Math642 Introduction to Machine Learning
Spring 2020

Final Project

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Introduction

Online auction has been one of the most popular methods to buy and sell items online. It is part of the growing body of eCommerce system and can be further apply to other competitive markets. EBay, Yahoo! Auctions, Amazon Marketplace have started this auction methods a long time ago, recent year there are new sites who focus on creating a second-hand and sustainable environment also adopted this bidding system. The activities of online auction produce a large number of bidding histories and transaction data that can be utilized to provide insights into e-commerce system.

Forecasting the auction outcome is beneficial to all auction parties. Bidders can use price forecasts to make more informed bidding decisions based on their budgets. Since it's difficult for a bidder to monitor all the active auctions, it might be difficult for a bidder to decide which of these numerous auctions to participate in and to place a bid that does not end up overpaid or failed to win the auction.

Sellers can use price forecasts to determine when to post their listing. There are companies that re-sell materials on eBay could also make use of price forecasting. These stores sell materials on behalf of individuals who do not want to use eBay directly. Ongoing price forecasting would allow these stores to pay their customers for their merchandise before the close of the auction, in effect offering faster liquidation for their customers.

Online auction platform providers and auction houses can use forecasts for long-term budgeting and planning purposes or even real-time adjustments during the bidding process. Another way to think in the online setting is that a dynamic price scoring would allow auctions to be ranked by lowest expected price, which would in turn allow purchasers to focus their time and energy on just those few auctions that promise the lowest price. Thus, a well-functioning forecasting system could be adaptive to accommodate and incorporate change in the dynamic setting.

The study of the mechanics of online auction is not only useful in respect to the existing online auction platforms, but also can be applicable to other emerging markets such as search driven online advertising business. By 2018, more than 80% of digital display ads are bought in the form of real-time bidding. Thus, understanding the challenge and of online auction data provide practical solutions for these problems may help us design a better and more efficient programmatic bidding system.

Problem Description

Modelling and predicting the bidding price of online auction in real-time can be a challenging task. The price dynamics change throughout the auction and directly reflect the bidding behavior of all parties participated.

First of all, the online auction data has a relatively complicated structure. The dataset contains two different types of data, the bid history and the auction related information. The bid history is a sequence of bids placed over time, which can be considered as a time series type of data. One the other hand, the auction related information is a cross-sectional type of data. It contains product information, information about the bidders and the sellers and auction type information that does not change during the auction. Thus, creating a model that can capture both temporal and static information is challenging.

Moreover, most of the time series methods typically assume that events arrive at equally spaced time intervals, for example in stock market, the timestamp of price is placed equal length. However, in the case for online auction data, the timestamp is generated by user activities, where events are not equally spaced. Therefore, each auction has different staring times, different

ending times, different durations and different bidding decisions are recorded. This is the first time I tried to handle a dataset like this.

The main challenge I encounter in this project is defining the machine learning object of my model. Although it's our initiation to predict the end-price of an auction, which can be done easily using given information on its category, starting bid value and so on, the real challenge here is how can we predict the future price with ongoing information. In this project I am trying to build a model is capable for real-time predicting along with the auction. I will address more on this issue in the next section.

Objective and Methods

The main goal is to develop a model that can predict the auction outcomes, that is the auction price at a future time (T+1) using information from the present (T) and the past (T-1, T-2,…,1). Assuming the estimated model parameters did not change from time T-1 to time T, at time T (present), the model can make a prediction about the future price at time T + 1.

First I consider static models (information given before the start of the auction) that predicting the end-price of an auction using multiple linear regression, KNN and XGboost. Then I have generated some new variables to capture the price dynamics, using the information at present time T to predict the future time T+1. The final approach involves information from a period of time (T, T-1, T-2 etc.) by implementing LSTM neural network to predict the future price(T+1).

Since predicting future price is a regression problem instead of a classification problem, all model performances are evaluated by RMSE (Mean Absolute Error).

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n} |\mathrm{yi} - \hat{\mathrm{y}}\mathrm{i}|}{n}$$

The smaller the mean absolute error, the better the model predicts.

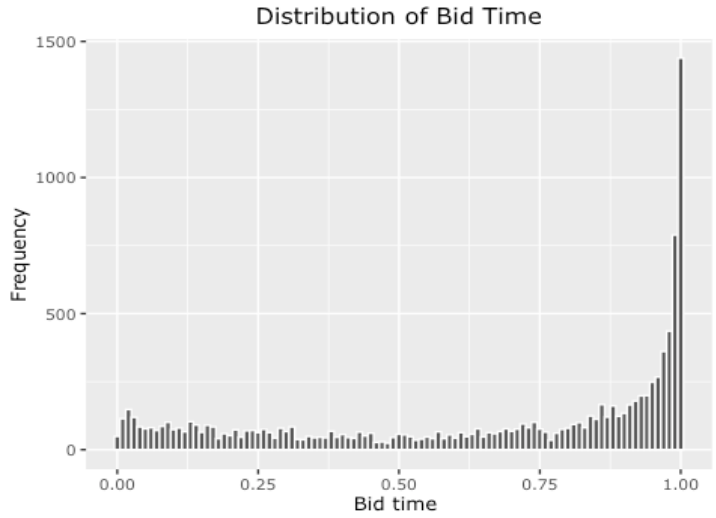
Data Description

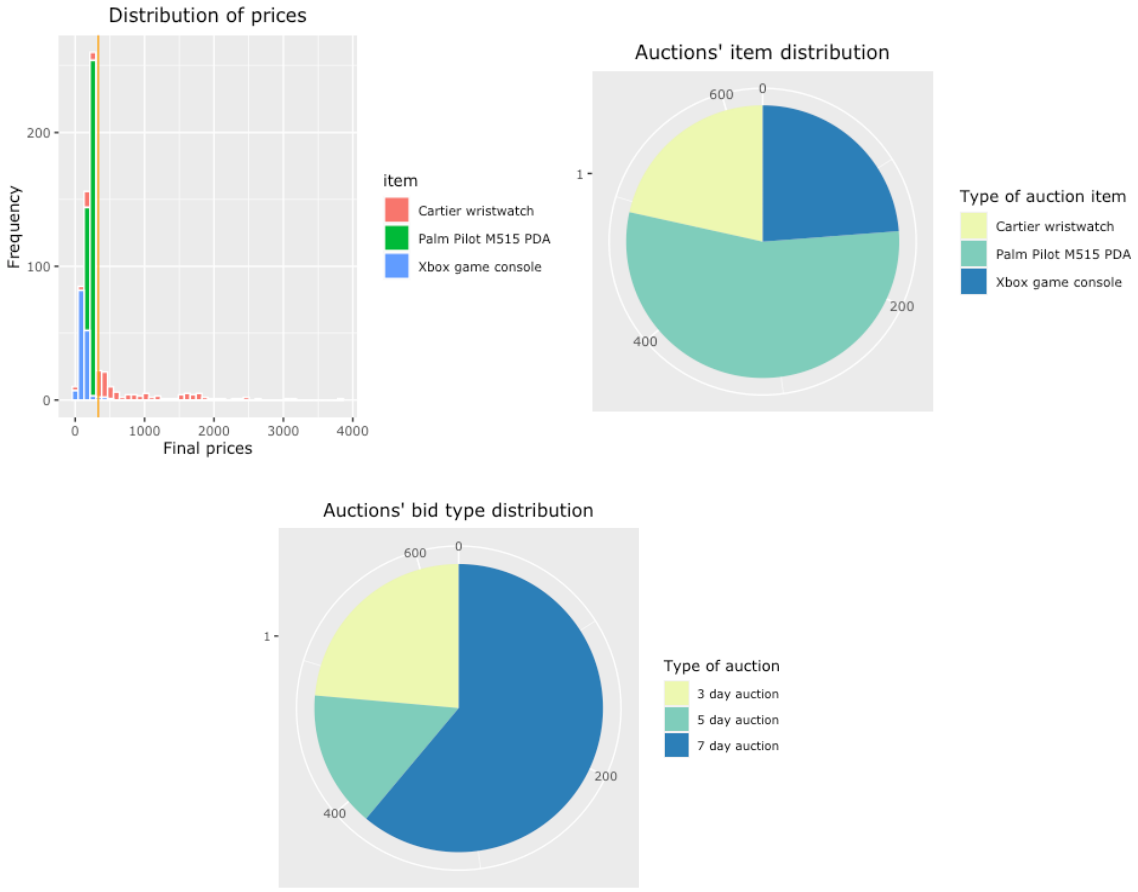
The original data set is from Kaggle (https://www.kaggle.com/onlineauctions/online-auctions-dataset), which was collected from Ebay for the book Modeling Online Auctions, by

Wolfgang Jank and Galit Shmueli (Wiley and Sons, ISBN: 978-0-470-47565-2, July 2010). The dataset includes 10681 observations and 9 variables.

auctionid	the unique identifier of an auction
bid	the proxy bid placed by a bidder
bidtime	the time in days at which the bid was placed (from the start of the auction)
bidder	username of the bidder
bidderrate	bidder's rating
openbid	the opening bid (set by the seller)
price	the price that the item was sold for (equivalent to the second highest bid + an increment)

It includes the complete bidding records for 627 auctions on Cartier wristwatches, Palm Pilot M515 PDAs, Xbox game consoles. The auction durations vary from 3 days to 7 days. While analyzing the original dataset, I found that the last bid is not necessarily the highest bid, nor the last bidder is necessarily the one who win the auction. It's important to understand how eBay auction policy works before conducting further analysis.





People have to bid higher than current price, which is the second highest bid increment. If someone places a bid much higher than current price, eBay will only use part of his/her bid (second highest bid increment) as the new current price. People can bid as long as their bid is higher than current price. Therefore, as bid time increases, we sometimes see bids that are lower than previous bids. People will be notified when they are outbid, so they may increase their max bids to remain the higher bidder. So the real-time price one sees on the website do not directly reflect the latest bid, and it could be misleading to using the bidding value as the real-time auction price.

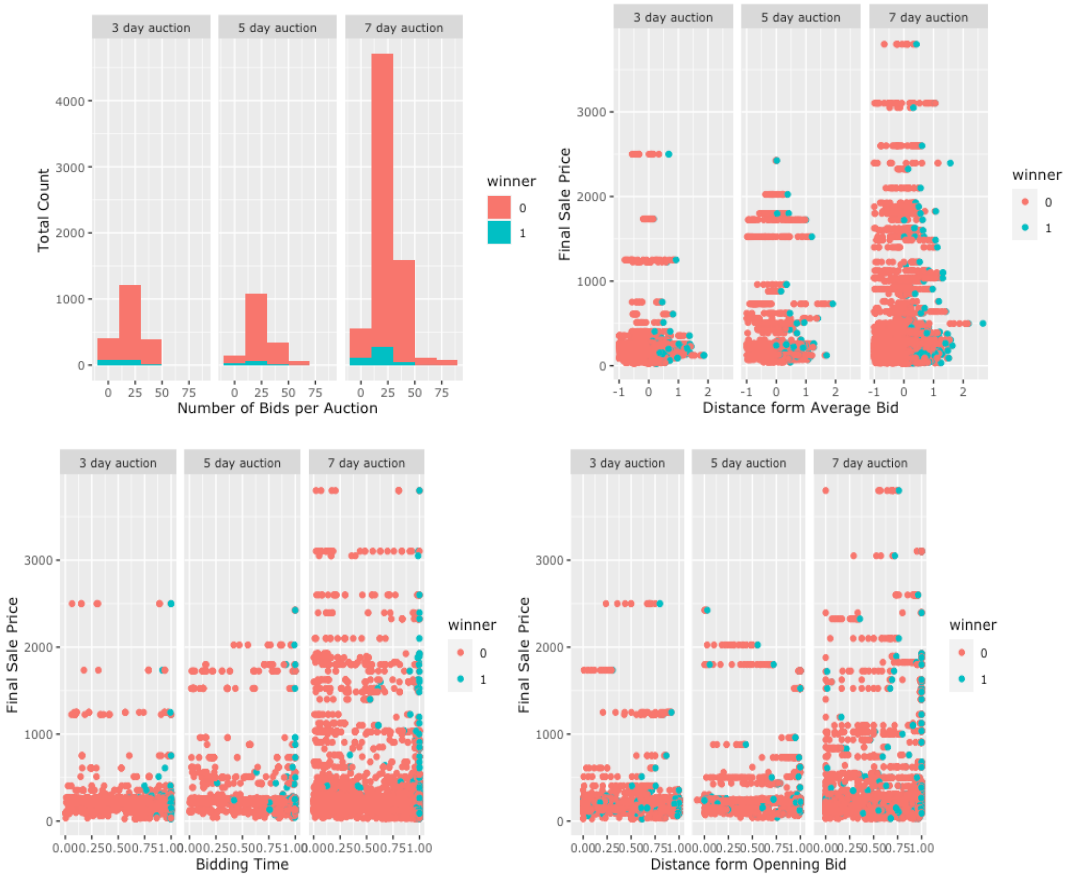
To simulate the real-time auction price, I create a function following the logic of EBay auction displaying policy and transformed a sequence of bidding values to the sequence of 'current price' that could have been displayed on the website. The method is described in the book *Modeling Online Auctions*.

Since the auctions has different auction durations, I normalized the bid time by dividing the bid time over the auction length. So that the sample auctions are on the same time scale from

0 to 1. This also makes it easier to add features such as the bid distance from the open bid and the bid distance from the average bid.

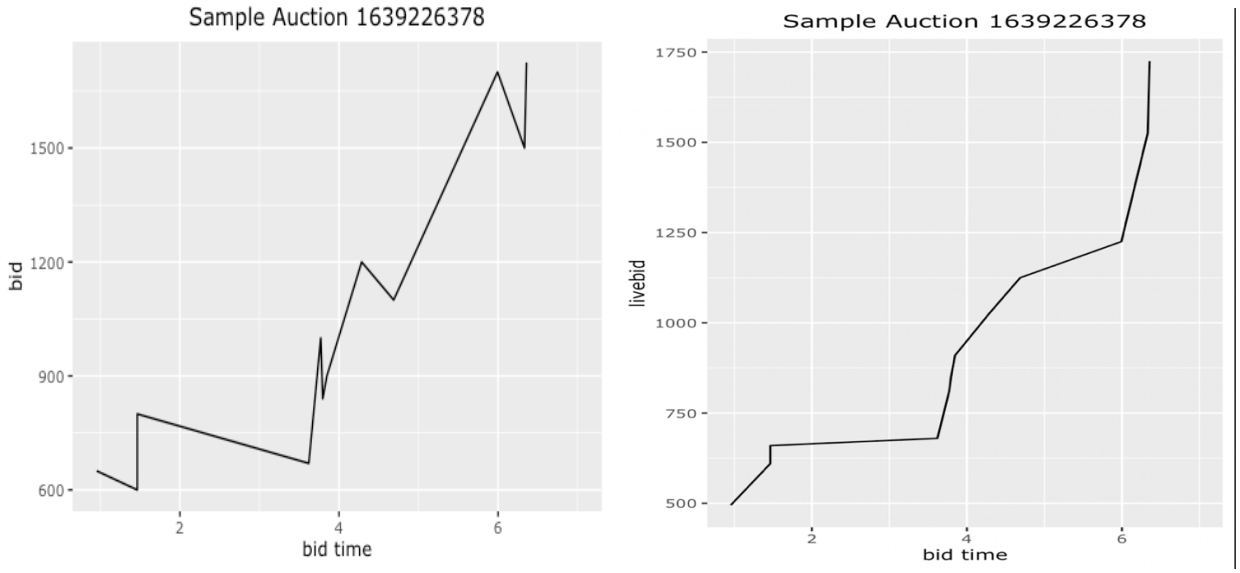
For each auction, I add the number of bidders, average bidding value, the standard deviation of the bidding value, the bid time difference and the bid value difference between each bid and the price velocity to capture the on-going features of an auction procedure.

Finally, I transformed the characteristic variables to dummy variables to keep the text information. The dataset after the data manipulation and cleaning the outliers, turns out to have 10677 observations and 30 variables. The distribution of the end-price is affected a lot by the product type, the distribution of the frequency of the bid time increases drastically for all auctions despite the different auction lengths. As I have explained before, the last few bids are not necessarily the winning bid. However, looking at the distribution of the winning bids, we can find that the winning bids are most likely to appear at the last 75% of the auction duration.



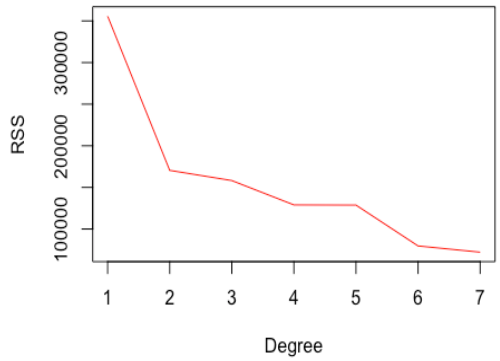
1. Modeling Individual Auctions

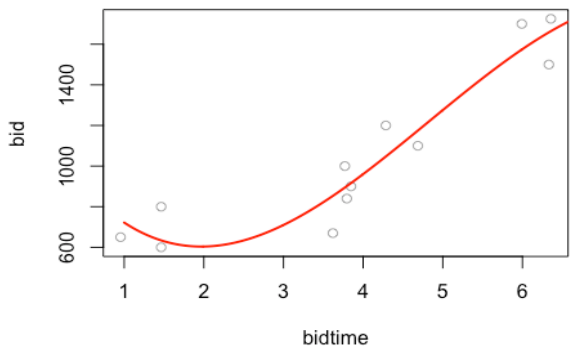
The plot shows an example of the raw bidding history of a sample auction and the recreated livebid of the same auction. We can see that the livebid is a non-decreasing curve that represents the auction price that was displaying on the auction website during the auction. Since the for the auction data set the timestamp is generated by user activities, where events are not equally spaced, my intension here is to use the unsupervised modeling methods to turn a discrete time series to a continuous function so that I can recreate equal length time series for each auction. In this section I applied polynomial function, regression spline, smoothing spline and GAM to model an individual auction.



i. Polynomial Fits

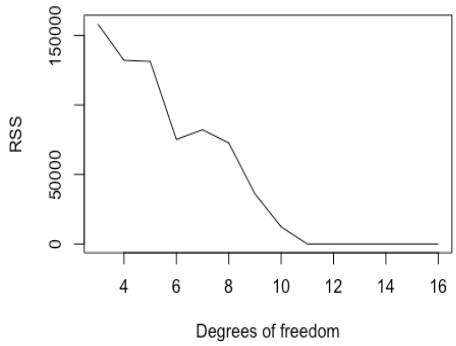
I applied the polynomial fit for the current price using the bid time. The RSS decreases as the degree of the polynomial decreases. It achieves its minimum at degree 7. However, it diverges as the degree of polynomial increases. Thus, from the plot we choose the degree of 3. All three degrees are significant with a p-value of 0.0001607.

