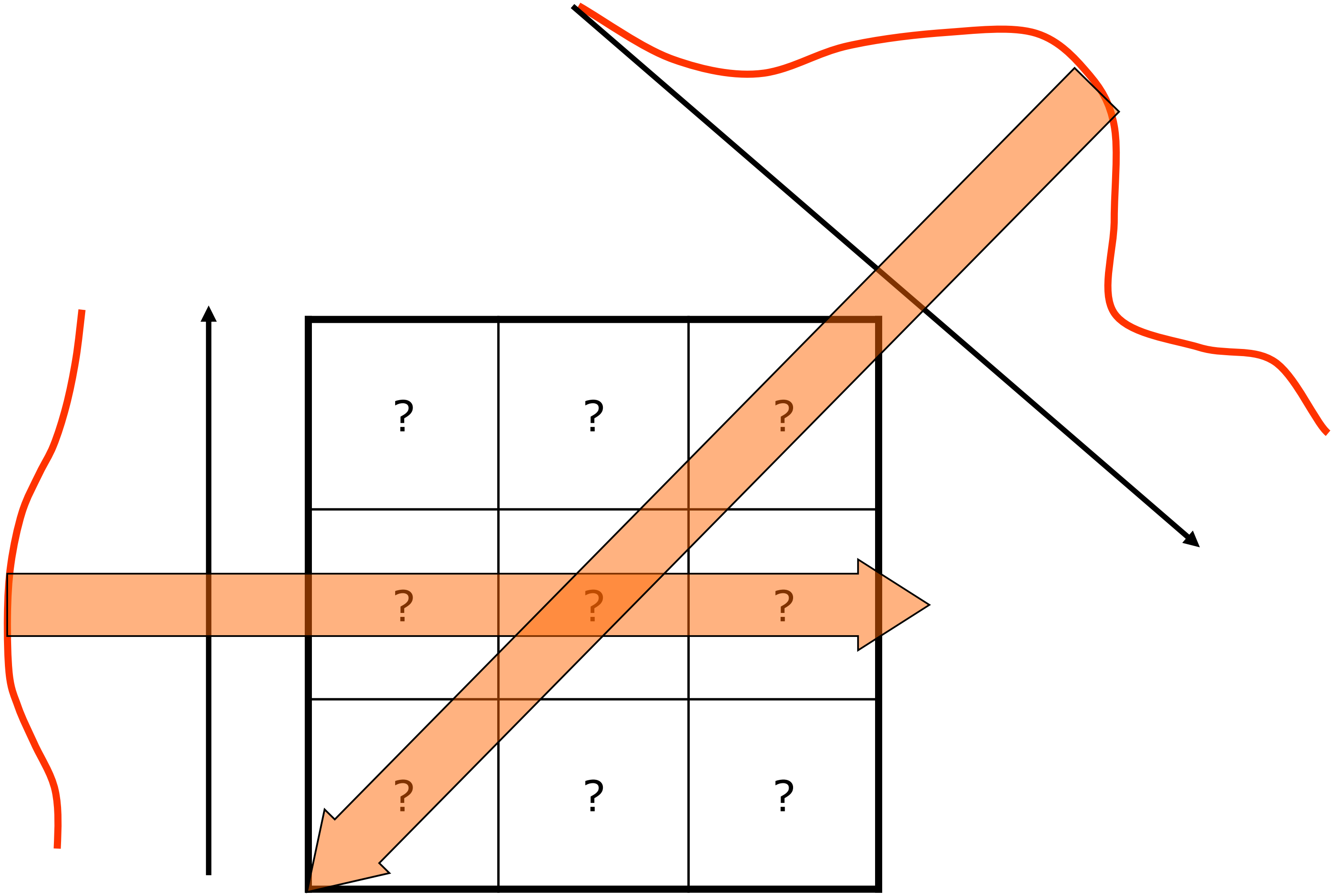


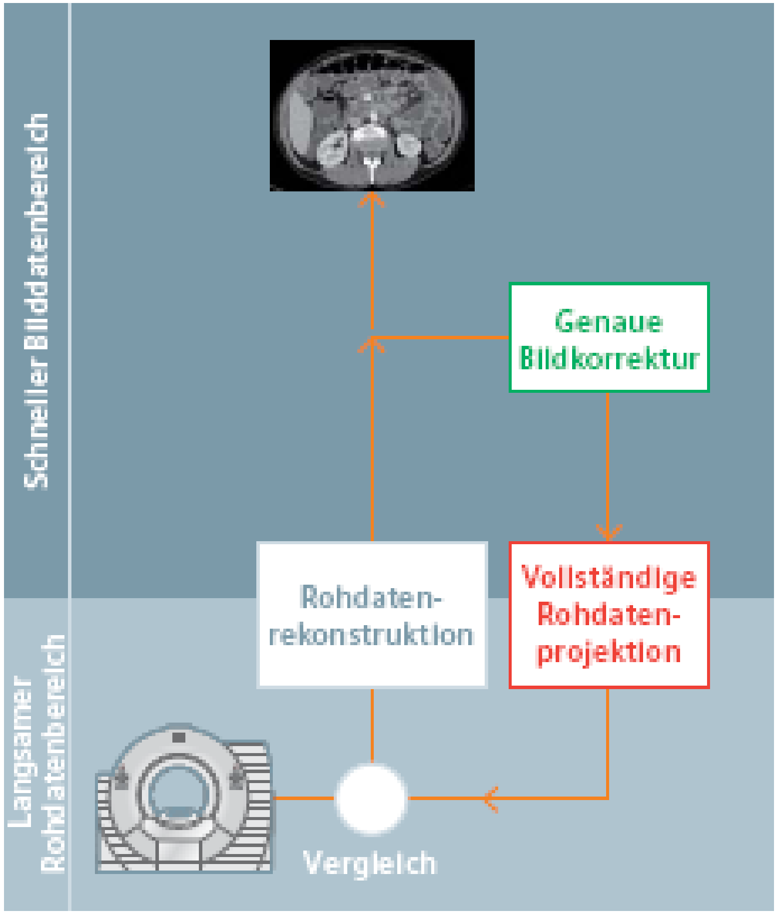
Image reconstruction - iterative

How can we calculate an image from the measurements?

Approximation of the image:

1. Estimate
2. Correction
3. Iteration

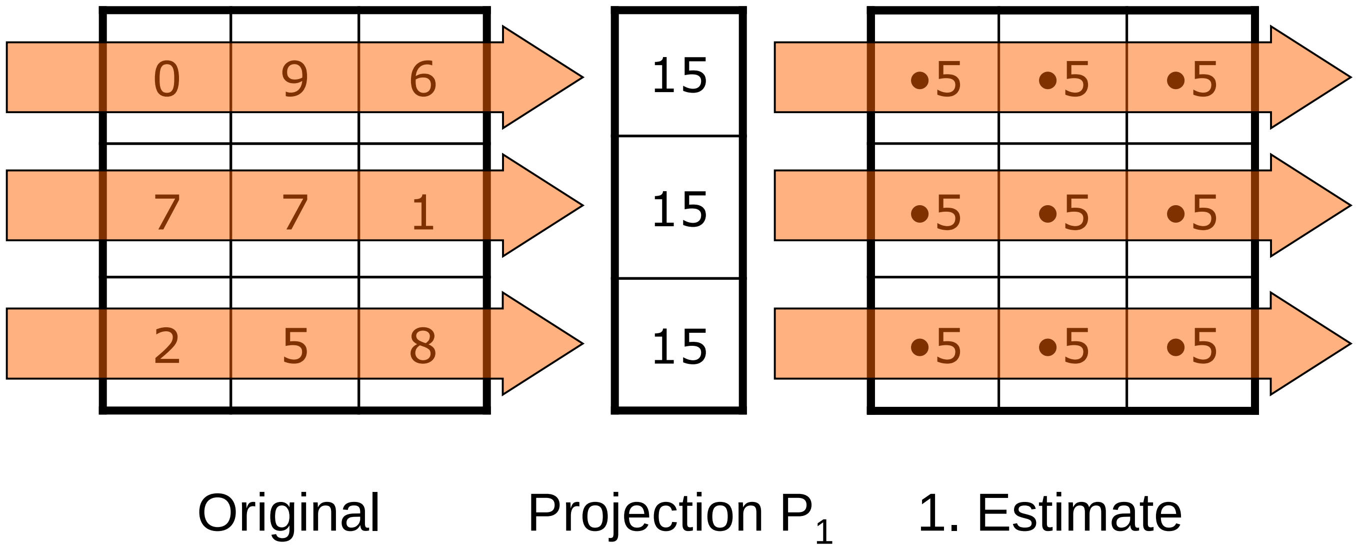
- First Approximation is derived from direction 1
- Creation of correction profiles
- Stop criterion: by specifying an error measure or maximum number of iterations



Quelle: Siemens

Estimate

1. Projection direction: 1. Estimate of the matrix



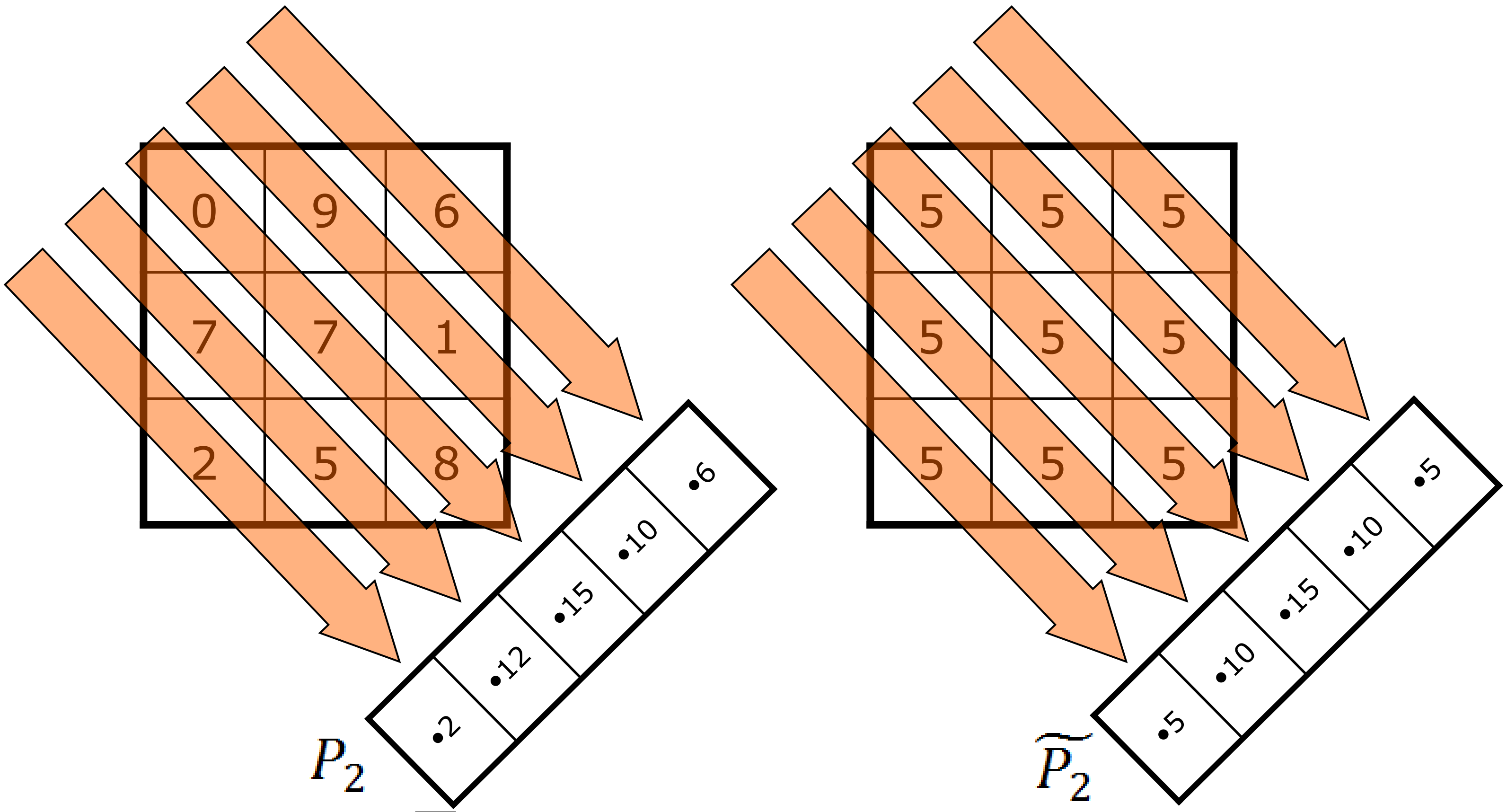
Correction

- In the second step, an attempt is made to correct the estimate.
- The 1st estimate is corrected with the 2nd projection P_2
- This is the difference

$$\tilde{P}_2 - P_2$$

  between the projection of the estimated values and the actual measured projection
- This value is then distributed to the voxels. The distribution depends on the lengths at which the projection beam cuts the individual voxels.
- Simplification: Length share is always set to 1

