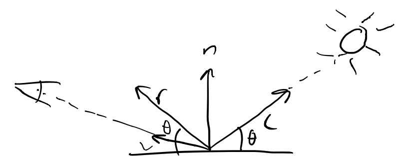
1) a) $I = k_a I_A + \left(k_D (n.l) I_D + k_s (v.r)^{q} I_s\right) \dfrac{\Phi}{\pi d^2}$



k_a is the ambient coefficient,
I_A is the ambient colour

k_D is the diffuse coefficient
I_D is the diffuse illumination

k_s is the specular coefficient
I_s is the specular illumination

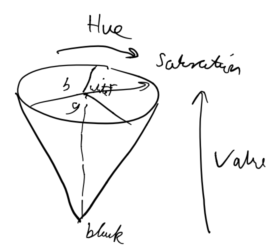
n is the normal to the surface at a fragment
l is the normalised vector in the direction of the light source
v is the normalised vector in the direction of the viewer
r is l reflected about n
q is the shininess of the object

d is the distance from the surface to the light source
Φ is the radiosity of the light source

b) If the light source and viewer are placed infinitely far away, the diffuse and specular terms disappear and only the ambient illumination will remain.

c) i) Saturation

ii) Value



d) i)

	M_1	x	y	z
(1,0,0)	Red	0.6	0.3	0.1
(0,1,0)	Green	0.2	0.6	0.2
(0,0,1)	Blue	0.1	0.1	0.8

$$M_1 = \begin{pmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.2 & 0.8 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0.6 & 0.1 & 0.15 \\ 0.25 & 0.7 & 0.1 \\ 0.15 & 0.2 & 0.75 \end{pmatrix}$$

ii) First convert the rgb into xyz, then back to rgb on the other monitor

$$rgb_2 = M_2^{-1} M_1 \, rgb_1$$

iii)

$$= M_2^{-1} \begin{pmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.2 & 0.8 \end{pmatrix} \begin{pmatrix} 0.7 \\ 0.1 \\ 0.1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.82 & -0.16 & -0.34 \\ -0.62 & 1.54 & -0.08 \\ -0.20 & -0.38 & 1.42 \end{pmatrix} \begin{pmatrix} 0.45 \\ 0.28 \\ 0.17 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7164 \\ 0.1386 \\ 0.045 \end{pmatrix}$$

2) a) i) $p(t) = at^3 + bt^2 + ct + d$

$p(0) = d$ $\quad$ $p(1) = a+b+c+d$ $\quad$ $p(1) = a+b+p(0)+p'(0)$

$p'(0) = c$ $\quad$ $p'(1) = 3a+2b+c$ $\quad$ $p'(1) = 3a+2b+p'(0)$

$$\begin{pmatrix} p(0) \\ p'(0) \\ p(1) \\ p'(1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$a = p(1) - b - p(0) - p'(0)$

$p'(1) = 3(p(1) - b - p(0) - p'(0)) + 2b + p'(0)$

$p'(1) = -b - 2p'(0) + 3p(1) - 3p(0)$

$b = -3p(0) - 2p'(0) + 3p(1) - p'(1)$

$a = 2p(0) + p'(0) - 2p(1) + p'(1)$

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} p(0) \\ p'(0) \\ p(1) \\ p'(1) \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p(0) \\ p'(0) \\ p(1) \\ p'(1) \end{pmatrix}$$

ii) At P_1, $p(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $\qquad$ P_2, $p(1) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$P_1' = \frac{P_2 - P_0}{2} = \frac{\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{2} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$

$$P_2' = \frac{P_3 - P_1}{2} = \frac{\begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}}{2} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$

$d = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$c = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$

$a+b+c+d = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$3a+2b+c = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$

$3a+2b = 0$

$a+b+\begin{pmatrix} 1 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$a+b = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}$

$a = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix} - b$

$$3\left(\begin{pmatrix}0\\-0.5\end{pmatrix} - b\right) + 2b = 0$$

$$-b = \begin{pmatrix}0\\1.5\end{pmatrix}$$

$$b = \begin{pmatrix}0\\-1.5\end{pmatrix}$$

$$a = \begin{pmatrix}0\\1\end{pmatrix}$$

b) i)

$p(0) = d$ $\qquad$ $p(1) = a + b + c + d$

$p'(0) = c$

$p''(0) = 2b$

$a = P(1) - \frac{P''(0)}{2} \cdot P'(0) - P(0)$

$b = P''(0)/2$

$c = P'(0)$

$d = P(0)$

ii)

$$p(0) = \begin{pmatrix}2\\2\end{pmatrix}$$

$$p'(0) = \begin{pmatrix}1\\0.5\end{pmatrix}$$

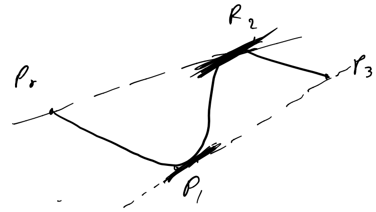
$$P''(0) = \frac{P_2' - P_0'}{2}$$

$$b = \frac{(P_2' - P_0')}{4}$$

$$c = P_1'$$

$$d = P_1$$

$$a = P_2 - \frac{P_2' - P_0'}{4} - P_1' - P_1$$



Don't know the gradient at P_0', but the curve will be the same as in a) surely?

c) Since we have set the gradients to be equal, the curve is C^1 and C^0 continuous since we set the positions to be equal. This implies the curve is also G^0 and G^1 continuous since G^0 continuous is the same as C^0 and if the gradients are equal, they are also proportional so the curve is also G^1 continuous.

