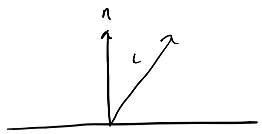
1)



Diffuse shading is how close the light vector is to the normal

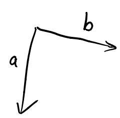
a) At each corner of the polygon, determine the normal vector, n and the light vector, L which is $-$ position of the vertex.

At each corner calculate a diffuse intensity as
$k_{DIFFUSE} \times n.L$
Use this to calculate the colour at each vertex
Interpolate the colour of each pixel from the colour of each corner

b) Instead of interpolating the colour of each pixel after shading interpolate the normal at each pixel and use this to calculate the shading.

c) Gouraud Shading will have artefacts especially around Specular highlights which Phong shading will not.

d)



$$a = P_3 - P_0$$
$$b = P_1 - P_0$$

$$a = \begin{pmatrix} 0 \\ 2 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ 10 \end{pmatrix}$$
$$= \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

$a \times b$ gives a normal

$$n = a \times b = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}$$

$$\begin{array}{cc} -1 & 5 \\ -4 & 0 \\ 2 & 0 \\ -1 & 5 \\ -4 & 0 \end{array}$$

Check that n is toward viewpoint

$$n \cdot (P_1 - V) \qquad V = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix}$$

> 0, so the normal is in the wrong direction

$$n_{OUTER} = \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$$

e) $n_{outer} = \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$

Center of quadrilateral is $\dfrac{P_0 + P_1 + P_2 + P_3}{4}$

$$= \begin{pmatrix} \frac{14}{4} \\ \frac{14}{4} \\ \frac{21}{2} \end{pmatrix}$$

$$L = -\begin{pmatrix} \frac{14}{4} \\ \frac{14}{4} \\ \frac{21}{2} \end{pmatrix}$$

$$n.L = \left(\frac{140}{4} + \frac{420}{2}\right) / 10$$

$$= (35 + 210)/10$$

$$= 24.5$$

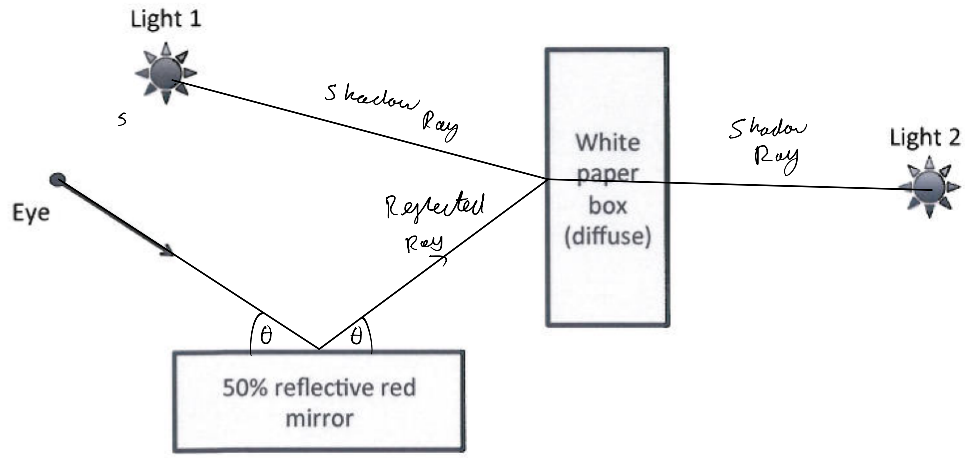
$k_d = 1$

$$|n||L| = \sqrt{1^2 + 2^2} \sqrt{\left(\tfrac{14}{4}\right)^2 + \left(\tfrac{14}{4}\right)^2 + \left(\tfrac{21}{2}\right)^2} = 25.95$$

$$\frac{n.L}{|n||L|} = 0.944$$

$$0.944 \times (255, 255, 255) = (241, 241, 241)$$

2) a) i)



ii) Colour = 0.5 (255, 0, 0) + 0.5 Reflected

Reflected = Diffuse * (255, 0, 0) * Shadow

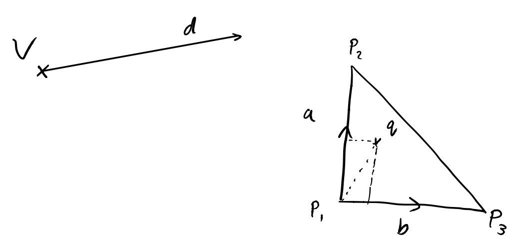
Shadow = 0.5 * Light1 + 0.5 Light2

Light1 = 1
Light 2 = 0

b) Ray tracing allows effects such as reflections, refraction, transparency and depth of field.