Iterative Reconstruction

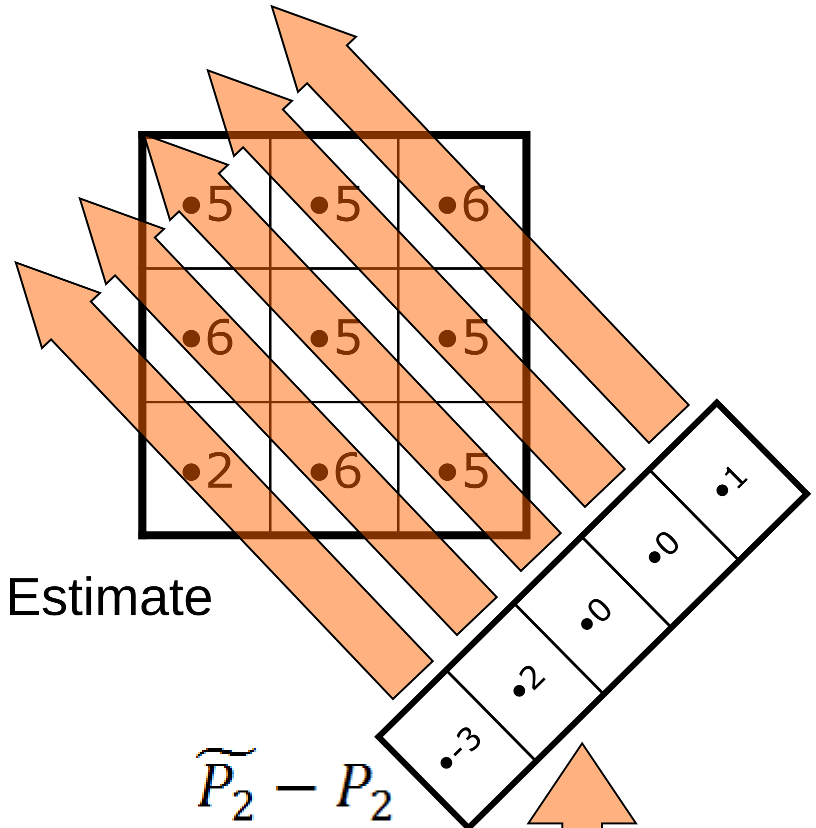


2	12	15	10	6
5	10	15	10	5
-3	2	0	0	1

NCT

Iterative Reconstruction

0	9	6
7	7	1
2	5	8



2. Estimate

Lengths normalized to 1

$$\widetilde{P_2} - P_2$$

2	12	15	10	6
5	10	15	10	5
-3	2	0	0	1

Iteration

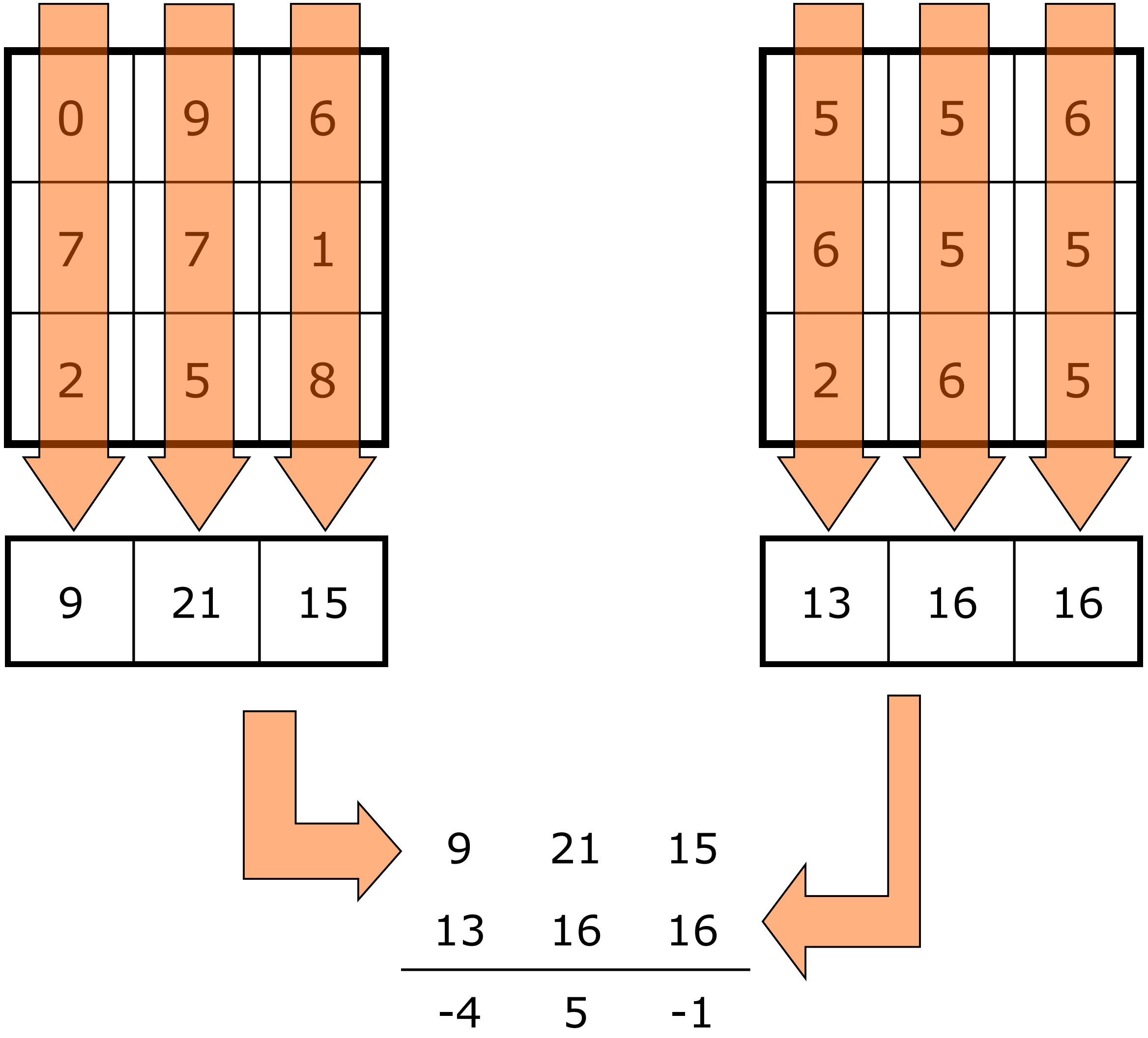
The estimation and correction of all projections is repeated until the norm between measured and estimated projection vector

$$\left\|\tilde{P}_n - P_n\right\| < \varepsilon$$

falls below a certain minimum after n iterations or a predetermined maximum number of iterations is exceeded.

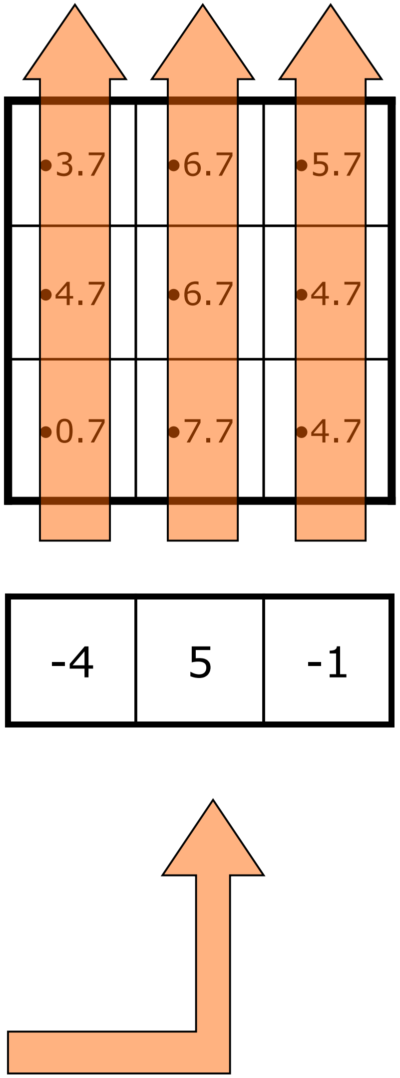
The image is traversed multiple times if necessary.

Iterative reconstruction



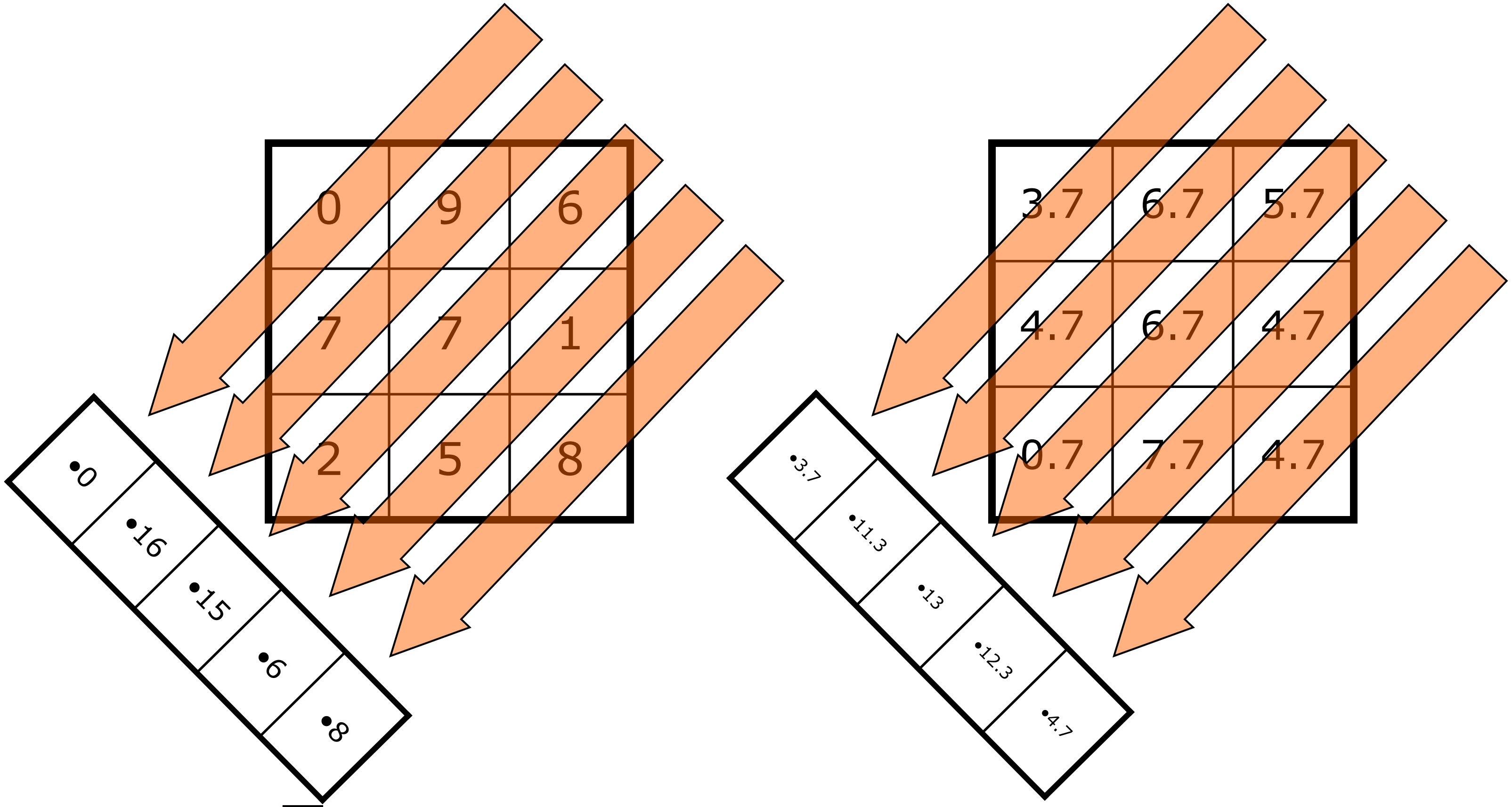
Iterative reconstruction

0	9	6
7	7	1
2	5	8



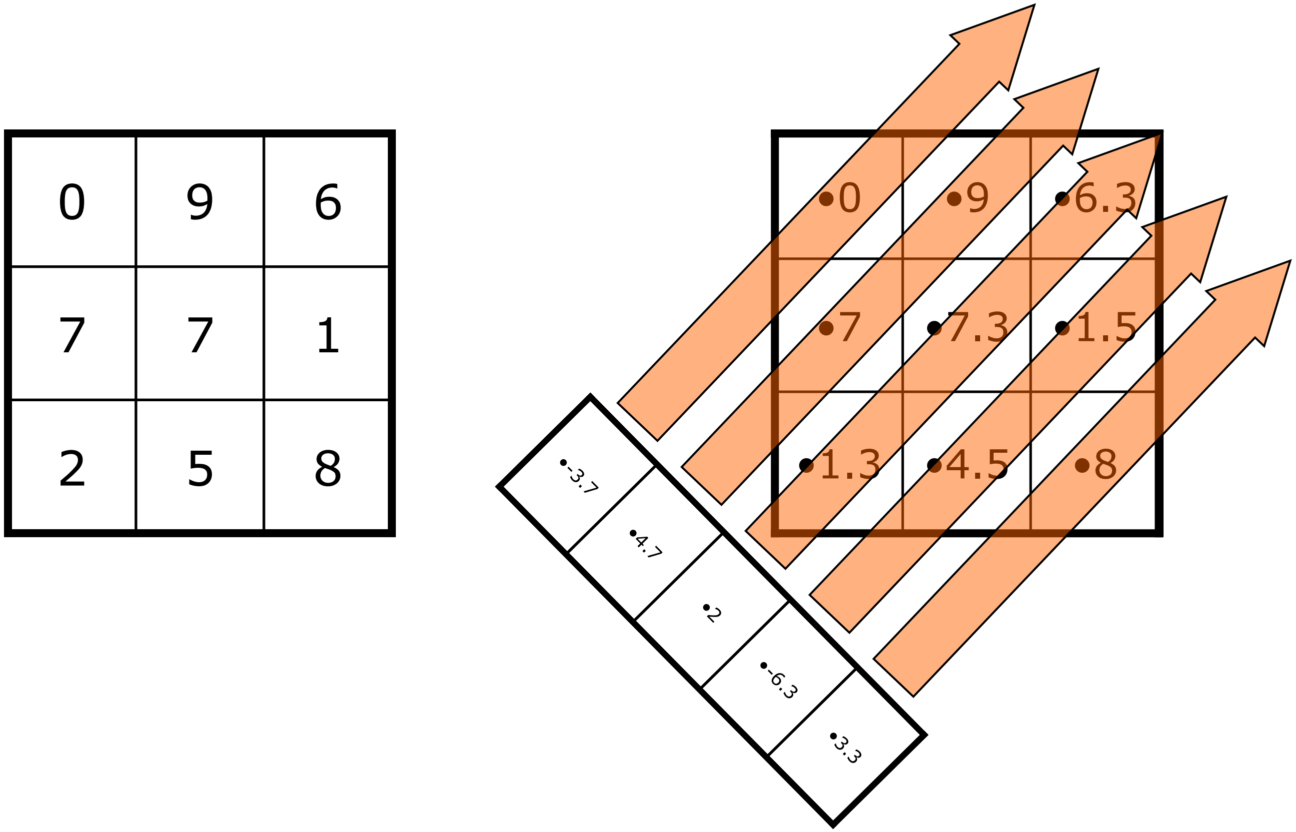
9	21	15
13	16	16
-4	5	-1

Iterative reconstruction



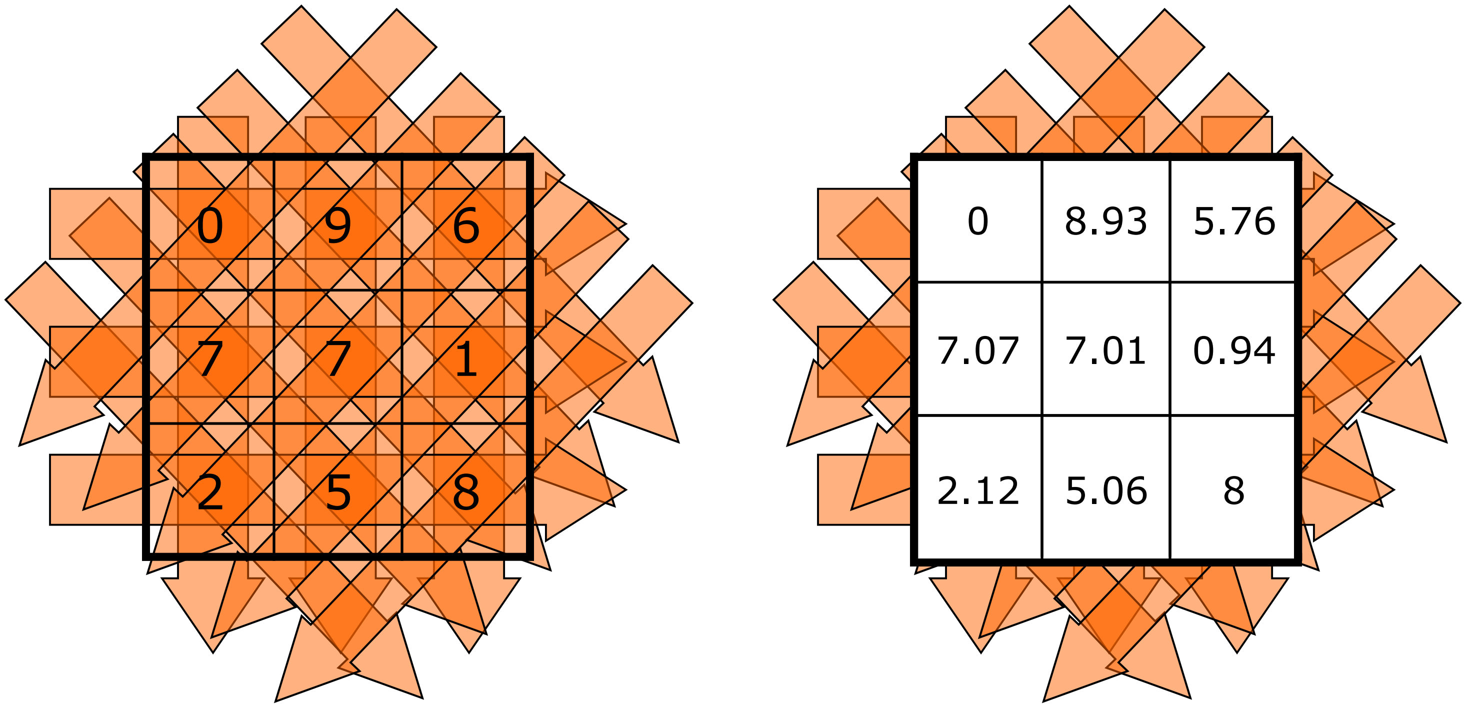
0	16	15	6	8
3.7	11.3	13	12.3	4.7
-3.7	4.7	2	-6.3	3.3

