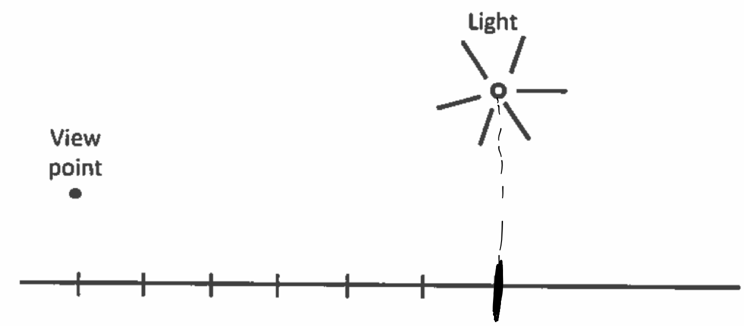
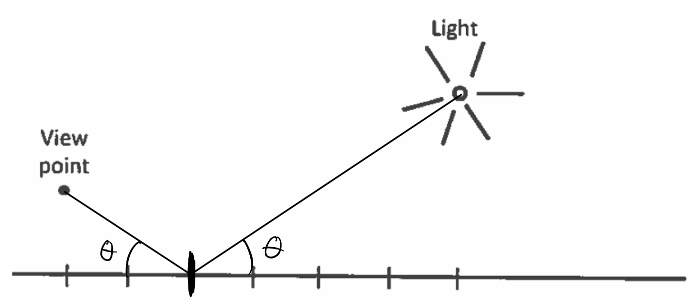
1) a) i)



n.l gives the diffuse term, at this point, the light is in the same direction as the normal, so the diffuse term will have the highest intensity

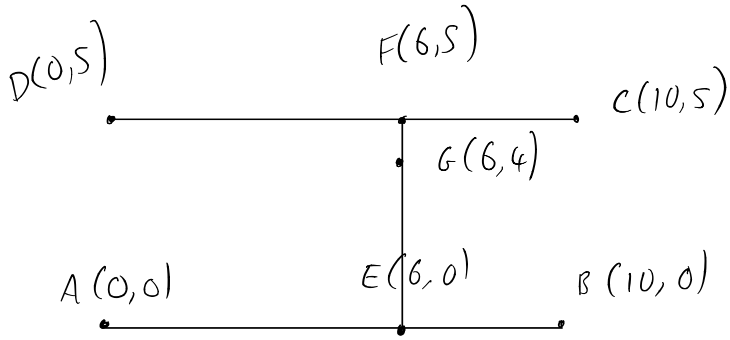
ii)



Specular component is given by how close the vector to the viewpoint is to the reflected ray. At this point, the vectors are in the same direction so the specular component will be largest

iii) The location of the brightest point will remain the same for the diffuse component because it is still brightest at the point when the surface normal points towards the light as this is the closest point on the surface to the light.

b)



i) $E = \frac{6}{10} B + \frac{4}{10} A$

ii) $F = \frac{6}{10} C + \frac{4}{10} D$

iii) $G = \frac{4}{5} F + \frac{1}{5} E$

iv) $G = \frac{4}{5}\left(\frac{6}{10} C + \frac{4}{10} D\right) + \frac{1}{5}\left(\frac{6}{10} B + \frac{4}{10} A\right)$

$$G = \frac{4}{5}\left(\begin{pmatrix}0\\150\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\end{pmatrix}\right) + \frac{1}{5}\left(\begin{pmatrix}150\\150\\0\end{pmatrix} + \begin{pmatrix}100\\0\\0\end{pmatrix}\right)$$

$$= \frac{4}{5}\begin{pmatrix}0\\150\\0\end{pmatrix} + \frac{1}{5}\begin{pmatrix}250\\150\\0\end{pmatrix}$$

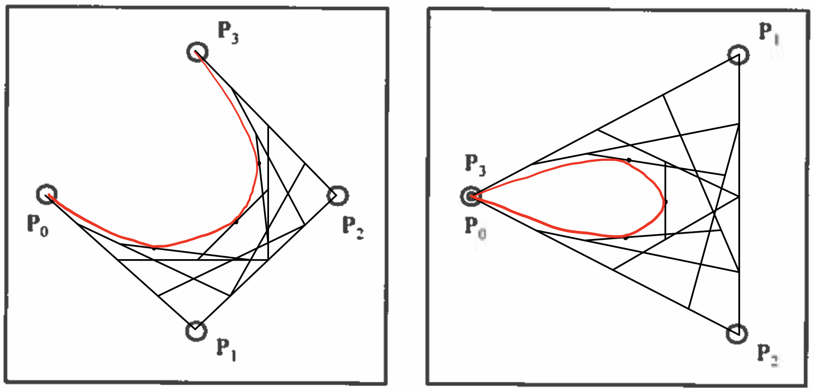
$$= \begin{pmatrix}50\\150\\0\end{pmatrix}$$