Ray tracing mimics a ray of light for each pixel, Radiosity models light energy for patches in a scene.

c)



Define the plane containing the triangle

$n = (P_2 - P_1) \times (P_3 - P_1)$ and then normalise!

$n \cdot p = P_1 \cdot n$ describes the plane

Ray eqn $\quad p = V + \lambda d$

$n \cdot (V + \lambda d) = P_1 \cdot n$ solve for λ

$n \cdot V + \lambda\, n \cdot d = P_1 \cdot n$

$$\lambda = \frac{n \cdot P_1 - n \cdot V}{n \cdot d} = \frac{n \cdot (P_1 - V)}{n \cdot d}$$

Use this to find the intersection point, q

$$q = V + \left(\frac{P_1 - n \cdot V}{n \cdot d} \right) d$$

Then determine if this lies in the triangle

Define $a = P_2 - P_1 \qquad b = P_3 - P_1$

Project $(q - P_1)$ onto a and b

Let $\alpha = \dfrac{(q - P_1) \cdot a}{|a||a|}$, $\beta = \dfrac{(q - P_1) \cdot b}{|b||b|}$

If
$0 < \alpha < 1$
$0 < \beta < 1$
$0 < \alpha + \beta < 1$

Then q lies within the triangle

d) $n = (P_2 - P_1) \times (P_3 - P_1)$

$$= \left(\begin{pmatrix} 12 \\ 6 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} \right) \times \left(\begin{pmatrix} 6 \\ 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}$$

$$\begin{array}{ccc} 6 & & 0 \\ 0 & & 4 \\ 0 & \times & -5 \\ 6 & \times & 0 \\ 0 & \times & 4 \end{array}$$

$$= \begin{pmatrix} 0 \\ -30 \\ 24 \end{pmatrix} \qquad n_{NORMAL} = \begin{pmatrix} 0 \\ 0.78 \\ 0.62 \end{pmatrix}$$

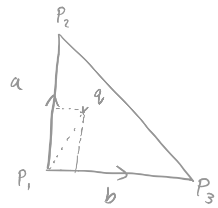
$$\lambda = \frac{n \cdot (P_1 - V)}{n \cdot d} \qquad P_1 - V = \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} - \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix}$$

$$n \cdot (V - P_1) = \begin{pmatrix} 0 \\ 0.78 \\ 0.62 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 10 \end{pmatrix} = 5.42 \qquad = \begin{pmatrix} -1 \\ -1 \\ 10 \end{pmatrix}$$

$$n \cdot d = \begin{pmatrix} 0 \\ 0.78 \\ 0.62 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0.62$$

$$q = V + \underbrace{\left(\frac{P_1 - n \cdot V}{n \cdot d} \right)}_{\lambda} d$$

$$\lambda = \frac{5.42}{0.62} = 8.74$$



$$q = \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$q = \begin{pmatrix} 7 \\ 7 \\ 8.74 \end{pmatrix}$$

Is this in the triangle?

$$a = P_2 - P_1 = \begin{pmatrix} 12 \\ 6 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$$

$$b = P_3 - P_1 = \begin{pmatrix} 6 \\ 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}$$

Project q onto a:

$$\alpha = \frac{(q - P_1) \cdot a}{|a||a|} \qquad q - P_1 = \begin{pmatrix} 1 \\ 1 \\ -1.26 \end{pmatrix}$$

$$= \frac{6}{6 \times 6}$$

$$= \frac{1}{6}$$

$$\beta = \frac{(q - P_1) \cdot b}{|b| \, |b|}$$

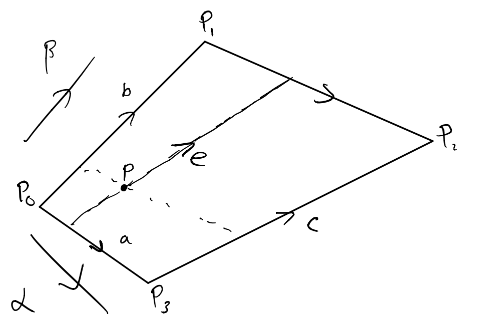
$$= \frac{4 + 6.3}{41}$$

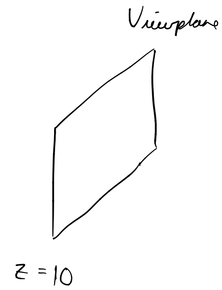
$$= \frac{10.3}{41}$$

$$= 0.25$$

$0 < \alpha < 1$ and $0 < \beta < 1$ and $0 < \alpha + \beta < 1$
so the point is in the triangle