Iterative reconstruction

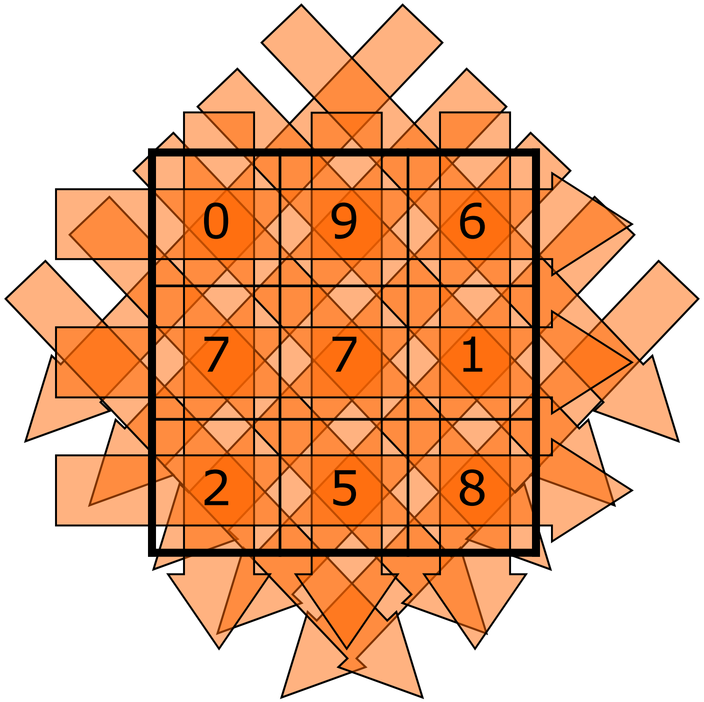


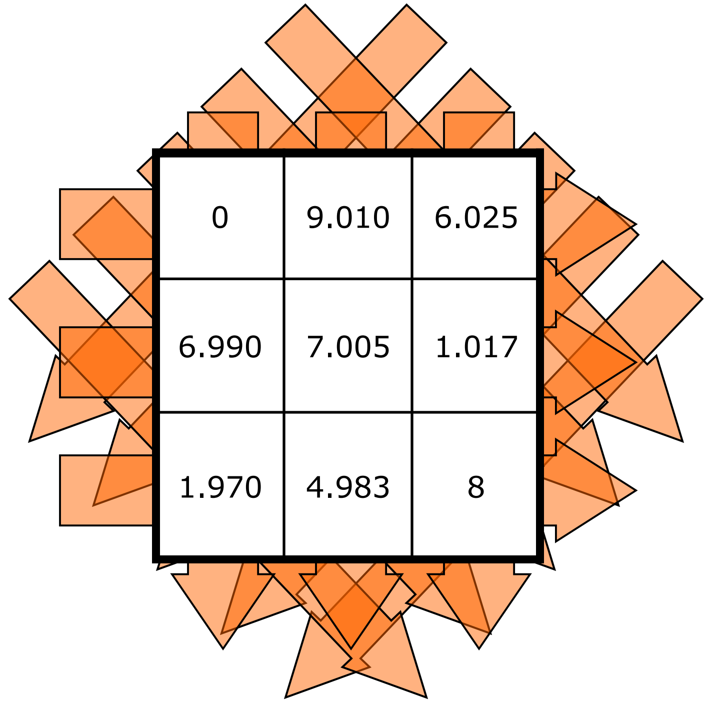
0	16	15	6	8
3.7	11.3	13	12.3	4.7
-3.7	4.7	2	-6.3	3.3

Result of first iteration



Result after terminating





Evaluation iterative reconstruction

- Method converges always
- Regularization through the use of prior knowledge: modeling of local noise
  → Decoupling the spatial resolution from image noise
  → Smoothing in low-contrast areas, keeping contrast borders
  → Very good signal-to-noise ration
- Exact mathematical description of the device needed
- Computationally expensive because of many iterations
  →Iterative Reconstruction in Image Space (IRIS):

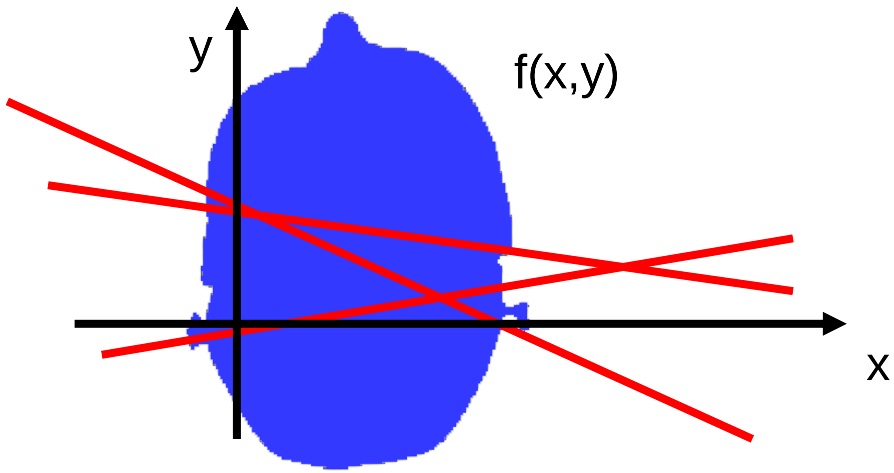
Reduction of the effort of iterating steps while maintaining image quality

Fourier Reconstruction

Radon Transformation

Mathematical Basis: Radon-Transformation

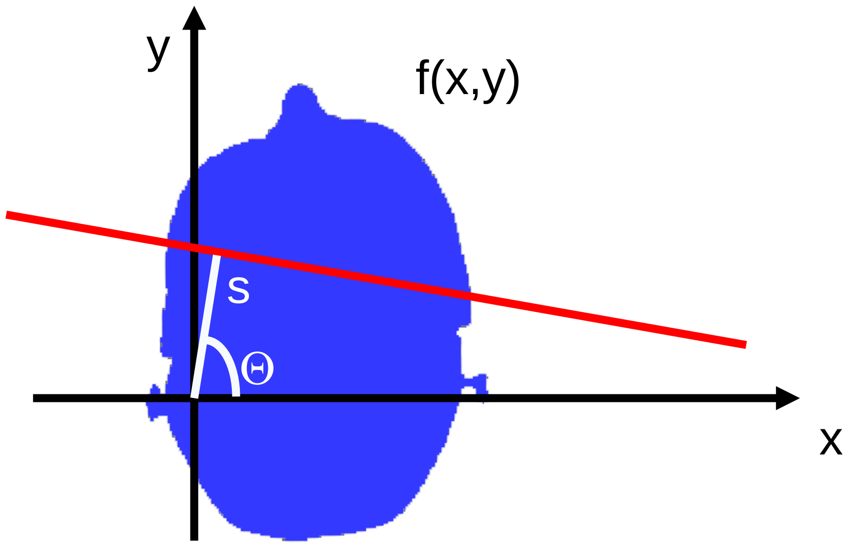
- Any integrable function f (x, y) is described by ALL line integrals over the domain of f
- The 2D distribution of an object property can be described exactly if there is an infinite number of line integrals



Radon

Radon Transformation

- Simplification by ordering scheme, so that all line integrals occur only once
- Angle Θ : 0..180°
- $s_{min} \leq s \leq s_{max}$

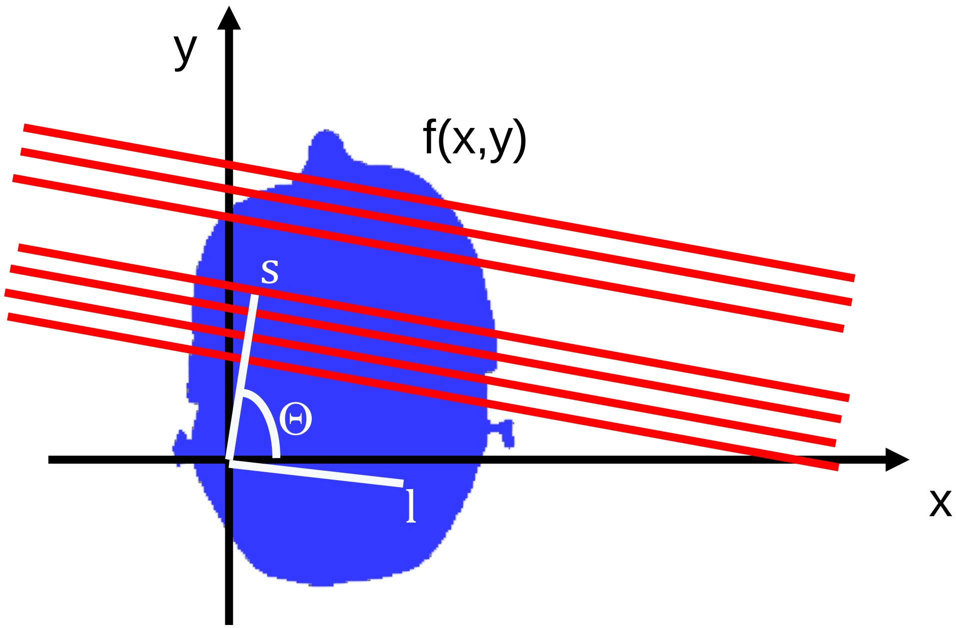


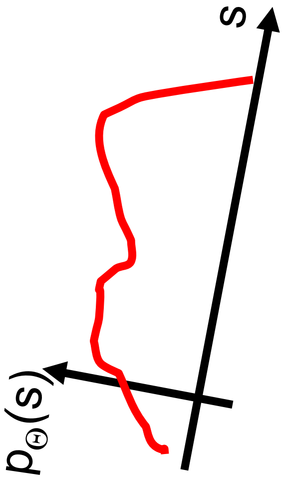
Radon Transformation

For a specific angle Θ all line integrals are calcualted

The result is can be presented as $p_\Theta(s)$ as a function of s (Projection of Θ)

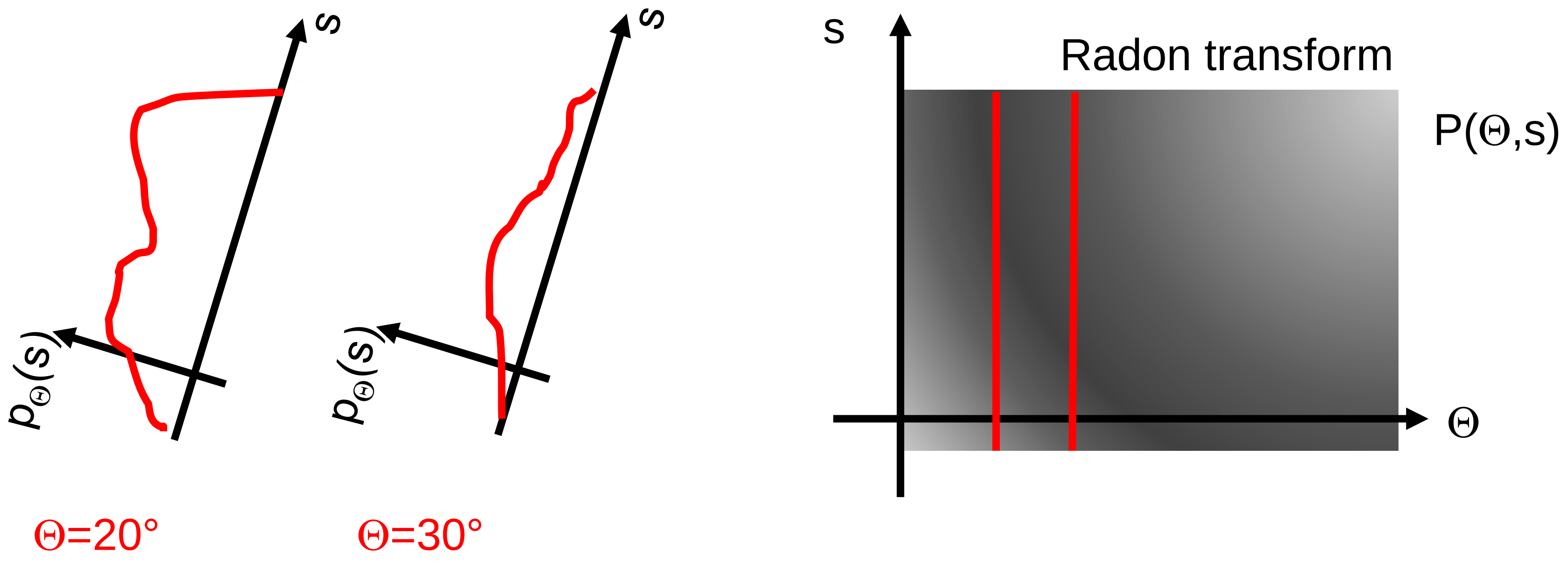
$$p_\Theta(s) = \int_{-\infty}^{\infty} f(x,y)\,dl$$





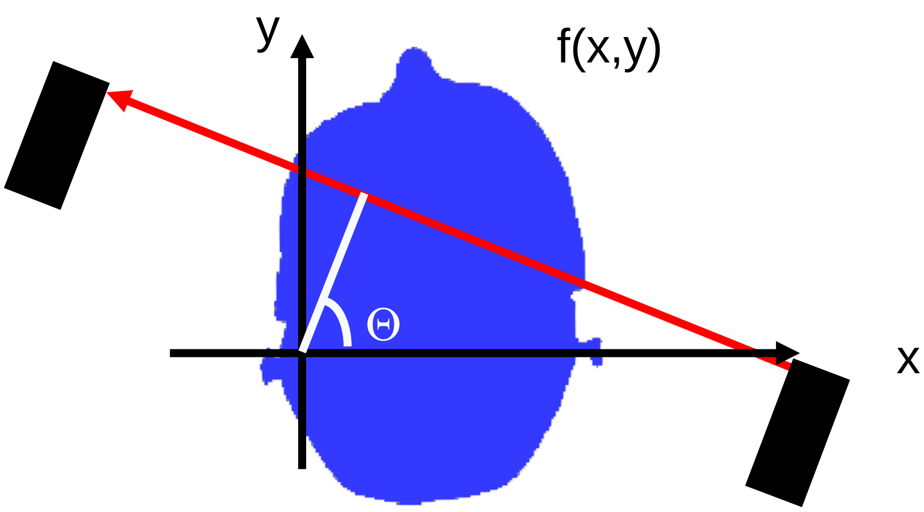
Radon Transformation

Based on the functions of several angles Θ: 0..180° the 2D radon transformation of the original function f(x,y) can be calculated



Radon Transform - CT

What is the relationship between the measured data of a CT and the radon transformation?