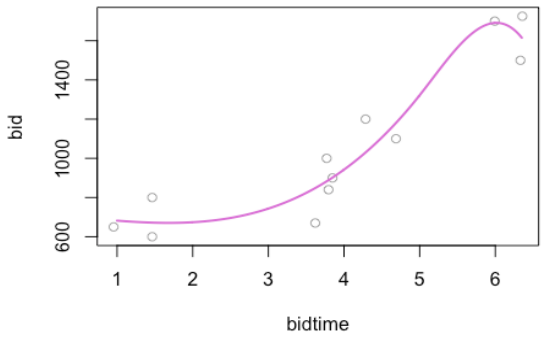




ii. Smoothing splines

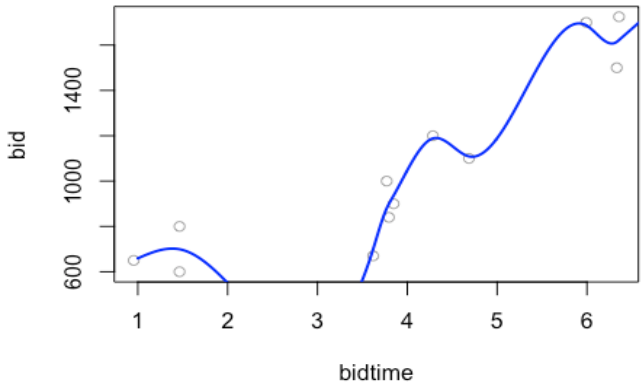
We want to find some function that fit a smooth curve to the set of data by automatically choosing the smoothing parameter λ. Here I applied smoothing spline to this sample data for a range of degrees of freedom and plot the resulting fits. RSS decreases as the degrees of freedom increases at 6. Then I select the smoothness level by cross-validation; this results in a value of λ that yields 7 degrees of freedom. The regression fits the data well with a p-value of 0.0005362.





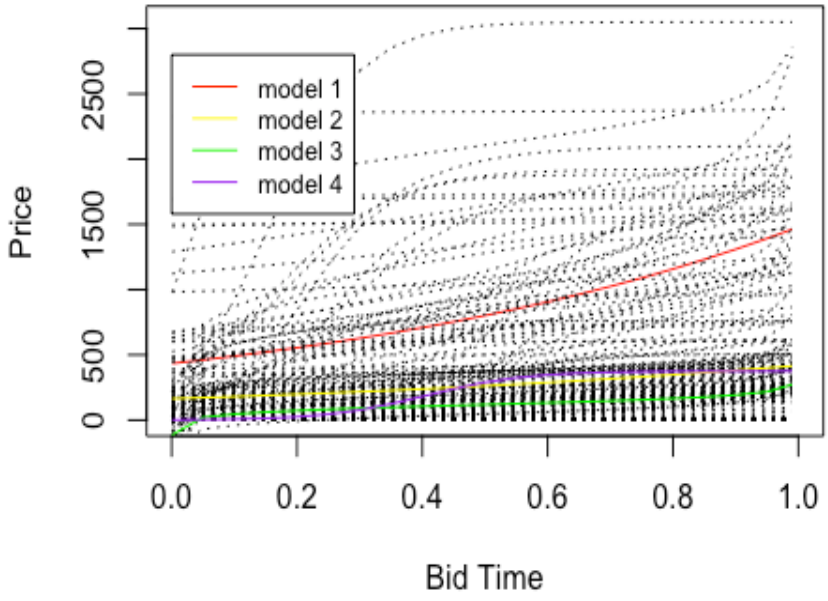
iii. GAM

The method of GAM allows us to fit a non-linear model to the set of data, so that we can automatically model non-linear relationships that standard linear regression will miss without manually try out many different transformations on each variable individually. The GAM fit is said to be more flexible than the Splines. Here for this sample set it returns a flexible curve with 11 degrees of freedom, which might results in overfitting.



2. Modeling Multiple Auctions

Since the auction dataset does not provide me with an actual time, for my study I assumed that these auctions happened simultaneously so that I can use all of the auctions as my sample data. I normalized the bidding time and the auction length so that all of the 628 auctions can be displayed on the same time scale. I fit the live bidding history for each auction with the unsurprised models I described above and classified the 628 models into 4 groups based on the smallest SSE. The method has been discussed in detail in the Chapter 4 of the book *Modeling Online Auctions*. I could have applied k-nearest neighbors but since creating a distance matrix for a time series is relatively challenging, I think I could complete this idea in the future work.




```
table(models)
```

```
## models
##   1   2   3   4
## 301   5  86 212
```

Supervised Modeling of Online Auction

1. Predicting the end price with Static Model

In this section I could discuss how I build models to predict the end price of the item would be auctioned for. I implement machine learning methods such as Multiple Linear Regression and KNN.

Since the auction dataset is sequenced by individual auctions and bidding history for each auction, random sampling for the entire observations does not make sense. Thus, I grouped bidding history information by auction id and split the 627 unique auctions into training and testing set. We have 369 auctions with 7557 observations in the training dataset and 258 auctions with 3120 observations in the testing dataset.

For the baseline model I use the static features, that is the information are given before a certain auction starts and do not change during the auction. The static information includes:

- opening price of the current auctions

- duration of the current auctions

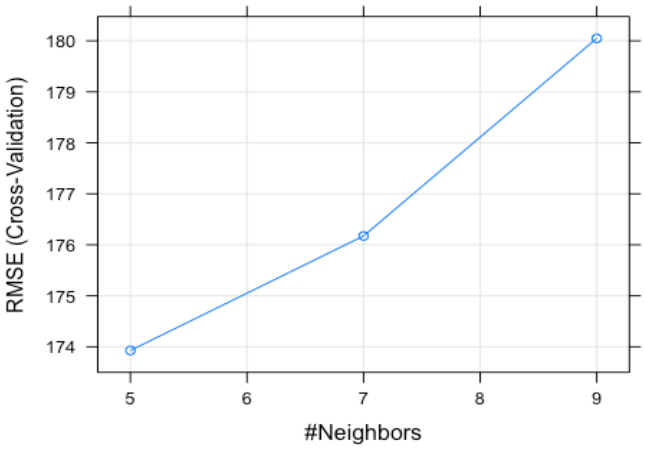
- sellers' ratting of the current auctions

I trained the multiple linear model with the training data and 10-folds validation method.

	RMSE	Rsquared	MAE
Training	268.2168	0.5408452	132.4348
Testing	303.4196504	0.3509475	184.0715687

For the KNN model, the k was chosen automatically by the cross-validation. RMSE was used to select the optimal model using the smallest value. The final value used for the model was k = 5.

k	RMSE	RSquared	MAE
5	203.5048	0.7333769	76.95662
7	208.5114	0.7198521	82.10603
9	217.1711	0.6960207	87.56857

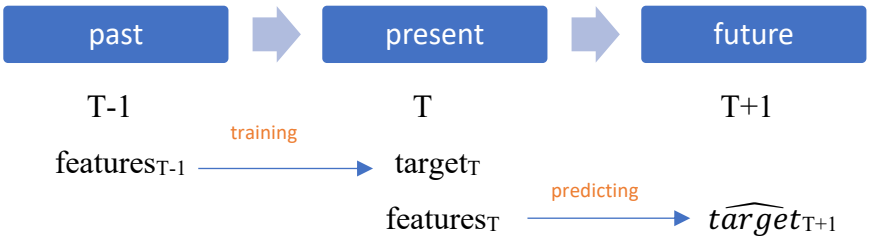


With k=5 the KNN model prediction on the testing set gives RMSE of 95.92843 and MAE of 67.50273, which is better than the multiple linear regression model. We can see that with limited information, predicting the end-price of an auction is quite difficult.

2. Predicting T+1 price with Static Model

We want to build a model that can predict the auction outcomes, that is the auction price at a future time (T+1) using information from the present (T) and the past (T-1, T-2,…,1). Assuming the estimated model parameters did not change from time T-1 to time T, at time T (present), the model can make a prediction about the future price at time T + 1.

A simple illustrate of the structure of the model would be:

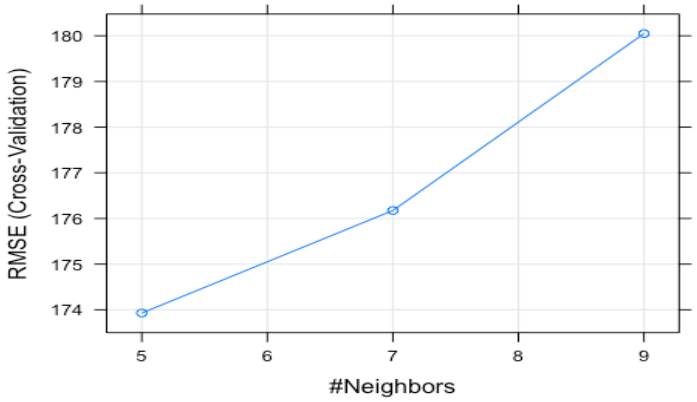


In order to follow the dynamic model scheme of the forecasting model, we want to prepare a training dataset that contains features available at T-1, and target, that is the livebid price at T.

BY changing the predicting target, using the same static features and the same modeling methods as in the previous section. The results remains the same with a slightly smaller MAE.

	RMSE	Rsquared	MAE
Training	210.1425	0.4695338	112.7726
Testing	232.015148	0.436801	110.296938

k	RMSE	RSquared	MAE
5	173.9282	0.6382034	85.05969
7	176.1739	0.6288147	87.21982
9	180.0484	0.6126760	89.90720



3. Predicting T+1 price with Dynamic Model

To capture the process of bidding I created a livebid with a 1 day lag as a feature for the prediction. The dynamic features also includes the number of bidders along the auction, average bidding value, the standard deviation of the bidding value, the bid time difference and the bid value difference between each bid and the price velocity to capture the on-going features of an auction procedure. Adding this evolving information, we can create several dynamic models.

LM

	RMSE	Rsquared	MAE
Training	210.1425	0.4695338	112.7726
Testing	96.08527	0.436801	67.37456

KNN

	RMSE	RSquared	MAE
Training	173.9282	0.6382034	85.05969
Testing	181.56717	0.6288147	85.78306

XGBOOST

```
     iter train_rmse
  1:    1   93.70865
  2:    2   75.34668
  3:    3   66.75747
  4:    4   60.80652
  5:    5   56.34924
 ---
496:  496   35.94705
497:  497   35.94705
498:  498   35.94699
499:  499   35.94699
500:  500   35.94699
```

	RMSE	RSquared	MAE
Testing	34.1539519	0.9766968	11.8855213

With 500 iterations, the XGboost linear predictor gives the lowest testing MAE.

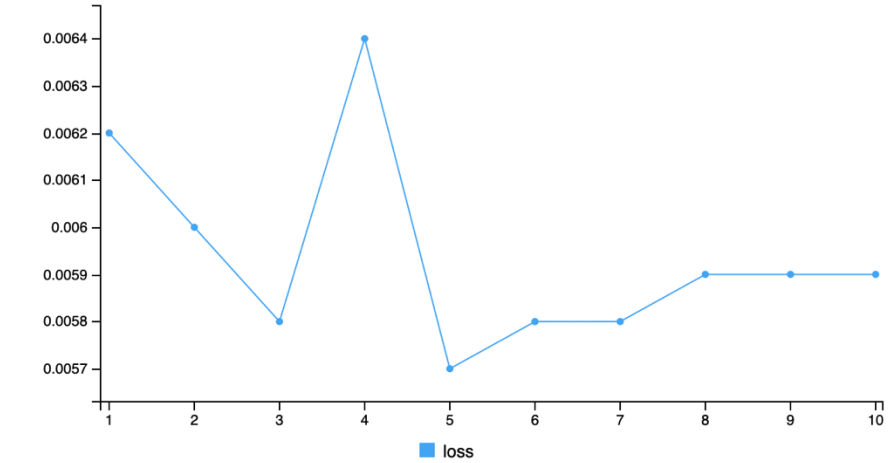
LSTM Neural Network

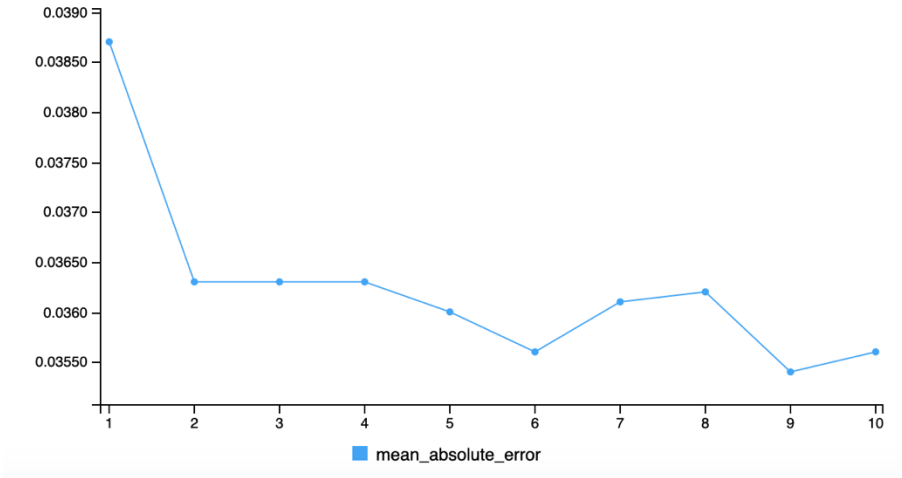
My final approach for this project is Long Short-Term Memory (LSTM) neural network. LSTM is a specific type of recurrent neural network which is more adequate when it comes to modeling long-range dependencies. The dynamic models from previous section only use the information from the T-1. The memory is very short. Any value that is output in one-time step becomes input in the next, but unless that same value is output again, it is lost at the next tick. To solve this, we can form a neural network with a Long Short-Term Memory (LSTM) layer. The LSTM introduces a "memory cell" value that is passed down for multiple time steps and being sent in parallel with the activation output.

We can set a time window, a period of time to look back and predict the next target. The neural network I built has three layers, the first one is a LSTM layer, the second is a dense layer

with 32 knot and the third layer is a dropout layer to prevent overfitting and the fourth layer is a dense layer for the output.

```
Model: "sequential_88"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
lstm_77 (LSTM)               (1, 128)                  66560
_________________________________________________________________
dense_115 (Dense)            (1, 32)                   4128
_________________________________________________________________
dropout_28 (Dropout)         (1, 32)                   0
_________________________________________________________________
dense_116 (Dense)            (1, 1)                    33
=================================================================
Total params: 70,721
Trainable params: 70,721
Non-trainable params: 0
_________________________________________________________________
```





	RMSE	RSquared	MAE
Testing	223.4743	NA	112.7578

Just like in any other neural network model, we have to rescale the input data to the range of the activation function. The default activation function for LSTM is sigmoid function whose range is [-1, 1]. Since the input of the neural network has to be normalized, the training error was very small. To compare the LSTM performance with other models, I revert the predicted values to the original scale and the MAE is 112.7578.

Conclusions

In this project I have implement several different machine learning methods to the online auction dataset. Just like other real-world data, the online auction dataset is a combination of time series type of data and cross-sectional type of data. So when building the learning models, there are many things to consider.

Our baseline model using Multiple Linear Regression models to predict the end-price with static features did not perform satisfactorily with a testing MAE of 184.0715687. Adding the on-going information such as the current price of concurrent auctions or the number of bidders in concurrent auctions to the model makes the performance much more accurate, where the time series played an importance role. With these dynamic features adding to the model, the testing MAE can be reduced to 11.8855213. The LSTM Neural Network did not return the best result part of the reason might be the insufficient amount of data that I have used for training. The unequal spaced time stamp might influence the result as well.

Future Works

An ensemble of unsupervised clustering results and linear regression might have higher predictive capabilities, since the method of XGboost linear predictor also combines the tree classification models and the linear models.

My future works also includes the practical application of auction predicting models that helps the buyers making bidding decisions. Making bidding is further complicated since there existence of many auctions that offer the same or similar item simultaneously. Thus we might need to consider the competitive bidding behaviors as well.

Final_Project_FyonaSun

Fyona Sun

2/10/2020

```
library(readr)
auction=read_csv('./online-auctions-dataset/auction.csv')
```

```
## Parsed with column specification:
## cols(
##   auctionid = col_double(),
##   bid = col_double(),
##   bidtime = col_double(),
##   bidder = col_character(),
##   bidderrate = col_double(),
##   openbid = col_double(),
##   price = col_double(),
##   item = col_character(),
##   auction_type = col_character()
## )
```

```
head(auction)
```

```
## # A tibble: 6 x 9
##    auctionid   bid bidtime bidder   bidderrate openbid price item   auctio
n_type
##        <dbl> <dbl>   <dbl> <chr>         <dbl>   <dbl> <dbl> <chr>  <chr>

## 1 1638893549  175    2.23  schaden…          0      99  178. Carti… 3 day
aucti…
## 2 1638893549  100    2.60  chuik              0      99  178. Carti… 3 day
aucti…
## 3 1638893549  120    2.60  kiwisst…           2      99  178. Carti… 3 day
aucti…
## 4 1638893549  150    2.60  kiwisst…           2      99  178. Carti… 3 day
aucti…
## 5 1638893549  178.   2.91  eli.fli…           4      99  178. Carti… 3 day
aucti…
## 6 1639453840    1    0.356 bfalconb           2       1  355  Carti… 3 day
aucti…
```

```
dim(auction)
```

```
## [1] 10681     9
```

```
sapply(auction, class)
```

```
##    auctionid          bid      bidtime       bidder   bidderrate      open
bid
##    "numeric"    "numeric"    "numeric"  "character"    "numeric"    "numer
ic"
##        price         item auction_type
##    "numeric"  "character"  "character"
```

The data set includes 10681 observations and 9 variables auctionid: unique identifier of an auction bid: the proxy bid placed by a bidder bidtime: the time in days that the bid was placed, from the start of the auction bidder: eBay username of the bidder bidderrate: eBay feedback rating of the bidder openbid: the opening bid set by the seller price: the closing price that the item sold for (equivalent to the second highest bid + an increment) item: auction item auction_type

Understanding eBay auction policy It's important to understand how eBay auction policy works before conducting further analysis. I found this explanation easy to understand.

