CS100
Introduction to Programming

Lecture 10. Recursion

Learning Objectives

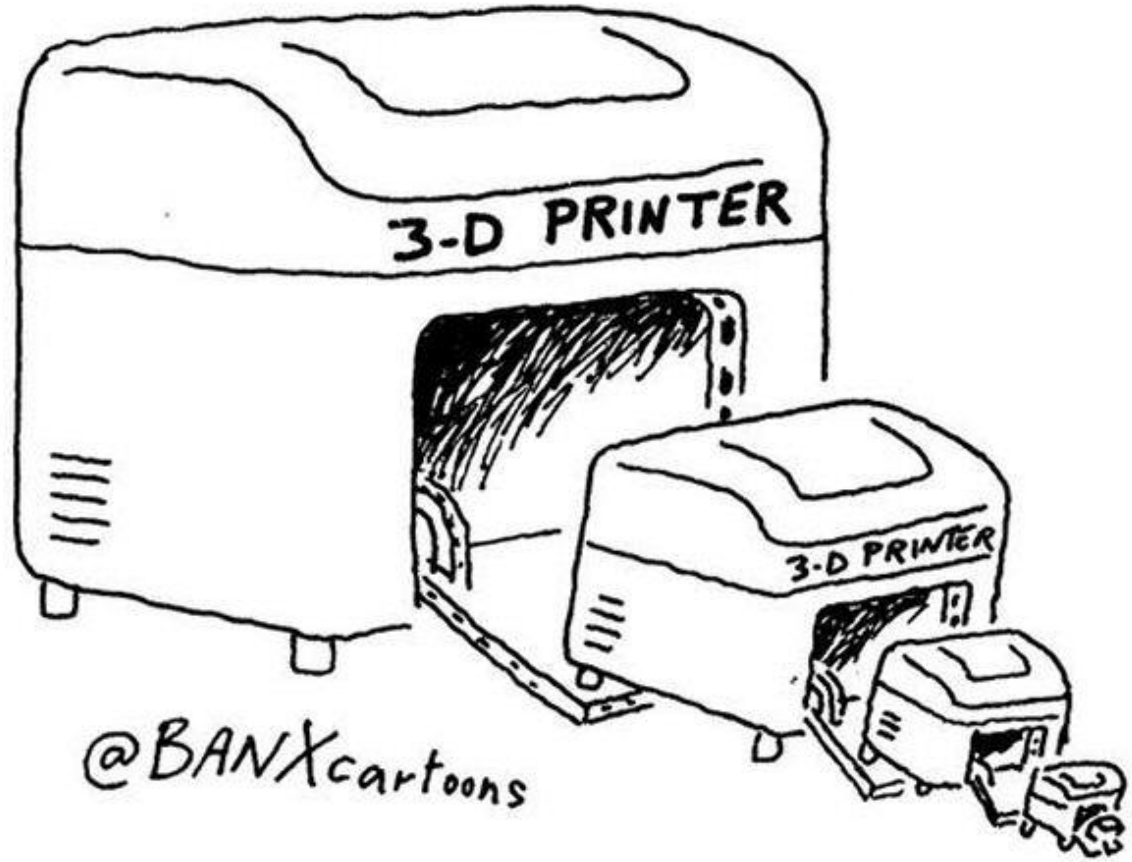
- At the end of this lecture, you should be able to understand the following:
  - Recursive Methods
  - Recursive Cheers
  - Recursive PrintSomething
  - Recursive Factorials
  - Recursive Multiplication by Addition
  - Recursive PrintDigits
  - Recursive SumArray
  - Tower of Hanoi

What Is Recursion?

- The method in which a problem is solved by reducing it to smaller cases of the same problem.
- Recursion is the name for the case when
  - a function calls itself, or
  - calls a sequence of other functions, one of which eventually calls the first function again.
- Two parts
  - **Base case** (with terminating condition)
  - **Recursive case** (with recursive condition)

Recursion in Art and Life

Recursion in Art and Life



Recursion in Art and Life

FROM THE MIRROR.

Life Mirror Recursive (1909),
by Coles Phillips (1880 – 1927)

Recursion in Art and Life

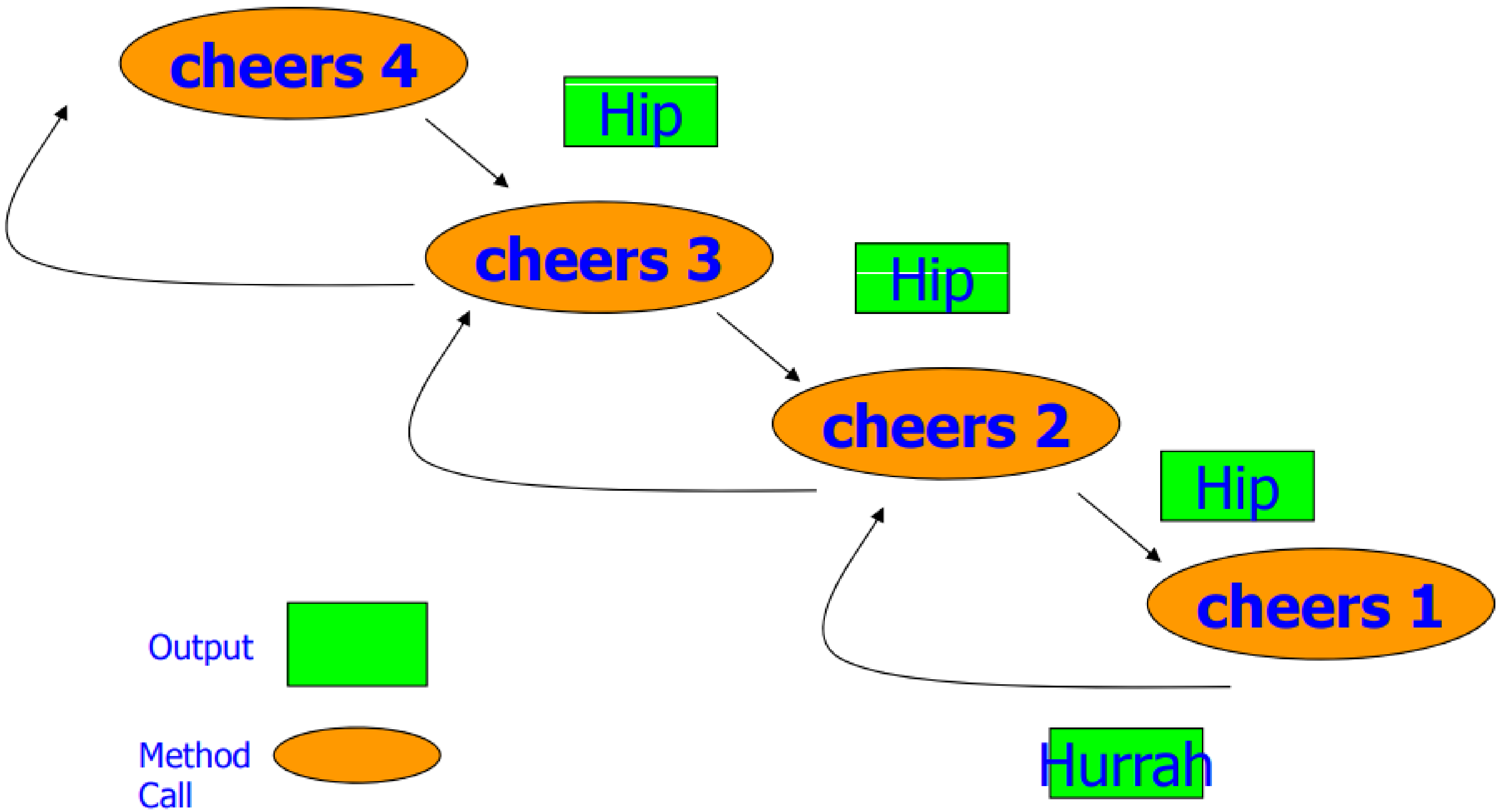
Example 1: Cheers

What does the following **recursive** method print when called with cheers(4)?

```
void cheers(int n)
{
   if (n <= 1)
      printf("Hurrah \n");
   else {
      printf("Hip \n");
      cheers(n-1);
   }
}
```

Output:
Hip
Hip
Hip
Hurrah

Recursive Cheers – Tracing



Recursive Cheers

```
void cheers(int n)
{
  if (n <= 1)
     printf("Hurrah \n");
  else {
     printf("Hip \n");
     cheers(n-1);
  }
}
```

terminating condition

recursive condition (n > 1)

Example 2: PrintSomething

What does the following **recursive** method print when called with printSomething(3)?

