



Review

- 连续点与间断点

初等函数的连续性

- 当 $x \rightarrow 0$ 时, $\arcsin x \sim x$, $\arctan x \sim x$.

- 有界闭区间上连续函数的性质

零点定理 介值定理 有界性定理

最大最小值定理 一致连续性定理

- f 在 I 上非一致连续 \Leftrightarrow

$$\exists \varepsilon_0 > 0, \exists x_n, y_n \in I, \lim_{n \rightarrow \infty} (x_n - y_n) = 0, \text{ s.t. } |f(x_n) - f(y_n)| \geq \varepsilon_0.$$



§ 1. 导数

Def. (导数, 左、右导数)

$$(1) f'(x_0) \triangleq \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0};$$

$$(2) f'_-(x_0) \triangleq \lim_{\Delta x \rightarrow 0^-} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \rightarrow x_0^-} \frac{f(x) - f(x_0)}{x - x_0};$$

$$(3) f'_+(x_0) \triangleq \lim_{\Delta x \rightarrow 0^+} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \rightarrow x_0^+} \frac{f(x) - f(x_0)}{x - x_0}.$$

Question. 导数的几何意义? 切线的斜率.

Question. 可导的几何意义? 光滑性

Remark. 导函数 $f'(x)$.



Ex. (1) $c' = 0$, (2) $(\sin x)' = \cos x$, (3) $(\cos x)' = -\sin x$,

(4) $(a^x)' = a^x \ln a$, (5) $(\log_a x)' = \frac{1}{x \ln a}$, (6) $(x^\alpha)' = \alpha x^{\alpha-1}$.

Proof. (1) $f(x) \equiv c$, 则

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{c - c}{\Delta x} = 0.$$

$$\begin{aligned} (2) (\sin x)' &= \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h} = \lim_{h \rightarrow 0} \frac{2 \sin \frac{h}{2} \cos(x + \frac{h}{2})}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin \frac{h}{2}}{h} \cdot \lim_{h \rightarrow 0} \cos(x + \frac{h}{2}) = 1 \cdot \cos x = \cos x. \end{aligned}$$



$$(3)(\cos x)' = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{h}{2} \sin(x + \frac{h}{2})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{h}{2}}{h} \cdot \lim_{h \rightarrow 0} \sin(x + \frac{h}{2}) = -\sin x.$$

$$(4)(a^x)' = \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h} = a^x \ln a.$$

特别地, $(e^x)' = e^x$.



$$\begin{aligned}(5)(\log_a x)' &= \lim_{h \rightarrow 0} \frac{\log_a (x+h) - \log_a x}{h} \\&= \frac{1}{\ln a} \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln x}{h} \\&= \frac{1}{x \ln a} \lim_{h \rightarrow 0} \frac{\ln(1+h/x)}{h/x} = \frac{1}{x \ln a}.\end{aligned}$$

特别地, $(\ln x)' = \frac{1}{x}$.



$$(6)(x^\alpha)' = \lim_{h \rightarrow 0} \frac{(x+h)^\alpha - x^\alpha}{h}$$

$$x \neq 0 \text{ 时, } (x^\alpha)' = x^\alpha \lim_{h \rightarrow 0} \frac{(1+h/x)^\alpha - 1}{h} = x^\alpha \lim_{h \rightarrow 0} \frac{\alpha h/x}{h} = \alpha x^{\alpha-1}.$$

$$x = 0 \text{ 时, } f'(0) = \lim_{h \rightarrow 0} \frac{h^\alpha - 0^\alpha}{h} = \lim_{h \rightarrow 0} h^{\alpha-1}$$

$$= \begin{cases} 1 & \alpha = 1 \\ \text{不存在} & \alpha < 1 \\ 0 & \alpha > 1 \text{ 且 } x^{\alpha-1} \text{ 在 } (-\delta, \delta) \text{ 有定义} \\ \text{不存在} & \alpha > 1 \text{ 且 } x^{\alpha-1} \text{ 在 } (-\delta, 0) \text{ 无定义} \end{cases}$$

综上, $(x^\alpha)' = \alpha x^{\alpha-1}$ ($x^{\alpha-1}$ 有意义时成立).