The website helps to answer the following questions:

What's the relationship between bids and bidtime? Poeple have to bid higher than current price, which is the second highest bid+increment. If someone places a bid much higher than current price, eBay will only use part of his/her bid(second highest bid+increment) as the new current price. People can bid as long as their bid is higher than current price. Therefore, as bidtime increases, we sometimes see bids that are lower than previous bids. People will be notified when they are outbid, so they may increase their max bids to remain the higher bidder.

What's the relationship between bids and closing price? Closing price=second highest bid+increment or highest bid in some cases when second highest bid+increment is greater than the thrid highest bid. For example, if third highest bid is 8 and second highest bid is 10, you can bid 10.01 and if no one's bid is higher than yours, you get the item by paying 10.01 instead of 10+increment(which is 10.5)

Person who placed the highest bid gets the item but how much he/she needs to pay in most cases depends on the second highest bid. ## Exploring the dataset

```
## function bid.incr(price)
bid.incr <-
       function(price)
           { bidint <- c(0,0.99,4.99,24.99,99.99,249.99,499.99,999.99,2499.9
9,4999.99)
             ## interval of prices
             inc <- c(0.05,0.25,0.5,1,2.5,5,10,25,50,100)
             ## increments of price for particular interval
             inc[findInterval(price,bidint)]
           }
############################################################
##########            make livebids              ###########
############################################################
```

```
## function recover.livebids(bids,bidder,openbid,winningbid)
## This function reconstructs the sequence of livebids by
## inputing the raw bids,bidder IDs and opening bid

recover.livebids <-
       function(bids,bidder,openbid,winningbid)
           { nbid <- length(bids)
             if (nbid==1) { live <- openbid
                            live
                           }
             else {
                   if (nbid==2) {live <- numeric(nbid)
                                 live[1] <- openbid
                                 live[nbid] <- winningbid
                                 live}

                   else { live <- numeric(nbid)
                          maxbid <- bids[1]
                          live[1] <- openbid
                          live[nbid] <- winningbid
                          currentprice <- bids[1]
                          maxbidder <- bidder[1]
                          for (i in 2:(nbid-1))
                                { newbid <- bids[i]
                                  newbidder <- bidder[i]
                                  if (newbid > maxbid) { if (!(identical(newb
idder,maxbidder)))
                                                           { incr <- bid.in
cr(maxbid)
                                                             if (newbid < m
axbid+incr)  currentprice <- newbid
                                                             else currentpr
ice <- maxbid+incr
                                                             maxbid <- newb
id
                                                             maxbidder <- n
ewbidder
                                                            }
                                                       else {maxbid <- newb
id
                                                             maxbidder <- m
axbidder
                                                             }
                                                     }
                                  else {if (newbid < maxbid) { incr <- bid.in
cr(newbid)
                                                             if (newbid+inc
r>=maxbid) currentprice <- maxbid
                                                             else currentpr
ice <- newbid+incr
```

```
                                                                    maxbid <- maxb
id
                                                                    maxbidder <- m
axbidder
                                                                  }
                                                 else { if (!(identical(newbidder,max
bidder)))

{currentprice <- maxbid

maxbid <- maxbid

maxbidder <- maxbidder
                                                                                  }
                                                       else {currentprice <- current
price
                                                             maxbid <- maxbid
                                                             maxbidder <- maxbidder
                                                             }

                                                    }
                                                }
                                    live[i] <- currentprice
                                  }
                           live
                          }
                  }
              }

auction=read_csv('./online-auctions-dataset/auction.csv')

## Parsed with column specification:
## cols(
##   auctionid = col_double(),
##   bid = col_double(),
##   bidtime = col_double(),
##   bidder = col_character(),
##   bidderrate = col_double(),
##   openbid = col_double(),
##   price = col_double(),
##   item = col_character(),
##   auction_type = col_character()
## )

swarovski=read_csv('./online-auctions-dataset/swarovski.csv')

## Parsed with column specification:
## cols(
##   Seller = col_double(),
##   Bidder = col_double(),
##   Weight = col_double(),
```

```
##   Bidder.Volume = col_double(),
##   Seller.Volume = col_double()
## )
```

```
head(auction)
```

```
## # A tibble: 6 x 9
##    auctionid   bid bidtime bidder   bidderrate openbid price item   auctio
n_type
##        <dbl> <dbl>   <dbl> <chr>         <dbl>   <dbl> <dbl> <chr>  <chr>

## 1 1638893549  175    2.23  schaden…          0      99  178. Carti… 3 day
aucti…
## 2 1638893549  100    2.60  chuik              0      99  178. Carti… 3 day
aucti…
## 3 1638893549  120    2.60  kiwisst…           2      99  178. Carti… 3 day
aucti…
## 4 1638893549  150    2.60  kiwisst…           2      99  178. Carti… 3 day
aucti…
## 5 1638893549  178.   2.91  eli.fli…           4      99  178. Carti… 3 day
aucti…
## 6 1639453840    1    0.356 bfalconb           2       1  355  Carti… 3 day
aucti…
```

```
head(swarovski)
```

```
## # A tibble: 6 x 5
##      Seller    Bidder Weight Bidder.Volume Seller.Volume
##       <dbl>     <dbl>  <dbl>         <dbl>         <dbl>
## 1 332874919 718577508      2             3           547
## 2 594667804 399983466      5             6           183
## 3 663070601 655828811      1             4           274
## 4 309608641 599835541      3             8          3986
## 5 201729374 693022555      1             2          4681
## 6 332874919 622536435      2             4           547
```

```
dim(auction)
```

```
## [1] 10681     9
```

```
dim(swarovski)
```

```
## [1] 200   5
```

```
summary(auction)
```

```
##    auctionid              bid              bidtime            bidder

##  Min.   :1.639e+09   Min.   :   0.01   Min.   :0.000567   Length:10681

##  1st Qu.:3.015e+09   1st Qu.:  72.00   1st Qu.:1.949931   Class :character
```

```
##  Median :3.021e+09   Median : 140.00   Median :4.140833   Mode  :character

##  Mean   :4.136e+09   Mean   : 207.59   Mean   :3.979628

##  3rd Qu.:8.212e+09   3rd Qu.: 210.00   3rd Qu.:6.448060

##  Max.   :8.216e+09   Max.   :5400.00   Max.   :6.999990

##
```

```
##    bidderrate         openbid            price            item
##  Min.   :  -4.00   Min.   :   0.01   Min.   :  26.0   Length:10681
##  1st Qu.:   1.00   1st Qu.:   1.00   1st Qu.: 186.5   Class :character
##  Median :   5.00   Median :   4.99   Median : 228.5   Mode  :character
##  Mean   :  31.94   Mean   :  52.25   Mean   : 335.0
##  3rd Qu.:  21.00   3rd Qu.:  50.00   3rd Qu.: 255.0
##  Max.   :3140.00   Max.   :5000.00   Max.   :5400.0
##  NA's   :11
##  auction_type
##  Length:10681
##  Class :character
##  Mode  :character
##
##
##
##
```

```
summary(swarovski)
```

```
##      Seller              Bidder              Weight       Bidder.Volume
##  Min.   :    24273   Min.   :   221355   Min.   : 1.000   Min.   : 1.00
##  1st Qu.: 94870114   1st Qu.: 80389757   1st Qu.: 1.000   1st Qu.: 4.00
##  Median :201729374   Median :191077522   Median : 2.000   Median : 6.00
##  Mean   :198089751   Mean   :276467688   Mean   : 3.085   Mean   : 9.26
##  3rd Qu.:309608641   3rd Qu.:449791350   3rd Qu.: 3.000   3rd Qu.:11.00
##  Max.   :663070601   Max.   :723365710   Max.   :56.000   Max.   :94.00
##  Seller.Volume
##  Min.   :   2.0
##  1st Qu.: 382.5
##  Median :1791.0
##  Mean   :1966.3
##  3rd Qu.:3986.0
##  Max.   :4681.0
```

```
levels(as.factor(auction$item))
```

```
## [1] "Cartier wristwatch"  "Palm Pilot M515 PDA" "Xbox game console"
```

```
#"Cartier wristwatch"  "Palm Pilot M515 PDA" "Xbox game console"
levels(as.factor(auction$auction_type))
```

```
## [1] "3 day auction" "5 day auction" "7 day auction"

#"3 day auction" "5 day auction" "7 day auction"
```

data manipulation

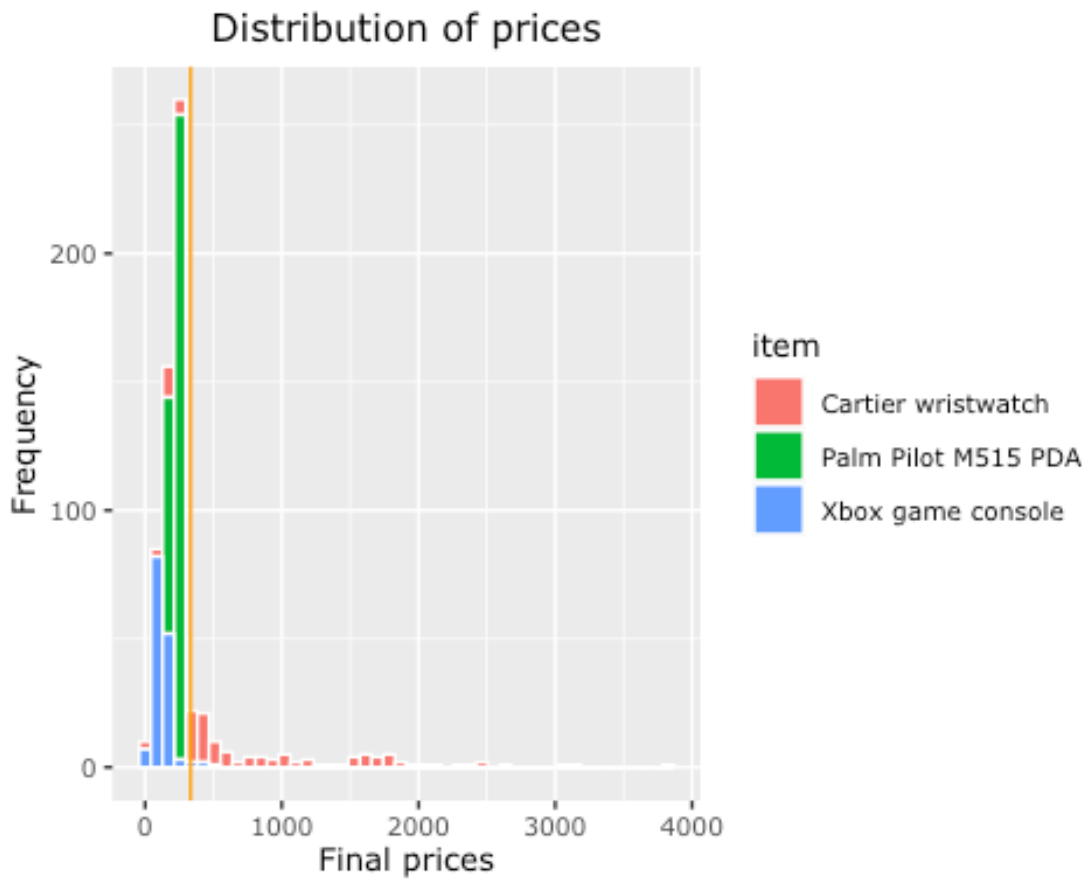
```
library(tidyverse)

## ── Attaching packages ─────────────────────────────────────── tidyverse 1.
3.0 ──

## ✓ ggplot2 3.3.0     ✓ dplyr   0.8.5
## ✓ tibble  2.1.3     ✓ stringr 1.4.0
## ✓ tidyr   1.0.2     ✓ forcats 0.5.0
## ✓ purrr   0.3.3

## ── Conflicts ────────────────────────────────────────── tidyverse_conflict
s() ──
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

library(zoo)

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

#change characteristic varibale to dummy variables
auction$is_3day<- ifelse(auction$auction_type=='3 day auction',1,0)
auction$is_5day<- ifelse(auction$auction_type=='5 day auction',1,0)
auction$is_7day<- ifelse(auction$auction_type=='7 day auction',1,0)
auction$is_catier<- ifelse(auction$item=='Cartier wristwatch',1,0)
auction$is_palm<- ifelse(auction$item=='Palm Pilot M515 PDA',1,0)
auction$is_xbox<- ifelse(auction$item=='Xbox game console',1,0)
#add a few statistical variables
## winning bid, number of bids & average bid per auction
u<-unique(auction$auctionid)
winner<-rep(0,nrow(auction))
livebid<- rep(0,nrow(auction))
average<-rep(0,nrow(auction))
sd<- rep(0,nrow(auction))
num.bids<-rep(0,nrow(auction))

for (i in u){
        indexes<-which(auction$auctionid == i)
        cur.average<-sum(auction$bid[indexes])/length(indexes)
        cur.dev<- sd(auction$bid[indexes])
        current<- recover.livebids(auction$bid[indexes],auction$bidder[indexe
s],auction$openbid[indexes],auction$price[indexes])
```

```
        j<-0
        for (k in indexes){
                j<- j+1
                winner[k]<-ifelse(auction$bid[k]==auction$price[k], 1 , 0)
                num.bids[k]<-length(indexes)
                average[k]<-cur.average
                sd[k]<- cur.dev
                livebid[k]<- current[j]
        }

}

auction$livebid.lag<- na.fill(auction$livebid.lag,0)
```

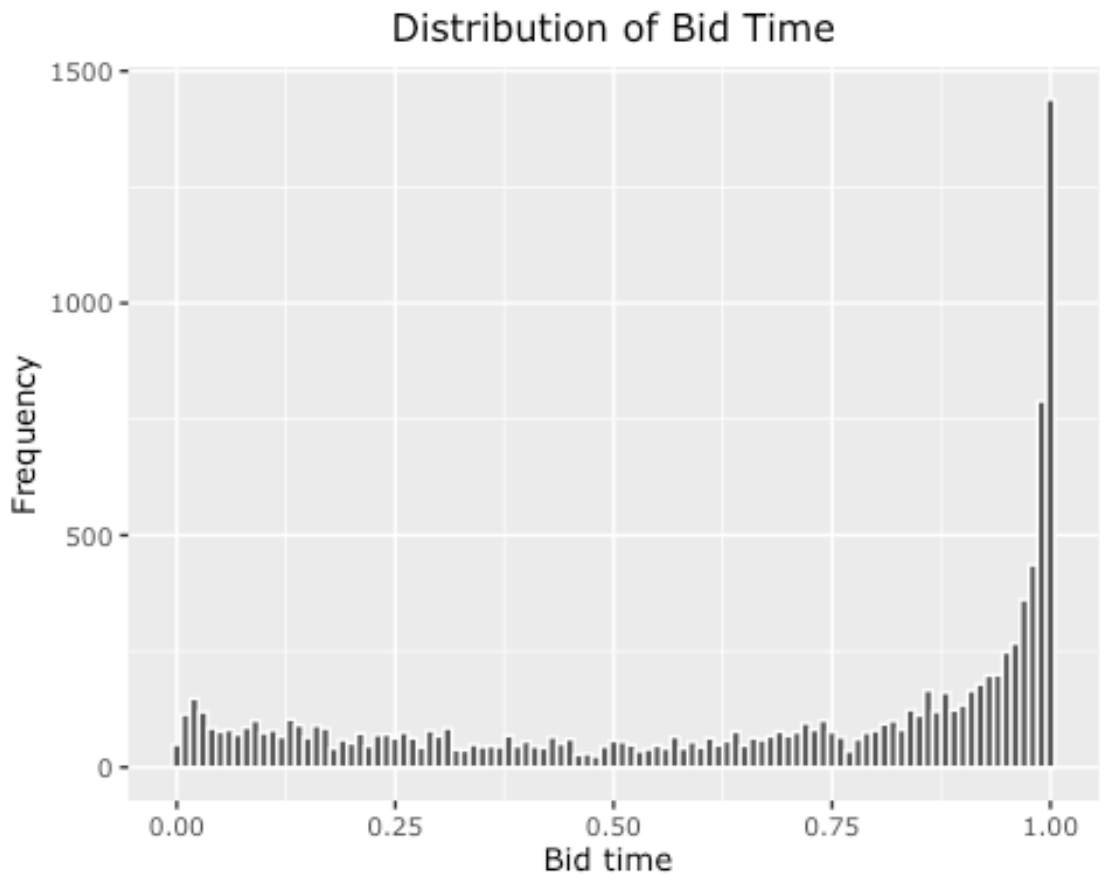
Exploring the data

There are 627 item in the data set in total. Since the bidders do not know what others have bid and might be placing mulptiple bids after being notified that they were outbid, the last bid is not necessaily the one who get the item.