3)





a) Given a texture coordinate

$$p = \alpha a + \beta e + P_0$$
$$e = b + \alpha(c - b)$$

$$p = \alpha a + \beta(b + \alpha(c-b)) + P_0$$
$$p = \alpha a + \beta b + \alpha\beta(c-b) + P_0$$

This is almost more intuitive as it is

$$a = P_3 - P_0$$
$$b = P_1 - P_0$$
$$c = P_2 - P_3$$

$$a = \begin{pmatrix} 20 \\ -20 \\ 20 \end{pmatrix} - \begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix}$$

$$c - b = \begin{pmatrix} 0 \\ 20 \\ 40 \end{pmatrix} - \begin{pmatrix} 0 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$b = \begin{pmatrix} -20 \\ 0 \\ 60 \end{pmatrix} - \begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 40 \end{pmatrix}$$

$\therefore$ Shape is a parallelogram!

$$c = \begin{pmatrix} 20 \\ 0 \\ 60 \end{pmatrix} - \begin{pmatrix} 20 \\ -20 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 40 \end{pmatrix}$$

$$p = \begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix}\alpha + \begin{pmatrix} 0 \\ 20 \\ 40 \end{pmatrix}\beta + \begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix}$$

b) $p = \begin{pmatrix} -15 \\ -5 \\ 50 \end{pmatrix}$

$$40\alpha - 20 = -15$$
$$40\alpha = 5$$
$$\alpha = \frac{5}{40} = \frac{1}{8}$$

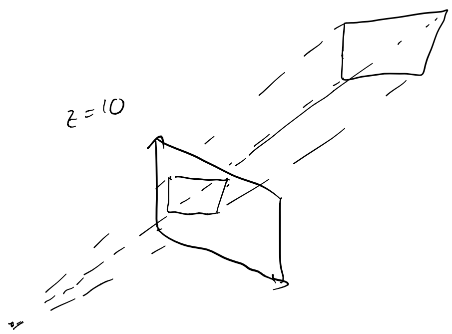
$$20\beta - 20 = -5$$
$$40\beta + 20 = 50$$

$$u = \left(\frac{v}{z} \Big/ \frac{1}{z} \right)$$

$$\beta = \frac{30}{40}$$

$$\beta = \frac{3}{4}$$

c)



For each point, draw line from $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ to the point, find intersection with plane at $z = 10$

View plane normal $= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot p = 10$ describes the viewing plane

$p = \lambda q$ is eqn of line to point

$\lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot q = 10$ and solve for λ

$$\lambda = \frac{10}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot q}$$

$p = \dfrac{10}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot q} q$ is the point of intersection with the viewing plane

	Point, P	Projected Point, P'
P_0	$\begin{pmatrix}-20\\-20\\20\end{pmatrix}$	$\begin{pmatrix}-10\\-10\\10\end{pmatrix}$
P_1	$\begin{pmatrix}-20\\0\\60\end{pmatrix}$	$\begin{pmatrix}-\frac{10}{3}\\0\\10\end{pmatrix}$
P_2	$\begin{pmatrix}20\\0\\60\end{pmatrix}$	$\begin{pmatrix}\frac{10}{3}\\0\\10\end{pmatrix}$
P_3	$\begin{pmatrix}20\\-20\\20\end{pmatrix}$	$\begin{pmatrix}10\\-10\\10\end{pmatrix}$
P_{int}	$\begin{pmatrix}-15\\5\\50\end{pmatrix}$	$\begin{pmatrix}-3\\1\\10\end{pmatrix}$

d)

