Problem 1 (20 pts)

The swap test circuit is as follows:

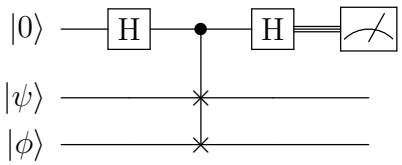

Figure 1: Swap test circuit

The first register is a single ancilla qubit, and the second and third registers are n-qubit quantum states. The middle gate is the *controlled-swap* gate: for any standard basis states $|i\rangle, |j\rangle$,

$$|0\rangle|i\rangle|j\rangle \mapsto |0\rangle|i\rangle|j\rangle$$
$$|1\rangle|i\rangle|j\rangle \mapsto |1\rangle|j\rangle|i\rangle$$

Show that the measurement outcome of this circuit can be used to estimate the overlap $|\langle\psi|\phi\rangle|$.

Problem 2 (30 pts)

Write a brief proposal (less than one page) for your final project. You may choose one of two options; the specific proposal requirements for each option are outlined below

1. *Original research for an open problem*
    - Clearly state the problem you aim to solve. The problem should be theoretical.
    - Justify that it remains open (e.g., referring to a recent paper that states it as an open problem).
    - List some possible approaches that might lead to progress and that you may want to try.
2. *Technical review for a recent result*
    - Your review should focus on a theoretical topic. Clearly state this topic in your proposal.
    - List the paper whose techniques you plan to review in your project. Briefly state the main result of this paper in the proposal.
    - List closely related references that you plan to discuss and compare **in depth** in your review.