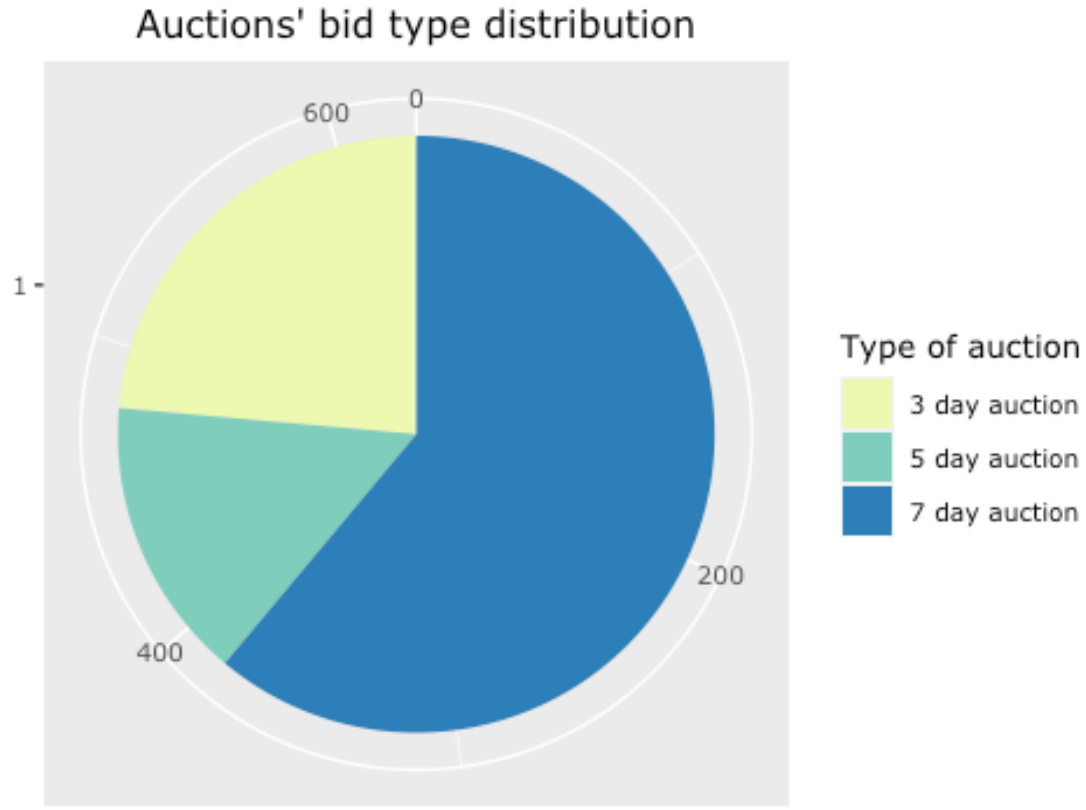
```
library(dplyr)
#get rid of the outlier
#1639672910
auction<-auction[auction$auctionid!="1639672910",]

closing_price<- auction %>% select(auctionid,openbid,price,auction_type,ite
m)%>%
  group_by(auction$auctionid) %>%
  summarise(openbid=max(openbid),price=max(price),auction_type=max(auction_ty
pe),item=max(item))
# text theme for every plot in this project
text_theme <- theme(text = element_text(size = 10,
                                family = "Verdana",
                                face = "plain"),
                    plot.title = element_text(hjust = 0.5))

# distribution of prices
ggplot(closing_price,
       aes(x = price)) +
  geom_histogram(binwidth = 85,
                 aes(fill = item),
                 col = "white") +
  geom_vline(aes(xintercept = mean(auction$price)),
             col = "orange",
             size = 0.5) +
  labs(title = "Distribution of prices",
       x = "Final prices",
       y = "Frequency") +
  text_theme
```



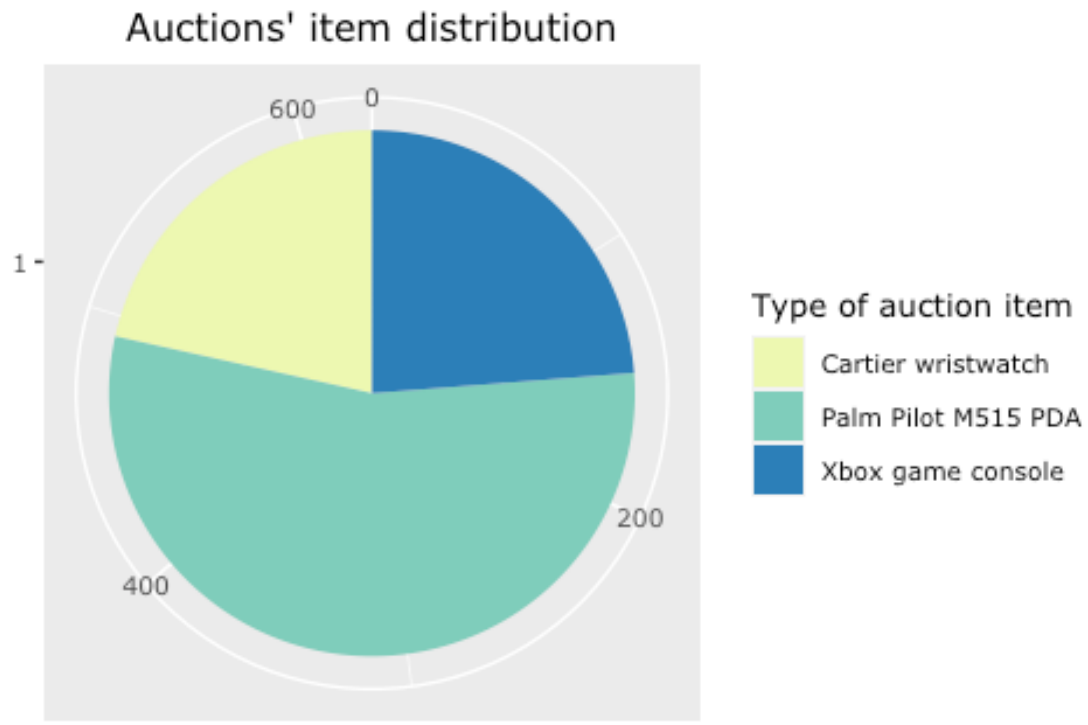

```
ggplot(auction,
       aes(x = time.fraction)) +
  geom_histogram(binwidth = 0.01,
                 col = "white") +
  labs(title = "Distribution of Bid Time",
       x = "Bid time ",
       y = "Frequency") +
  text_theme
```



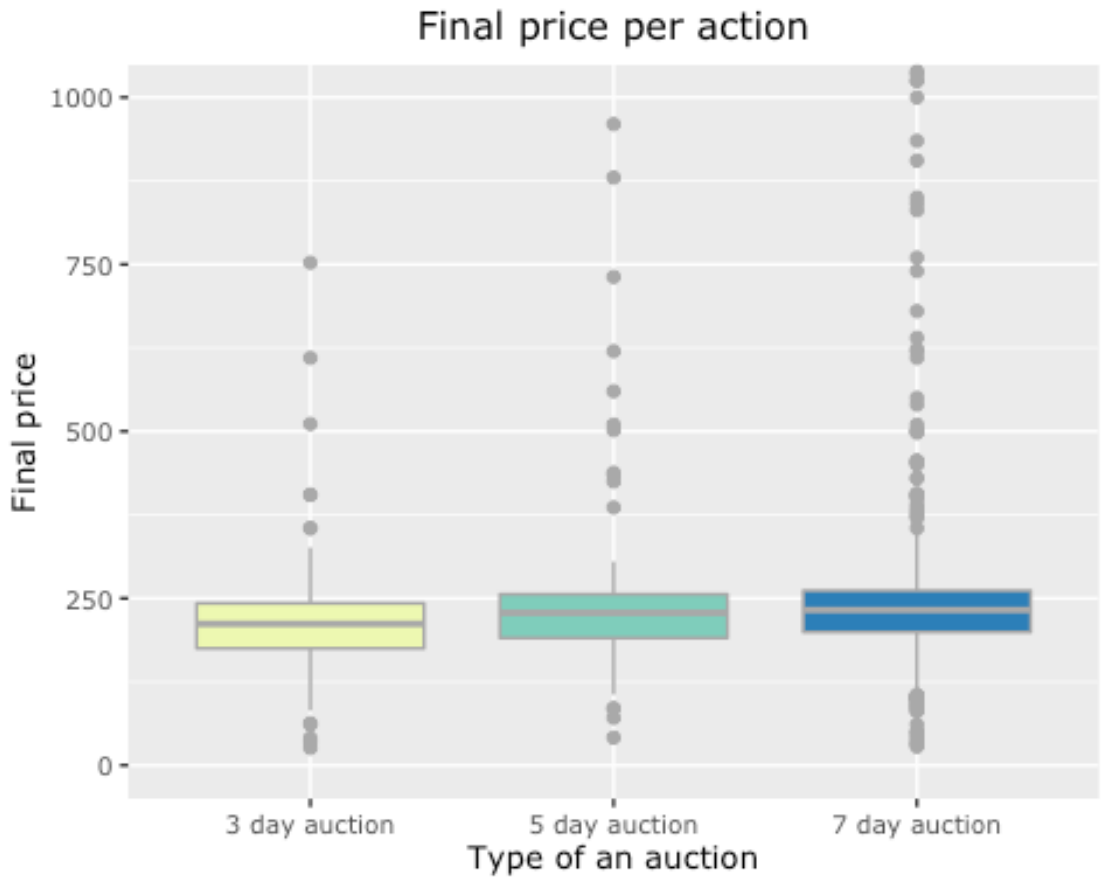

```
# distribution of auction types
ggplot(closing_price,
       aes(x = factor(1),
           fill = closing_price$auction_type)) +
  geom_bar(width = 3) +
  ggtitle("Auctions' bid type distribution") +
  coord_polar(theta = "y") +
  labs(fill= "Type of auction") +
  scale_fill_brewer(palette = "YlGnBu") +
  xlab(NULL) + ylab(NULL) +
  text_theme

## Warning: Use of `closing_price$auction_type` is discouraged. Use `auction_
type`
## instead.
```




```
# few ggplot complements
bar_auction_type <- geom_bar(aes(fill = auction$auction_type),
                             stat = "count")

ggplot(closing_price,
       aes(x = factor(1),
           fill = item)) +
  geom_bar(width = 3) +
  ggtitle("Auctions' item distribution") +
  coord_polar(theta = "y") +
  labs(fill= "Type of auction item") +
  scale_fill_brewer(palette = "YlGnBu") +
  xlab(NULL) + ylab(NULL) +
  text_theme
```




```
# few ggplot complements
bar_auction_type <- geom_bar(aes(fill = auction$auction_type),
                             stat = "count")

# relationship between final price per auction and auction type
ggplot(closing_price,
       aes(x = auction_type, y = price)) +
  geom_boxplot(aes(fill = auction_type),
           color = "darkgrey")  +
  labs(title = "Final price per action",
       x = "Type of an auction",
       y = "Final price") +
  guides(fill = F) +
  scale_fill_brewer(palette = "YlGnBu") +
  text_theme +
  coord_cartesian(ylim = c(0, 1000)) # few outliers masked
```



data manipulation

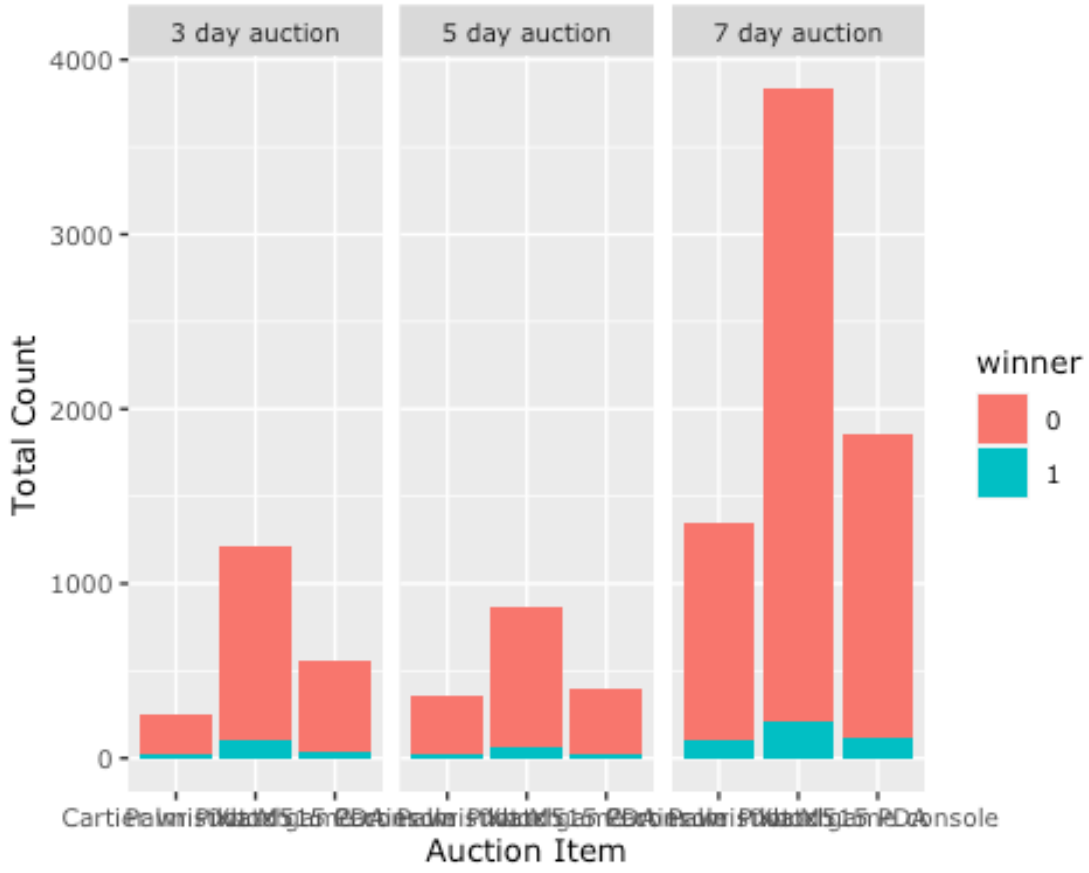
```
library(tidyverse)
library(zoo)
#change characteristic varibale to dummy variables
auction$is_3day<- ifelse(auction$auction_type=='3 day auction',1,0)
auction$is_5day<- ifelse(auction$auction_type=='5 day auction',1,0)
auction$is_7day<- ifelse(auction$auction_type=='7 day auction',1,0)
auction$is_catier<- ifelse(auction$item=='Cartier wristwatch',1,0)
auction$is_palm<- ifelse(auction$item=='Palm Pilot M515 PDA',1,0)
auction$is_xbox<- ifelse(auction$item=='Xbox game console',1,0)
#add a few statistical variables
## winning bid, number of bids & average bid per auction
u<-unique(auction$auctionid)
winner<-rep(0,nrow(auction))
livebid<- rep(0,nrow(auction))
average<-rep(0,nrow(auction))
sd<- rep(0,nrow(auction))
num.bids<-rep(0,nrow(auction))

for (i in u){
        indexes<-which(auction$auctionid == i)
        cur.average<-sum(auction$bid[indexes])/length(indexes)
        cur.dev<- sd(auction$bid[indexes])
```

```
        current<- recover.livebids(auction$bid[indexes],auction$bidder[indexe
s],auction$openbid[indexes],auction$price[indexes])
        j<-0
        for (k in indexes){
                j<- j+1
                winner[k]<-ifelse(auction$bid[k]==auction$price[k], 1 , 0)
                num.bids[k]<-length(indexes)
                average[k]<-cur.average
                sd[k]<- cur.dev
                livebid[k]<- current[j]
        }

}

auction$winner<-winner
auction$avg.bid<-average
auction$sd.bid<- sd
auction$num.bids<-num.bids
auction$livebid<- livebid

## length of auction (number of days)
auction$length<-rep(0,nrow(auction))
idx3<-which(auction$auction_type == "3 day auction")
auction$length[idx3]<-3
idx5<-which(auction$auction_type == "5 day auction")
auction$length[idx5]<-5
idx7<-which(auction$auction_type == "7 day auction")
auction$length[idx7]<-7

## normalized bidding time (when bid was placed over auction length)
auction$time.fraction<-auction$bidtime/auction$length

## normalized bidding price (bid over winning price)
auction$bid.fraction<-auction$bid/auction$price

## bid distance from open bid
auction<-mutate(auction,dist.open=(bid-openbid)/bid)

## bid distance from average bid
auction<-mutate(auction,dist.avg=(bid-avg.bid)/avg.bid)

## add sequence of each bid
auction$num_current_bids <- ave(auction$bidtime, auction$auctionid, FUN = seq
_along)

## add time difference
auction$time.diff <- ave(auction$bidtime, auction$auctionid, FUN = function
(x) c(0,diff(x)))
auction$time.diff[is.na(auction$time.diff)]<- 0
```

```
## add bid difference
auction$bid.diff <- ave(auction$bid, auction$auctionid, FUN = function(x) c
(0,diff(x)))
auction$bid.diff[is.na(auction$bid.diff)]<- 0

## add bid velocity
auction$velocity<- auction$bid.diff/auction$time.diff
auction$velocity[is.infinite(auction$velocity)]<- 0
auction$velocity[is.nan(auction$velocity)]<- 0

##add lag
auction$livebid.lag<- ave(auction$livebid, factor(auction$auctionid), FUN=fun
ction(x) c(NA,x[1:(length(x)-1)]))

auction$livebid.lag<- na.fill(auction$livebid.lag,0)

head(auction,50)

## # A tibble: 50 x 30
##    auctionid    bid bidtime bidder bidderrate openbid price item  auction_
type
##        <dbl>  <dbl>   <dbl> <chr>       <dbl>   <dbl> <dbl> <chr> <chr>

##  1    1.64e9 175      2.23  schad…          0      99  178. Cart… 3 day au
cti…
##  2    1.64e9 100      2.60  chuik           0      99  178. Cart… 3 day au
cti…
##  3    1.64e9 120      2.60  kiwis…          2      99  178. Cart… 3 day au
cti…
##  4    1.64e9 150      2.60  kiwis…          2      99  178. Cart… 3 day au
cti…
##  5    1.64e9 178.     2.91  eli.f…          4      99  178. Cart… 3 day au
cti…
##  6    1.64e9   1      0.356 bfalc…          2       1  355  Cart… 3 day au
cti…
##  7    1.64e9   1.25   0.485 sbord           1       1  355  Cart… 3 day au
cti…
##  8    1.64e9   1.5    0.493 bfalc…          2       1  355  Cart… 3 day au
cti…
##  9    1.64e9  25      0.495 sbord           1       1  355  Cart… 3 day au
cti…
## 10    1.64e9   2      0.511 bfalc…          2       1  355  Cart… 3 day au
cti…
## # … with 40 more rows, and 21 more variables: is_3day <dbl>, is_5day <db
l>,
## #   is_7day <dbl>, is_catier <dbl>, is_palm <dbl>, is_xbox <dbl>, winner <
dbl>,
## #   avg.bid <dbl>, sd.bid <dbl>, num.bids <dbl>, livebid <dbl>, length <db
l>,
## #   time.fraction <dbl>, bid.fraction <dbl>, dist.open <dbl>, dist.avg <db
```