Thm. $f'(x_0)$ 存在 $\Leftrightarrow f'_-(x_0), f'_+(x_0)$ 均存在且相等.

f 在 x_0 可导时, $f'(x_0) = f'_-(x_0) = f'_+(x_0)$.

Ex. $f(x) = \begin{cases} x+1, & x \leq 0, \\ e^x, & x > 0. \end{cases}$ 求 $f'(0)$.

$$f'_-(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^-} \frac{x+1-1}{x} = 1.$$

$$f'_+(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^+} \frac{e^x - 1}{x} = 1 = f'_-(0).$$

故 $f'(0) = 1$. \square



Thm. f 在 x_0 可导 $\Rightarrow f$ 在 x_0 连续

Proof. f 在 x_0 可导, 则

$$\begin{aligned}\lim_{x \rightarrow x_0} (f(x) - f(x_0)) &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \cdot \lim_{x \rightarrow x_0} (x - x_0) \\ &= f'(x_0) \cdot 0 = 0.\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow x_0} f(x) &= \lim_{x \rightarrow x_0} (f(x) - f(x_0)) + \lim_{x \rightarrow x_0} f(x_0) \\ &= 0 + f(x_0) = f(x_0),\end{aligned}$$

故 f 在 x_0 连续. \square



Ex. $f(x) = x^2 D(x)$ 的可导性质? $D(x)$ 为Dirichlet函数.

解:
$$f'(0) = \lim_{x \rightarrow 0} \frac{x^2 D(x) - 0}{x} = 0.$$

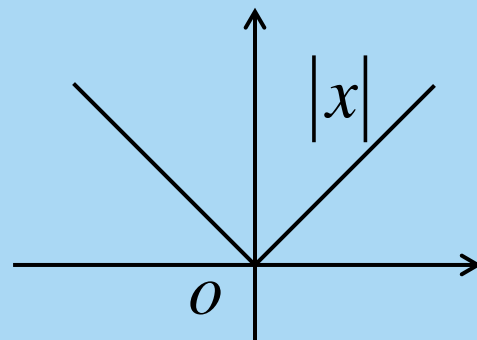
$f(x)$ 在任意 $x_0 \neq 0$ 处不连续, 因而不可导. \square

Ex. $f(x) = |x|$ 在 $x_0 = 0$ 是否可导?

连续 ~~不可~~ 可导

$$\lim_{x \rightarrow 0^+} \frac{|x| - 0}{x} = 1,$$

$f(x) = |x|$ 在 $x_0 = 0$ 不可导.



Remark. 利用级数可以构造处处连续处处不可导的例子.



Question. 导数的物理意义?

t	$f(t)$	$f'(t)$
时间	位移	速度
时间	速度	加速度

Def. 记 $\Delta f(x_0) = f(x_0 + \Delta x) - f(x_0)$, 若存在常数 α , s.t.

$$\Delta f(x_0) = \alpha \Delta x + o(\Delta x) \quad (\Delta x \rightarrow 0),$$

则称 f 在 x_0 可微, 并称 $df(x_0) \triangleq \alpha \Delta x \triangleq \alpha dx$ 为 f 在点 x_0 的微分.



Thm. f 在 x_0 可微 $\Leftrightarrow f$ 在点 x_0 可导.