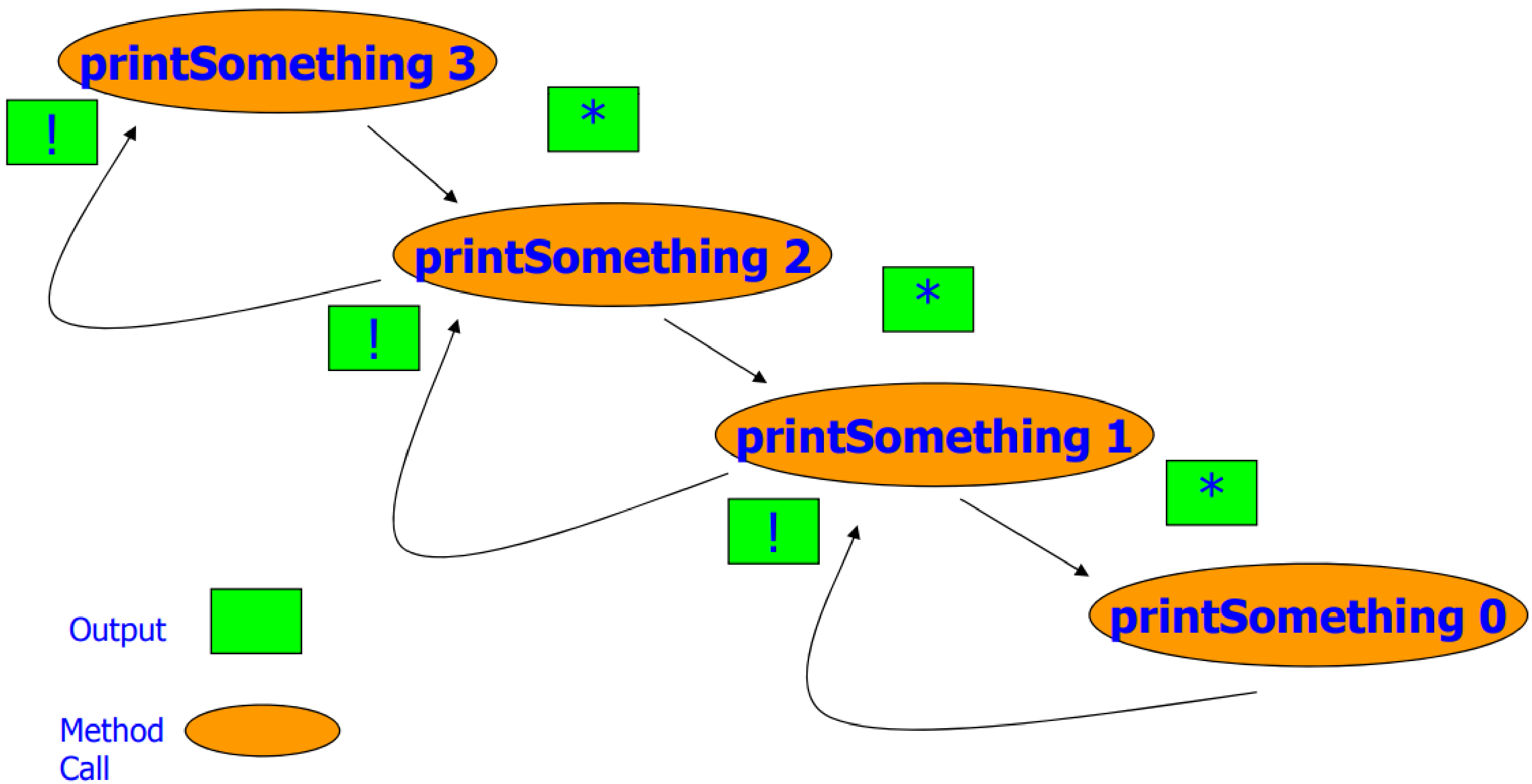
```
void printSomething(int n)
{
   if (n > 0) {
      printf("*");
      printSomething(n-1);
      printf("!");
   }
}
```

What condition is this?
Continue looping condition

Output:
***!!!

Where is the other condition?

Recursive PrintSomething – Tracing



Example 3: Factorials

- **Problem**: Find the factorial of a non-negative integer number.
- The factorial function of a positive integer is usually defined by the formula

  $n! = n \times (n - 1) \times \ldots\ldots \times 1$
- A more precise definition (recursive definition)

  $n! = 1$ if $n = 0$

  $n! = n \times (n - 1)!$ if $n > 0$

Non-recursive Factorials

The following program will do the job, using a for loop:

```
#include <stdio.h>
int factorial(int n);
int main(void)
{
    int num;
    printf(“Enter an integer:”);
    scanf(“%d”, &num);
    printf(“n! = %d\n”,
          factorial(num));
    return 0;
}
```

```
int factorial(int n)
{
    int i;
    int temp = 1;
    for (i=n; i > 0; i--)
        temp *= i;
    return temp;
}
```

Output:

Enter an integer: ***4***

n! = 24

Recursive Factorials

- Suppose we wish to calculate 4!

  As $4 > 0$, $4! = 4 \times 3!$
- But we do not know what is 3!

  As $3 > 0$, $3! = 3 \times 2!$

  As $2 > 0$, $2! = 2 \times 1!$

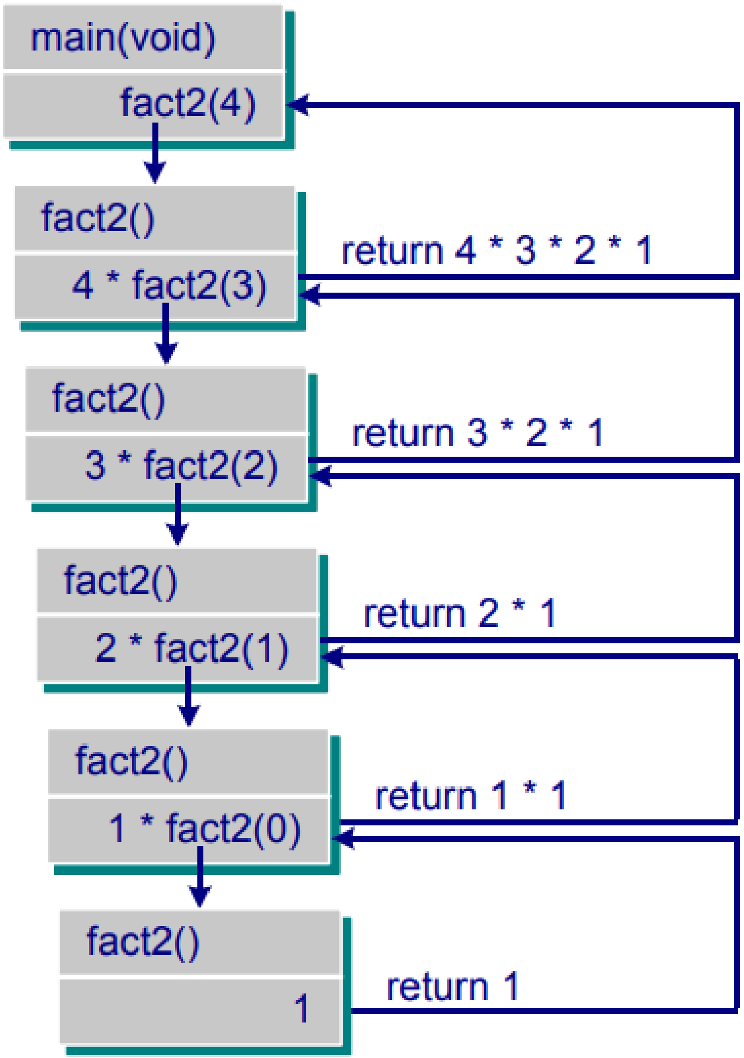
  As $1 > 0$, $1! = 1 \times 0!$
- What is 0! equal to?