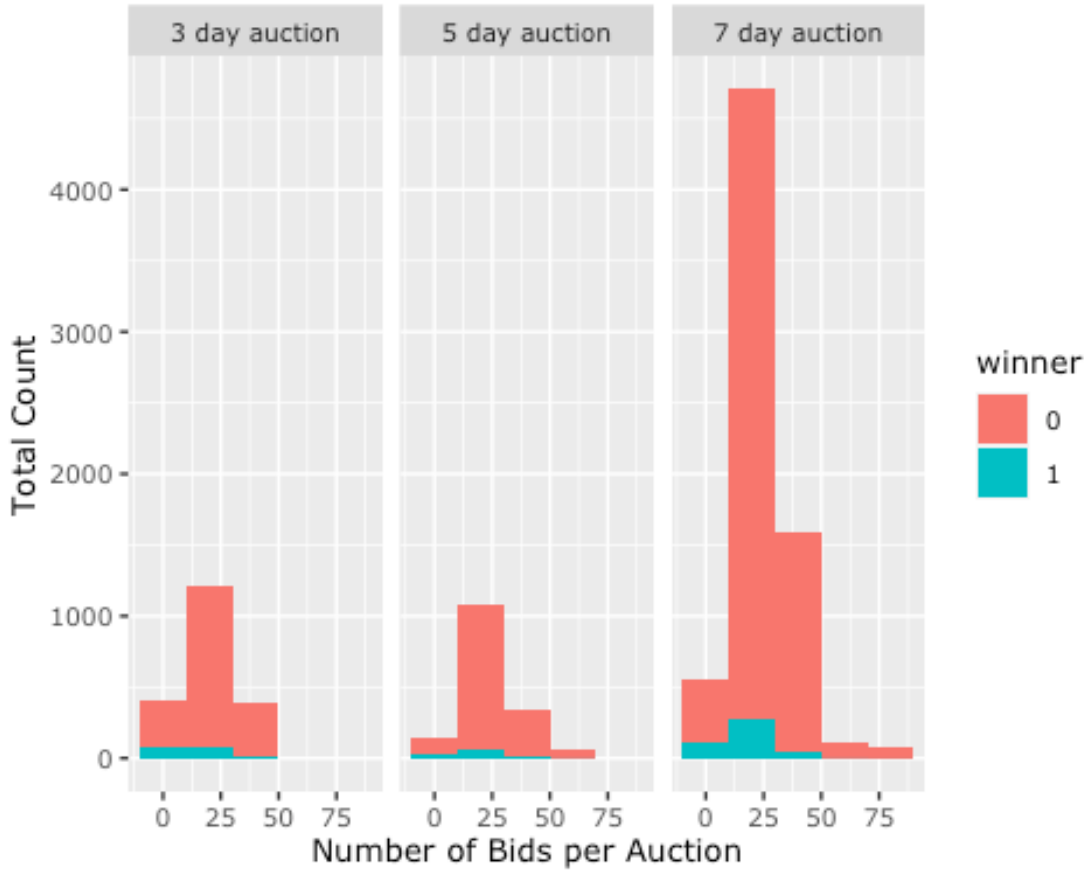
```
l>,
## #   num_current_bids <dbl>, time.diff <dbl>, bid.diff <dbl>, velocity <db
l>,
## #   livebid.lag <dbl>
```

```
auction$winner<-as.factor(auction$winner)
######## PLOTS
# text theme for every plot in this project
text_theme <- theme(text = element_text(size = 10,
                                family = "Verdana",
                                face = "plain"),
                    plot.title = element_text(hjust = 0.5))

# distribution of bids over item and aution type
ggplot(auction,aes(x =item, fill= winner))+
        geom_bar()+
        facet_wrap(~auction_type)+
        xlab("Auction Item")+ ylab("Total Count")+
        text_theme
```



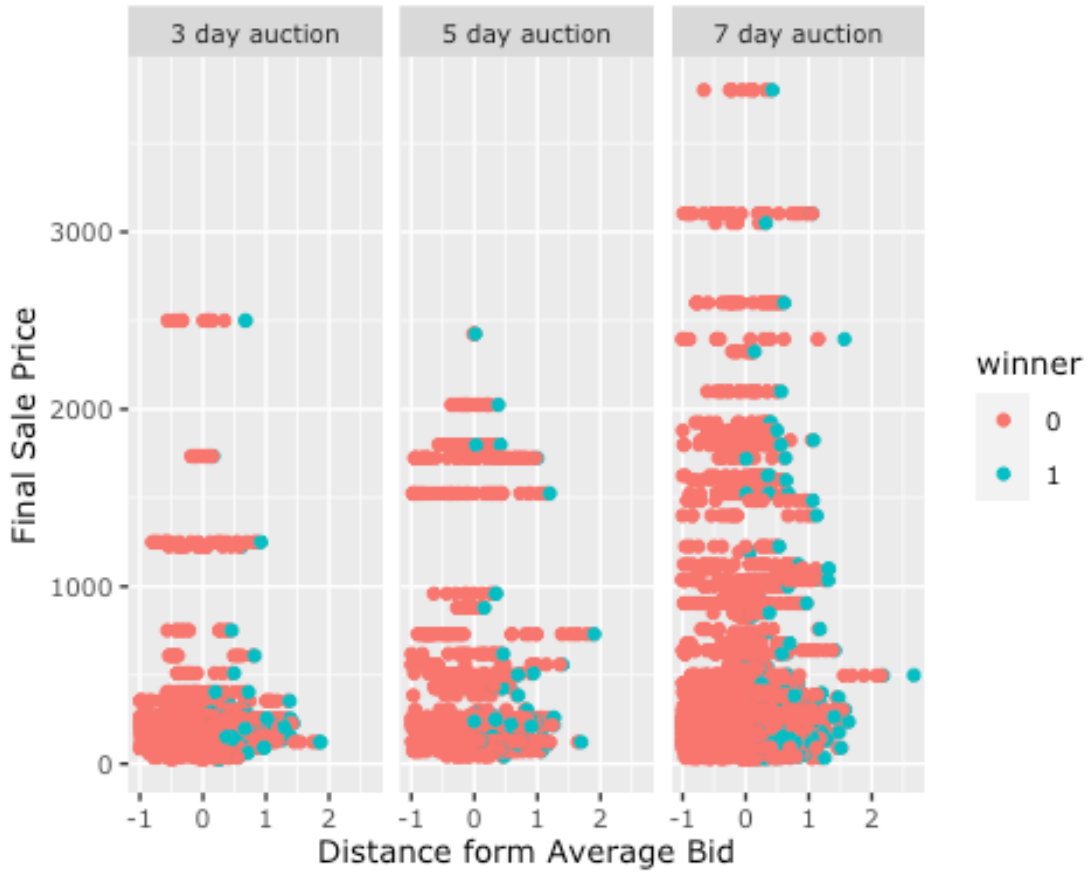

```
# check if no. of bids has predictive power
# hypothesis - As number of bids per item increases chance of winning drops
```

```
ggplot(auction,aes(x = num.bids, fill= winner))+
        geom_histogram(binwidth = 20)+
        facet_wrap(~auction_type)+
        xlab("Number of Bids per Auction")+ ylab("Total Count")+
        text_theme
```




```
# check if distance from open bid has predictive power
# hypothesis - Bids much higher than openning bid have higher chance of winni
ng
ggplot(auction)+
        geom_point(aes(x = dist.open,y=price, color= winner))+
        facet_wrap(~auction_type)+
        xlab("Distance form Openning Bid")+ ylab("Final Sale Price")+
        text_theme
```

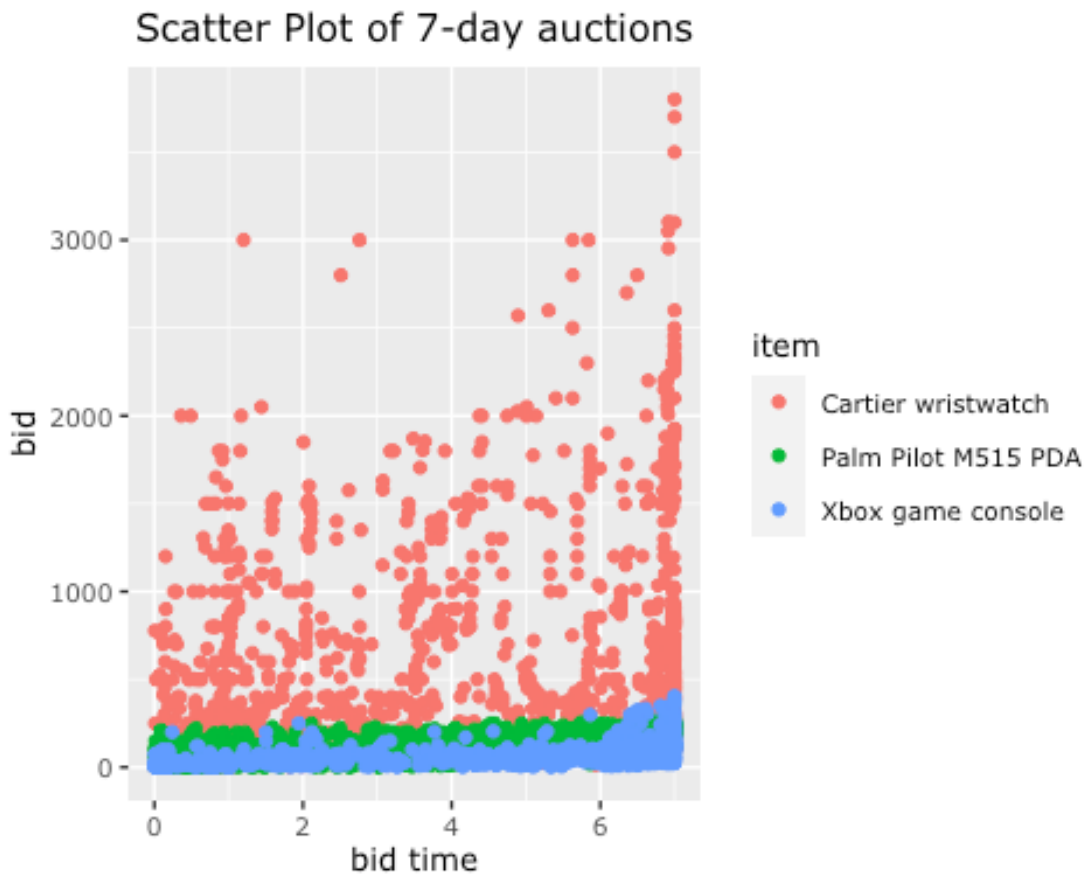

```
# check if distance from average bid has predictive power
# hypothesis - Bids below the average bid have zero chance of winning

ggplot(auction)+
        geom_point(aes(x =dist.avg,y=price, color= winner))+
        facet_wrap(~auction_type)+
        xlab("Distance form Average Bid")+ ylab("Final Sale Price")+
        text_theme
```



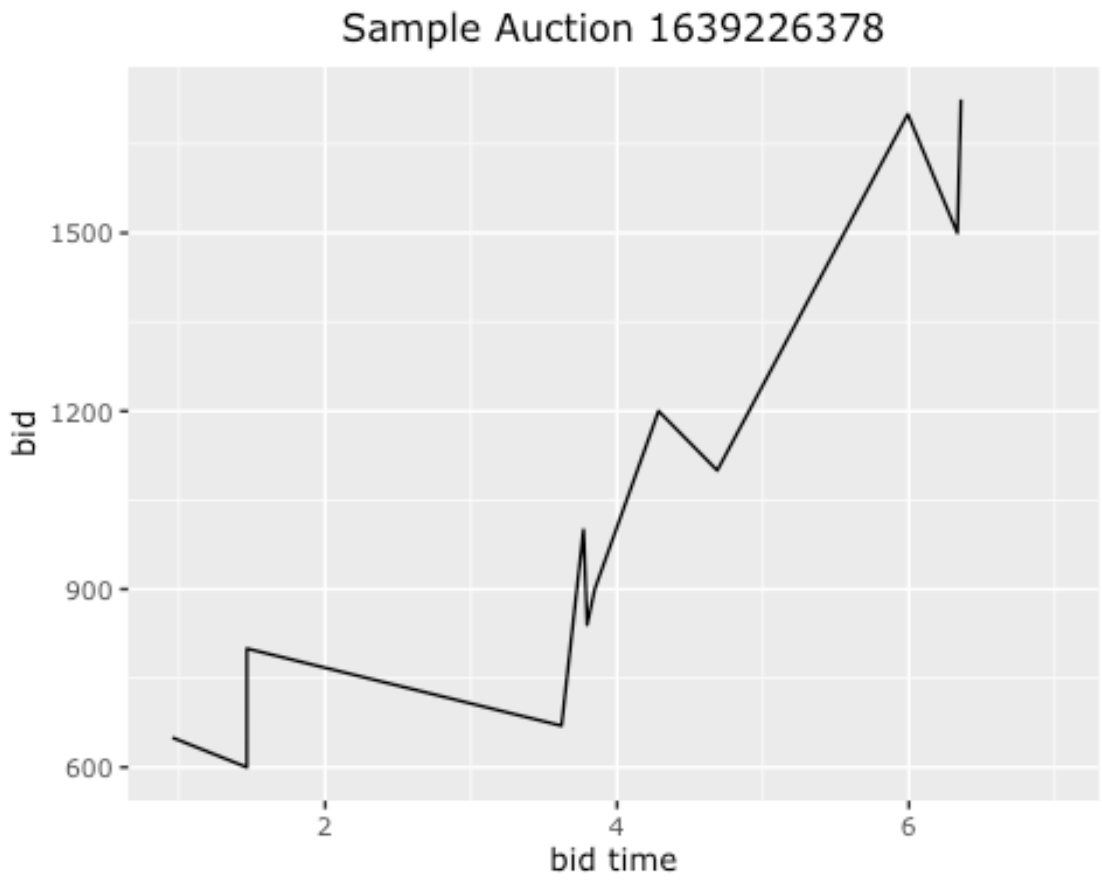

```
# check if time fraction has predictive power
# hypothesis - Bids placed toward the end of the auction have higher chance o
f winning

ggplot(auction)+
        geom_point(aes(x =time.fraction,y=price, color= winner))+
        facet_wrap(~auction_type)+
        xlab("Bidding Time")+ ylab("Final Sale Price")+
        text_theme
```

modeling a single auction using Smoothing splines

```
#Example of a 7-day auction
sevenday<- auction[which(auction$length==7),]
ggplot(sevenday, aes(x=bidtime, y=bid, color= item)) +
   geom_point() +
   scale_fill_brewer(palette = "YlGnBu") +
   xlab("bid time") +
   ggtitle("Scatter Plot of 7-day auctions")+
   expand_limits(x=7)+
   text_theme
```

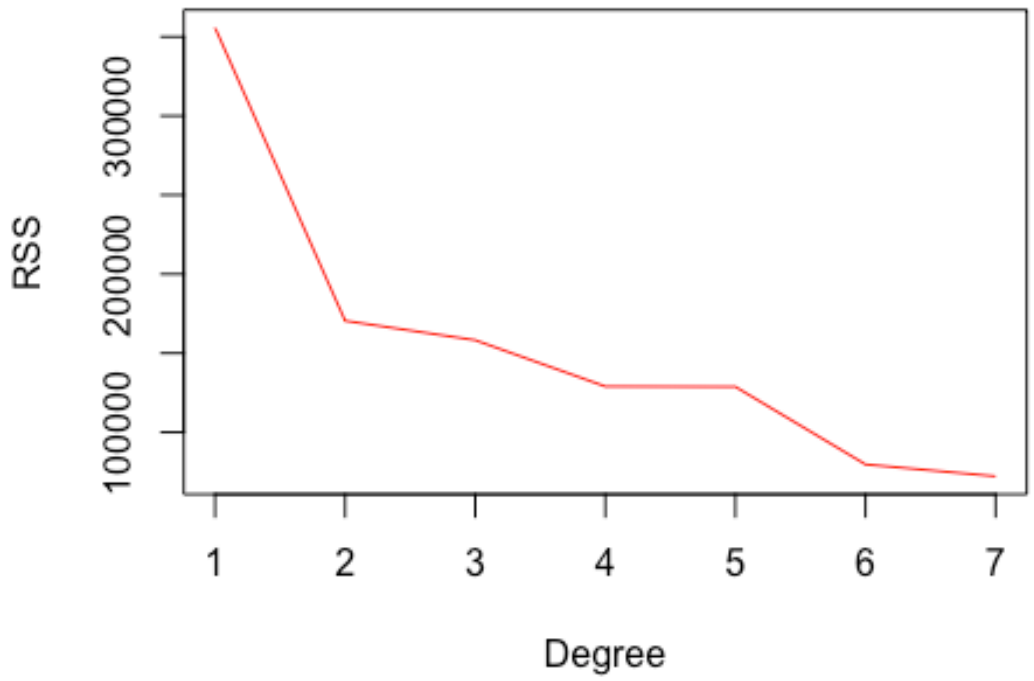



```
sample3<- auction[which(auction$auctionid=='1639226378'),]
ggplot(sample3, aes(x=bidtime, y=bid)) +
   geom_line() +
   xlab("bid time") +
   ggtitle("Sample Auction 1639226378")+
   expand_limits(x=7)+
   text_theme
```