v) $n_G = \frac{4}{5}\left(\frac{6}{10} n_C + \frac{4}{10} n_D\right) + \frac{1}{5}\left(\frac{6}{10} n_B + \frac{4}{10} n_A\right)$

$$= \frac{12}{25} n_C + \frac{8}{25} n_D + \frac{3}{25} n_B + \frac{2}{25} n_A$$

c) Blinn phong uses the halfway vector between the viewer and the lightsource. This is computationally faster than using the angle of the reflected ray.

2) a)



b) $P(\mu) = \sum_{i=0}^{3} a_i(\mu) P_i$

$P(0) = P_0$

$P(1) = P_3$

$P(\mu) = a_0(\mu) P_0 + a_1(\mu) P_1 + a_2(\mu) P_2 + a_3(\mu) P_3$

$a_0(0) = 1$
$a_1(0) = 0$
$a_2(0) = 0$
$a_3(0) = 0$

At all other points,

$a_0(\mu) + a_1(\mu) + a_2(\mu) + a_3(\mu) = 1$

$a_0(1) = 0$
$a_1(1) = 0$
$a_2(1) = 0$
$a_3(0) = 0$

$a_0(\mu) = (1-\mu)^3 \quad a_1(\mu) = 3\mu(1-\mu)^2 \quad a_2(\mu) = 3\mu^2(1-u) \quad a_3(\mu) = \mu^3$

c) Differentiating the above formula with respect to μ,

$P(\mu) = (1-\mu)^3 P_0 + 3\mu(1-\mu)^2 P_1 + 3\mu^2(1-\mu) P_2 + \mu^3 P_3$

$P'(\mu) = -3(1-\mu)^2 P_0 + 3[(1-\mu)^2 - 2\mu(1-\mu)] P_1 + 3[2\mu(1-\mu) - \mu^2] P_2 + 3\mu^2 P_3$

$$P'(0) = -3P_0 + 3P_1$$

$$P'(1) = -3P_2 + 3P_3$$

Setting $P_0 = P_3$

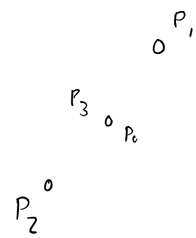
$$P'(1) = -3P_2 + 3P_0$$

So for these gradients to be equal,

$$-3P_0 + 3P_1 = -3P_2 + 3P_0$$

$$6P_0 = 3P_1 + 3P_2$$

$$P_0 = \frac{P_1 + P_2}{2}$$



This would require P_0, P_3 lie on the line of $\overrightarrow{P_2 P_1}$ which creates a bezier curve which is simply a line and is therefore not closed.

d) i) $0 \leq \mu \leq 1$

$$0 \geq -\mu \geq -1$$

$$1 \geq 1-\mu \geq 0$$

$$0 \leq 1-\mu \leq 1$$

$$0 \leq (1-\mu)^i \leq 1$$

$$(1-\mu)^i \geq 0 \qquad \mu^i \geq 0 \qquad \binom{3}{i} \geq 0$$

$$a_i(\mu) = \binom{3}{i}(1-\mu)^{3-i}\mu^i$$

$$\therefore \quad a_i(\mu) \geq 0$$

ii) $$\sum_{i=0}^{3} a_i(\mu) = \sum_{i=0}^{3} \mu^i (1-\mu)^{3-i}$$

$$= \mu^3 + 3\mu^2(1-\mu) + 3\mu(1-\mu)^2 + (1-\mu)^3$$

$$= \mu^3 + 3\mu^2 - 3\mu^3 + 3\mu(1-2\mu+\mu^2) + 1 - 3\mu + 3\mu^2 - \mu^3$$

$$= \cancel{\mu^3} + \cancel{3\mu^2} \cancel{- 3\mu^3} \cancel{+ 3\mu} - \cancel{6\mu^2} + \cancel{3\mu^3} + 1 \cancel{- 3\mu} \cancel{+ 3\mu^2} - \cancel{\mu^3}$$

$$= 1$$

iii) Implies that all points in the curve must lie in between all four of the control points.

3) a) for every screen point (x, y)
Initialise z buffer to the back-plane culling distance
for every fragment to be rendered
project fragment to screen point (x, y)
If z value of the fragment < z value in buffer at (x, y)
Place the fragment at (x, y)
Set the z buffer at (x, y) to the z value of the fragment

b) An extra colour channel, alpha representing the transparency of the fragments is required.