$$
\begin{aligned}
4! &= 4 \times 3! \\
&= 4 \times (3 \times 2!) \\
&= 4 \times (3 \times (2 \times 1!)) \\
&= 4 \times (3 \times (2 \times (1 \times 0!))) \\
&= 4 \times (3 \times (2 \times (1 \times 1))) \\
&= 4 \times (3 \times (2 \times 1)) \\
&= 4 \times (3 \times 2) \\
&= 4 \times 6 \\
&= 24
\end{aligned}
$$

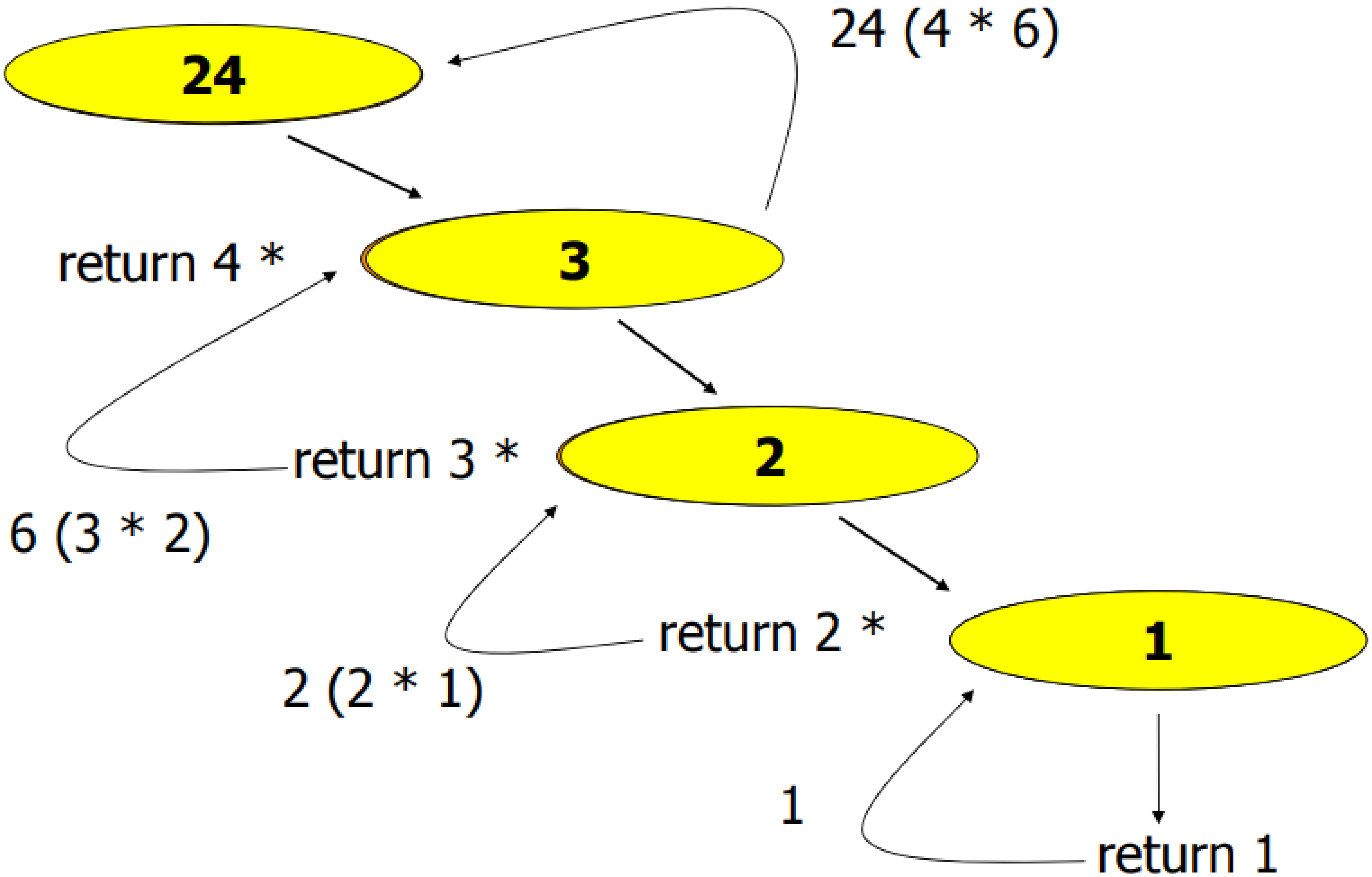
Recursive Factorials

```
int factorial(int n)
{
   if (n == 0) {
   // teminating condition
      return 1;
   } else {
   // recursive condition
      return n*factorial(n-1);
   }
}
```

Output:
Enter an integer: ***4***
n! = 24



Recursive Factorials: Tracing



Example 4: Multiplication by Addition

- The multiplication operation

$$\text{multi}(m, n) = m \times n$$

can be defined **recursively** as

$$\text{multi}(m, n) = m \qquad \text{if } n = 1$$

$$\text{multi}(m, n) = m + \text{multi}(m, n-1) \qquad \text{if } n > 1$$

Using Call by Value

```
/* multiplication by addition, pass parameter using
call by value */
#include <stdio.h>
int multi1(int, int);
int main(void)
{
   printf("5 * 3 = %d\n", multi1(5, 3));
   return 0;
}
int multi1(int m, int n)
{
   if (n == 1)                // terminating condition
      return m;
   else                       // recursive condition
      return m + multi1(m, n-1);
}
```

Output:
5 * 3 = 15

Using Call by Reference

```
/* multiplication by addition, pass parameter using call by
reference */
#include <stdio.h>
void multi2(int, int, int*);
int main(void)
{
    int result;
    multi2(5, 3, &result);
    printf("5 * 3 = %d\n", result);
    return 0;
}
void multi2(int m, int n, int *product)
{
    if (n == 1)                 // terminating condition
        *product = m;
    else {                      // recursive condition
        multi2(m, n-1, product);
        *product = *product + m;
    }
}
```

Output:
5 * 3 = 15

Recursive Call by Reference

```
#include <stdio.h>
void fn(int x, int y, int *z);

int main(void)
{
    int n1=20, n2=15, n3=0;
    fn(n1, n2, &n3);
    printf("n1 = %d, n2 = %d, n3 = %d\n", n1, n2, n3);
    return 0;
}
void fn(int x, int y, int *z)
{
    if (x > y) {
        x = x - 2;
        y = y - 1;
        *z = x * y;
        fn(x, y, z);
    }
    printf("x = %d, y = %d, *z = %d\n", x, y, *z);
}
```

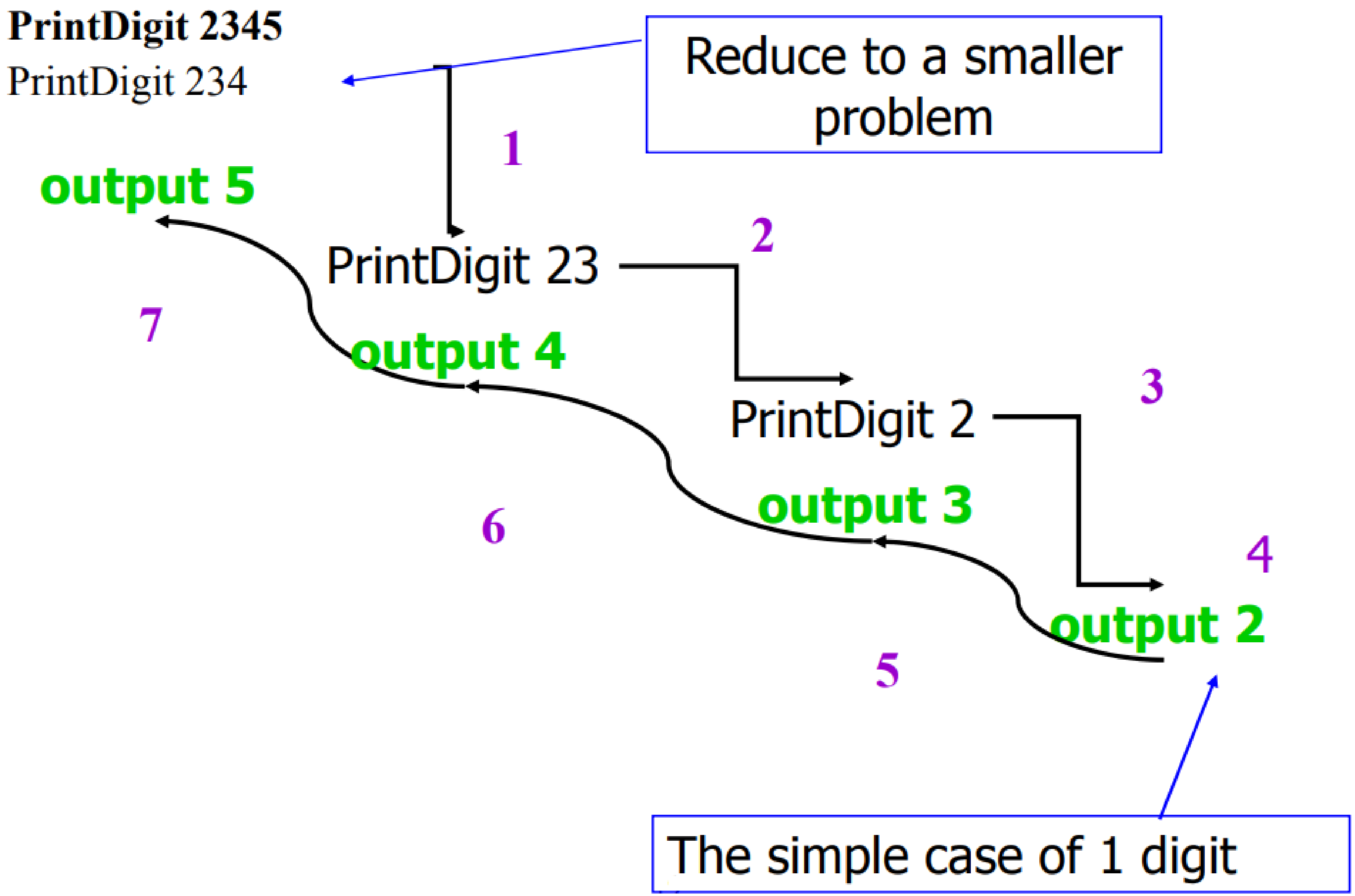
Output:

x = 10, y = 10, *z = 100
x = 10, y = 10, *z = 100
x = 12, y = 11, *z = 100
x = 14, y = 12, *z = 100
x = 16, y = 13, *z = 100
x = 18, y = 14, *z = 100
n1 = 20, n2 = 15, n3 = 100

Example 5: Printing Digits

- **Problem**: Given a number, say 2345, print each digit of the number, in the order from left to right, one per line.
- Recursive Solution:
  - Look for the simplest case (**terminating condition**). If the number is a single digit, just print that digit.
  - Look into reducing the problem into a smaller but same problem (**recursive condition**).

Printing Digits



Printing Digits: Recursive Solution

```
#include <stdio.h>
void printDigit(int);

int main(void) {
    int num;
    printf("Enter a number: ");
    scanf("%d", &num);
    printDigit(num);
    return 0;
}

void printDigit(int num) {
    if (num < 10)               // terminating condition
        printf("%d\n", num);
    else {                      // recursive condition
        printDigit(num/10);
        printf("%d\n", num%10);
    }
}
```

Output:

Enter a number: ***2345***

2

3

4

5

printDigit(2345);

```
printDigit( 2345 )
if (num < 10 )
  …
else
  printDigit ( 2345/10 )
  output 2345 % 10
```

```
printDigit( 234 )
if ( num < 10 )
  …
else
  printDigit ( 234 / 10)
  output 234 % 10
```

```
printDigit( 23 )
if ( num < 10 )
  …
else
  printDigit ( 23 / 10)
  output 23 % 10
```

```
printDigit( 2 )
if ( num < 10 )
  output 2
else
  …
```

Recursion: Summary

- To obtain the answer to a larger problem, a general method is used that
  - reduces the large problem to one or more problems of a similar nature but a smaller size.
- Recursion continues until the size of the problem is reduced to the smallest, i.e. base case
  - where the solution is given directly without using further recursion.
- As such, recursive methods consist of two parts:
  - A smallest, **base case** that is processed without recursion (**terminating condition**).
  - A general method that reduces a particular case to one or more of the smaller cases (**recursive condition**).

Recursion: Summary

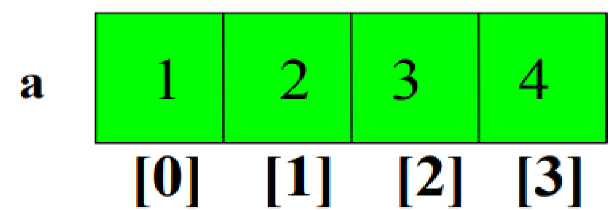
- Each function makes a call to itself with an argument which is closer to the terminating condition.
- Each call to a function has its own set of values/arguments for the formal arguments and local variables.
- When a recursive call is made, control is transferred from the calling point to the first statement of the recursive function. When a call at a certain level is finished, control returns to the calling point one level up.

Example 6: Summing Array of Integers

```
int sumArray(int a[], int size)
{
    int sum = 0;
    for (int i = 0; i < size; i++)
    {
        sum = sum + a[i];
    }
    return sum;
}
int recursiveSum(int a[], int size)
{
    if (size == 1)
        return a[0];
    else
        return a[size - 1] +
            recursiveSum(a, size - 1);

    }
}
```

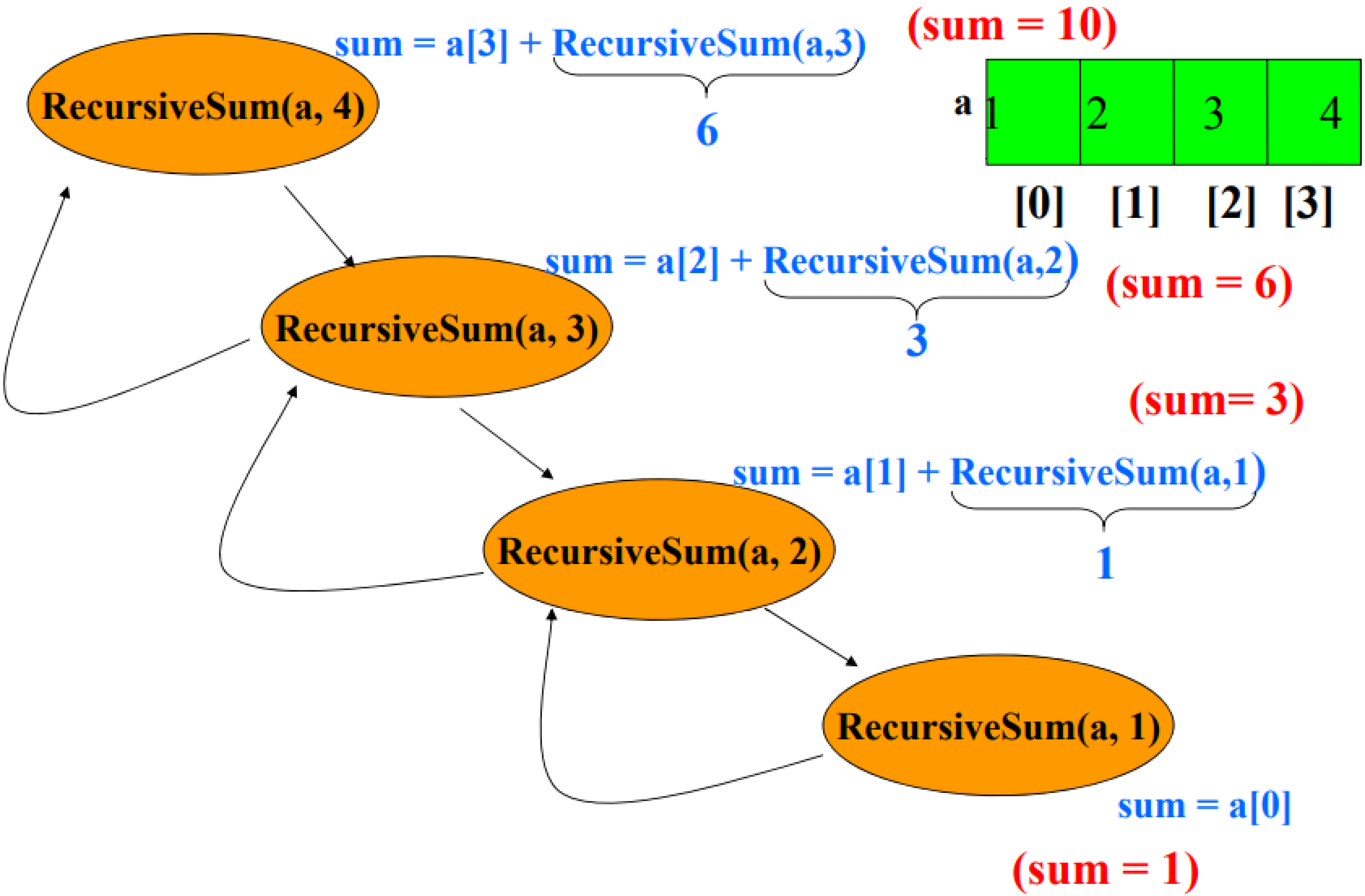
- Given an array of integers, write a method to calculate the sum of all the integers.