Proof. 设 f 在 x_0 可微, 则 $\exists \alpha \in \mathbb{R}, s.t.$

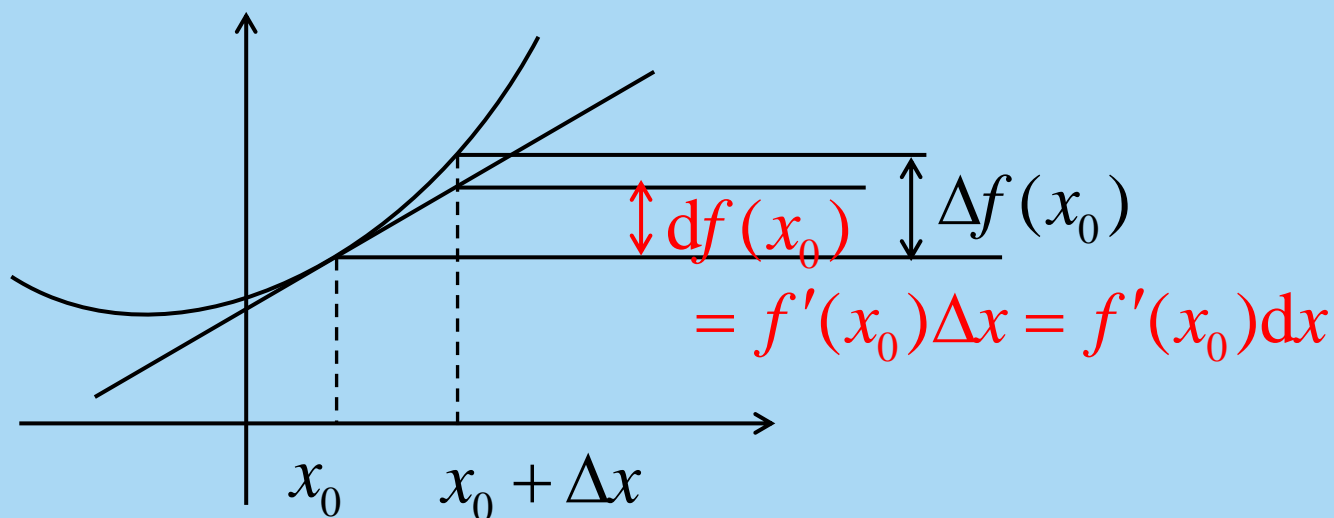
$$\Delta f(x_0) = \alpha \Delta x + o(\Delta x) \quad (\Delta x \rightarrow 0).$$

$$\text{因此 } f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = \alpha + \lim_{\Delta x \rightarrow 0} \frac{o(\Delta x)}{\Delta x} = \alpha.$$

设 f 在 x_0 可导. $\rho(x) \triangleq \frac{f(x) - f(x_0)}{x - x_0} - f'(x_0)$, 则 $\lim_{x \rightarrow x_0} \rho(x) = 0$,

$$\begin{aligned} f(x) - f(x_0) &= f'(x_0)(x - x_0) + \rho(x)(x - x_0) \\ &= f'(x_0)(x - x_0) + o(x - x_0) \quad (x \rightarrow x_0). \end{aligned}$$

故 f 在 x_0 可微. \square



Remark. $y = f(x)$ 在 x_0 可微,

$$\Delta f(x_0) = df(x_0) + o(\Delta x) \quad (\Delta x \rightarrow 0),$$

$$dy = df(x_0) = \alpha \Delta x = \alpha dx$$

$$\text{则 } f'(x_0) = \alpha = \frac{df}{dx}(x_0) = \frac{dy}{dx}(x_0).$$



Remark. f 在 x_0 可微, 则

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x.$$

Question. 可导与可微等价, 为什么需要给两个定义?

可微的概念是“以直代曲”, 便于推广到多元函数.



§ 2. 求导法则

Thm. f, g 在 x_0 可导, $c \in \mathbb{R}$, 则

$$(1) (f + g)'(x_0) = f'(x_0) + g'(x_0);$$

$$(2) (cf)'(x_0) = cf'(x_0);$$

$$(3) (fg)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0);$$

$$(4) g(x_0) \neq 0 \text{ 时}, \left(\frac{f}{g}\right)'(x_0) = \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{g^2(x_0)}.$$

$$\text{特别地, } g(x_0) \neq 0 \text{ 时}, \left(\frac{1}{g}\right)'(x_0) = \frac{-g'(x_0)}{g^2(x_0)}.$$

$$d(f + g) = df + dg$$

$$d(cf) = cdf$$

$$d(fg) = gdf + fdg$$

$$d\left(\frac{f}{g}\right) = \frac{gdf - fdg}{g^2}$$



$$\text{Proof. (3)} (fg)'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h)g(x_0 + h) - f(x_0)g(x_0)}{h}$$