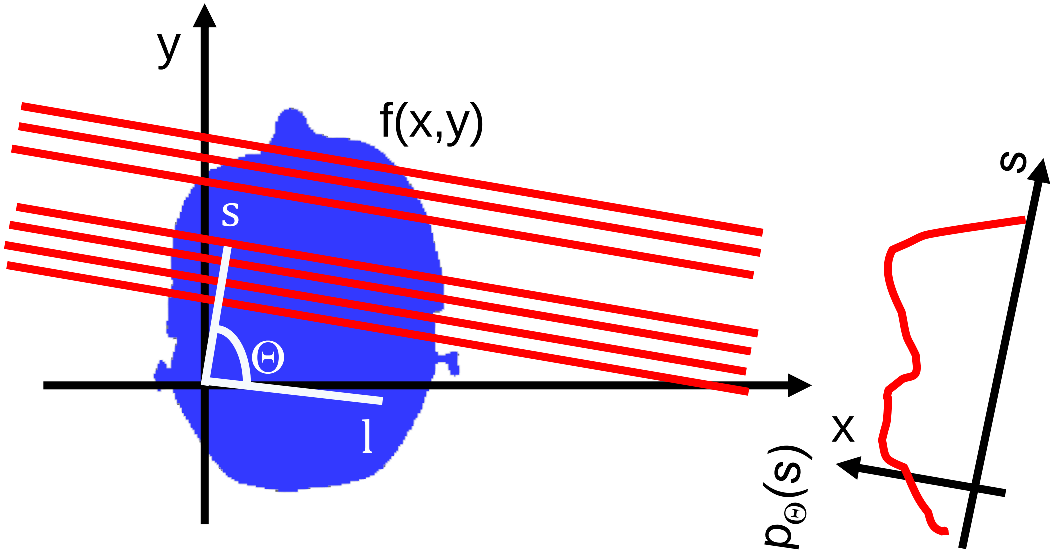


- Transmitter and detector rotate 180° around the patient
- Projections of the X-ray attenuation are measured

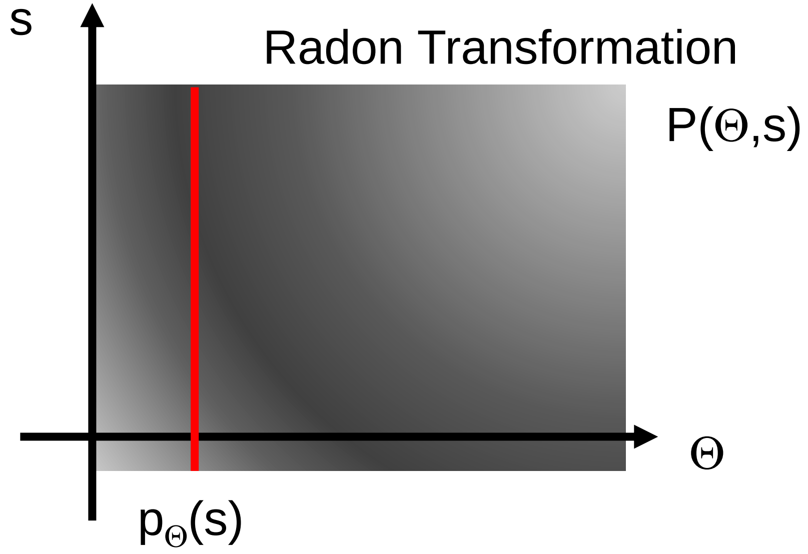
$$p_{\Theta}(s) = \ln \frac{N_0}{N(\Theta, s)}$$

This corresponds to a description of the slice image of a patient f (x, y) by line integrals with a specific ordering scheme

Radon Transformation - CT

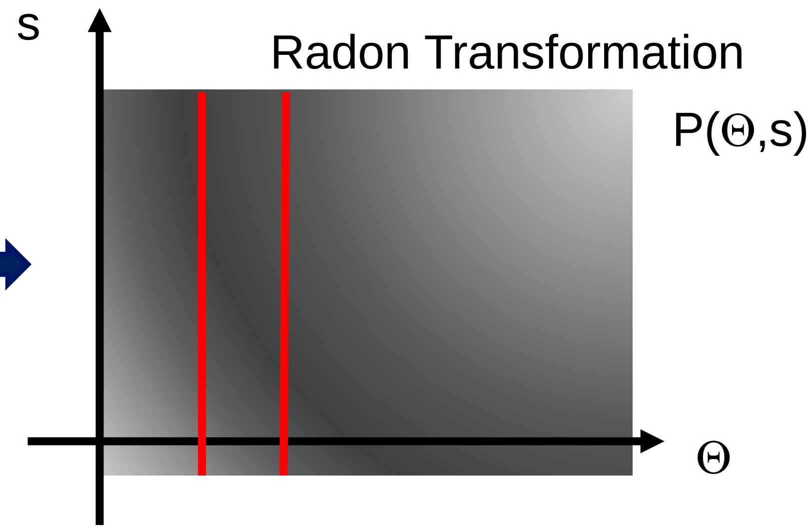


A line in the radon transformation with Θ = *const* is called a projection $p_{\Theta}(s)$



Radon Transformation - CT

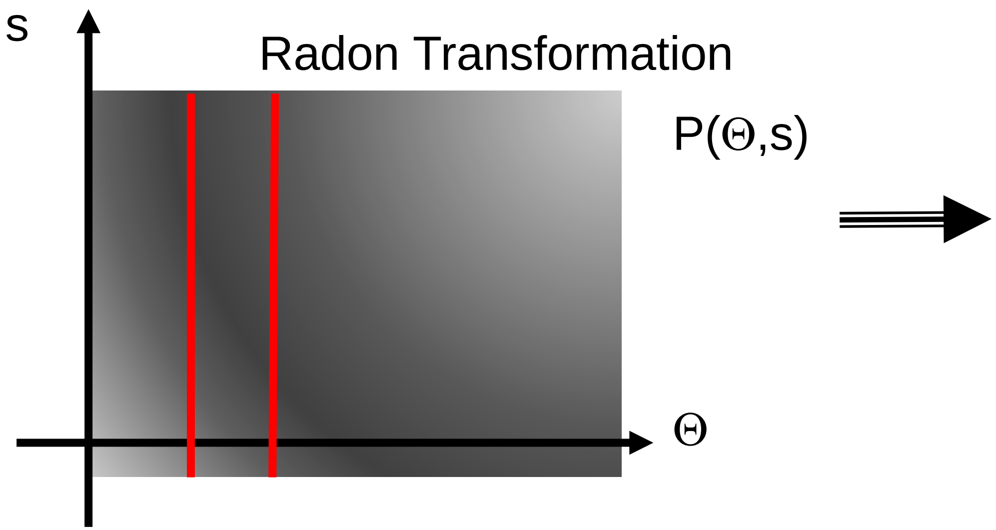
What is the relationship between the measured data of a CT and the radon transformation?



The CT calculates the radon transformation

Reconstruction

- How can the function f(x,y) be determined from the measured data (radon transformation)?



Slice Image f(x,y)?

Idea: use relationship between Radon- und Fourier-Transformation

Inverse Radon Transformation

Fourier-Slice-Theorem

- Given: *f(x,y)* and the 2D Fourier transformation *F(u,v)*

$$f(x,y) \circ\!\xrightarrow{\text{2D-FT}}\!\bullet \; \mathrm{F}(\mathrm{u},\mathrm{v}) = \iint_{-\infty}^{\infty} \mathrm{f}(\mathrm{x},\mathrm{y})\, \mathrm{e}^{-\mathrm{j}2\pi(\mathrm{ux}+\mathrm{vy})}\, \mathrm{dxdy}$$

- Let $p_\Theta(s)$ be the projection from f(x,y) with a specific angle Θ and $P_\Theta(w)$ the 1D Fourier transformation

$$p_\Theta(s) \circ\!\xrightarrow{\text{1D-FT}}\!\bullet \; P_\Theta(w) = \int_{-\infty}^{\infty} p_\Theta(s)\, \mathrm{e}^{-\mathrm{j}2\pi\mathrm{ws}}\, \mathrm{ds}$$

- Then $P_\Theta(w)$ describes the function F(u,v) along the Θ through the origin, where:

$$F(w,\Theta) = P_\Theta(w)$$

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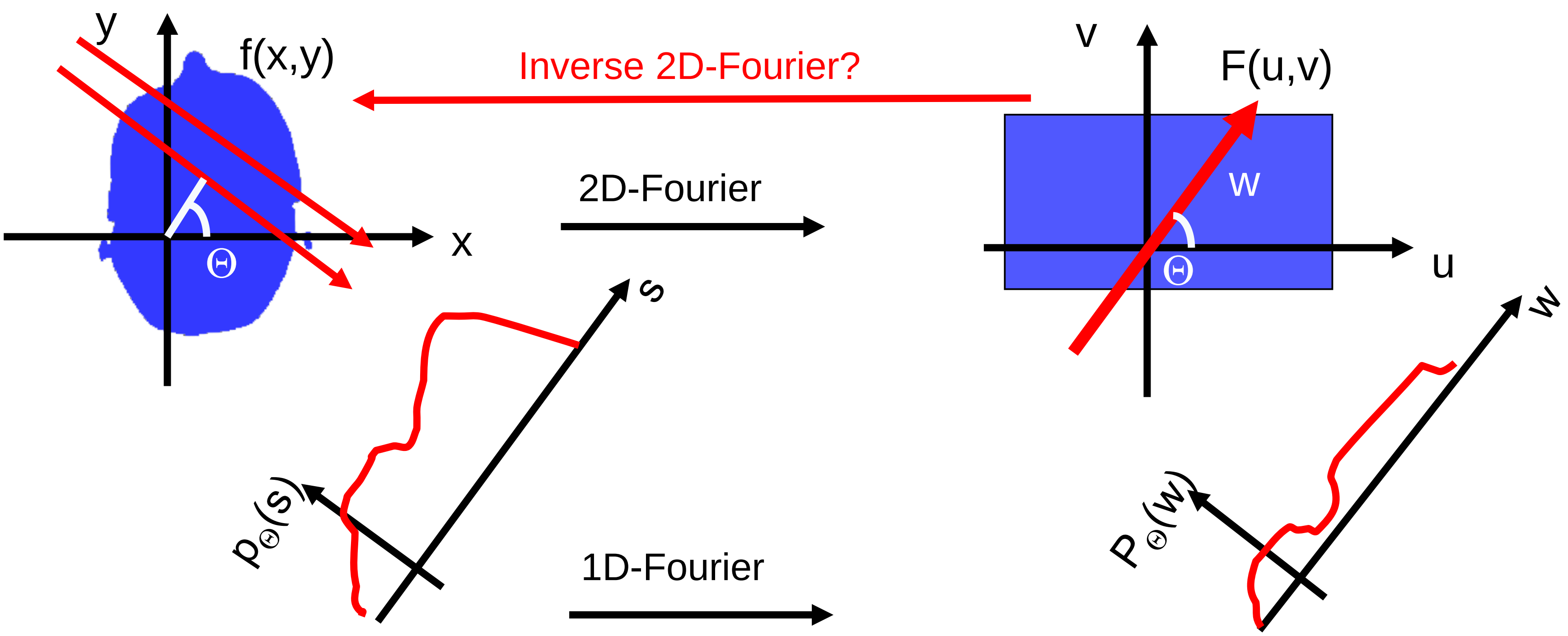
Fourier Slice Theorem

Fourier Slice Theorem (general angle Θ)

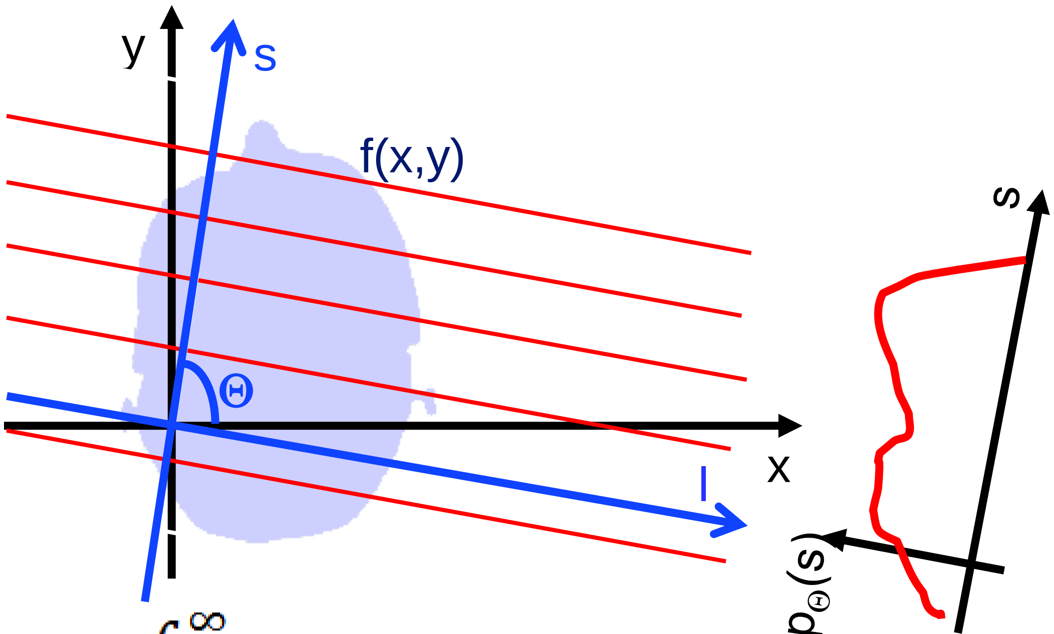


Coordinate systems of the projection

Goal: geometric relationship between projection $p_\Theta(s)$ and slice f(x,y)