Input:
a[] = {1,2,3,4}

Output:
Sum = 10

Recursive Sum – Tracing

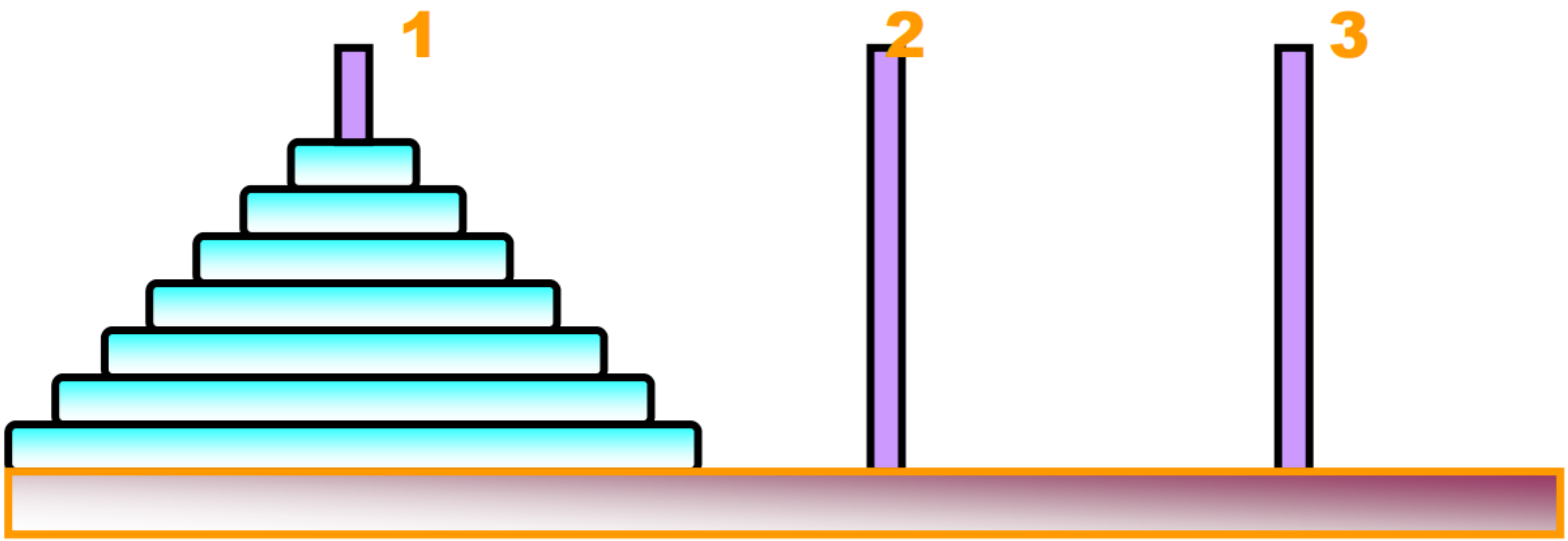


Example 7: Tower of Hanoi

- In the 19th century, a game called the **Towers of Hanoi** appeared in Europe.
- The game represents a task underway in the Temple of Brahma.
- At the creation of the world, the priests were given a brass platform on which were 3 diamond needles.
- On the first needle were stacked 64 golden disks.
- Each one slightly smaller than the one under it.
- The priest were assigned the task of moving all the golden disks from the first needle to the third
  - **Condition**: Only one disk can be moved at a time, and no disk is allowed to be placed on top of a smaller disk.

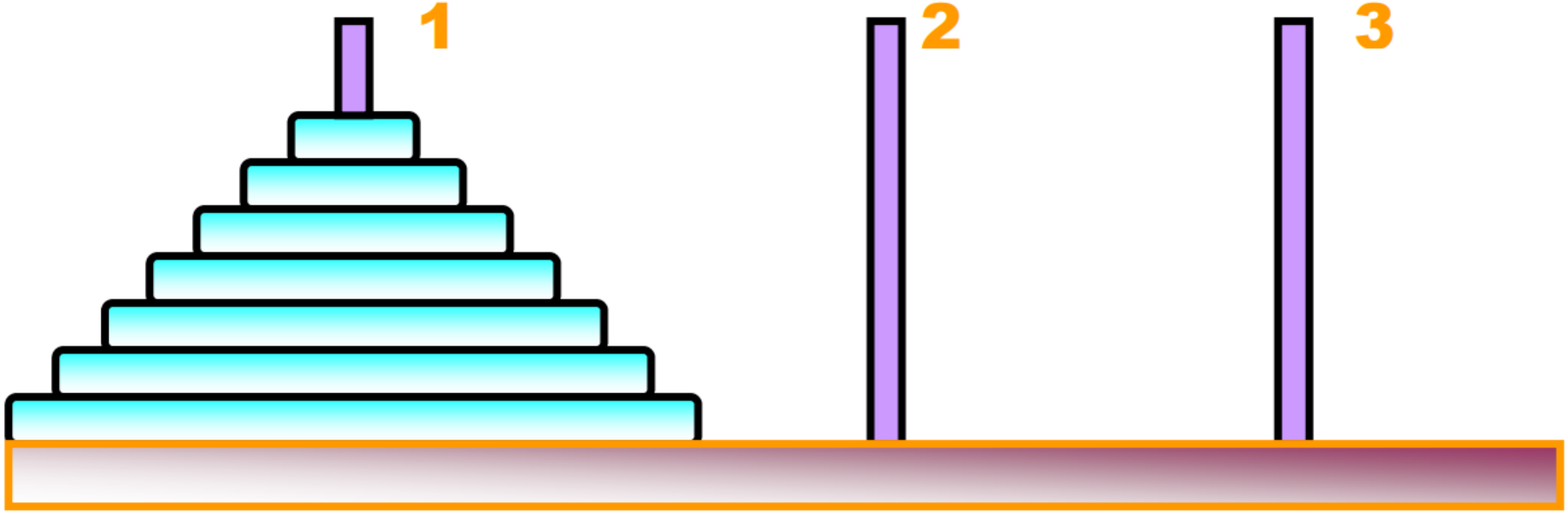
Tower of Hanoi

- **Problem:** The priests were told that, when they had finished moving the 64 disks from tower 1 to tower 3, it would signify the end of the world!!!



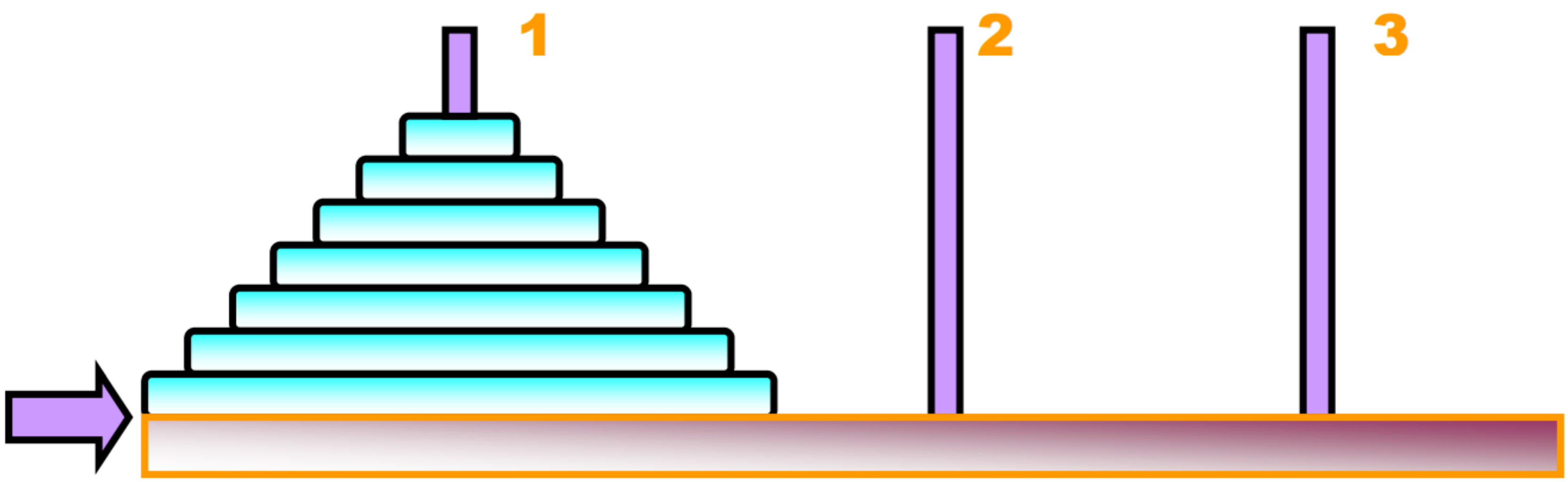
The Problem

- Write a computer program that will type out a list of instructions for the priests
- **move(64, 1, 3, 2)**
  - Move 64 disks from tower 1 to tower 3 using tower 2 as temporary storage