3) a) First project the points onto the viewing plane

$$a_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.5 & -2.5 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix}0.4\\0.2\\-2\\1\end{pmatrix} = \overset{\text{BEFORE}}{\begin{pmatrix}0.4\\0.2\\0.5\\2\end{pmatrix}} = \overset{\text{AFTER}}{\begin{pmatrix}0.2\\0.1\\0.25\\1\end{pmatrix}}$$

$$b_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.5 & -2.5 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix}1.75\\0.25\\-2.5\\1\end{pmatrix} = \overset{\text{BEFORE}}{\begin{pmatrix}1.75\\0.25\\1.25\\2.5\end{pmatrix}} = \overset{\text{AFTER}}{\begin{pmatrix}0.7\\0.1\\0.5\\1\end{pmatrix}}$$

Why is z different !?

Angled viewing plane ? Must be...

$$c_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.5 & -2.5 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix}0.6\\1.5\\-3\\1\end{pmatrix} = \overset{\text{BEFORE}}{\begin{pmatrix}0.6\\1.5\\2\\3\end{pmatrix}} = \overset{\text{AFTER}}{\begin{pmatrix}0.2\\0.5\\0.67\\1\end{pmatrix}}$$

b) $\beta, \gamma, n_x, n_y, n_z, t_x, t_y, z$

9 values

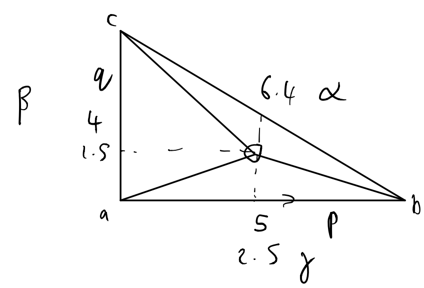
if the fragment processor calculates perspective correction, also pass the interpolated z value

c) The normal, the screen position and the texture coordinate

	normal	z value	screen position, perspective corrected
a	$(-0.5, -0.3, 1)$	-2	$\begin{pmatrix}0.2\\0.1\\0.25\end{pmatrix} \quad \frac{1}{-2}\begin{pmatrix}4.375\\3.375\end{pmatrix}$
b	$(0.5, -0.3, 1)$	-2.5	$\begin{pmatrix}0.7\\0.1\\0.5\end{pmatrix} = \frac{1}{-2.5}\begin{pmatrix}11.75\\5.75\end{pmatrix}$
c	$(-0.5, 0.5, 1)$	-3	$\begin{pmatrix}0.2\\0.5\\0.67\end{pmatrix} = \frac{1}{-3}\begin{pmatrix}8.365\\11.365\end{pmatrix}$

$$\begin{pmatrix}10 & 0 & 9.5\\ 0 & 10 & 9.5 \\ 0 & 0 & 1\end{pmatrix}$$

At circled point



Find the barycentric coordinate

$p = (b - a)$

$q = (a - c)$

$p = \begin{pmatrix}5\\0\end{pmatrix}$

$q = \begin{pmatrix}0\\4\end{pmatrix}$

Barycentric coordinates are $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0.125 \\ 0.375 \\ 0.5 \end{pmatrix}$

Interpolate the normal

$$n = \begin{pmatrix} -0.5 & 0.5 & -0.5 \\ -0.3 & -0.3 & 0.5 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0.125 \\ 0.375 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} -0.125 \\ 0.1 \\ 1 \end{pmatrix}$$

$$z \text{ value} = \begin{pmatrix} -2 & -2.5 & -3 \end{pmatrix} \begin{pmatrix} 0.125 \\ 0.375 \\ 0.5 \end{pmatrix}$$

$$= -2.6875$$

$$\text{Screen position} = \begin{pmatrix} 4.375 & 11.75 & 8.365 \\ 3.375 & 5.75 & 11.365 \end{pmatrix} \begin{pmatrix} 0.125 \\ 0.375 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 9.136 \\ 8.261 \end{pmatrix}$$

d) We have interpolated z, and interpolated

Interpolate between perspective corrected points

perspective corrected points

$$a = \begin{pmatrix} -2.1875 \\ -1.6875 \end{pmatrix}$$
$$b = \begin{pmatrix} -4.7 \\ -2.3 \end{pmatrix}$$
$$c = \begin{pmatrix} -2.78 \\ -3.78 \end{pmatrix}$$

perspective correct the incoming position $= \begin{pmatrix} -3.399 \\ -3.674 \end{pmatrix}$

Interpolate between these to get texture position

?

e) UGH! Not today, please please not today.

I would definitely not do this question

BELEN IS READING THIS ☺

4) a) i) ?

ii) ?

b) i) for each pixel in screen
fire ray from origin through the viewing plane

while ray depth < max ray depth
ray depth = ray depth + 1
closest intersection . distance = ∞
for each object in scene
if ray intersects object and ray intersection distance less than closest intersection distance
closest intersection = ray intersection
output colour += closest intersection . colour
if ray didn't intersect any objects
break

ii) Whether or not the ray intersected any of the objects and the depth of the ray tracing

c) i) Fire an additional ray from each intersection to the light source. If this ray intersects any objects, then the intersection is in a shadow.

ii) These artefacts occur due to floating point rounding error, the ray may be sent slightly from inside the object causing an intersection with the object itself and therefore a shadow to be rendered where it shouldn't be. This can be avoided by moving the ray origin a small amount in the direction of the normal of the intersection.

iii) Multiple shadow rays could be sent to random points within an area light source. The average colour of these shadow rays could then be used. This however will be much slower as multiple rays would have to be sent.