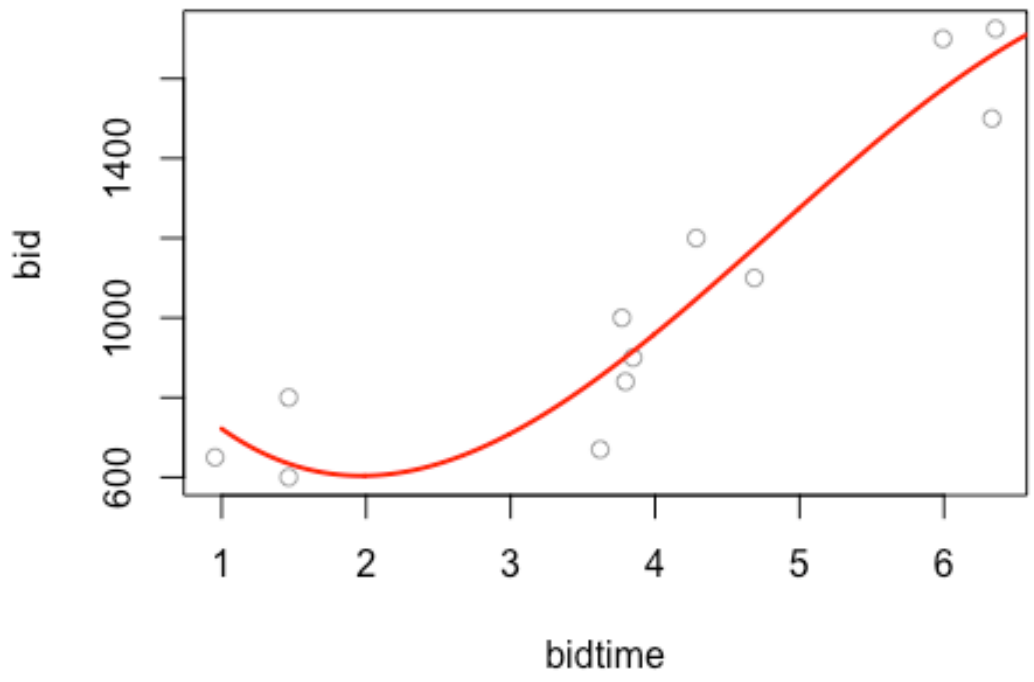
Model one single auction using non-linear polunomial regression

```
library(splines)
library(gam)

## Loading required package: foreach

##
## Attaching package: 'foreach'

## The following objects are masked from 'package:purrr':
##
##     accumulate, when

## Loaded gam 1.16.1

rss <- rep(0, 7)
for (i in 1:7) {
    fit <- lm(bid ~ poly(bidtime, i), data = sample3)
    rss[i] <- sum(fit$residuals^2)
}
plot(1:7, rss, xlab = "Degree", ylab = "RSS", type = "l", col='red')
```




```
fit.lm <- lm(bid ~ poly(bidtime,3), data = sample3)
summary(fit.lm)

##
## Call:
## lm(formula = bid ~ poly(bidtime, 3), data = sample3)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -183.30  -77.02  -25.10  110.84  165.86
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)         1057.1       40.6  26.039 5.08e-09 ***
## poly(bidtime, 3)1   1178.1      140.6   8.377 3.13e-05 ***
## poly(bidtime, 3)2    430.3      140.6   3.060   0.0156 *
## poly(bidtime, 3)3   -110.7      140.6  -0.787   0.4540
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 140.6 on 8 degrees of freedom
## Multiple R-squared:  0.9093, Adjusted R-squared:  0.8752
## F-statistic: 26.72 on 3 and 8 DF,  p-value: 0.0001607
```

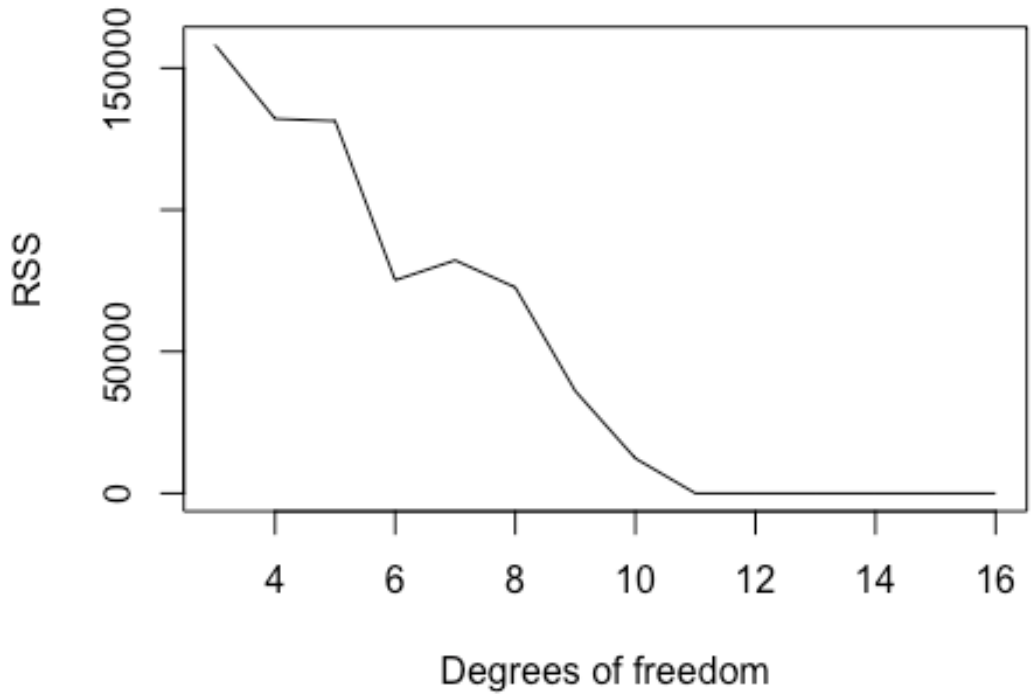
```
bidtime.new <- seq(1, 7, by = 0.01)
pred <- predict(fit.lm, list(bidtime = bidtime.new))
plot(bid ~ bidtime, data = sample3, col = "darkgrey")
lines(bidtime.new,pred, col = "red", lwd = 2)
```



The RSS decreases as the degree of the polynomial decreases. It achieves its minimum at degree 7. However, it diverges as the degree of polynomial increases. Thus from the plot we choose the degree of 3.

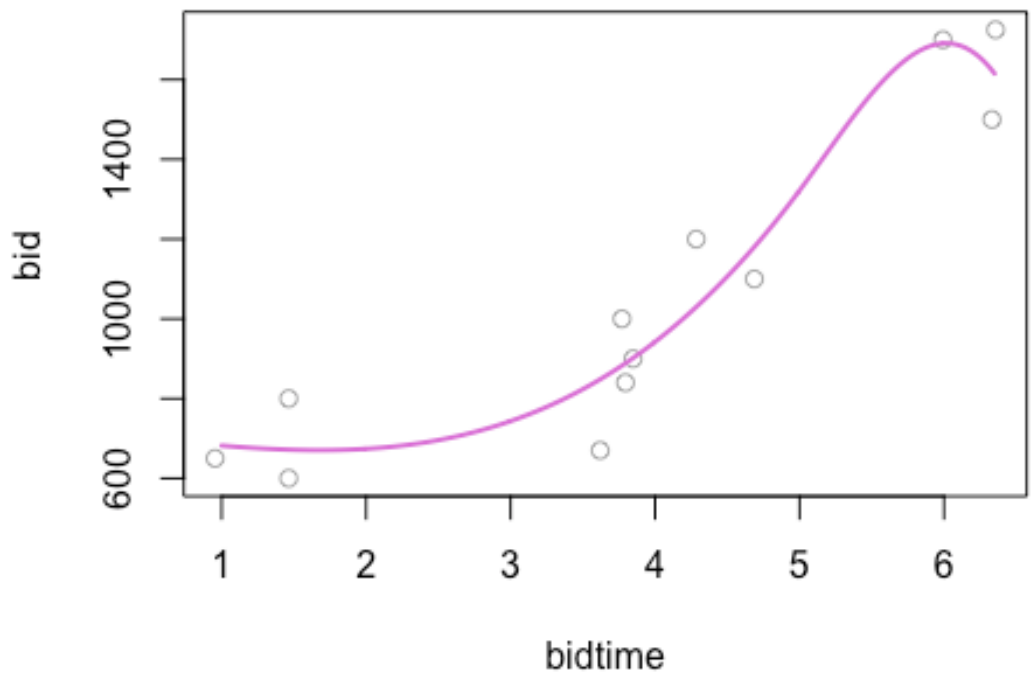
Soomthing splines and gam

```
rss <- rep(0, 16)
for (i in 3:16) {
    fit <- lm(bid ~ bs(bidtime, df = i), data = sample3)
    rss[i] <- sum(fit$residuals^2)
}
plot(3:16, rss[-c(1, 2)], xlab = "Degrees of freedom", ylab = "RSS", type = "
l")
```



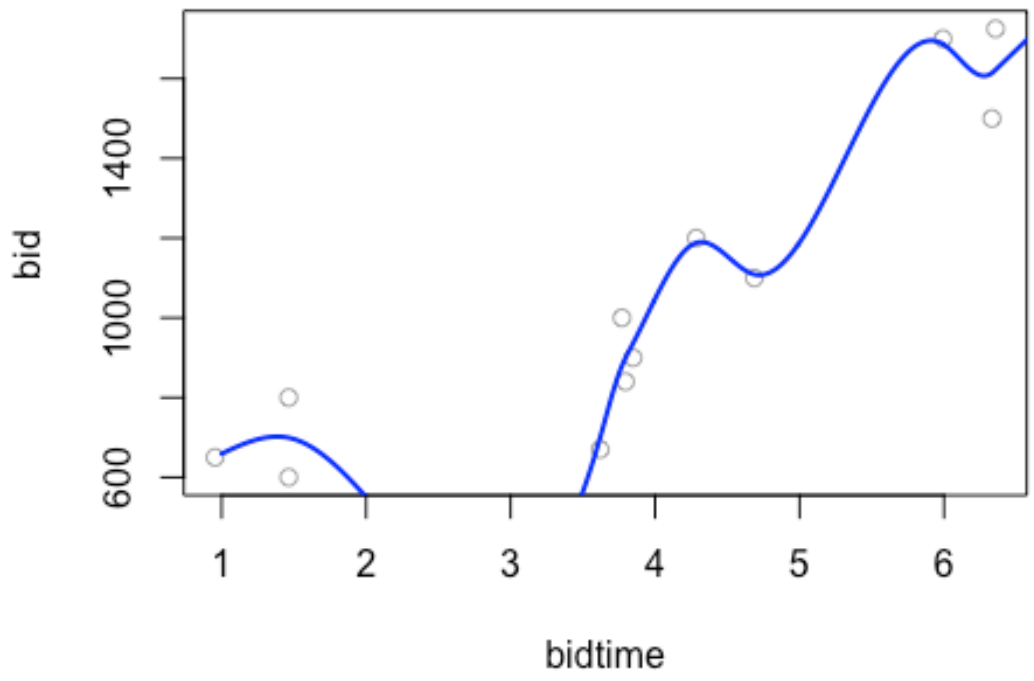

```
fit.spline <- lm( bid~ bs(bidtime, knots = c(5, 14)), data = sample3)
summary(fit.spline)

##
## Call:
## lm(formula = bid ~ bs(bidtime, knots = c(5, 14)), data = sample3)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -178.36  -74.05  -17.51  114.28  170.18
##
## Coefficients: (1 not defined because of singularities)
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       683.35     130.03   5.255 0.001179 **
## bs(bidtime, knots = c(5, 14))1    -41.55     506.34  -0.082 0.936893
## bs(bidtime, knots = c(5, 14))2    -10.57     303.66  -0.035 0.973216
## bs(bidtime, knots = c(5, 14))3   1150.59     312.44   3.683 0.007833 **
## bs(bidtime, knots = c(5, 14))4    928.39     161.69   5.742 0.000704 ***
## bs(bidtime, knots = c(5, 14))5        NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 138.5 on 7 degrees of freedom
```

```
## Multiple R-squared:  0.923,  Adjusted R-squared:  0.879
## F-statistic: 20.97 on 4 and 7 DF,  p-value: 0.0005362

pred.spline <- predict(fit.spline, list(bidtime = bidtime.new))

## Warning in bs(bidtime, degree = 3L, knots = c(5, 14), Boundary.knots =
## c(0.954039352, : some 'x' values beyond boundary knots may cause ill-condi
tioned
## bases

## Warning in predict.lm(fit.spline, list(bidtime = bidtime.new)): prediction
 from
## a rank-deficient fit may be misleading

plot(bid ~ bidtime, data = sample3, col = "darkgrey")
lines(bidtime.new, pred.spline, col = "orchid", lwd = 2)
```




```
bid<- sample3$bid
bidtime<- sample3$bidtime
fit.sspline <- smooth.spline(bid, bidtime,df=3)
fit.gam<- gam(bid ~ s(bidtime,7))
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts arg
ument
## ignored

pred.sspline<- predict(fit.sspline, bidtime.new)
pred.gam <- predict(fit.gam, list(bidtime = bidtime.new))
plot(bid ~ bidtime, data = sample3, col = "darkgrey")
lines(bidtime.new,pred.sspline$y, col = "red", lwd = 2)
lines(bidtime.new,pred.gam, col = "blue", lwd = 2)
```



Add bidding pattern

```
#split by auction
duration<-1
sLive <- split(auction$livebid,auction$auctionid)
sTime <- split(auction$time.fraction,auction$auctionid)
#create variables with opening and closing bid added
numauct<-length(sLive)
sLiveEnds<-list(0);length(sLiveEnds)<-numauct
sTimeEnds<-list(0); length(sTimeEnds)<-numauct
sLiveStart<-list(0); length(sLiveStart)<-numauct
sTimeStart<-list(0); length(sTimeStart)<-numauct
sTimeEndsTemp<-list(0); length(sTimeStart)<-numauct
sTimeStartTemp<-list(0); length(sTimeStartTemp)<-numauct
```

```
for (j in 1:numauct){
     sLiveEnds[[j]]<-c(min(sLive[[j]]),sLive[[j]],max(sLive[[j]]))
     sTimeEnds[[j]]<-c(0,sTime[[j]],duration)
     sLiveStart[[j]]<-c(min(sLive[[j]]),sLive[[j]])
     sTimeStart[[j]]<-c(0,sTime[[j]])
     sTimeEndsTemp[[j]]<-c(0.000001,sTime[[j]],duration)
     sTimeStartTemp[[j]]<-c(0.000001,sTime[[j]])
     }

delta<- 0.01
epsilon<-0.000001
num.mod<-4  ##number of models
w.x<-.5
w.y<-.5


########################
##SSE for Diffent Models
########################
n<-length(sLive)  ##number of auctions I'm looking at
wSSE.range<-array(0,c(n,num.mod))
wSSE.var<-array(0,c(n,num.mod))
models.range<-array(0,n)
models.var<-array(0,n)

for (i in 1:n){

## Exponential Model (Model 1)
x<-seq(0,duration,by=.01)
reg1<-lm(log(sLiveEnds[[i]])~ sTimeEnds[[i]])
fit1y<-exp(as.vector(reg1$coefficients)[1])*exp(as.vector(reg1$coefficients)
[2]*sTimeEnds[[i]])
fit1x<-log(sLiveEnds[[i]]/exp(as.vector(reg1$coefficients)[1]))/as.vector(reg
1$coefficients)[2]
fit1.SSEy<- sum((fit1y-sLiveEnds[[i]])^2)
fit1.SSEx<- sum((fit1x-sTimeEnds[[i]])^2)
wSSE.range[i,1]<- (w.y*fit1.SSEy)/diff(range(sLive[[i]]))^2 + (w.x*fit1.SSEx)
/diff(range(sTimeEnds[[i]]))^2
wSSE.var[i,1]<- (w.y*fit1.SSEy)/var(sLive[[i]]) + (w.x*fit1.SSEx)/var(sTimeEn
ds[[i]])

## Invere Exponential Model (Model 2)
x<-seq(0,max(sLive[[i]]),by=.1)
reg2<-lm(log(sTimeEndsTemp[[i]])~ sLiveEnds[[i]])
fit2y<-log(sTimeEndsTemp[[i]]/exp(as.vector(reg2$coefficients)[1]))  /as.vect
or(reg2$coefficients)[2]
fit2x<-exp(as.vector(reg2$coefficients)[1]) * exp(as.vector(reg2$coefficient
s)[2]*sLiveEnds[[i]])
fit2.SSEy<- sum((fit2y-sLiveEnds[[i]])^2)
fit2.SSEx<- sum((fit2x-sTimeEnds[[i]])^2)
```

```
wSSE.range[i,2]<- (w.y*fit2.SSEy)/diff(range(sLive[[i]]))^2 + (w.x*fit2.SSEx)
/diff(range(sTimeEnds[[i]]))^2
wSSE.var[i,2]<- (w.y*fit2.SSEy)/var(sLive[[i]]) + (w.x*fit2.SSEx)/var(sTimeEn
ds[[i]])

## Logistic Growth (Model 3)
x<-seq(0,duration,by=.01)
reg3<- lm( log((max(sLiveStart[[i]])+delta)/sLiveStart[[i]]-1) ~ sTimeStart
[[i]] )
fit3y<- (max(sLiveEnds[[i]])+delta)/(1+exp(as.vector(reg3$coefficients)[1])*e
xp(as.vector(reg3$coefficients)[2]*sTimeEnds[[i]]))
fit3x<- (log( (max(sLiveEnds[[i]])+delta)/sLiveEnds[[i]] - 1) - log (exp(as.v
ector(reg3$coefficients)[1])))/as.vector(reg3$coefficients)[2]
fit3.SSEy<- sum((fit3y-sLiveEnds[[i]])^2)
fit3.SSEx<- sum((fit3x-sTimeEnds[[i]])^2)
wSSE.range[i,3]<- (w.y*fit3.SSEy)/diff(range(sLive[[i]]))^2 + (w.x*fit3.SSEx)
/diff(range(sTimeEnds[[i]]))^2
wSSE.var[i,3]<- (w.y*fit3.SSEy)/var(sLive[[i]]) + (w.x*fit3.SSEx)/var(sTimeEn
ds[[i]])

## Inverse Logistic Growth (Model 4)
x<-seq(0,max(sLive[[i]]),by=.1)
reg4<- lm( log((duration+epsilon)/sTimeEndsTemp[[i]]-1) ~ sLiveEnds[[i]])
fit4y<- (log(((duration+epsilon)/sTimeEndsTemp[[i]])-1) - as.vector(reg4$coef
ficients)[1]) /as.vector(reg4$coefficients)[2]
fit4x<- (duration+epsilon)/(1+exp(as.vector(reg4$coefficients)[1])*exp(as.vec
tor(reg4$coefficients)[2]*sLiveEnds[[i]]))
fit4.SSEy<- sum((fit4y-sLiveEnds[[i]])^2)
fit4.SSEx<- sum((fit4x-sTimeEnds[[i]])^2)
wSSE.range[i,4]<- (w.y*fit4.SSEy)/diff(range(sLive[[i]]))^2 + (w.x*fit4.SSEx)
/diff(range(sTimeEnds[[i]]))^2
wSSE.var[i,4]<- (w.y*fit4.SSEy)/var(sLive[[i]]) + (w.x*fit4.SSEx)/var(sTimeEn
ds[[i]])

}

for (k in 1:n){
     wSSE.range[k,][wSSE.range[k,]=="NaN"]=(min(na.omit(wSSE.range[k,])+1))
     wSSE.var[k,][wSSE.var[k,]=="NaN"]=(min(na.omit(wSSE.var[k,])+1))
     models.range[k]<-(1:num.mod)[min(wSSE.range[k,])==wSSE.range[k,]]
     models.var[k]<-(1:num.mod)[min(wSSE.var[k,])==wSSE.var[k,]]
}

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
```

```
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
```

```
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
```

```
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length
```

