Course Information

CS 58500 – Theoretical Computer Science Toolkit

Time/Location: TTH 3:00 PM - 4:15 PM at HAAS G066

Instructor: Ruizhe Zhang
Office hours: By appointment

Teaching Assistant: Xiuyu Ye
Office hours: TBD

Prerequisites: Mastery of the material covered in Calc III (Math 261), Linear Algebra (Math265), Probability (STAT 416), Foundations of CS (CS 182), and Analysis of Algorithms (CS 381or CS 580). Undergraduates interested in taking the course should contact the instructor for permission.

Webpage: https://ruizhezhang.com/course_fall_2026.html

1 Course Description

This course covers fundamental techniques and a range of mathematical tools that underlie today's research in theoretical computer science and are essential knowledge for students pursuing research in theoretical computer science or machine learning theory. The course targets current graduate and undergraduate students interested in pursuing research in these areas. Topics will be chosen from four core areas:

- Concentration Inequalities and their Applications
- Selected Topics in Convex Analysis and Optimization
- Foundations of Spectral Methods
- Discrete Fourier Analysis

Depending on student interest, additional topics will be chosen. The topics may include applied analysis for learning theory, coding theory, probabilistic proofs, graph algorithms, and sampling.

Students will read research papers to gain an additional perspective on applying these techniques to different areas and their use and extensions in recent research results.

2 Course Materials

We will not be following any particular textbook, but the following resources could be helpful for or complementary to various parts of the course:

- Hemanta K. Maji-Paul Valiant. Previously offered CS 58500 and CS 59200 and useful materials.
- Stéphane Boucheron-Gabor Lugosi-Pascal Massart. Concentration Inequalities: A Nonasymptotic Theory of Independence.
- Joel A. Tropp. An Introduction to Matrix Concentration Inequalities.
- Joel A. Tropp. Probability in High Dimensions.
- Ryan O'Donnell. Analysis of Boolean Functions.
- Kevin Tian. Continuous Algorithms.
- Kuikui Liu. Algorithmic Counting and Sampling.
- Gil Cohen. Free Probability and Ramanujan Graphs.
- Max Hopkins. Fourier Analysis on HDX (talks at the Simons Institute).

3 Grading

The grading breakdown is as follows:

- Problem sets (30%)
- Midterm (25%)
- Class Participation (5%)
- Final Group Presentation (40%)

Problem sets There will be three to four problem sets. All submissions must be in PDF format. You are allowed to discuss problems with other students, but the final submission must be written individually.

Participation In-personal attendance at lectures is **mandatory** to receive full participation credit. If you have any unavoidable conflict (illness, travel, etc.), you should try to let me know in advance whenever possible.

Final project Students will work individually or in pairs to complete a written final project and give an oral presentation. The final project should: (1) focus on a research frontier in at least one of the topics discussed in class; (2) include a thorough literature review; (3) have a strong technical component, either in the form of original research or in the form of insightful exposition of existing work. A list of suggested topics and papers will be provided; however, students are encouraged to propose their own topics, subject to the instructor's approval.

4 Tentative List of Topics

Mathematical Basics

- Inequalities: Jensen's Inequality and consequences
- Summations/Integrals
- Stirling Approximation

Concentration Inequalities

- Markov Inequality, Chebyshev Inequality
- Chernoff-Hoeffding Bound
- Azuma's Inequality
- Talagrand Inequality
- Non-Commutative Khintchine Inequality
- Matrix Chernoff/Bernstein Bound
- Gaussian Process, Dudley's Inequality, and Generic chaining (optional)
- The BBvH Matrix Concentration Inequality (optional)

Selected Topics in Convex Analysis and Optimization

- Convexity, logconcavity, and continuous algorithms
- Strong convexity and oracle complexity of gradient descent
- Polynomial approximation (optional)
- Linear regression (optional)
- Convex geometry, localization, and isoperimetry
- Stochastic localization, sampling, and volume estimation (optional)

Foundations of Spectral Methods

- Positive semidefiniteness
- Spectral and singular value decompositions
- Courant–Fischer–Weyl minimax theorems
- Perron–Frobenius theory

- Matrix norms and perturbation theory
- Free probability, interlacing family, and bipartite Ramanujan graphs (optional)
- Expander graphs and Zig-Zag product (optional)

Discrete Fourier Analysis

- BLR Linearity Testing
- Hypercontractivity
- KKL Theorem
- Fourier analysis on high-dimensional expanders (HDXs) (optional)

5 Course Policies[1]

Late Policy Problem sets may be submitted up to 3 days late, with a 10% deduction in credit. Late submissions of the final project report will not be accepted under any circumstances.

Academic Dishonesty Purdue prohibits "dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty." Furthermore, the University Senate has stipulated that "he commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during exams) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest." While it is all right to discuss homework in this class with other students in general terms, do not copy another student's homework or let anyone copy your homework. These discussions should be appropriately acknowledged. When two identical homeworks are found, both are sent to the Dean of Students to determine who copied from whom. The use of generative AI for assignments is strongly discouraged. However, if a student chooses to use it, they must submit a complete transcript of all prompts and ensure that the final solutions are written entirely in their own words. Penalties for academic dishonesty range from a 0 grade on one assignment to a failing grade in the class and even expulsion from the University. A hearing at the Dean of Students' Office can ruin your whole day.

Violent Behavior Policy Purdue University is committed to providing a safe and secure campus environment for members of the university community. Purdue strives to create an educational environment for students and a work environment for employees that promote educational and career goals. Violent behavior impedes such goals. Therefore, violent behavior is prohibited in or on any University facility or while participating in any university activity.

[1] Most of the following material is based on the policy followed by Prof. Samuel Wagstaff in his cryptography courses.

Students with Disabilities Purdue University is required to respond to the needs of the students with disabilities as outlined in both the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990 through the provision of auxiliary aids and services that allow a student with a disability to fully access and participate in the programs, services, and activities at Purdue University. The ODOS Testing Center is an excellent place for students with disabilities to take exams for this class. Please tell the instructor at least one week in advance if you wish to take the exams there. If you have a disability that requires other special accommodation, please tell the instructor early in the semester. It is the student's responsibility to notify the Disability Resource Center of an impairment/condition that may require accommodations and/or classroom modifications.

Emergencies In the event of a major campus emergency (such as a tornado, earthquake, flu epidemic or terrorist attack), course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructor via email or phone. You should read your Purdue email frequently.

Nondiscrimination Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. Purdue University prohibits discrimination against any member of the University community on the basis of race, religion, color, sex, age, national origin or ancestry, marital status, parental status, sexual orientation, disability or status as a veteran. The University will conduct its programs, services and activities consistent with applicable federal, state and local laws, regulations and orders and in conformance with the procedures and limitations as set forth in Executive Memorandum No. D-1, which provides specific contractual rights and remedies. The instructor agrees completely with all Purdue policies mentioned in this document.

Privacy The Federal Educational Records Privacy Act (FERPA) protects information about students, such as grades. If you apply for a job and wish to use the instructor as a reference, you should tell the instructor beforehand. Otherwise, the instructor cannot say anything about you to a prospective employer who might call. The instructor is happy to provide references and to write letters of recommendation for his students as needed.

Disclaimer.

This syllabus and grading scheme are subject to change.