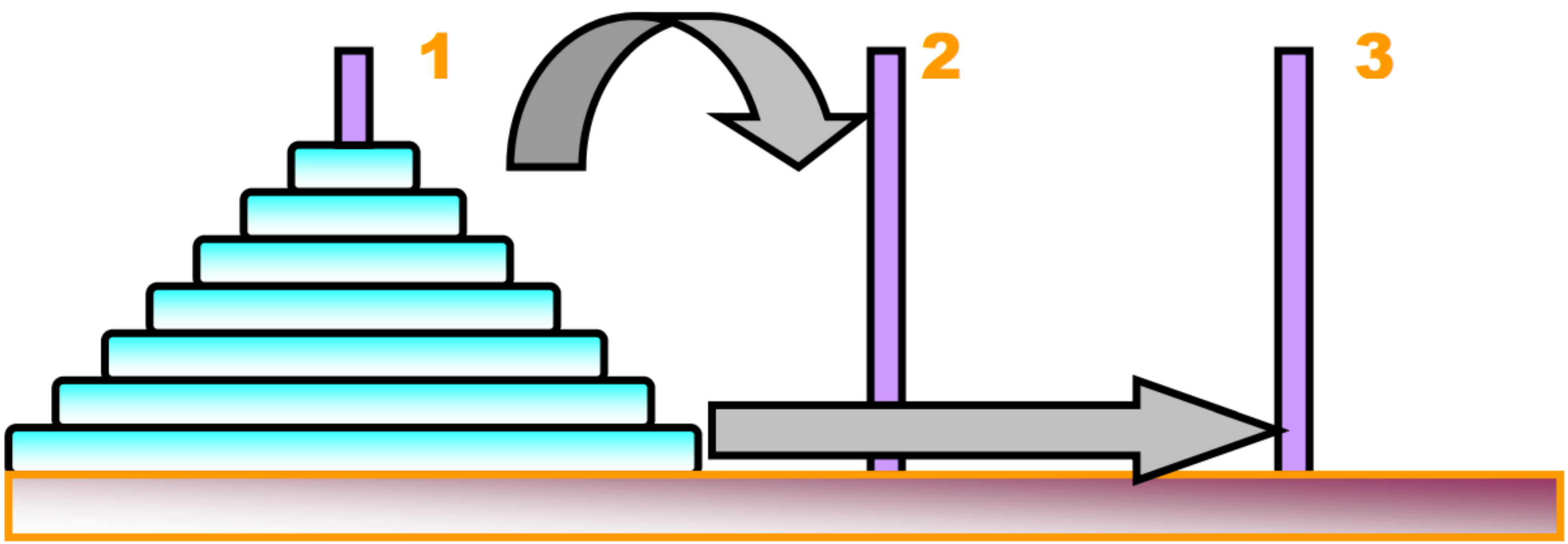
The Solution

- Concentrate our attention not on the first step
  - – Move the top disk anywhere
- Rather, on the hardest step
  - – Moving the bottom disk



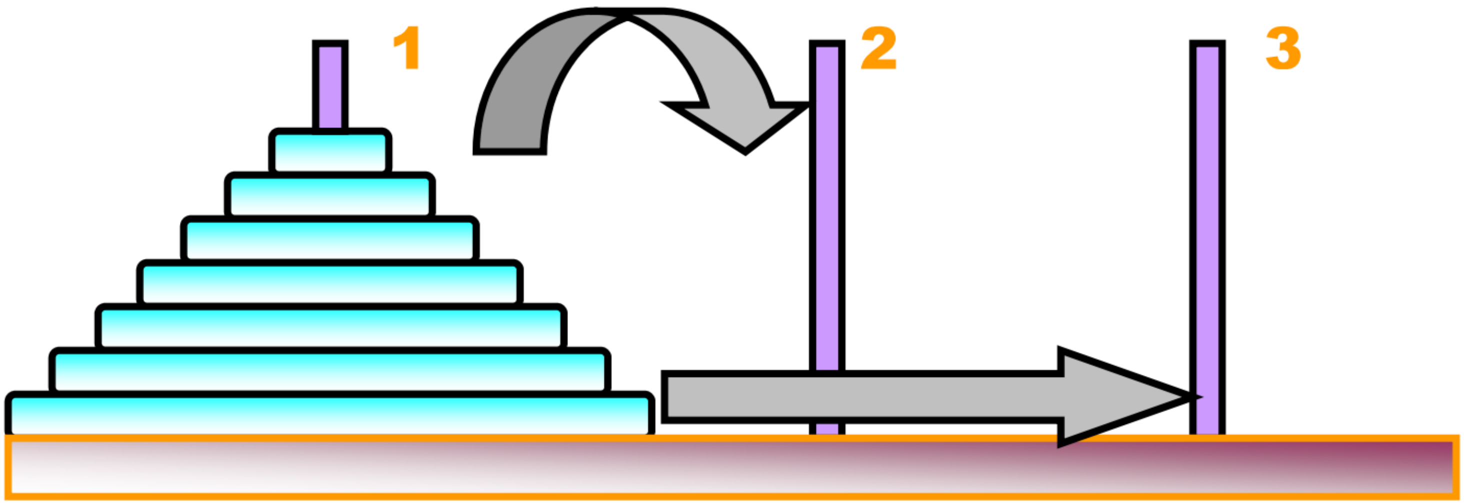
The Solution

- How to reach the bottom disk?
  - The top 63 disks have to be moved to tower 2
  - The bottom disk can then be moved from tower 1 to 3
- There cannot be any other disk in tower 3