$a = P_1 - P_0 = \begin{pmatrix}\frac{20}{3}\\10\\0\end{pmatrix}$ $\quad b = P_3 - P_0 = \begin{pmatrix}20\\0\\0\end{pmatrix}$

$$p = \alpha a + \beta b + P_0$$

$$p = \begin{pmatrix}\frac{20}{3}\\10\\0\end{pmatrix}\alpha + \begin{pmatrix}20\\0\\0\end{pmatrix}\beta + \begin{pmatrix}-10\\-10\\10\end{pmatrix} = \begin{pmatrix}-3\\1\\10\end{pmatrix}$$

$$\frac{20}{3}\alpha + 20\beta - 10 = -3$$

$$10\alpha - 10 = 1$$

$$10\alpha = 11$$

$$\alpha = \frac{11}{10} \qquad ?$$

$$\frac{22}{3} + 20\beta - 10 = -3$$

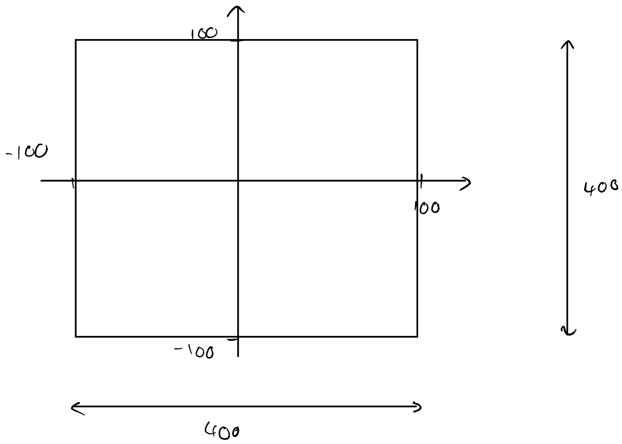
$$20\beta = 7 - \frac{22}{3}$$

$$\beta = \frac{7 - \frac{22}{3}}{20}$$

Reason they are different is because of perspective texture mapping, a correction needs to be applied.

$$\beta = \frac{-1}{60} \; ?$$

e)



$$200 + \frac{x}{200} \times 400 \quad 200 + \frac{y}{200} \times 400$$

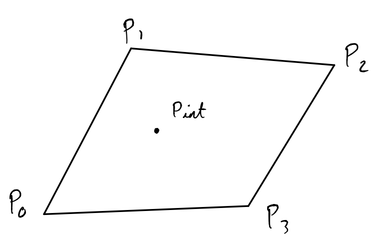
$$(-100, -100) = (0, 0)$$

$$\begin{pmatrix} x_{pix} \\ y_{pix} \end{pmatrix} = \begin{pmatrix} 200 \\ 200 \end{pmatrix} + 2 \begin{pmatrix} x \\ y \end{pmatrix}$$

P_0	$\begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix}$	$\begin{pmatrix} -10 \\ -10 \\ 10 \end{pmatrix}$
P_1	$\begin{pmatrix} -20 \\ 0 \\ 60 \end{pmatrix}$	$\begin{pmatrix} -\frac{10}{3} \\ 0 \\ 10 \end{pmatrix}$
P_2	$\begin{pmatrix} 20 \\ 0 \\ 60 \end{pmatrix}$	$\begin{pmatrix} \frac{10}{3} \\ 0 \\ 10 \end{pmatrix}$
P_3	$\begin{pmatrix} 20 \\ -20 \\ 20 \end{pmatrix}$	$\begin{pmatrix} 10 \\ -10 \\ 10 \end{pmatrix}$
P_{int}	$\begin{pmatrix} -15 \\ 5 \\ 50 \end{pmatrix}$	$\begin{pmatrix} -3 \\ 1 \\ 10 \end{pmatrix}$

Point	Pixel Coords
P_0	$\begin{pmatrix} 180 \\ 180 \end{pmatrix}$
P_1	$\begin{pmatrix} 193.3 \\ 200 \end{pmatrix}$
P_2	$\begin{pmatrix} 206.7 \\ 200 \end{pmatrix}$
P_3	$\begin{pmatrix} 220 \\ 180 \end{pmatrix}$
P_{int}	$\begin{pmatrix} 194 \\ 202 \end{pmatrix}$

e)



Interpote (α, β) from the pixel coordinates

$$P_{int} \simeq \alpha(P_1 - P_0) + \beta(P_3 - P_0) + \alpha\beta(P_2 - P_3 - P_1 + P_0)$$

? Texture problem sheet?