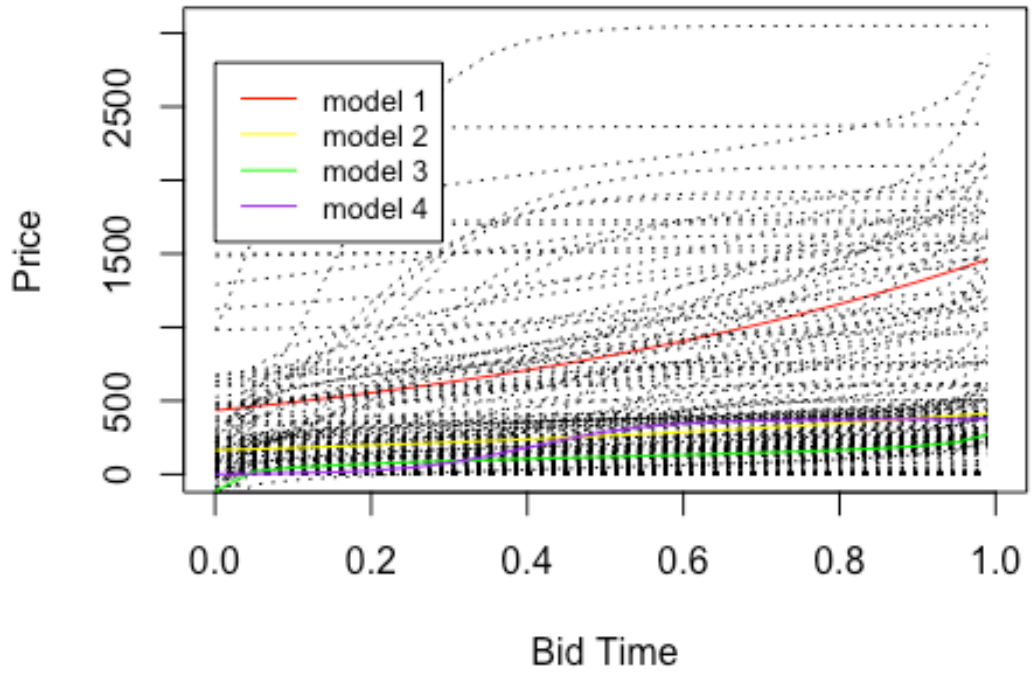
```
## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length
```

```
## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length
```

```
## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

## Warning in min(na.omit(wSSE.var[k, ]) + 1): no non-missing arguments to mi
n;
## returning Inf

## Warning in models.range[k] <- (1:num.mod)[min(wSSE.range[k, ]) ==
## wSSE.range[k, : number of items to replace is not a multiple of replacemen
t
## length

## Warning in models.var[k] <- (1:num.mod)[min(wSSE.var[k, ]) == wSSE.var[k,
 :
## number of items to replace is not a multiple of replacement length

(1:n)[models.range!=models.var]

##  [1]  13  14  17  42  55  65  71  84 143 145 152  NA 177 179  NA  NA 204
NA 211
## [20]  NA 238  NA  NA  NA 306  NA  NA  NA 345  NA  NA  NA  NA  NA 414  NA
NA  NA
## [39]  NA 449  NA  NA  NA 487 514 527 557 559 587 615

table(models.range)

## models.range
##   1   2   3   4
## 301   5  86 212

table(models.var)

## models.var
##   1   2   3   4
## 296   6  92 210

numauct<-length(sLive)
x<-seq(0,duration,by=.1)
x[1]<-min(x)+.001
x[length(x)]<-max(x)-.01
numplot<-length(x)
ypred <- yfdpred7 <- ysdpred7 <- array(0,c(numauct,numplot))
```

```
models<- models.range


#####################################################################
##Creating Fitted Data Based on Selected (by SSE criteria) Model
#####################################################################
## Matrix of data
numauct<-length(sLive)
x<-seq(0,duration,by=.05)
x[1]<-min(x)+.001
x[length(x)]<-max(x)-.01
numplot<-length(x)
ypred <- yfdpred7 <- ysdpred7 <- array(0,c(numauct,numplot))

models7<- models<- models.range   ###IMPT: THIS IS THE SSE CRITERIA THAT I AM
 USING (either models.range or models.var)

for (i in 1:n){
if (models[i]==1){
reg1<-lm(log(sLiveEnds[[i]])~ sTimeEnds[[i]])
ypred[i,]<-exp(as.vector(reg1$coefficients)[1])*exp(as.vector(reg1$coefficien
ts)[2]*x)
yfdpred7[i,]<-exp(as.vector(reg1$coefficients)[1])*as.vector(reg1$coefficient
s)[2]*exp(as.vector(reg1$coefficients)[2]*x)
ysdpred7[i,]<-exp(as.vector(reg1$coefficients)[1])*(as.vector(reg1$coefficien
ts)[2])^2*exp(as.vector(reg1$coefficients)[2]*x)
}

if (models[i]==2){
reg2<-lm(log(sTimeEndsTemp[[i]])~ sLiveEnds[[i]])
ypred[i,]<-log(x/exp(as.vector(reg2$coefficients)[1]))  /as.vector(reg2$coeff
icients)[2]
yfdpred7[i,]<-1/(as.vector(reg2$coefficients)[2]*x)
ysdpred7[i,]<- -1/(as.vector(reg2$coefficients)[2]*x^2)
}

if (models[i]==3){
reg3<- lm( log((max(sLiveStart[[i]])+delta)/sLiveStart[[i]]-1) ~ sTimeStart
[[i]] )
ypred[i,]<- (max(sLive[[i]])+delta)/(1+exp(as.vector(reg3$coefficients)[1])*e
xp(as.vector(reg3$coefficients)[2]*x))
yfdpred7[i,]<- -(max(sLive[[i]])+delta)*exp(as.vector(reg3$coefficients)[1])*
as.vector(reg3$coefficients)[2]*exp(as.vector(reg3$coefficients)[2]*x)/
      (1+exp(as.vector(reg3$coefficients)[1])*exp(as.vector(reg3$coefficient
s)[2]*x))^2
ysdpred7[i,]<- -(max(sLive[[i]])+delta)*exp(as.vector(reg3$coefficients)[1])*
(as.vector(reg3$coefficients)[2])^2*exp(as.vector(reg3$coefficients)[2]*x)*
           (1-exp(as.vector(reg3$coefficients)[1])*exp(as.vector(reg3$coeffic
ients)[2]*x))/
```

```
       (1+exp(as.vector(reg3$coefficients)[1])*exp(as.vector(reg3$coefficient
s)[2]*x))^3
}

if (models[i]==4){
reg4<- lm( log((duration+epsilon)/sTimeEndsTemp[[i]]-1) ~ sLiveEnds[[i]])
ypred[i,]<- (log(((duration+epsilon)/x)-1) - as.vector(reg4$coefficients)[1])
 /as.vector(reg4$coefficients)[2]
yfdpred7[i,]<- -(duration+epsilon)/(as.vector(reg4$coefficients)[2]*(x^2)*
(((duration+epsilon)/x)-1))
ysdpred7[i,]<- ((duration+epsilon)*((duration+epsilon)-2*x))/(as.vector(reg4
$coefficients)[2]*(x^4)*(((duration+epsilon)/x)-1)^2)
}
}

## Error in if (models[i] == 1) {: missing value where TRUE/FALSE needed

plot(x,ypred[1,],type="l",xlim=c(0,1),ylim=c(0,max(ypred)),xlab="Bid Time",yl
ab="Price",col='red')
for (i in 2:numauct)
lines(x,ypred[i,], lty=3)

lines(x,ypred[86,], col='yellow')
lines(x,ypred[61,], col='green')
lines(x,ypred[9,], col='purple')

legend(0, 2800, legend=c("model 1", "model 2", "model 3", "model 4"),
       col=c("red", "yellow", 'green', 'purple'), lty=1, cex=0.8)
```




```
table(models)
```

```
## models
##   1   2   3   4
## 301   5  86 212
```

Static model

```
library(dplyr)
library(caret)
```

```
## Loading required package: lattice
```

```
##
## Attaching package: 'caret'
```

```
## The following object is masked from 'package:purrr':
##
##     lift
```

```
# Split data into training and testing
set.seed(123)
Train <- createDataPartition(u, p = 0.7, list = FALSE)
Train.auction<- auction$auctionid %in% u[Train]
```

```
training <- auction[Train.auction,]
testing <- auction[!Train.auction,]
```

static prediction

Static competition features are known at the outset of the auction and do not change during the auction; - opening price of concurrent auctions - duration of concurrent auctions - sellers' feedback of concurrent auctions

```
library(ggplot2)
library(caret)

#######Static Model Predicting the End Price######
set.seed(123)
train_control<- trainControl(method = 'cv',number=10,repeats = 3)

## Warning: `repeats` has no meaning for this resampling method.

#fit auction with linear model
static.lm <- train(price~ openbid+length+bidderrate+is_catier+is_palm+is_xbox
+is_3day+is_5day+is_7day, data=training, trControl=train_control, method="lm
",na.action=na.exclude)

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
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## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
static.lm
```

```
## Linear Regression
##
## 7562 samples
##    9 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6795, 6796, 6796, 6797, 6795, 6798, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   268.2168  0.5408452  132.4348
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
static.pred.lm<- predict(static.lm, testing)
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
postResample(pred = static.pred.lm, obs = testing$livebid)
```

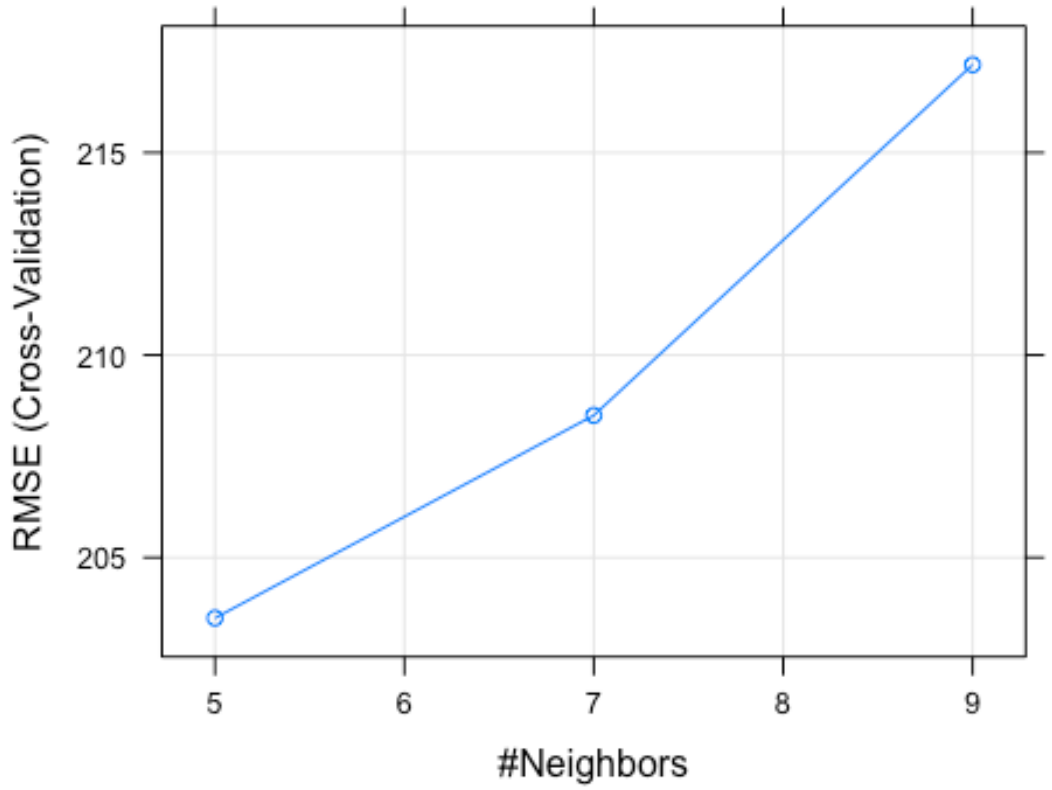
```
##        RMSE    Rsquared         MAE
## 303.4196504   0.3509475 184.0715687
```

```
##fit auction with knn model
static.knn<- train(price ~ openbid+length+bidderrate+is_catier+is_palm+is_xbo
x+is_3day+is_5day+is_7day, data=training, trControl=train_control, method="kn
n",na.action=na.exclude)
static.knn
```

```
## k-Nearest Neighbors
##
## 7562 samples
##    9 predictor
##
## No pre-processing
```

```
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6797, 6796, 6796, 6795, 6795, 6796, ...
## Resampling results across tuning parameters:
##
##   k  RMSE      Rsquared   MAE
##   5  203.5048  0.7333769  76.95662
##   7  208.5114  0.7198521  82.10603
##   9  217.1711  0.6960207  87.56857
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was k = 5.

plot(static.knn)
```




```
static.pred.knn<- predict(static.knn, testing)
postResample(pred = static.pred.knn, obs = testing$livebid)

##       RMSE   Rsquared        MAE
## 325.888647   0.268136 181.203730

library(ggplot2)
library(caret)

#######Static Model######
```

```r
set.seed(123)

train_control<- trainControl(method = 'cv',number=10,repeats = 3)

## Warning: `repeats` has no meaning for this resampling method.

#fit auction with linear model
static.lm <- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_x
box+is_3day+is_5day+is_7day, data=training, trControl=train_control, method="
lm",na.action=na.exclude)

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
static.lm
```

```
## Linear Regression
##
## 7562 samples
##    9 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6796, 6796, 6796, 6796, 6795, 6796, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   210.1425  0.4695338  112.7726
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
static.pred.lm<- predict(static.lm, testing)
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

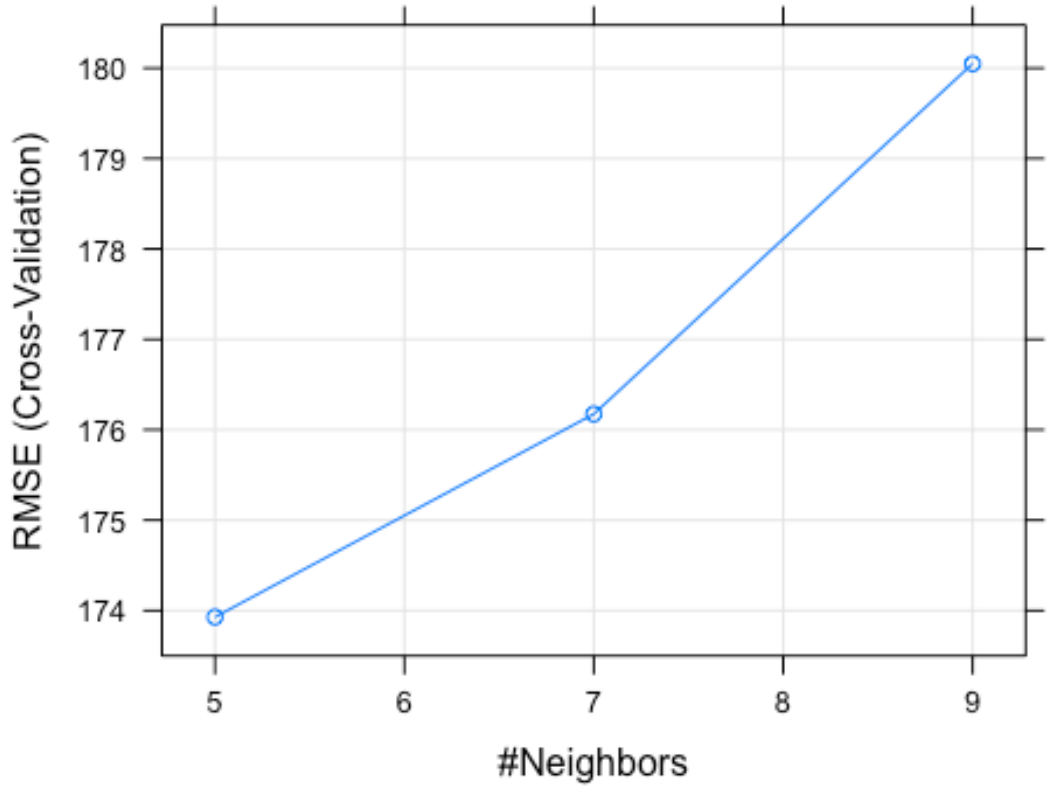
```
postResample(pred = static.pred.lm, obs = testing$livebid)
```