The Solution

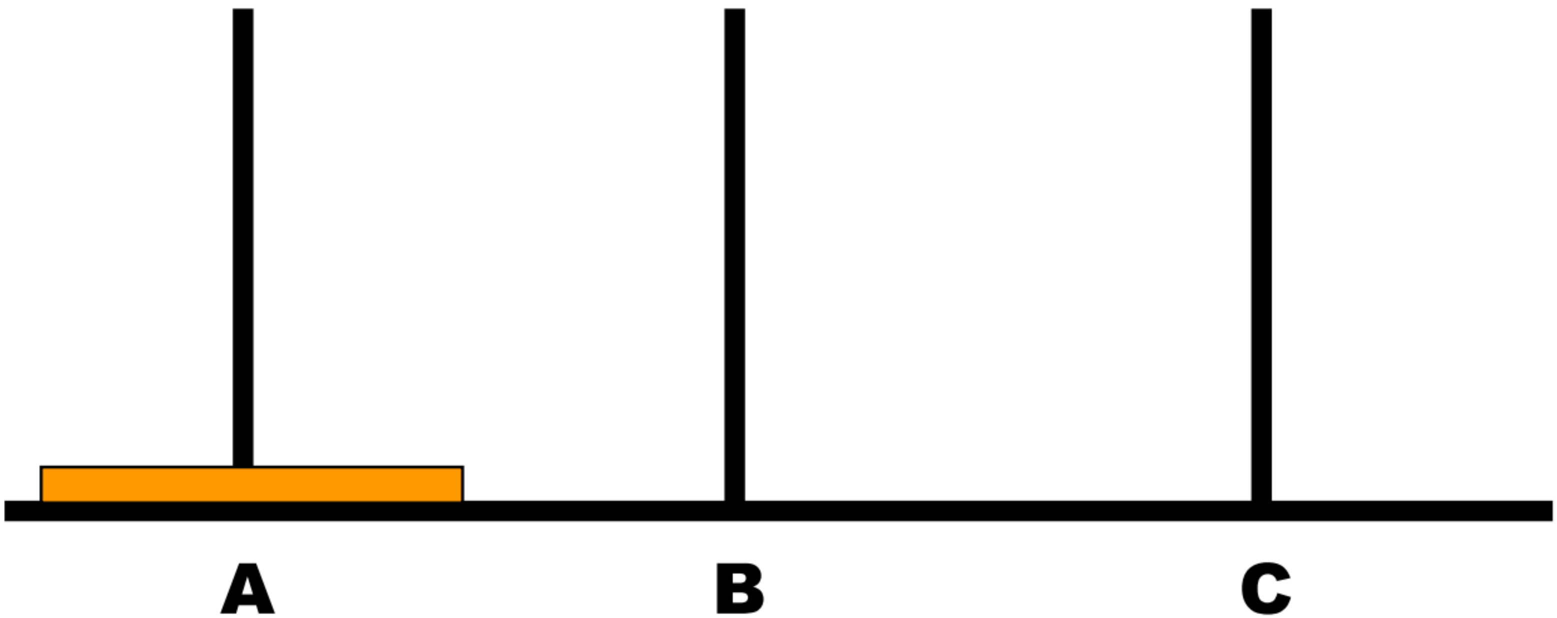
- move(63, 1, 2, 3); // Move 63 disks from tower 1 to tower 2 (using tower 3 as temporary)
- Display “Move disk 64 from tower 1 to 3”
- move (63, 2, 3, 1); // Move 63 disks from tower 2 to tower 3 (using tower 1 as temporary)



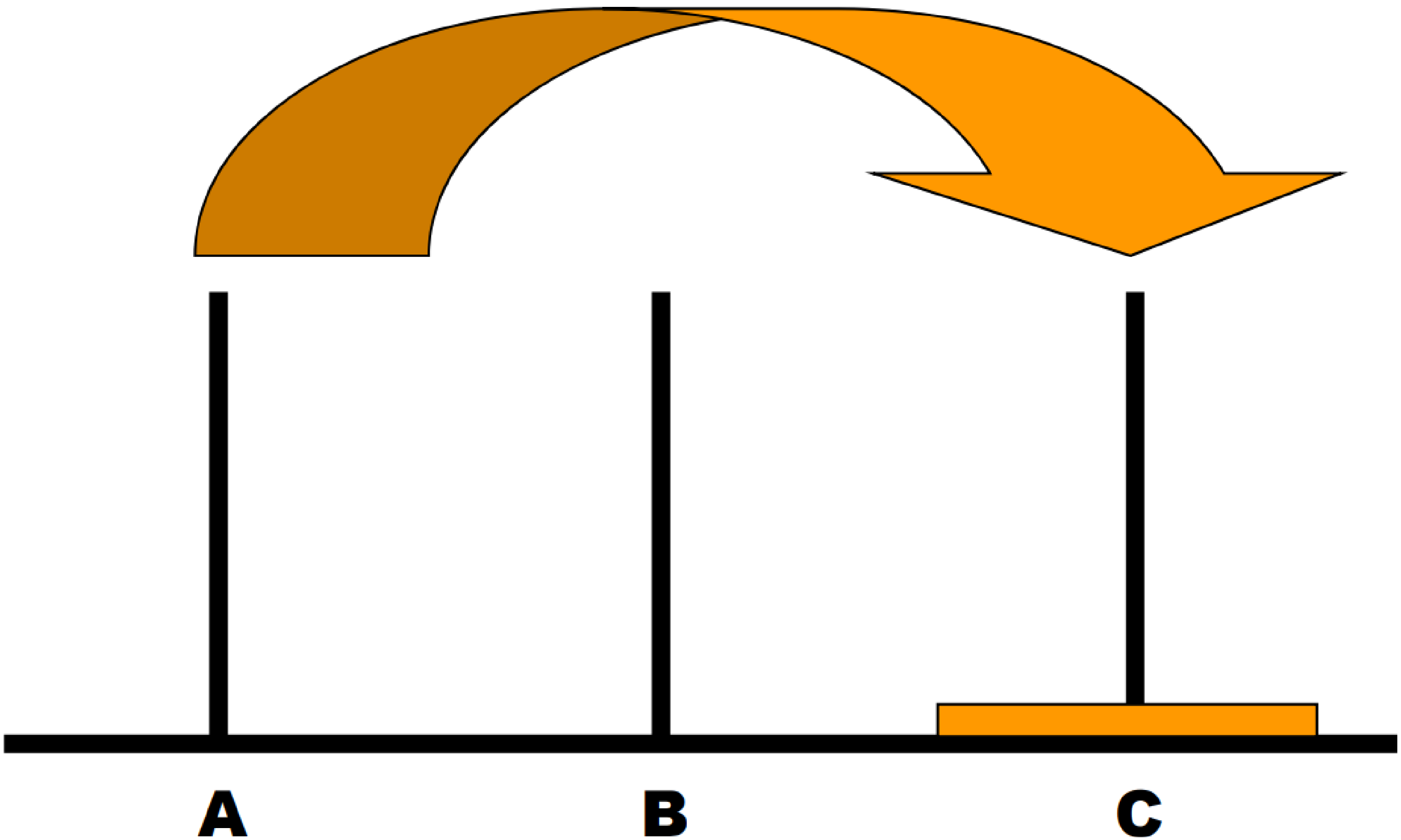
The Solution

- The remaining 63 disks can be moved in the same way
- Divide and conquer
  – To solve a problem we split it into smaller parts which are easier to handle.