4) $P(\mu, v)$

At $P(0,0)$

$$\frac{\partial P}{\partial \mu} = \frac{P(1,0) - P(-1,0)}{2} = \frac{\begin{pmatrix}2\\14\\1\end{pmatrix} - \begin{pmatrix}0\\10\\1\end{pmatrix}}{2} = \begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\frac{\partial P}{\partial v} = \frac{P(0,1) - P(0,-1)}{2} = \frac{\begin{pmatrix}1\\13\\2\end{pmatrix} - \begin{pmatrix}1\\11\\0\end{pmatrix}}{2} = \begin{pmatrix}0\\1\\1\end{pmatrix}$$

At $P(0,1)$

$$\frac{\partial P}{\partial u} = \frac{P(1,1) - P(-1,1)}{2} = \frac{\begin{pmatrix}2\\15\\2\end{pmatrix} - \begin{pmatrix}0\\11\\2\end{pmatrix}}{2} = \begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\frac{\partial P}{\partial v} = \frac{P(0,2) - P(0,0)}{2} = \frac{\begin{pmatrix}1\\13\\3\end{pmatrix} - \begin{pmatrix}1\\12\\1\end{pmatrix}}{2} = \begin{pmatrix}0\\\frac{1}{2}\\1\end{pmatrix}$$

At $P(1,0)$

$$\frac{\partial P}{\partial u} = \frac{P(2,0) - P(0,0)}{2} = \frac{\begin{pmatrix}3\\16\\1\end{pmatrix} - \begin{pmatrix}1\\12\\1\end{pmatrix}}{2} = \begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\frac{\partial P}{\partial v} = \frac{P(1,1) - P(1,-1)}{2} = \frac{\begin{pmatrix}2\\15\\2\end{pmatrix} - \begin{pmatrix}2\\12\\0\end{pmatrix}}{2} = \begin{pmatrix}0\\\frac{3}{2}\\0\end{pmatrix}$$

At $P(1,1)$

$$\frac{\partial P}{\partial c} = \frac{P(2,1) - P(0,1)}{2} \qquad\qquad \frac{\partial P}{\partial v} = \frac{P(1,2) - P(1,0)}{2}$$

$$= \frac{\begin{pmatrix}3\\19\\2\end{pmatrix} - \begin{pmatrix}1\\13\\2\end{pmatrix}}{2} \qquad\qquad = \begin{pmatrix}2\\14\\3\end{pmatrix} - \begin{pmatrix}2\\14\\1\end{pmatrix} \Big/ 2$$

$$= \begin{pmatrix}1\\3\\0\end{pmatrix} \qquad\qquad = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

b) $P(\mu, 0) \Rightarrow P_0 = P(0,0)$
$P_1 = P(1,0)$

$$P_0 = \begin{pmatrix}1\\12\\1\end{pmatrix} \qquad P_1 = \begin{pmatrix}2\\14\\1\end{pmatrix}$$

$$P_0' = \frac{\partial P(0,0)}{\partial u} \qquad P_1' = \frac{\partial P(1,0)}{\partial \mu}$$

$$= \begin{pmatrix}1\\2\\0\end{pmatrix} \qquad = \begin{pmatrix}1\\2\\6\end{pmatrix}$$

$$\begin{pmatrix}a_0\\a_1\\a_2\\a_3\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -3 & -2 & 3 & -1\\ 2 & 1 & -2 & 1\end{pmatrix}\begin{pmatrix}1 & 12 & 1\\ 1 & 2 & 0\\ 2 & 14 & 1\\ 1 & 2 & 0\end{pmatrix}$$

$$= \begin{pmatrix}1 & 12 & 1\\ 1 & 2 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \qquad a_0 = \begin{pmatrix}1\\12\\1\end{pmatrix} \qquad a_1 = \begin{pmatrix}1\\2\\0\end{pmatrix}$$

a_2 and a_3 are $\underline{0}$

$$P(\mu, 0) = a_3\mu^3 + a_2\mu^2 + a_1\mu + a_0$$

$$P(\mu, 0) = \begin{pmatrix}1\\12\\1\end{pmatrix}\mu + \begin{pmatrix}1\\2\\0\end{pmatrix}$$