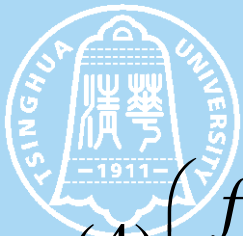
$$= \lim_{h \rightarrow 0} \frac{f(x_0 + h)g(x_0 + h) - \textcolor{red}{f(x_0 + h)g(x_0)}}{h}$$

$$+ \lim_{h \rightarrow 0} \frac{\textcolor{red}{f(x_0 + h)g(x_0)} - f(x_0)g(x_0)}{h}$$

$$= \lim_{h \rightarrow 0} f(x_0 + h) \cdot \lim_{h \rightarrow 0} \frac{g(x_0 + h) - g(x_0)}{h}$$

$$+ g(x_0) \cdot \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$= f(x_0)g'(x_0) + f'(x_0)g(x_0). \quad (\text{可导} \Rightarrow \text{连续})$$



$$\begin{aligned}(4) \left(\frac{f}{g} \right)'(x_0) &= \lim_{h \rightarrow 0} \left(\frac{f(x_0 + h)}{g(x_0 + h)} - \frac{f(x_0)}{g(x_0)} \right) / h \\&= \lim_{h \rightarrow 0} \frac{f(x_0 + h)g(x_0) - f(x_0)g(x_0 + h)}{h} \cdot \frac{1}{g(x_0)g(x_0 + h)} \\&= \frac{1}{g^2(x_0)} \lim_{h \rightarrow 0} \frac{f(x_0 + h)g(x_0) - f(x_0)g(x_0 + h)}{h} \\&= \frac{1}{g^2(x_0)} \left\{ \lim_{h \rightarrow 0} \frac{f(x_0 + h)g(x_0) - \textcolor{red}{f(x_0)g(x_0)}}{h} \right. \\&\quad \left. + \lim_{h \rightarrow 0} \frac{\textcolor{red}{f(x_0)g(x_0)} - f(x_0)g(x_0 + h)}{h} \right\} \\&= \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{g^2(x_0)}. \quad \square\end{aligned}$$



Ex. $(\tan x)' = \sec^2 x, \quad (\cot x)' = -\csc^2 x,$
 $(\sec x)' = \sec x \tan x, \quad (\csc x)' = -\csc x \cot x.$

Proof. $(\tan x)' = \left(\frac{\sin x}{\cos x} \right)' = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x}$
 $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x};$

$$(\sec x)' = \left(\frac{1}{\cos x} \right)' = \frac{-(\cos x)'}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \sec x \tan x.$$

.....□



Thm.(复合函数求导的链式法则) $\varphi(x)$ 在 x_0 可导, $f(u)$ 在

$u_0 = \varphi(x_0)$ 可导, 则 $h(x) = f(\varphi(x))$ 在 x_0 可导, 且

$$h'(x_0) = f'(\varphi(x_0)) \cdot \varphi'(x_0).$$

即 $df(\varphi(x)) = f'(\varphi(x))d\varphi(x) = f'(\varphi(x)) \cdot \varphi'(x)dx$.

Proof.

$$\text{令 } g(u) = \begin{cases} \frac{f(u) - f(u_0)}{u - u_0}, & u \neq u_0, \\ f'(u_0), & u = u_0. \end{cases} \quad \text{则 } \lim_{u \rightarrow u_0} g(u) = f'(u_0),$$

$$\begin{aligned} h'(x_0) &= \lim_{x \rightarrow x_0} \frac{f(\varphi(x)) - f(\varphi(x_0))}{x - x_0} = \lim_{x \rightarrow x_0} g(\varphi(x)) \cdot \frac{\varphi(x) - \varphi(x_0)}{x - x_0} \\ &= f'(\varphi(x_0)) \cdot \varphi'(x_0). \square \end{aligned}$$



Remark. $u = \varphi(x)$ 在 x 可导, $y = f(u)$ 在 $u = \varphi(x)$ 可导, 则 $y = f(\varphi(x))$ 在 x 可导, 且

$$y'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Remark. (一阶微分形式的不变性) $u = \varphi(x)$ 在 x_0 可微, $y = f(u)$ 在 $u_0 = \varphi(x_0)$ 可微, 则 $y = f(\varphi(x))$ 在 x_0 可微, 且

$$dy = f'(\varphi(x_0))\varphi'(x_0)dx = f'(u_0)du.$$

无论将 u 视为中间变量还是自变量, 都有 $dy = f'(u)du$.



Ex. $f(x) = \ln|x|$, 求 $f'(x)$.

解. $x > 0$ 时, $f(x) = \ln x$, $f'(x) = \frac{1}{x}$.

$x < 0$ 时, $f(x) = \ln(-x)$, $f(x)$ 是 $\ln u$ 与 $u = -x$ 的复合函数.