- p is represented in the coordinate system (s,l) with angle Θ
- Transformation from coordinate system (x,y) to (s,l):

$$\begin{pmatrix} s \\ l \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$s = \mathrm{x}\cos\Theta + \mathrm{y}\sin\Theta$$
$$l = -\mathrm{x}\sin\Theta + \mathrm{y}\cos\Theta$$

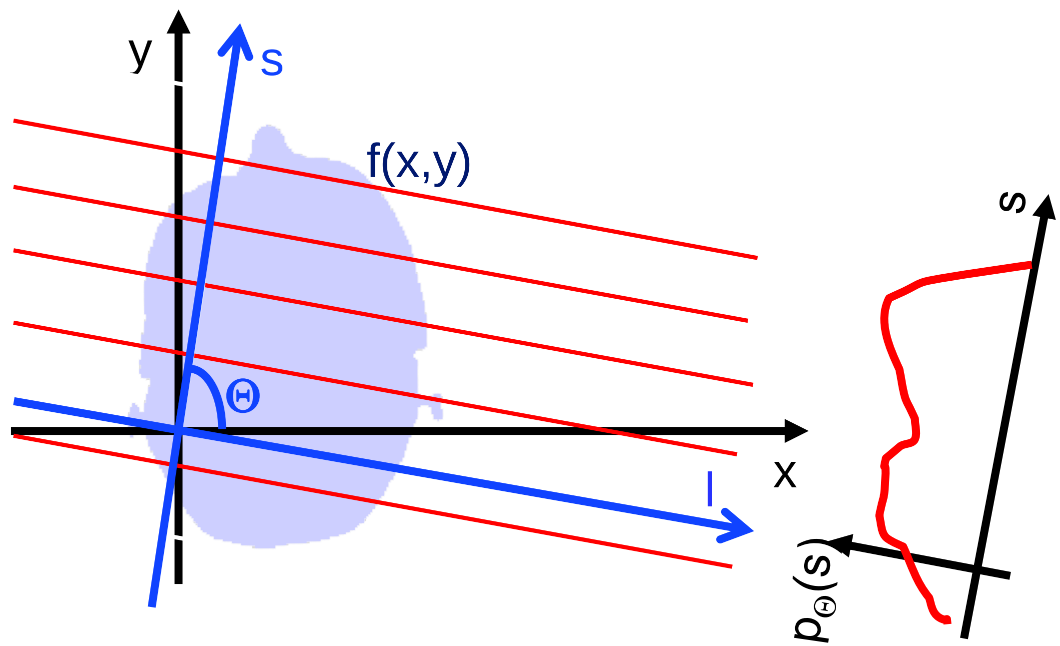
$$p_\Theta(s) = p_\Theta(\mathrm{x}\cos\Theta + \mathrm{y}\sin\Theta) = \int_{-\infty}^{\infty} f(x,y)\,\mathrm{dl}$$

Coordinate systems of the projection

Goal: geometric relationship between projection $p_\Theta(s)$ and slice f(x,y)

- p is represented in the coordinate system (s,l) with angle Θ
- Transformation from coordinate system (x,y) to (s,l):

$$\begin{pmatrix} s \\ l \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



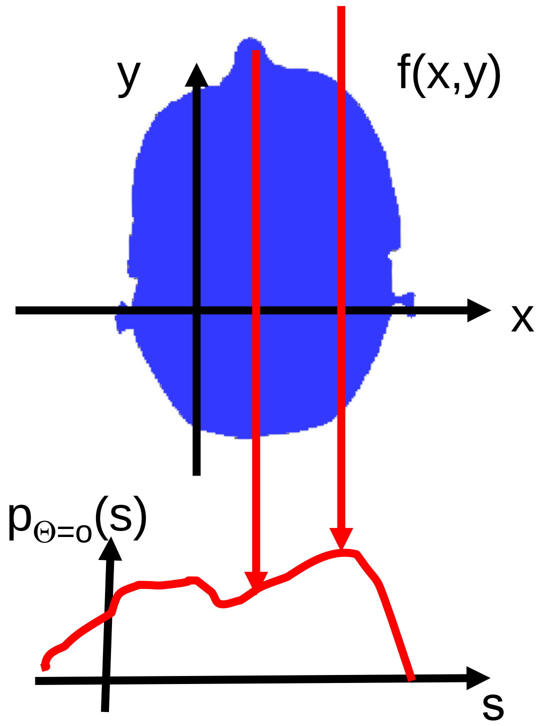
$$x = s\cos\Theta - l\sin\Theta$$

$$y = s\sin\Theta + l\cos\Theta$$

$$p_\Theta(s) = \int_{-\infty}^{\infty} f(x,y)\,dl = \int_{-\infty}^{\infty} f(s\cos\Theta - l\sin\Theta\,, s\sin\Theta + l\cos\Theta)\,dl$$

Projection equation

Fourier Slice Theorem (Proof for angle Θ = 0°)



general: $$p_\Theta(s) = \int_{-\infty}^{\infty} f(x,y)\,dl$$

here Θ = 0:

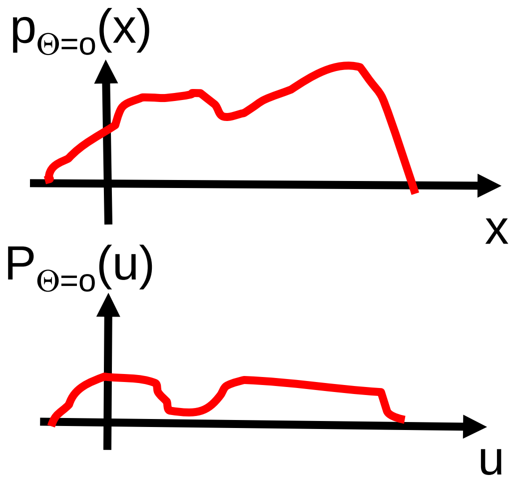
$$s = x\cos 0 + y\sin 0 = x$$

$$dl = dy$$

$$p_{\Theta=0}(s) = p_0(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy$$

Fourier Slice Theorem

Fourier Slice Theorem (angle Θ = 0°)



$$p_{\Theta=0}(s) = \boxed{p_0(x)} = \boxed{\int_{-\infty}^{+\infty} f(x,y)\,dy}$$

1D Fourier transformation

$$P_0(w) = P_0(u) = \int_{-\infty}^{+\infty} \boxed{p_0(x)} e^{-j2\pi ux}\,dx$$

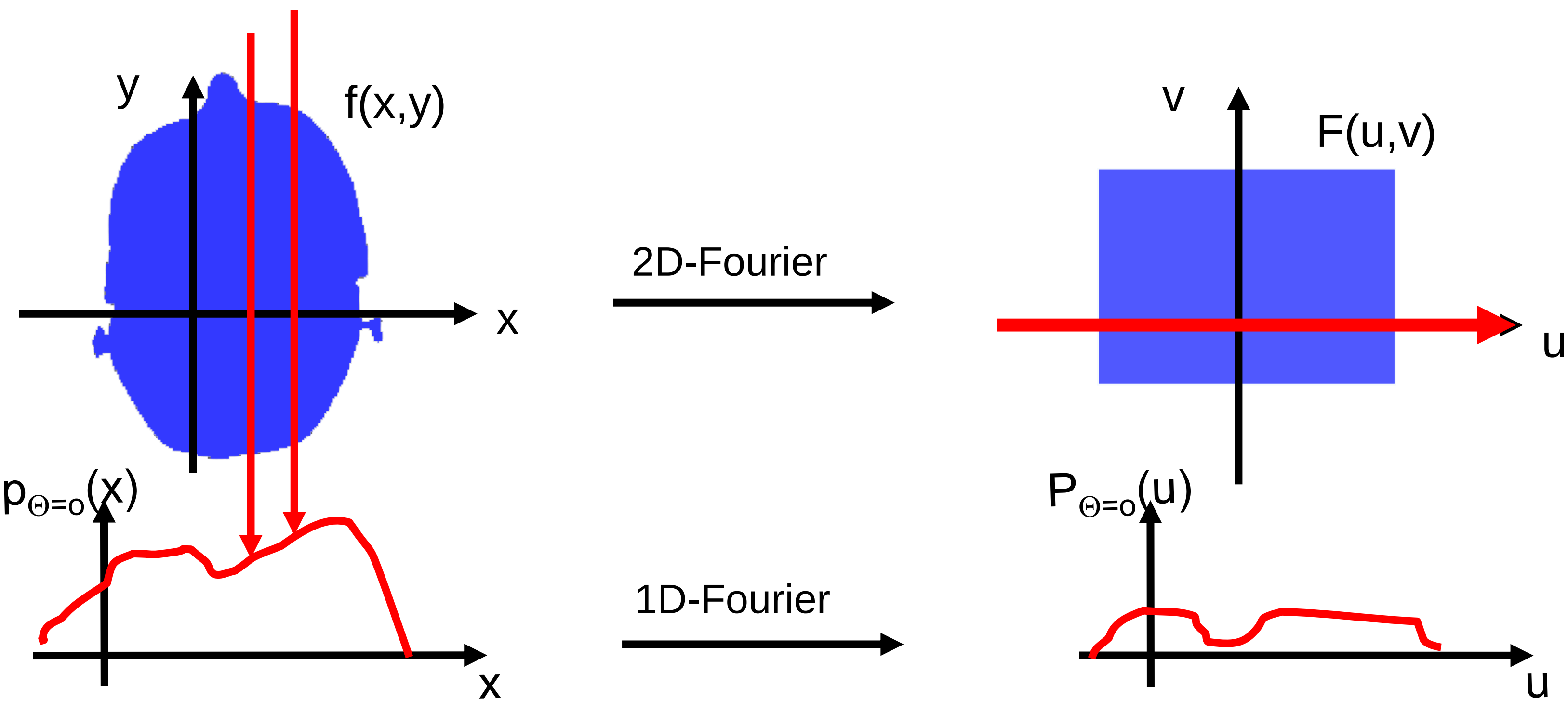
$$P_0(u) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y) e^{-j2\pi(ux+0y)}\,dx\,dy$$

$$P_0(u) = F(u,0)$$

Fourier Slice Theorem

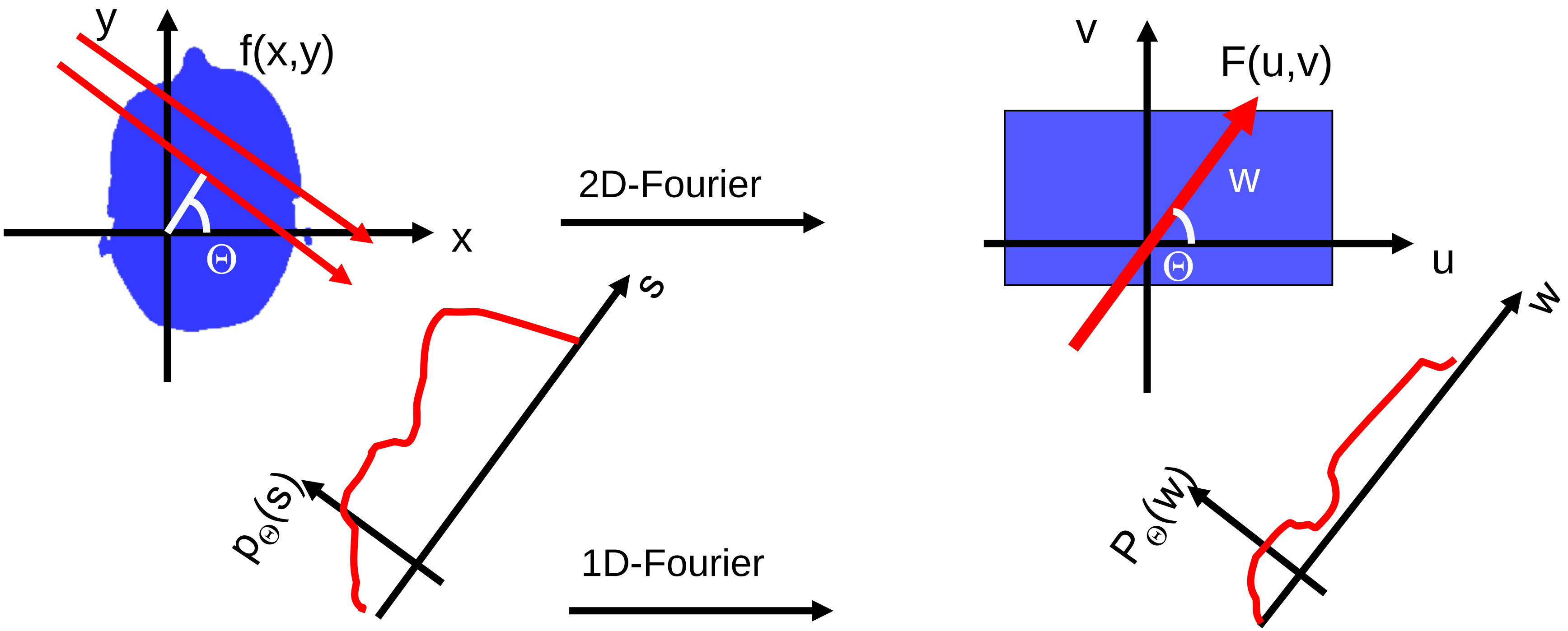
Fourier Slice Theorem (Angle Θ = 0°)

$$P_0(u) = F(u,0)$$



Fourier Slice Theorem

Fourier Slice Theorem (Angle Θ in general)



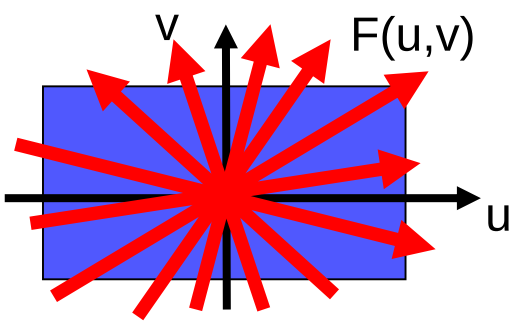
Summary Fourier Reconstruction

Reconstruktion of CT images:

- Measure projections $p_\Theta(s)$ for as many as possible Θ
- Calculate 1D Fourier transformation through: $P_\Theta(w)$
- Construct from all $P_\Theta(w)$ the matrix $F(u,v)$
- Calcualte $f(x,y)$ through inverse 2D Fourier transformation

Summary Fourier Reconstruction

- Problem: in fourier space the $P_{\Theta}(w)$ lie closer to the origin than in the margins



→ low frequencies are amplified

→ Interpolation of the values lead to subsequent errors (Data is presented in polar coordinates; FFT needs cartesian coordinates)

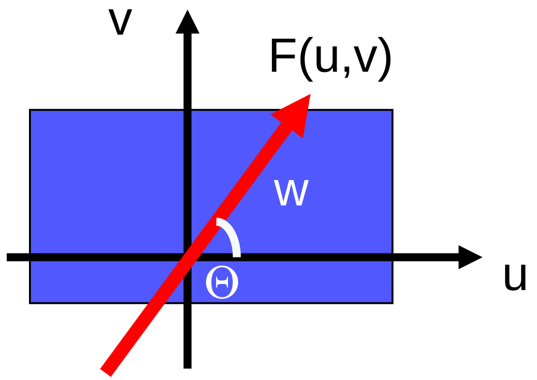
→Filtered backprojection

Filtered Backprojection

Reconstruction: Filtered Backprojection

Basic equation FFT

$$f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v) e^{j2\pi(ux+vy)} du dv$$



Derivation: How can you express (u,v) with (w,Θ) ?