Assuming this channel is present, the z buffer algorithm can be modified, the colour of screen point (x, y) is updated even if the z value of the fragment is higher than the value in the z buffer. It is a linear interpolation of the colour in the buffer and the colour of the new fragment, weighted using the transparency of the object in the buffer. The z buffer is then set to the value of the new fragment and the transparency is also updated.

c) One approach is to use several samples per pixel and then find the colour using a reconstruction filter. Another approach is to render the image at a high resolution and then downscale it (FSAA)

d) The advantage of a reconstruction filter is it is flexible and computationally cheap, however it can still lead to artefacts.

FSAA produces a nice result but is computationally expensive as it involves rendering the whole scene at a higher resolution.

e) i) Picket Fence Effect

ii) $(5,5)$ would be $\begin{pmatrix} 180 \\ 225 \\ 90 \end{pmatrix}$

$(3,4)$ would be $$\frac{\begin{pmatrix} 180 \\ 225 \\ 90 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 180 \end{pmatrix}}{2}$$

$$= \begin{pmatrix} 90 \\ 112.5 \\ 135 \end{pmatrix}$$

4) a) Perspective Projection e

Identity f

Orthographic Projection d

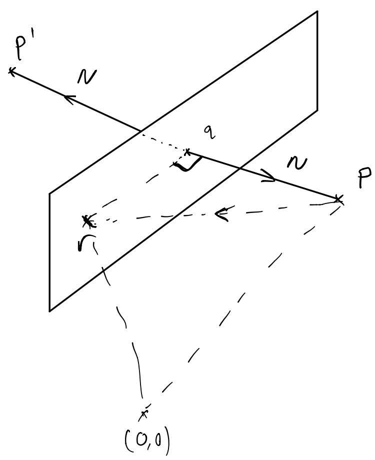
Uniform Scaling a

Non-Uniform Scaling b

Rotation c

b) $a = 0$ $b = 1$ $c = 0$ $d = y_0$

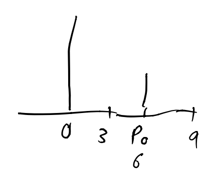
$$n \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -y_0 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



Project P-r onto N to find the point on the plane which is closest

$[(r-P)\cdot n]n$ gives the line to plane, double it

$$P' = P + 2[(r-P)\cdot n]n$$



$$r = \begin{pmatrix} 0 \\ -P_0 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$(y - P_0)$

let $P = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$P' = P + 2\left[\left(\begin{pmatrix} 0 \\ -P_0 \\ 0 \end{pmatrix} - P\right)\cdot n\right]n$$

$$P' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$P' = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + 2(-P_0 - y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$P' = \begin{pmatrix} x \\ -y - 2P_0 \\ z \end{pmatrix}$$

The matrix for this transformation is then

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -2P_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c) (a) $M \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

(b) $M \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

(c) $M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(d) $M \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$$M \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}^{-1}$$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

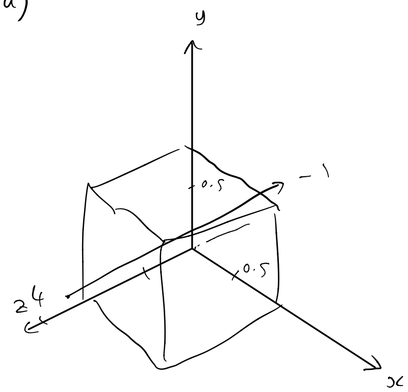
$$M = \begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Check against $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ for sanity

$$\begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ which looks correct!

d)



i) $f = 2$

Shift focal point to origin first, project then return

$$C = \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -0.5 & 0 \end{pmatrix}$$

Final matrix is ABC

$$= A \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & -0.5 & 2 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & -0.25 & 0.5 \\ 0 & 1 & -0.25 & 0.5 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & -0.5 & 2 \end{pmatrix}$$

ii) $$M \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.25 \\ 3 \\ 1.5 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 2 \\ 1 \end{pmatrix} \qquad M \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 1.25 \\ 3 \\ 1.5 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \\ \frac{5}{6} \\ 2 \\ 1 \end{pmatrix}$$

$$M\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}1.25\\1.25\\3\\1.5\end{pmatrix} = \begin{pmatrix}\frac{5}{6}\\\frac{5}{6}\\2\\1\end{pmatrix} \qquad M\begin{pmatrix}1\\0\\1\\1\end{pmatrix} = \begin{pmatrix}\frac{5}{6}\\\frac{1}{6}\\2\\1\end{pmatrix}$$