由链式法则,

$$f'(x) = \frac{1}{-x} (-x)' = \frac{1}{x}.$$

综上, $(\ln|x|)' = \frac{1}{x}$. \square



Ex. $f(x) = \left(\frac{x+1}{x-1}\right)^{3/2}$, 求 $f'(x)$. $x \in (-\infty, -1] \cup (1, +\infty)$

解. 令 $g(u) = u^{3/2}$, $h(x) = \frac{x+1}{x-1}$, 则 $f(x) = g(h(x))$,

$$g'(u) = \frac{3}{2}u^{1/2},$$

$$h'(x) = \left(1 + \frac{2}{x-1}\right)' = \frac{-2}{(x-1)^2}$$

$$f'(x) = g'(h(x))h'(x) = \frac{-3}{(x-1)^2} \left(\frac{x+1}{x-1}\right)^{1/2} \quad \square$$



Ex. $f(x) = \ln \left| x + \sqrt{x^2 \pm a^2} \right|$, 求 $f'(x)$.

$$\text{解. } f'(x) = \frac{\left(x + \sqrt{x^2 \pm a^2} \right)'}{x + \sqrt{x^2 \pm a^2}} = \frac{1 + \frac{2x}{2\sqrt{x^2 \pm a^2}}}{x + \sqrt{x^2 \pm a^2}} = \frac{1}{\sqrt{x^2 \pm a^2}}. \square$$

Ex. $f(x) = u(x)^{v(x)}$, $u(x) > 0$, $u(x), v(x)$ 可导, 求 $f'(x)$.

$$\begin{aligned} \text{解. } f'(x) &= \left(e^{v(x) \ln u(x)} \right)' = e^{v(x) \ln u(x)} \cdot \left(v(x) \ln u(x) \right)' \\ &= u(x)^{v(x)} \cdot \left(v'(x) \ln u(x) + v(x) \frac{u'(x)}{u(x)} \right) \\ &= u(x)^{v(x)} \ln u(x) \cdot v'(x) + v(x) u(x)^{v(x)-1} u'(x). \square \end{aligned}$$



Ex. $f(x) = f_1(x)f_2(x)\cdots f_n(x)$, 求 $f'(x)$.

对数求导法

解: $\ln|f(x)| = \ln|f_1(x)| + \ln|f_2(x)| + \cdots + \ln|f_n(x)|$,

两边对 x 求导, 得
$$\frac{f'(x)}{f(x)} = \frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} + \cdots + \frac{f_n'(x)}{f_n(x)}.$$

$$\begin{aligned} f'(x) &= f(x) \left(\frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} + \cdots + \frac{f_n'(x)}{f_n(x)} \right) \\ &= \sum_{k=1}^n f_1(x) \cdots f_{k-1}(x) f_k'(x) f_{k+1}(x) \cdots f_n(x). \square \end{aligned}$$

Remark. 多个因子连乘的函数求导时先取对数再两端求导可简化计算. $(f(x_0) = 0 \text{ 时结论仍成立? 如何处理?})$

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Ex. $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, 求 $f'(x)$.

解: $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$

$$f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}. \quad \square$$

Question. (1) f 在 $[a, b]$ 可导, f' 在 $[a, b]$ 上一定连续吗? **不一定.**

(2) $f \in C[a, b]$, f 在 (a, b) 可导, $f'_+(a)$ 与 $f'_-(b)$ 一定存在? **不一定.**

反例? (1) $x^2 \sin \frac{1}{x}$, (2) $x \sin \frac{1}{x}$.



Thm. (反函数求导) 设 f 在 (a, b) 严格单调且连续, $x_0 \in (a, b)$,

$f'(x_0) \neq 0$, 则 $x = f^{-1}(y)$ 在 $y_0 = f(x_0)$ 处可导, 且

$$(f^{-1})'(y_0) = 1 / f'(x_0).$$

Proof. f 在 (a, b) 严格单调且连续, 则其反函数 $x = f^{-1}(y)$ 也严格单调且连续. 当 $y \neq y_0, y \rightarrow y_0$ 时, 有 $x \neq x_0, x \rightarrow x_0$.

$$(f^{-1})'(y_0) = \lim_{y \rightarrow y_0} \frac{f^{-1}(y) - f^{-1}(y_0)}{y - y_0} = \lim_{x \rightarrow x_0} \frac{x - x_0}{f(x) - f(x_0)}$$

$$= \frac{1}{\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}} = \frac{1}{f'(x_0)}. \quad \square$$