Note that this is just a straight line

c) $\mu = v = 0.5$

$$\begin{aligned} P(0.5,0.5) = {} & 0.5\,P(0.5,0) + 0.5\,P(0.5,1) + 0.5\,P(0,0.5) \\ & + 0.5\,P(1,0.5) - 0.25\,P(0,0) - 0.25\,P(0,1) \\ & - 0.25\,P(1,0) - 0.25\,P(1,1) \end{aligned}$$

$$P(0.5,0) = \begin{pmatrix} \frac{3}{2} \\ 8 \\ \frac{1}{2} \end{pmatrix}$$

$$P(0.5,1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} 0.5^3 + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} 0.5^2 + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} 0.5 + \begin{pmatrix} 1 \\ 13 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1.5 \\ 13.875 \\ 2 \end{pmatrix}$$

$$P(0,0.5) = \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} 0.5^3 + \begin{pmatrix} 0 \\ 0.5 \\ 0 \end{pmatrix} 0.5^2 + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} 0.5 + \begin{pmatrix} 1 \\ 12 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 12.5625 \\ 1.5 \end{pmatrix}$$

$$P(1,0.5) = \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} 0.5^3 + \begin{pmatrix} 0 \\ 1.5 \\ 1 \end{pmatrix} 0.5 + \begin{pmatrix} 1 \\ 14 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 14.6875 \\ 1.5 \end{pmatrix}$$

$$P(0,0) = \begin{pmatrix} 1 \\ 12 \\ 1 \end{pmatrix} \qquad P(1,0) = \begin{pmatrix} 2 \\ 14 \\ 1 \end{pmatrix} \qquad P(0,1) = \begin{pmatrix} 1 \\ 13 \\ 2 \end{pmatrix} \qquad P(1,1) = \begin{pmatrix} 2 \\ 15 \\ 2 \end{pmatrix}$$

$$P(0.5,0.5) = 0.5\,P(0.5,0) + 0.5\,P(0.5,1) + 0.5\,P(0,0.5) + 0.5\,P(1,0.5) - 0.25\,P(0,0) - 0.25\,P(0,1) - 0.25\,P(1,0) - 0.25\,P(1,1)$$

Let's do all the x terms first...

$$P(0.5,0.5)_x = 0.5 \times 1.5 + 0.5 \times 1.5 + 0.5 \times 1 + 0.5 \times 1 - 0.25 - 0.5 - 0.25 - 0.5 = 1$$

$$P(0.5,0.5)_y = 0.5 \times 8 + 0.5 \times 13.875 + 0.5 \times 12.5625 + 0.5 \times 14.6875 - 3 - 3.5 - 3.25 - 3.75 = 11.0625$$

$$P(0.5,0.5)_z = 0.5 \times 0.5 + 0.5 + 0.75 + 0.75 - 0.25 - 0.25 - 0.5 - 0.5 = 0.75$$

$$P(0.5,0.5) = \begin{pmatrix} 1 \\ 11.0625 \\ 0.75 \end{pmatrix}$$

d) Find $P(\mu, 0.5)$ and differentiate

$$P_0 = P(0, 0.5) = \begin{pmatrix} 1 \\ 12.5625 \\ 1.5 \end{pmatrix} \qquad P_1 = P(1, 0.5) = \begin{pmatrix} 1 \\ 14.6875 \\ 1.5 \end{pmatrix}$$

$$P_0' = \frac{\frac{\partial P(0,0)}{\partial \mu} + \frac{\partial P(0,1)}{\partial \mu}}{2} = \frac{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}{2} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$P_1' = \frac{\frac{\partial P(1,0)}{\partial \mu} + \frac{\partial P(1,1)}{\partial \mu}}{2} = \frac{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}}{2} = \begin{pmatrix} 1 \\ 2.5 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 12.5625 & 1.5 \\ 1 & 2 & 0 \\ 1 & 14.6875 & 1.5 \\ 1 & 2.5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 12.5625 & 1.5 \\ 1 & 2 & 0 \\ 1 & 8.875 & 0 \\ 2 & 0.25 & 0 \end{pmatrix}$$

$$P(\mu, 0.5) = \begin{pmatrix} 2 \\ 0.25 \\ 0 \end{pmatrix} \mu^3 + \begin{pmatrix} 1 \\ 8.875 \\ 0 \end{pmatrix} \mu^2 + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \mu + \begin{pmatrix} 1 \\ 12.5625 \\ 1.5 \end{pmatrix}$$

$$P'(\mu, 0.5) = \begin{pmatrix} 6 \\ 0.75 \\ 0 \end{pmatrix} \mu^2 + \begin{pmatrix} 1 \\ 17.75 \\ 0 \end{pmatrix} \mu + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

At $\mu = 0.5$

$$P'(0.5, 0.5) = \begin{pmatrix} 3 \\ 11.0625 \\ 0 \end{pmatrix}$$