```
##       RMSE   Rsquared        MAE
## 232.015148   0.436801 110.296938
```

```
##fit auction with knn model
static.knn<- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_x
box+is_3day+is_5day+is_7day, data=training, trControl=train_control, method="
knn",na.action=na.exclude)
static.knn
```

```
## k-Nearest Neighbors
##
## 7562 samples
##    9 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6797, 6796, 6795, 6795, 6795, 6796, ...
## Resampling results across tuning parameters:
##
##   k  RMSE      Rsquared   MAE
##   5  173.9282  0.6382034  85.05969
##   7  176.1739  0.6288147  87.21982
##   9  180.0484  0.6126760  89.90720
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was k = 5.
```

```
plot(static.knn)
```




```
static.pred.knn<- predict(static.knn, testing)
postResample(pred = static.pred.knn, obs = testing$livebid)

##         RMSE     Rsquared          MAE
## 253.5584894   0.3333473 111.1309446

#######Static + Evolving info Model######
#current price, time left, current number of bids, current number of bidders

#fit auction with linear model
se.lm <- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_xbox+
is_3day+is_5day+is_7day+livebid.lag+num_current_bids+avg.bid+sd.bid+num.bids+
time.fraction, data=training, trControl=train_control, method="lm",na.action=
na.exclude)

static.lm

## Linear Regression
##
## 7562 samples
##    9 predictor
##
```

```
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6796, 6796, 6796, 6796, 6795, 6796, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   210.1425  0.4695338  112.7726
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
se.pred.lm<- predict(se.lm, testing)
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
postResample(pred = se.pred.lm, obs = testing$livebid)
```

```
## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length
```

```
## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length
```

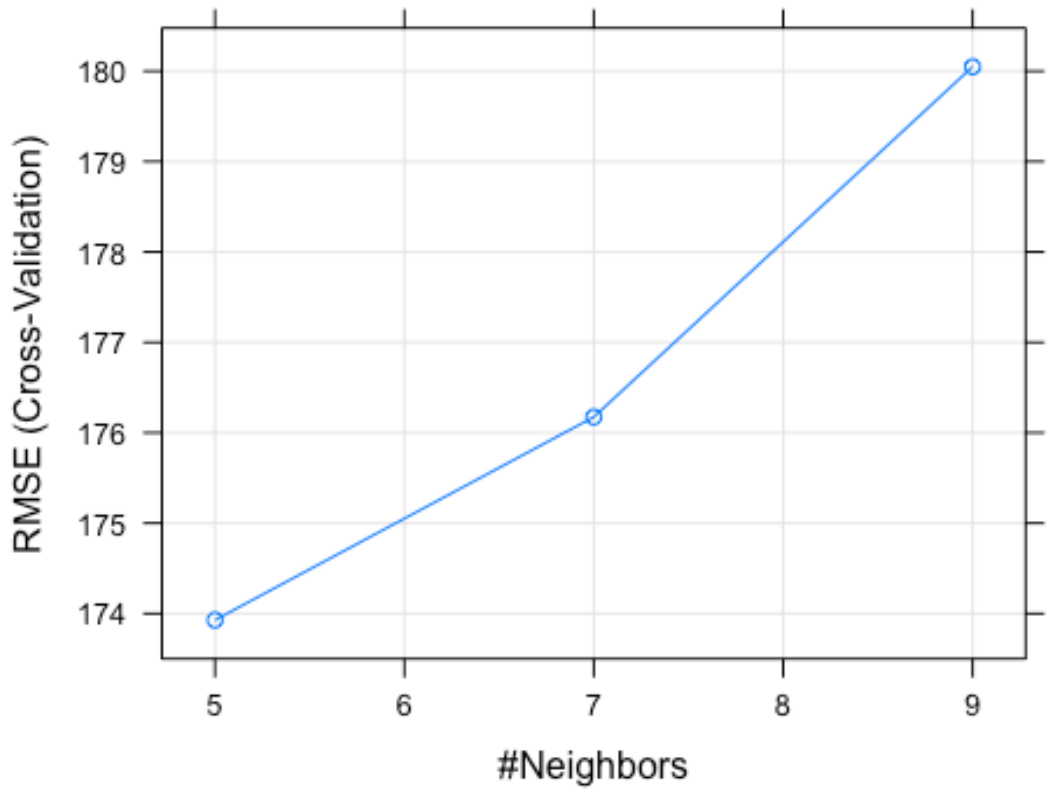
```
##     RMSE Rsquared      MAE
## 95.92843       NA 67.50273
```

```
##fit auction with knn model
se.knn<- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_xbox+
is_3day+is_5day+is_7day+livebid.lag+num_current_bids+avg.bid+sd.bid+num.bids+
time.fraction, data=training, trControl=train_control, method="knn",na.action
=na.exclude)
static.knn
```

```
## k-Nearest Neighbors
##
## 7562 samples
##    9 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6797, 6796, 6795, 6795, 6795, 6796, ...
## Resampling results across tuning parameters:
##
##   k  RMSE      Rsquared   MAE
##   5  173.9282  0.6382034  85.05969
##   7  176.1739  0.6288147  87.21982
##   9  180.0484  0.6126760  89.90720
##
```

```
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was k = 5.

plot(static.knn)
```




```
se.pred.knn<- predict(se.knn, testing)
postResample(pred = se.pred.knn, obs = testing$livebid)

## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length

## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length

##      RMSE Rsquared      MAE
## 99.06266       NA 62.78169

#######Static + Evolving info + dynamic Model######
#distance to

#fit auction with linear model
sed.lm <- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_xbox
```

```
+is_3day+is_5day+is_7day+livebid.lag+num_current_bids+avg.bid+sd.bid+num.bids
+time.fraction+bid.fraction+dist.open+dist.avg+time.diff+bid.diff+velocity, d
ata=training, trControl=train_control, method="lm",na.action=na.exclude)
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading

## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
static.lm
```

```
## Linear Regression
##
## 7562 samples
##    9 predictor
```

```
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6796, 6796, 6796, 6796, 6795, 6796, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   210.1425  0.4695338  112.7726
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
sed.pred.lm<- predict(sed.lm, testing)
```

```
## Warning in predict.lm(modelFit, newdata): prediction from a rank-deficient
 fit
## may be misleading
```

```
postResample(pred = sed.pred.lm, obs = testing$livebid)
```

```
## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length
```

```
## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length
```

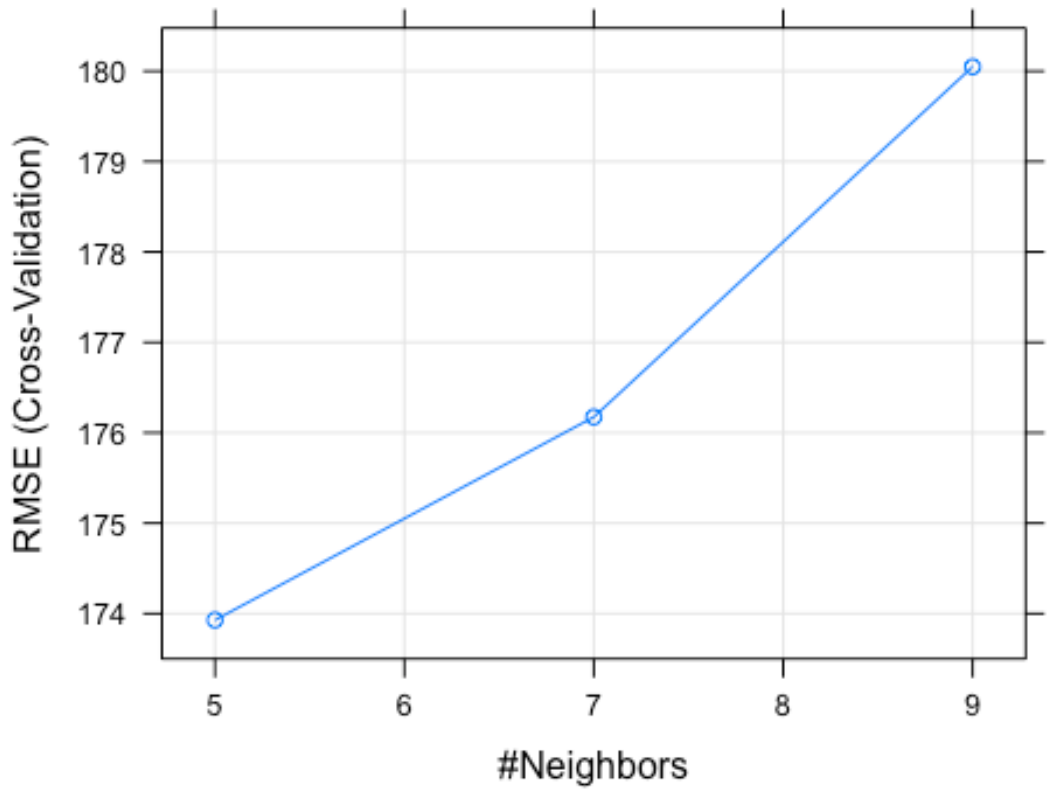
```
##     RMSE Rsquared      MAE
## 96.08527       NA 67.37456
```

```
##fit auction with knn model
sed.knn<- train(livebid ~ openbid+length+bidderrate+is_catier+is_palm+is_xbox
+is_3day+is_5day+is_7day+livebid.lag+num_current_bids+avg.bid+sd.bid+num.bids
+time.fraction+bid.fraction+dist.open+dist.avg+time.diff+bid.diff+velocity, d
ata=training, trControl=train_control, method="knn",na.action=na.exclude)
static.knn
```

```
## k-Nearest Neighbors
##
## 7562 samples
##    9 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 6797, 6796, 6795, 6795, 6795, 6796, ...
## Resampling results across tuning parameters:
##
##   k  RMSE      Rsquared   MAE
##   5  173.9282  0.6382034  85.05969
##   7  176.1739  0.6288147  87.21982
##   9  180.0484  0.6126760  89.90720
##
```

```
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was k = 5.

plot(static.knn)
```




```
sed.pred.knn<- predict(sed.knn, testing)
postResample(pred = sed.pred.knn, obs = testing$livebid)

## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length

## Warning in pred - obs: longer object length is not a multiple of shorter o
bject
## length

##      RMSE  Rsquared       MAE
## 181.56717        NA  85.78306

library(ggplot2)
library(xgboost)

##
## Attaching package: 'xgboost'
```

```
## The following object is masked from 'package:dplyr':
##
##     slice

library(caret)

#######XGboost Full model######
set.seed(123)

train_x<- as.matrix(training %>% select(-which(sapply(.,is.character))) %>%se
lect(-c(livebid,auctionid)))

train_y<- training$livebid

test_x<- as.matrix(testing %>% select(-which(sapply(.,is.character))) %>%sele
ct(-c(livebid,auctionid)))

test_y<- testing$livebid

full.xg<- xgboost(data = train_x,
                     label=train_y,
                  booster = "gblinear",
                  objective = "reg:linear",
                  max.depth = 8,
                  nround = 500,
                  lambda = 0,
                  lambda_bias = 0,
                  alpha = 0,
                  missing=NA,
                  verbose = 0)

## Error in xgb.DMatrix(data, label = label, missing = missing): 'data' has c
lass 'character' and length 189050.
##   'data' accepts either a numeric matrix or a single filename.

pred.test.xgboost <- predict(full.xg,test_x, missing=NA)

## Error in predict(full.xg, test_x, missing = NA): object 'full.xg' not foun
d

rmse <- function(error)
{
    sqrt(mean(error^2))
}

# Function that returns Mean Absolute Error
mae <- function(error)
{
    mean(abs(error))
}
```

```
error<- test_y-pred.test.xgboost

## Error in eval(expr, envir, enclos): object 'pred.test.xgboost' not found

postResample(pred = pred.test.xgboost, obs = testing$livebid)

## Error in postResample(pred = pred.test.xgboost, obs = testing$livebid): ob
ject 'pred.test.xgboost' not found
```

using LSTM to predicting the bid pattern

```
library(keras)

lags <- function(x, k){

    lagged =  c(rep(NA, k), x[1:(length(x)-k)])
    DF = as.data.frame(cbind(lagged, x))
    colnames(DF) <- c( paste0('x-', k), 'x')
    DF[is.na(DF)] <- 0
    return(DF)
  }

lstm.train = lags(training$bid.diff, 1)
lstm.test = lags(testing$bid.diff, 1)


normalize <- function(train, test, feature_range = c(0, 1)) {
    x = train
    fr_min = feature_range[1]
    fr_max = feature_range[2]
    std_train = ((x - min(x) ) / (max(x) - min(x)  ))
    std_test  = ((test - min(x) ) / (max(x) - min(x)  ))

    scaled_train = std_train *(fr_max -fr_min) + fr_min
    scaled_test = std_test *(fr_max -fr_min) + fr_min

    return( list(scaled_train = as.vector(scaled_train), scaled_test = as.vec
tor(scaled_test) ,scaler= c(min =min(x), max = max(x))) )

  }


Scaled = normalize(train, test, c(-1, 1))

## Error in min(x): invalid 'type' (closure) of argument

y_train = Scaled$scaled_train[, 2]

## Error in eval(expr, envir, enclos): object 'Scaled' not found

x_train = Scaled$scaled_train[, 1]
```

```
## Error in eval(expr, envir, enclos): object 'Scaled' not found

y_test = Scaled$scaled_test[, 2]

## Error in eval(expr, envir, enclos): object 'Scaled' not found

x_test = Scaled$scaled_test[, 1]

## Error in eval(expr, envir, enclos): object 'Scaled' not found

dim(x_train) <- c(length(x_train), 1, 1)

## Error in eval(expr, envir, enclos): object 'x_train' not found

dim(x_train)

## Error in eval(expr, envir, enclos): object 'x_train' not found

batch_size = 1
units = 1
# specify required arguments
model <- keras_model_sequential() %>%
  layer_lstm(units=128,batch_input_shape = c(batch_size, dim(x_train)[2], dim
(x_train)[3]),   stateful= TRUE)%>%
  layer_dropout(0.2) %>%
  layer_dense(units = 1)

## Error in normalize_shape(batch_input_shape): object 'x_train' not found

model %>% compile(
  optimizer_adam( lr= 0.02 , decay = 1e-6 ),
  loss = "mse",
  metrics = list("mean_absolute_error")
)

## Error in eval(lhs, parent, parent): object 'model' not found

summary(model)

> summary(model)

Model: "sequential_82"

_________________________________________________________________________
_____

Layer (type)                         Output Shape                      Param #


=========================================================================
======

lstm_71 (LSTM)                       (1, 128)                          66560
```

```
_________________________________________________________________
dropout_22 (Dropout)         (1, 128)                  0
_________________________________________________________________
dense_104 (Dense)            (1, 1)                    129
=================================================================
Total params: 66,689
Trainable params: 66,689
Non-trainable params: 0
_________________________________________________________________
>
>
> history <- model %>% fit(
+   x_train,y_train,
+   epochs = 10,
+   batch_size = 1
+ )
Train on 7474 samples
Epoch 1/10
7474/7474 [==============================] - 47s 6ms/sample - loss: 0.0070 -
mean_absolute_error: 0.0409
Epoch 2/10
7474/7474 [==============================] - 21s 3ms/sample - loss: 0.0059 -
mean_absolute_error: 0.0364
Epoch 3/10
7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0060 -
mean_absolute_error: 0.0360
```

```
Epoch 4/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0059 -
mean_absolute_error: 0.0369

Epoch 5/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0059 -
mean_absolute_error: 0.0361

Epoch 6/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0060 -
mean_absolute_error: 0.0359

Epoch 7/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0056 -
mean_absolute_error: 0.0355

Epoch 8/10

7474/7474 [==============================] - 21s 3ms/sample - loss: 0.0058 -
mean_absolute_error: 0.0361

Epoch 9/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0060 -
mean_absolute_error: 0.0359

Epoch 10/10

7474/7474 [==============================] - 22s 3ms/sample - loss: 0.0058 -
mean_absolute_error: 0.0357

>

> dim(x_test) <- c(length(x_test), 1, 1)

> yhat = model %>% predict(X, batch_size=1)

>

> postResample(pred = yhat, obs = y_test)

      RMSE   Rsquared        MAE

0.01653302         NA 0.01107428

>

history <- model %>% fit(
  x_train,y_train,
  epochs = 10,
  batch_size = 1
```

```
Model: "sequential_88"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
lstm_77 (LSTM)               (1, 128)                  66560
_________________________________________________________________
dense_115 (Dense)            (1, 32)                   4128
_________________________________________________________________
dropout_28 (Dropout)         (1, 32)                   0
_________________________________________________________________
dense_116 (Dense)            (1, 1)                    33
=================================================================
Total params: 70,721
Trainable params: 70,721
Non-trainable params: 0
_________________________________________________________________
>
>
> history <- model %>% fit(
+   x_train,y_train,
+   epochs = 10,
+   batch_size = 1
```

```
+ )

Train on 7557 samples

Epoch 1/10

7557/7557 [==============================] - 24s 3ms/sample - loss: 80823.051
9 - mean_absolute_error: 141.0224

Epoch 2/10

7557/7557 [==============================] - 21s 3ms/sample - loss: 96979.073
0 - mean_absolute_error: 158.2778

Epoch 3/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104691.39
47 - mean_absolute_error: 163.1122

Epoch 4/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104151.26
94 - mean_absolute_error: 161.5961

Epoch 5/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 103964.54
18 - mean_absolute_error: 161.6179

Epoch 6/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104444.34
82 - mean_absolute_error: 161.5754

Epoch 7/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104060.57
58 - mean_absolute_error: 162.5403

Epoch 8/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 103962.09
32 - mean_absolute_error: 161.4539

Epoch 9/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104125.07
15 - mean_absolute_error: 161.7306

Epoch 10/10

7557/7557 [==============================] - 22s 3ms/sample - loss: 104097.25
88 - mean_absolute_error: 162.4093

>
```

```
> dim(x_test) <- c(length(x_test), 1, 1)
> yhat = model %>% predict(X, batch_size=1)
>
> postResample(pred = yhat, obs = y_test)
    RMSE Rsquared      MAE
223.4743       NA 112.7578
```