Remark. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}.$



Ex. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad \arctan x = \frac{1}{1+x^2},$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}, \quad \operatorname{arccot} x = \frac{-1}{1+x^2}.$$

解: (1) $y = \arcsin x$ 与 $x = \sin y$ 互为反函数, 因此

$$(\arcsin x)' = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

(2) $y = \arctan x$ 与 $x = \tan y$ 互为反函数, 因此

$$(\arctan x)' = \frac{1}{(\tan y)'} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}.$$



Ex. 证明: 1) $2^x = ax + 2 (a \leq 0)$ 确定隐函数 $x = x(a)$;

2) $x(a)$ 在其定义域上连续;

3) $x(a)$ 在其定义域上可导, 并求 $x'(a)$.

证明: 1) 由 $2^x = ax + 2 (a \leq 0)$ 得

$$0 \geq a = a(x) = \frac{2^x - 2}{x}, \quad (0 < x \leq 1).$$

$2 - 2^x$ 与 $\frac{1}{x}$ 均在 $(0, 1]$ 上非负且严格单调递减, $a(x)$ 在 $(0, 1]$

上非正且严格单调递增, 故 $a = a(x)$ 有反函数 $x = x(a)$,

即 $2^x = ax + 2 (a \leq 0)$ 确定隐函数 $x = x(a)$.



2) $a(x) = \frac{2^x - 2}{x}$ 在 $(0, 1]$ 上严格单调递增且连续, 值域为

$(-\infty, 0]$, 其反函数 $x = x(a)$ 在 $(-\infty, 0]$ 上严格单调递增且值域为 $(0, 1]$, 因此 $x = x(a)$ 在 $[0, +\infty)$ 上连续.

3) $a(x)$ 可导, 且 $a'(x) = \frac{2^x(x \ln 2 - 1) + 2}{x^2} > 0$, 因此其反函数

$x = x(a)$ 可导. 在 $2^x = ax + 2$ 中视 $x = x(a)$, 两边对 a 求导, 得

$$2^x \ln 2 \cdot x'(a) = x + ax'(a), \quad x'(a) = \frac{x}{2^x \ln 2 - a}. \quad \square$$



Ex. 求曲线 $x^2 + y \cos x - 2e^{xy} = 0$ 在点 $M(0, 2)$ 处的切线方程.

解: 视方程 $x^2 + y \cos x - 2e^{xy} = 0$ 中 $y = y(x)$, 两边对 x 求导, 得

$$2x + y' \cos x - y \sin x - 2(y + xy')e^{xy} = 0,$$

$$y' = \frac{2ye^{xy} + y \sin x - 2x}{\cos x - 2xe^{xy}}.$$

将 $x = 0, y(0) = 2$ 代入, 得 $y'(0) = 4$.

故曲线在点 $M(0, 2)$ 处的切线方程为 $y = 4x + 2$. \square

Remark. $x^2 + y \cos x - 2e^{xy} = 0$ 在点 $M(0, 2)$ 附近隐函数 $y(x)$ 的存在性与可微性, 需要用下学期的隐函数定理来证明.



Ex. $x = \varphi(t)$ 严格单调且连续, 其反函数 $t = \varphi^{-1}(x)$ 也严格单调且连续. 于是, 参数方程 $\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases}$ 确定了函数

$$y = \psi(t) = \psi(\varphi^{-1}(x)).$$

若 $\varphi(t)$ 可导且 $\varphi'(t) \neq 0$, 求 $\frac{dy}{dx}$.

解: 由复合函数求导的链式法则,

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \psi'(t) \cdot (\varphi^{-1})'(x) = \frac{\psi'(t)}{\varphi'(t)}. \square$$



Ex. $y = y(x)$ 由参数方程 $\begin{cases} x = t + e^t \\ y = t^2 + e^{2t} \end{cases}$ 确定, 求 $\frac{dy}{dx}$.

解:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(t + e^{2t})}{1 + e^t}. \square$$



作业：

习题3.1 No. 5(4), 9, **14(1)**

习题3.2 No. 4(3)(6)(8), 6(1), 7(1),
8(4), 10(1), 11(4)