

**Due:** Monday 4/22/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

**Self assessment due:** Monday 4/29/2019 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope – be sure to delete any extra title pages that Gradescope attaches). For each subpart where your original answer was correct, write “correct.” Otherwise, write and explain the correct answer. **Do not leave any boxes empty.**

If you did not submit the homework (or skipped some questions) but wish to receive credit for the self-assessment, we ask that you first complete the homework without looking at the solutions, and then perform the self assessment afterwards.

**Policy:** Can be solved in groups (acknowledge collaborators) but must be written up individually

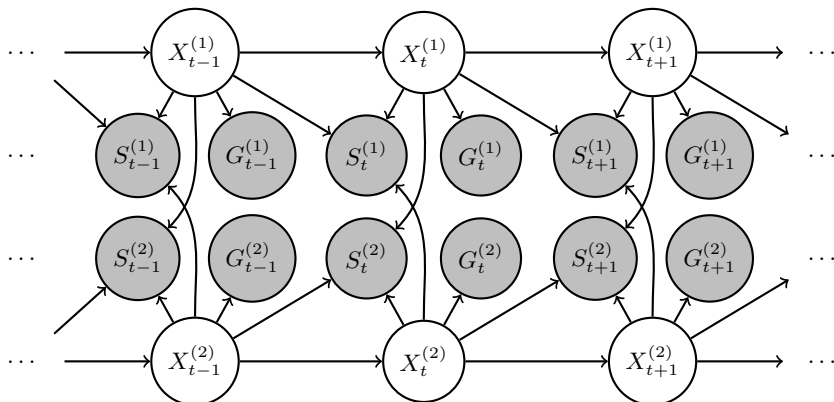
**Submission:** Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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# Q1. Particle Filtering: Where are the Two Cars?

As before, we are trying to estimate the location of cars in a city, but now, we model two cars jointly, i.e. car  $i$  for  $i \in \{1, 2\}$ . The modified HMM model is as follows:

- $X^{(i)}$  – the location of car  $i$
- $S^{(i)}$  – the noisy location of the car  $i$  from the signal strength at a nearby cell phone tower
- $G^{(i)}$  – the noisy location of car  $i$  from GPS



$d$	$D(d)$	$E_L(d)$	$E_N(d)$	$E_G(d)$
-4	0.05	0	0.02	0
-3	0.10	0	0.04	0.03
-2	0.25	0.05	0.09	0.07
-1	0.10	0.10	0.20	0.15
0	0	0.70	0.30	0.50
1	0.10	0.10	0.20	0.15
2	0.25	0.05	0.09	0.07
3	0.10	0	0.04	0.03
4	0.05	0	0.02	0

The signal strength from one car gets noisier if the other car is at the same location. Thus, the observation  $S_t^{(i)}$  also depends on the current state of the other car  $X_t^{(j)}$ ,  $j \neq i$ .

The transition is modeled using a drift model  $D$ , the GPS observation  $G_t^{(i)}$  using the error model  $E_G$ , and the observation  $S_t^{(i)}$  using one of the error models  $E_L$  or  $E_N$ , depending on the car's speed and the relative location of both cars. These drift and error models are in the table above. **The transition and observation models are:**

$$\begin{aligned}
 P(X_t^{(i)} | X_{t-1}^{(i)}) &= D(X_t^{(i)} - X_{t-1}^{(i)}) \\
 P(S_t^{(i)} | X_{t-1}^{(i)}, X_t^{(i)}, X_t^{(j)}) &= \begin{cases} E_N(X_t^{(i)} - S_t^{(i)}), & \text{if } |X_t^{(i)} - X_{t-1}^{(i)}| \geq 2 \text{ or } X_t^{(i)} = X_t^{(j)} \\ E_L(X_t^{(i)} - S_t^{(i)}), & \text{otherwise} \end{cases} \\
 P(G_t^{(i)} | X_t^{(i)}) &= E_G(X_t^{(i)} - G_t^{(i)}).
 \end{aligned}$$

Throughout this problem you may give answers either as unevaluated numeric expressions (e.g.  $0.1 \cdot 0.5$ ) or as numeric values (e.g. 0.05). The questions are decoupled.

(a) Assume that at  $t = 3$ , we have the single particle ( $X_3^{(1)} = -1, X_3^{(2)} = 2$ ).

- (i) What is the probability that this particle becomes ( $X_4^{(1)} = -3, X_4^{(2)} = 3$ ) after passing it through the dynamics model?

Answer: 0.025

- (ii) Assume that there are no sensor readings at  $t = 4$ . What is the joint probability that the *original* single particle (from  $t = 3$ ) becomes ( $X_4^{(1)} = -3, X_4^{(2)} = 3$ ) and then becomes ( $X_5^{(1)} = -4, X_5^{(2)} = 4$ )?

Answer: 0.025\*0.01

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

For the remaining of this problem, we will be using 2 particles at each time step.