→ Introduction of polar coordinates in Fourier space and coordinate transformation

$$u = w\cos\Theta \qquad v = w\sin\Theta \qquad du dv = w dw d\Theta$$

$$f(x,y) = \int_{\boxed{0}}^{\boxed{2\pi}}\int_{\boxed{0}}^{\boxed{+\infty}} F(w,\Theta) e^{j2\pi w(x\cos\Theta + y\sin\Theta)} w dw d\Theta$$

Reconstruction: Filtered Backprojection

Change of integration margins

$$f(x,y) = \boxed{\int_{o}^{\pi}\int_{-\infty}^{+\infty}} F(w,\Theta) e^{j2\pi w\left(\boxed{x\cos\Theta + y\sin\Theta}\right)} \boxed{|w|} dw\, d\Theta$$

Replace

$$\mathrm{x}\cos\Theta + \mathrm{y}\sin\Theta = \mathrm{s}$$

Fourier Slice Theorem:

$$F(w,\Theta) = P_{\Theta}(w)$$

Insert:

$$f(x,y) = \int_{0}^{\pi}\left[\int_{-\infty}^{+\infty} P_{\Theta}(w)|w| e^{j2\pi ws} dw\right] d\Theta$$

Filtered Backprojection

$$f(x,y) = \int_0^{\pi} \boxed{\left[\int_{-\infty}^{+\infty} P_{\Theta}(w) |w| e^{j2\pi ws} dw \right]} d\Theta$$

$$\tilde{p}_{\Theta}(s) = \int_{-\infty}^{\infty} P_{\Theta}(w)\, |\mathrm{w}| \mathrm{e}^{\mathrm{j}2\pi \mathrm{ws}}\, \mathrm{dw}$$

- Inverse 1D Fourier transformation of $P_{\Theta}(w)$ multiplied with |w|.
- Multiplication in fourier space corresponds to a convolution in time domain

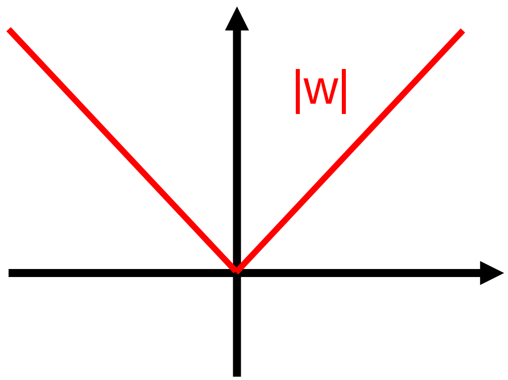
$$p_{\Theta}(s) \circ\!\!-\!\!\bullet\; P_{\Theta}(w)$$

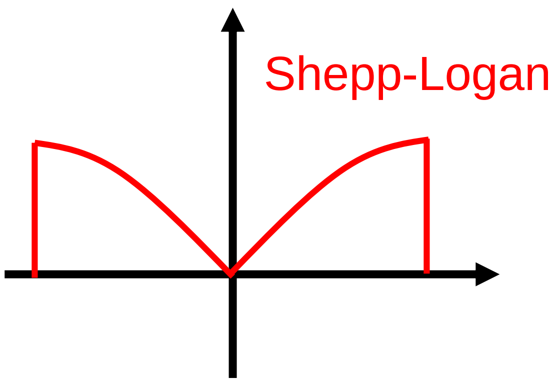
$$h(s) \circ\!\!-\!\!\bullet\; |w|$$

$$\tilde{p}_{\Theta}(s) = p_{\Theta}(s) * h(s) \circ\!\!-\!\!\bullet\; P_{\Theta}(w) \cdot |w|$$

Filtered Projection

- Filter of |w|: Amplification of high frequencies, supression of low frequencies
- In practice, this filter function is replaced (e.g. with Shepp & Logan function)
- Reason: limited sensor resolution, sampling theorem

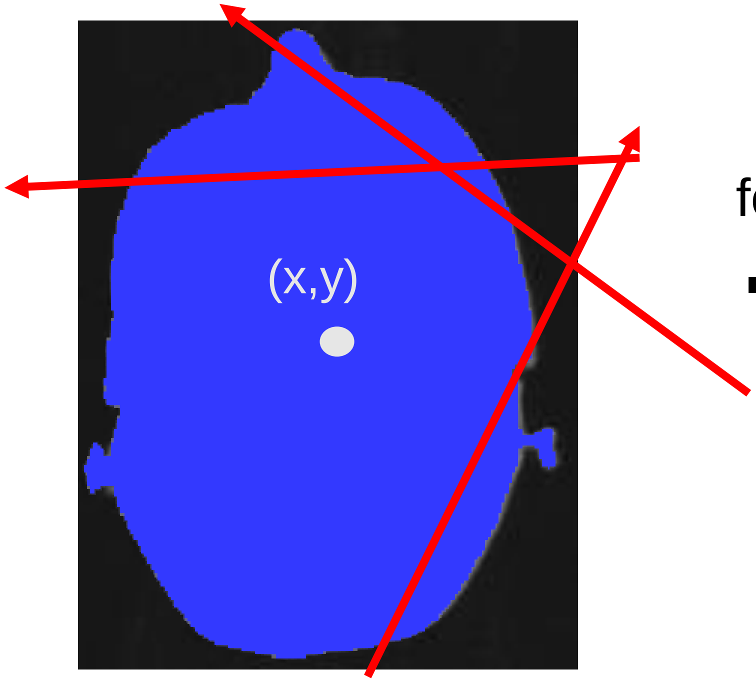




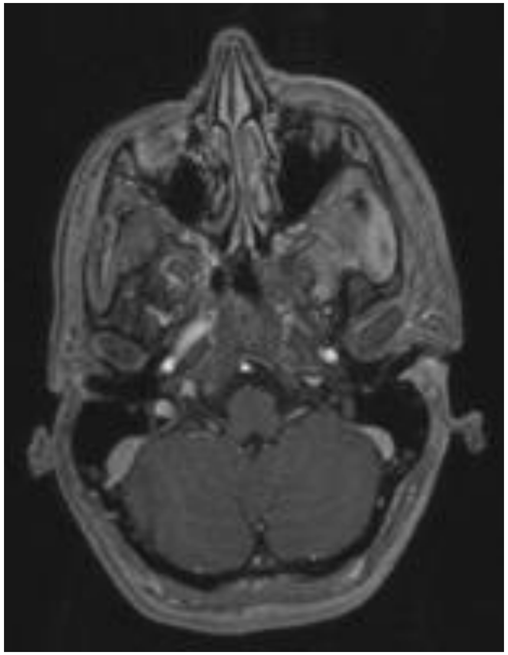
Backprojection

$$f(x,y) = \int_0^{\pi} \tilde{p}_\Theta(s)\, d\Theta = \int_0^{\pi} \tilde{p}_\Theta(x\cos\Theta + y\sin\Theta)\, d\Theta$$

Obtain for point (x,y) the value f(x,y), in which all filtered projections are summed up at the point $x\cos\Theta + y\sin\Theta$



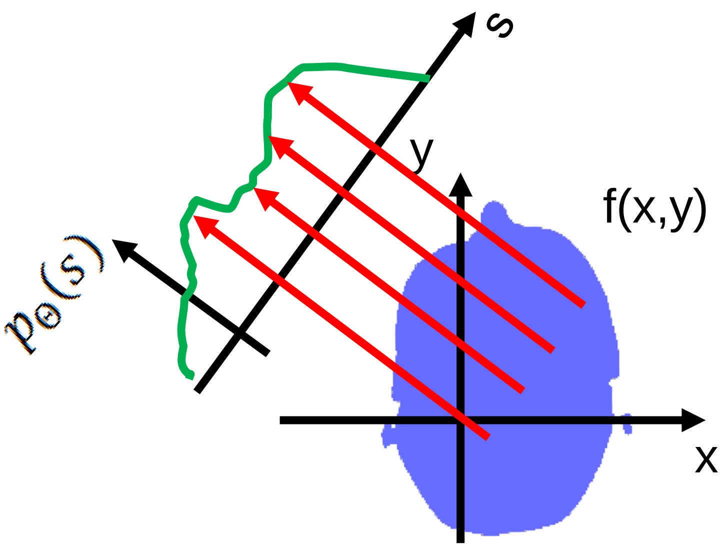
for all f(x,y) →

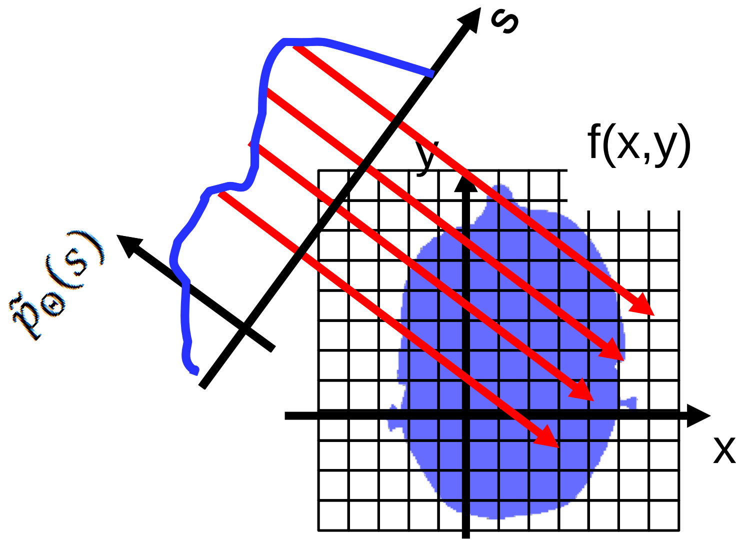


Summary filtered backprojection

- Filter all projections $p_{\Theta}(s)$ with filter function h(s) by transformation into the Fourier space $\tilde{p}_{\Theta}(s) = p_{\Theta}(s) * h(s)$
- Draw all $\tilde{p}_{\Theta}(s)$ like a comb along the angle Θ over the image matrix f(x,y), add the appropriate value on each pixel hit

$$\tilde{p}_{\Theta}(s') = \tilde{p}_{\Theta}(x_1 \cos\theta + y_1 \sin\theta)$$



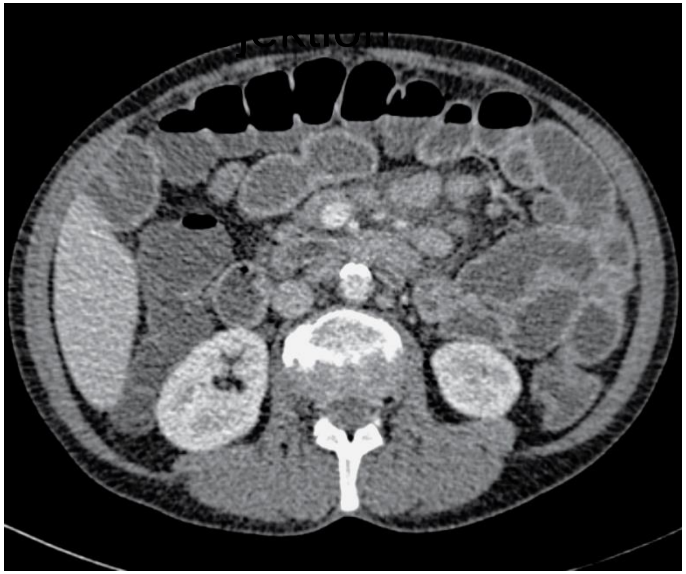


Evaluation filtered backprojection

- Standard method, established in clinical routine
- Very fast: result is directly calcluated
- Signal-to-noise ratio is worse compared to the iterative approach

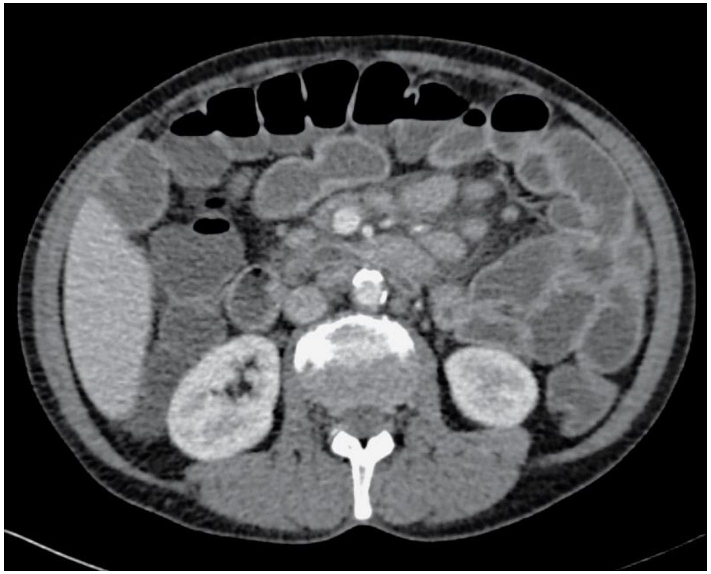
→ Reduction of dosis limited

Filtered Backprojection



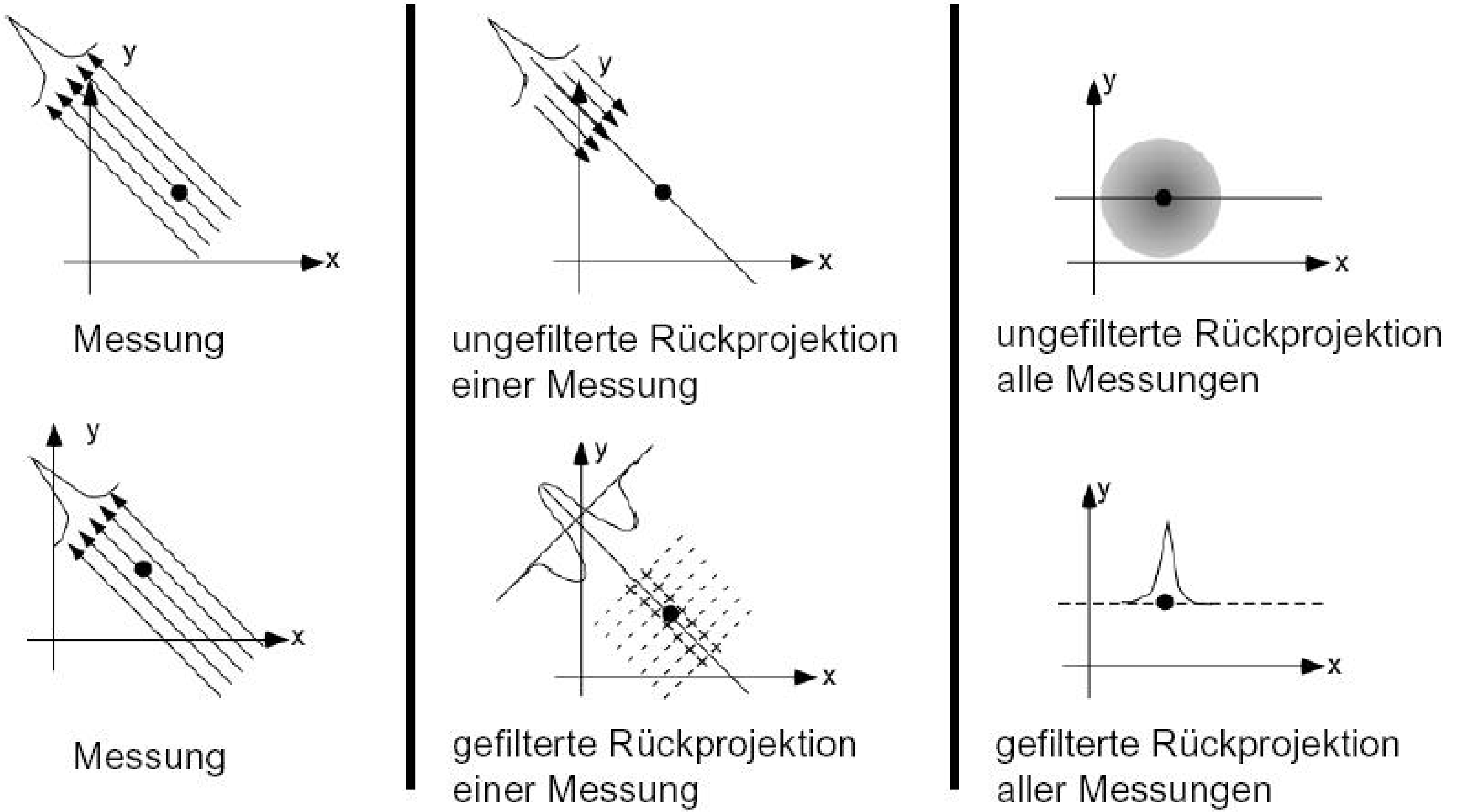
Quelle: Siemens

Iterative Reconstruction



Quelle: Siemens

Filtered vs. unfiltered backprojection



Quelle: O. Dössel: Bildgebende Verfahren in der Medizin

NCT

What is illustrated in the CT?

The spatial distribution of the attenuation coefficient $\mu(x,y)$ is measured

Problem:

- Not very descriptive measurement

Therefore:

- Relative indication of "CT numbers" (relative in terms of attenuation coefficient of water)

Hounsfield scale

In honor of Hounsfield

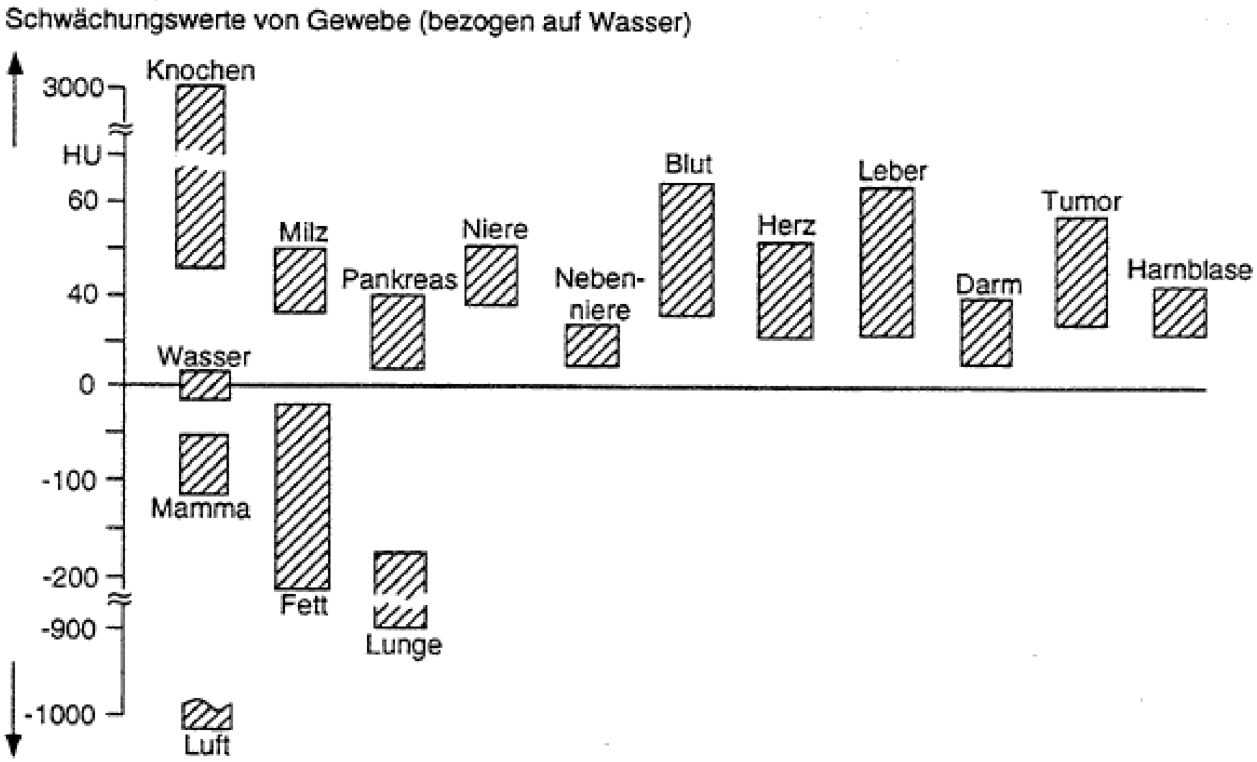
- Values between -1000 and +3000
- min. 12 Bit gray values necessary (=4096)

$$"CT - Zahl" = \frac{\mu - \mu_{Wasser}}{\mu_{Wasser}} \bullet 1000[HU]$$

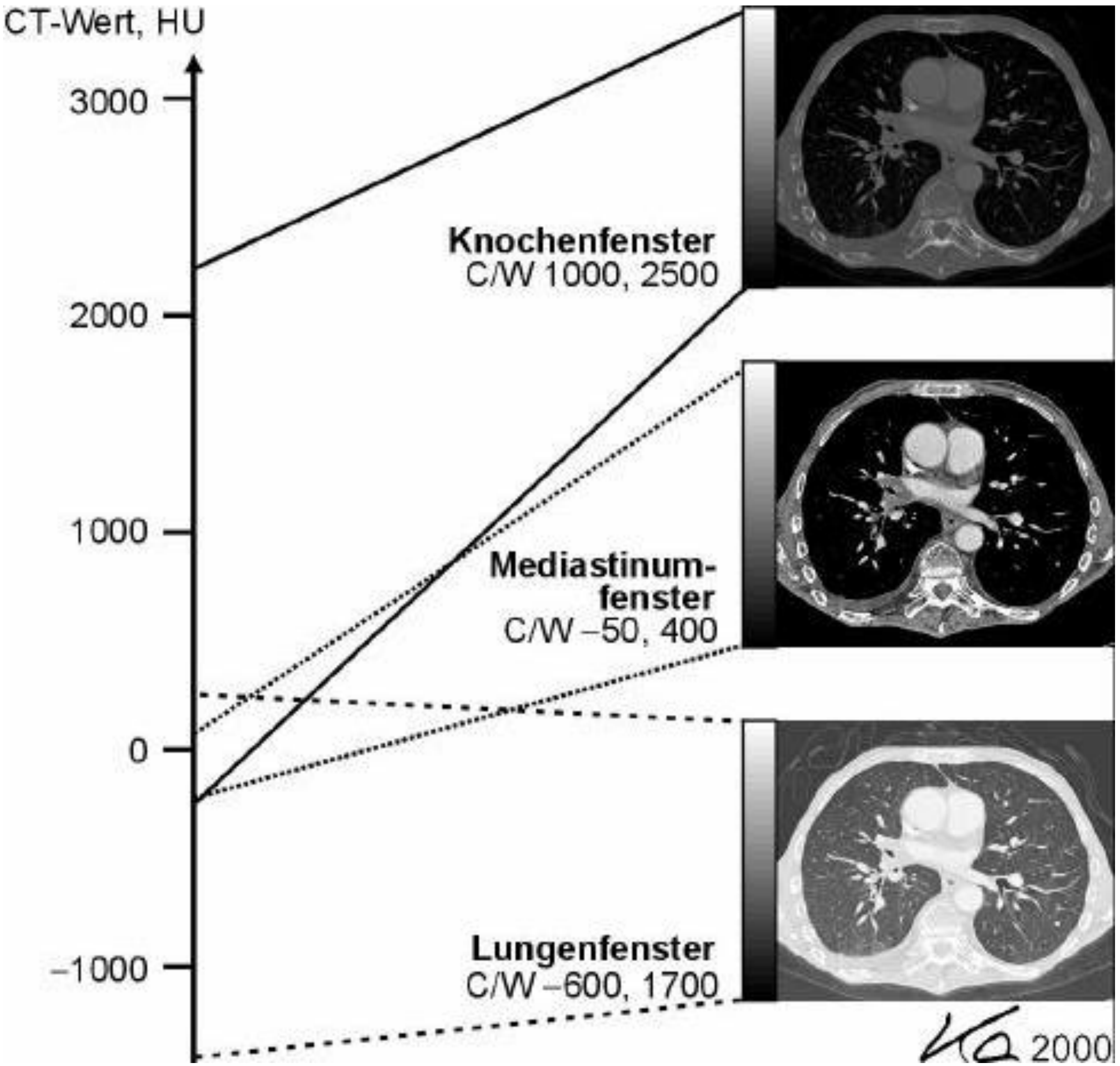
Hounsfield

Windowing for visualization necessary

Hounsfield scale



Windowing



Quelle: Epileptologie Bonn

Interpretation CT numbers

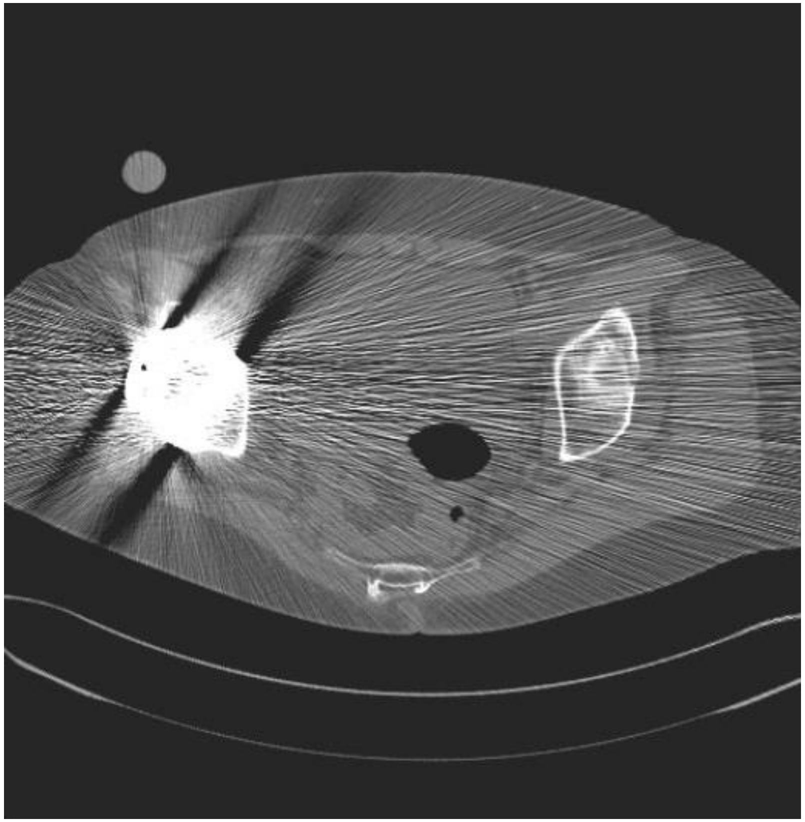
Interpretation

- Normally unique, since feedback to a physical effect (attenuation) is possible
- But: every volume element consists of different materials
- Ambigious diagnostic findings are possible

Artefact

Reasons

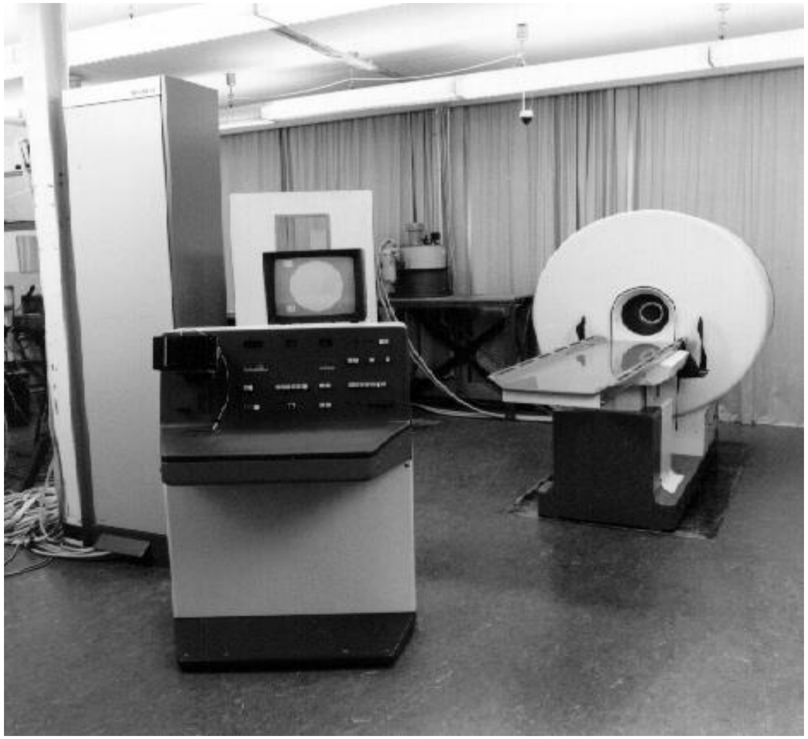
- Different tissue types in one pixel
- Deformation of organs
- metallic implants
- Imprecise approximation of slices
- Sampling errors
- Sensor failure
- …