- (b) At  $t = 6$ , we have particles  $[(X_6^{(1)} = 3, X_6^{(2)} = 0), (X_6^{(1)} = 3, X_6^{(2)} = 5)]$ . Suppose that after weighting, resampling, and transitioning from  $t = 6$  to  $t = 7$ , the particles become  $[(X_7^{(1)} = 2, X_7^{(2)} = 2), (X_7^{(1)} = 4, X_7^{(2)} = 1)]$ .

- (i) At  $t = 7$ , you get the observations  $S_7^{(1)} = 2, G_7^{(1)} = 2, S_7^{(2)} = 2, G_7^{(2)} = 2$ . What is the weight of each particle?

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	<b>0.7*0.5</b>
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	<b>0.2*0.15</b>

- (ii) Suppose both cars' cell phones died so you only get the observations  $G_7^{(1)} = 2, G_7^{(2)} = 2$ . What is the weight of each particle?

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	<b>0.5</b>
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	<b>0.15</b>

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

- (c) To decouple this question, assume that you got the following weights for the two particles.

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	0.09
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	0.01

What is the belief for the location of car 1 and car 2 at  $t = 7$ ?

Location	$P(X_7^{(1)})$	$P(X_7^{(2)})$
$X_7^{(i)} = 1$		
$X_7^{(i)} = 2$		
$X_7^{(i)} = 4$		

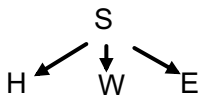
**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

## Q2. Naive Bayes

Your friend claims that he can write an effective Naive Bayes spam detector with only three features: the hour of the day that the email was received ( $H \in \{1, 2, \dots, 24\}$ ), whether it contains the word ‘viagra’ ( $W \in \{\text{yes}, \text{no}\}$ ), and whether the email address of the sender is Known in his address book, Seen before in his inbox, or Unseen before ( $E \in \{K, S, U\}$ ).

(a) Flesh out the following information about this Bayes net:

**Graph structure:**



**Parameters:**  $P(S)$ ,  $P(H|S)$ ,  $P(W|S)$ ,  $P(E|S)$

**Size of the set of parameters:**  $2 \cdot 24 \cdot 2 \cdot 3 = 288$

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

Suppose now that you labeled three of the emails in your mailbox to test this idea:

spam or ham?	$H$	$W$	$E$
spam	3	yes	S
ham	14	no	K
ham	15	no	K

(b) Use the three instances to estimate the maximum likelihood parameters.

$P(\text{spam})=0.33$ ,  $P(\text{ham})=0.67$

$P(H=3|\text{spam})=1$ ,  $P(W=\text{yes}|\text{spam})=1$ ,  $P(E=S|\text{spam})=1$

$P(H=14|\text{ham})=0.5$ ,  $P(H=15|\text{ham})=0.5$ ,  $P(W=\text{no}|\text{ham})=1$ ,  $P(E=K|\text{ham})=1$ , and else is 0

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

(c) Using the maximum likelihood parameters, find the predicted class of a new datapoint with  $H = 3$ ,  $W = \text{no}$ ,  $E = U$ .

Either spam or ham

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

- (d) Now use the three to estimate the parameters using Laplace smoothing and  $k = 2$ . Do not forget to smooth both the class prior parameters and the feature values parameters.

$$P(H=14|\text{ham})=3/50, P(H=15|\text{ham})=3/50, P(H=1|\text{ham})=1/25, P(H=2|\text{ham})=1/25...$$

$$P(H=3|\text{spam})=3/49, P(H=1|\text{spam})=2/49, P(H=2|\text{spam})=2/49...$$

$$P(W=\text{no}|\text{ham})=0.67, P(W=\text{yes}|\text{ham})=0.33, P(W=\text{no}|\text{spam})=0.4, P(W=\text{yes}|\text{spam})=0.6$$

$$P(E=K|\text{ham})=0.5, P(E=S|\text{ham})=0.25, P(E=U|\text{ham})=0.25, P(E=K|\text{spam})=2/7, P(E=U|\text{spam})=2/7, P(E=S|\text{spam})=3/7$$

$$P(\text{spam})=3/7, P(\text{ham})=4/7$$

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

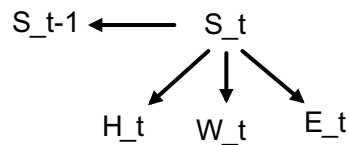
- (e) Using the parameters obtained with Laplace smoothing, find the predicted class of a new datapoint with  $H = 3$ ,  $W = \text{no}$ ,  $E = U$ .

$$0.0038 > 0.0030, \text{ so ham.}$$

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.

- (f) You observe that you tend to receive spam emails in batches. In particular, if you receive one spam message, the next message is more likely to be a spam message as well. Specify a new graphical that most naturally captures this phenomenon.

Graph structure:



Parameters:  $P(S_{t-1}|S_t), P(H_t|S_t), P(W_t|S_t), P(E_t|S_t)$

Size of the set of parameters: 576

**Self assessment** If correct, write “correct” in the box. Otherwise, write and explain the correct answer.