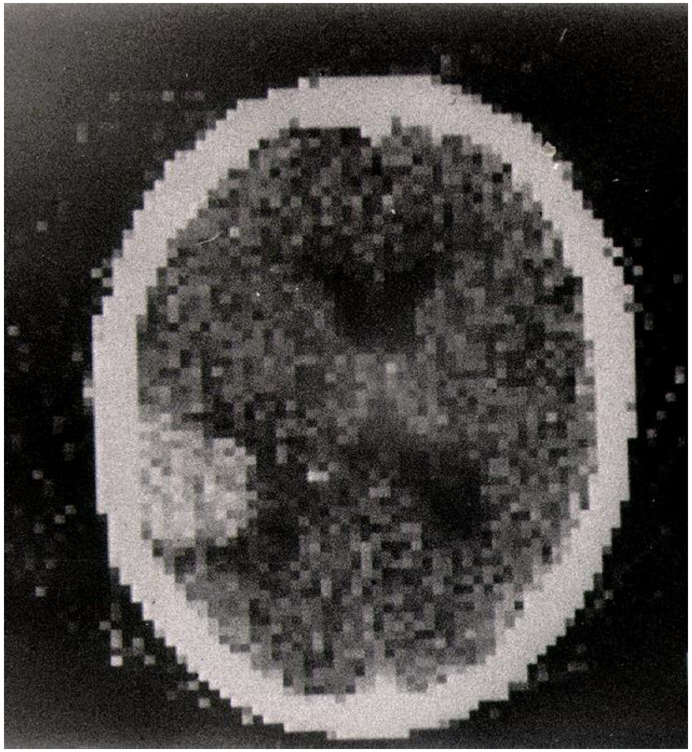


CT development

1974 Siemens Siretom

- 80x 80 Pixel, 300s recording time



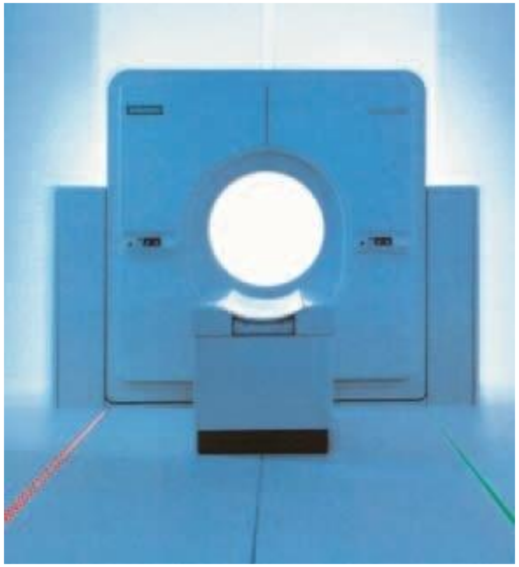


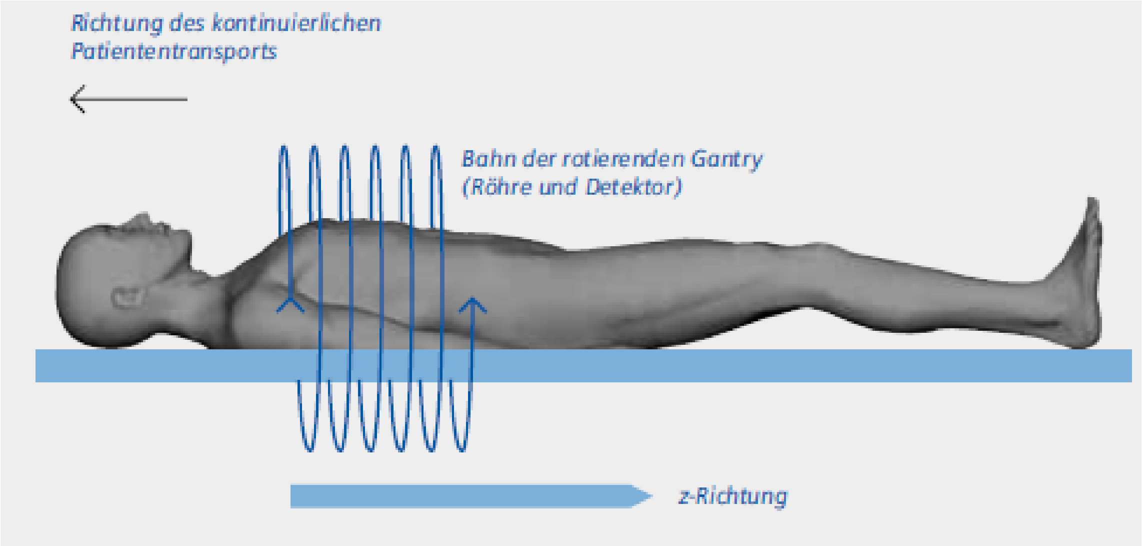
CT development

1988 Siemens Somatom Plus

- Spiral-CT with slip ring technology
- Continuous tube rotation, continuous feed

  shorter recording times, volume recording



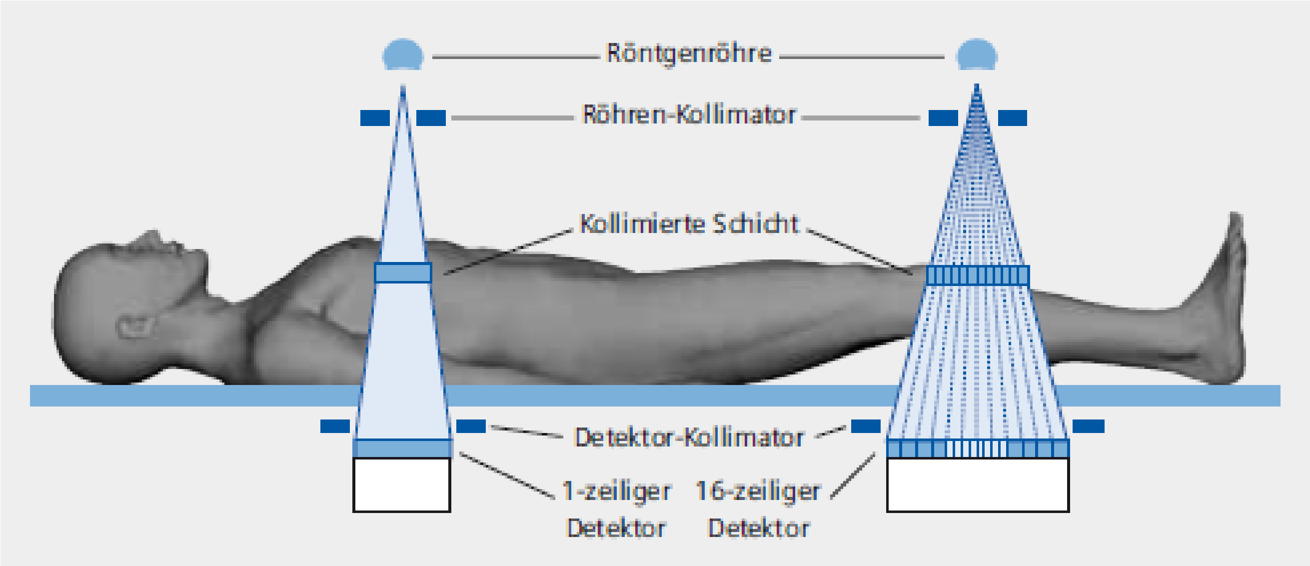


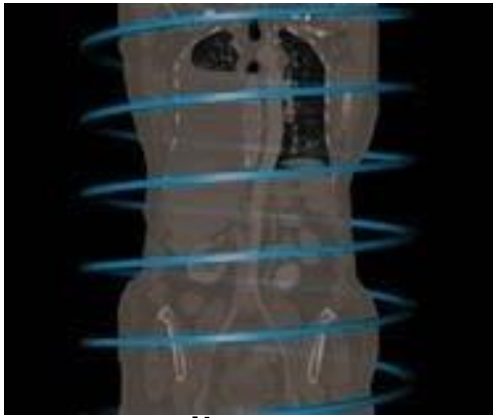
Quelle: Siemens

CT - Entwicklung

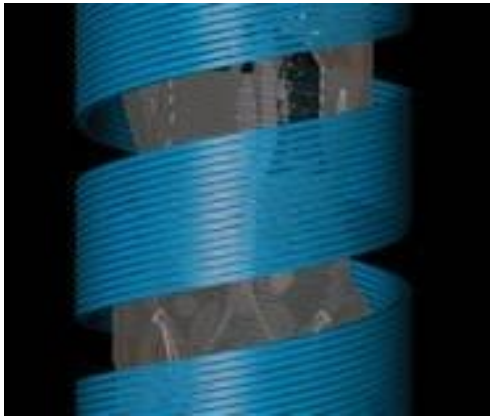
1998 Siemens Somatom Volume Zoom

- Multislice-Detector, several detector lines
- Recording of 4 slices simultaneously
- Rotation time 0.5 sec





1 slice



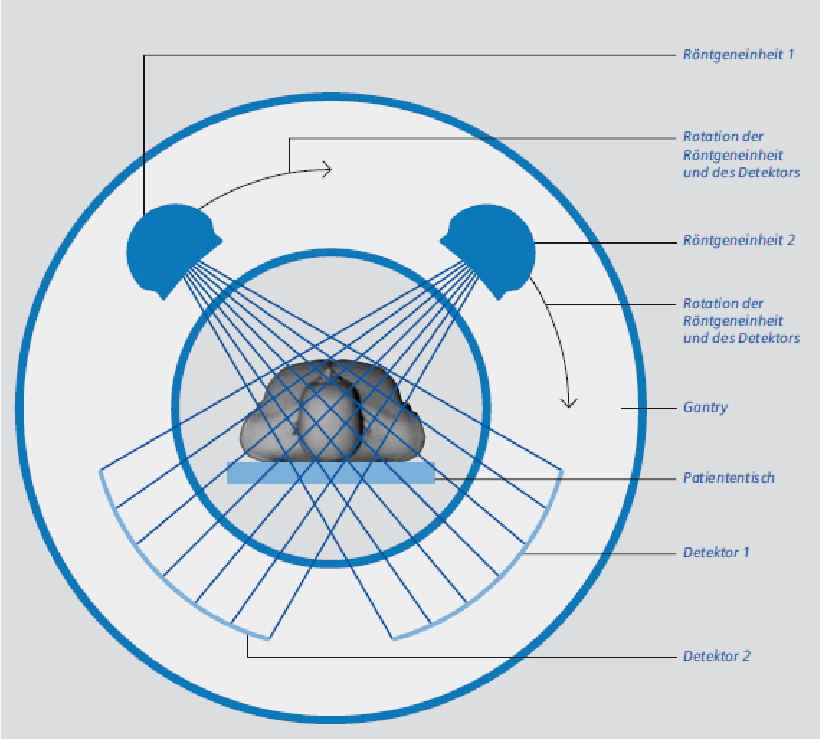
16 slices

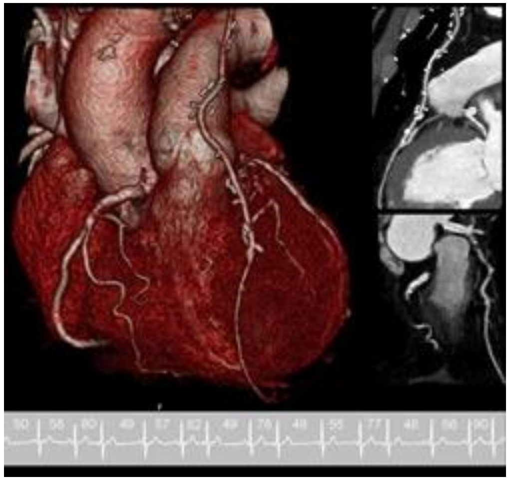
Quelle: Siemens

CT development

2006 Siemens Somatom Definition Dual Source CT

- Two X-ray sources, recording time halved



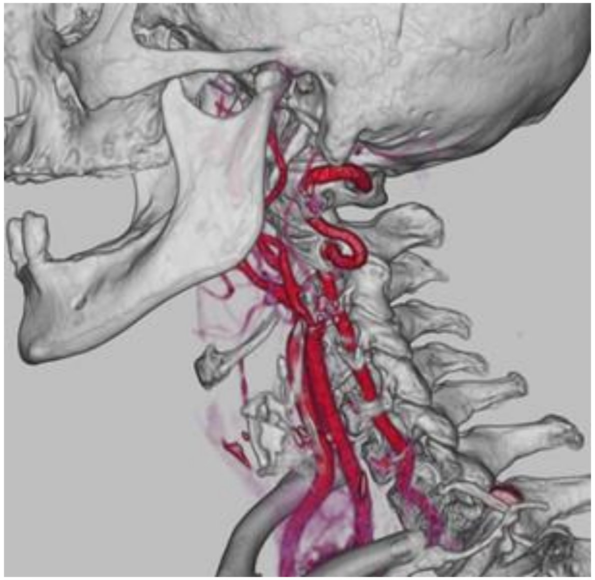


→ Better beating heart recording

CT development

2012 Toshiba Aquilion One Vision Volume Scanner

- 640 slices per recording, 16 cm wide volume
- Dosis reduction due to volume acquisition




Quelle: Toshiba

CT – Evaluation

Principle

- X-ray beams are used for Imaging
- Based on recordings from different perspectives a slice image is calculated

Advantage

- Good visualization of bones, …
- Very good resolution (Somatom Dual Source 0,24 mm)
- Very fast (Rotation Somatom Dual Source 0,33 sec, Aquillion One 0,35 sec)

Disadvantages

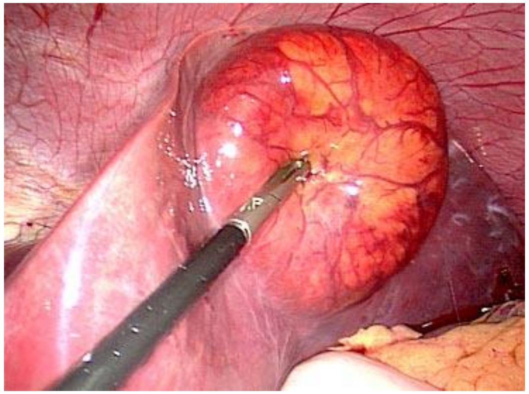
- Radiation exposure

  → Reduction through better hardware and rekonstruction techniques
- Intraoperative application costly

Outlook

Themen der nächsten Vorlesung:

- Magnetresonanztomographie
  - Physikalische Grundlagen: Kernspin
  - Signalerzeugung: Längs- und Quermagnetisierung
  - Signalmessung: Relaxationsprozesse
  - Signalkodierung: Gradientenfelder
  - Signalrekonstruktion
- Endoskopie
  - Systeme
  - Anwendungen



Literature

- O. Dössel, Bildgebende Verfahren in der Medizin, Springer
- W. Kalender, Computertomographie, Publics MCD Verlag
- Buzug, Einführung in die Computertomographie, Springer Verlag
- H. Handels, Medizinische Bildverarbeitung, Teubner
- T. Lehmann, Bildverarbeitung für die Medizin
- A. Dhawan, Medical Image Analysis
- Elektronisches Skriptum zum Thema Röntgen usw: http://www.emk.e-technik.tu-darmstadt.de/~ronblech/bmt/www_Bildarchiv.htm
- Fa. Siemens
- http://www.vms.ei.tum.de/publ/pdf/at0210_472.pdf
- http://www.slaney.org/pct/pct-toc.html
- http://www.medical.siemens.com/siemens/de_DE/rg_marcom_FBAs/files/Events/drk2010/CT_IRIS_04-2010.pdf