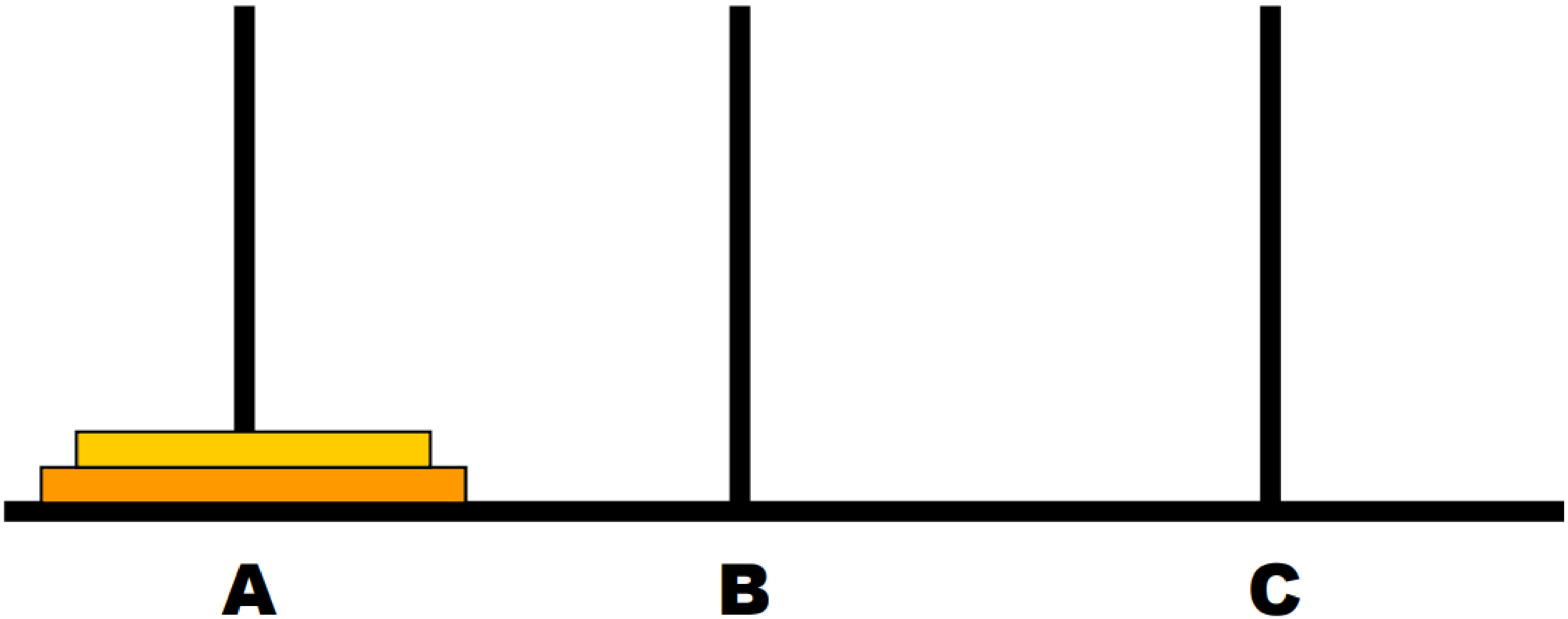
A Single Disk Tower



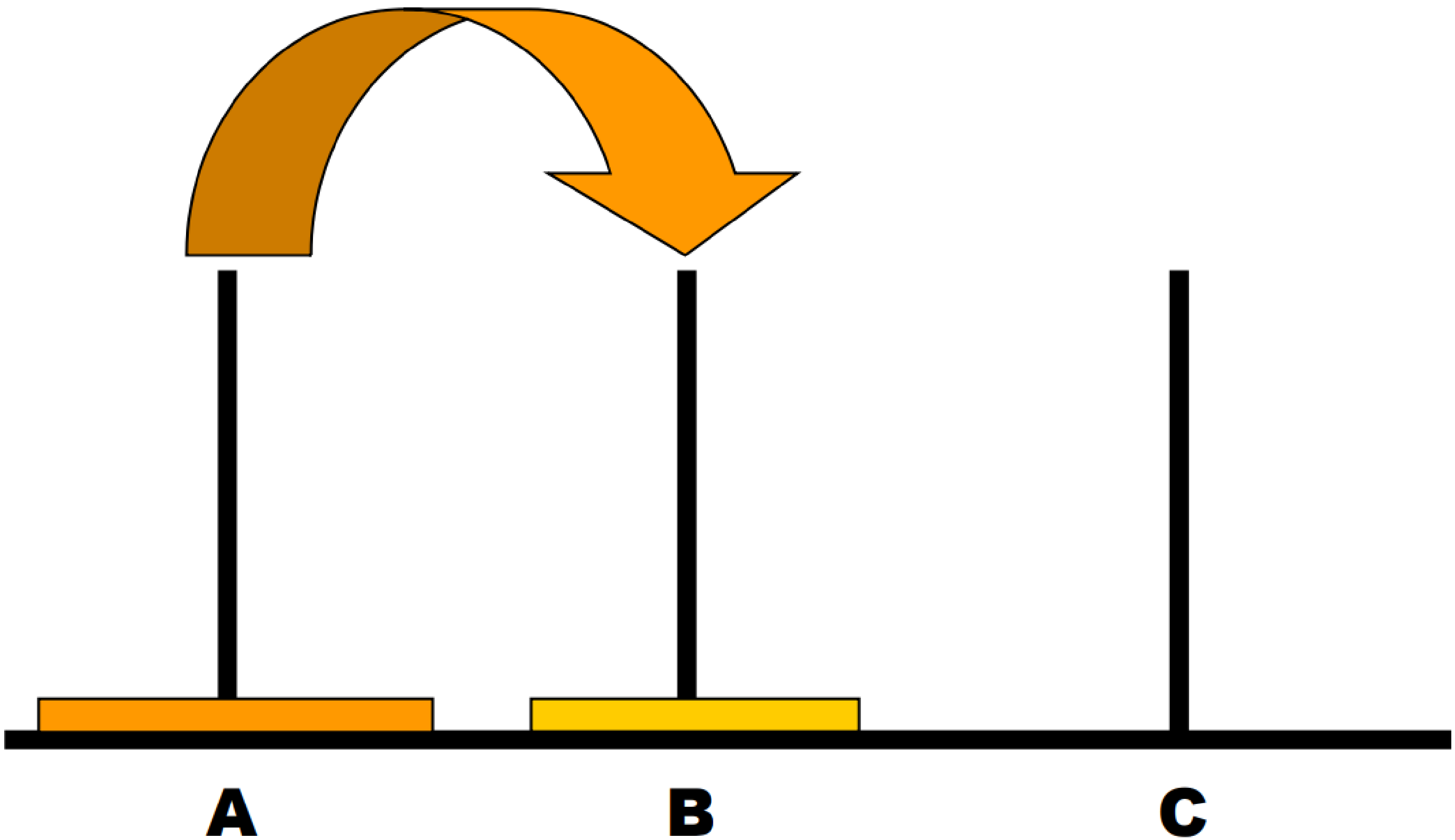
A Single Disk Tower



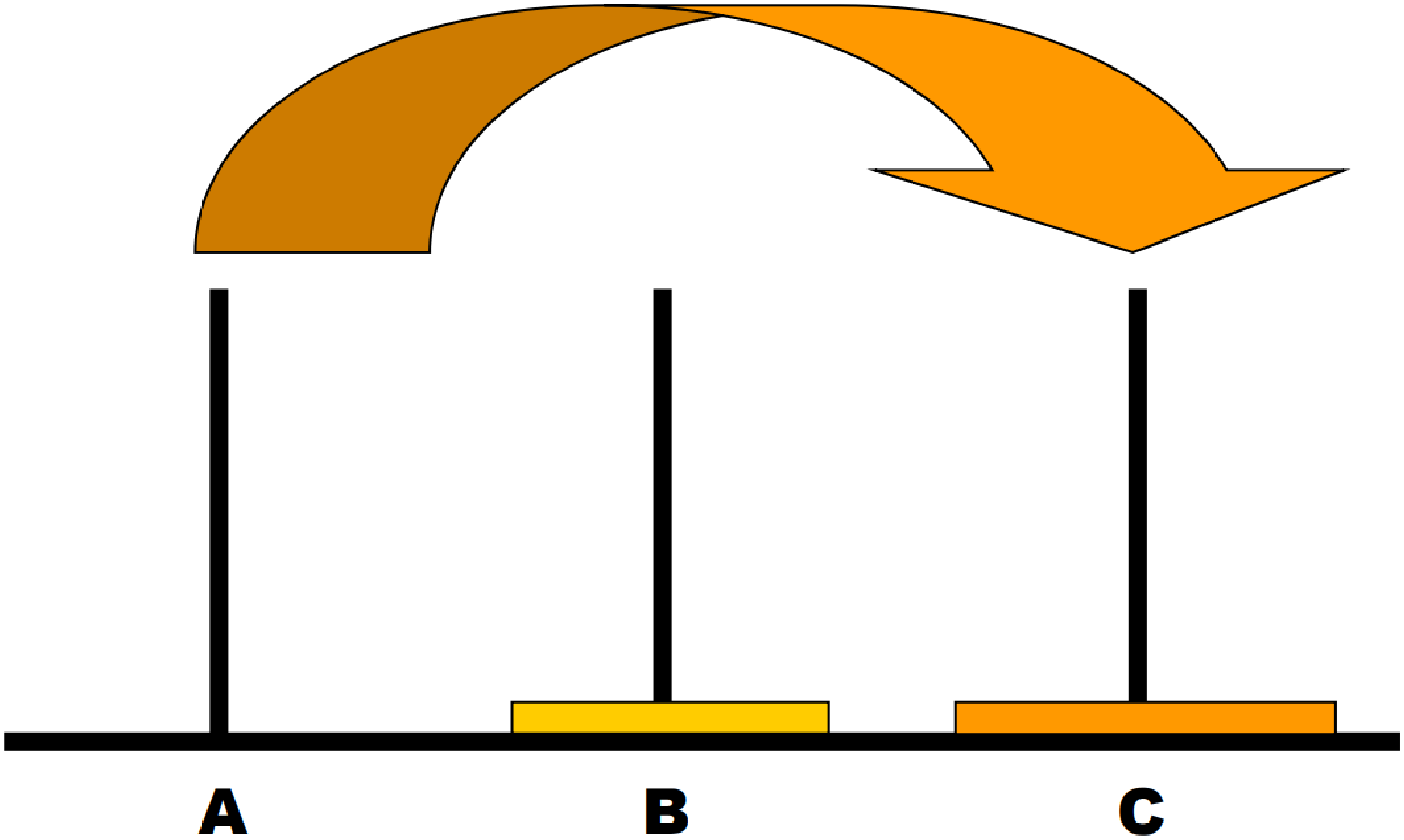
A 2-Disk Tower



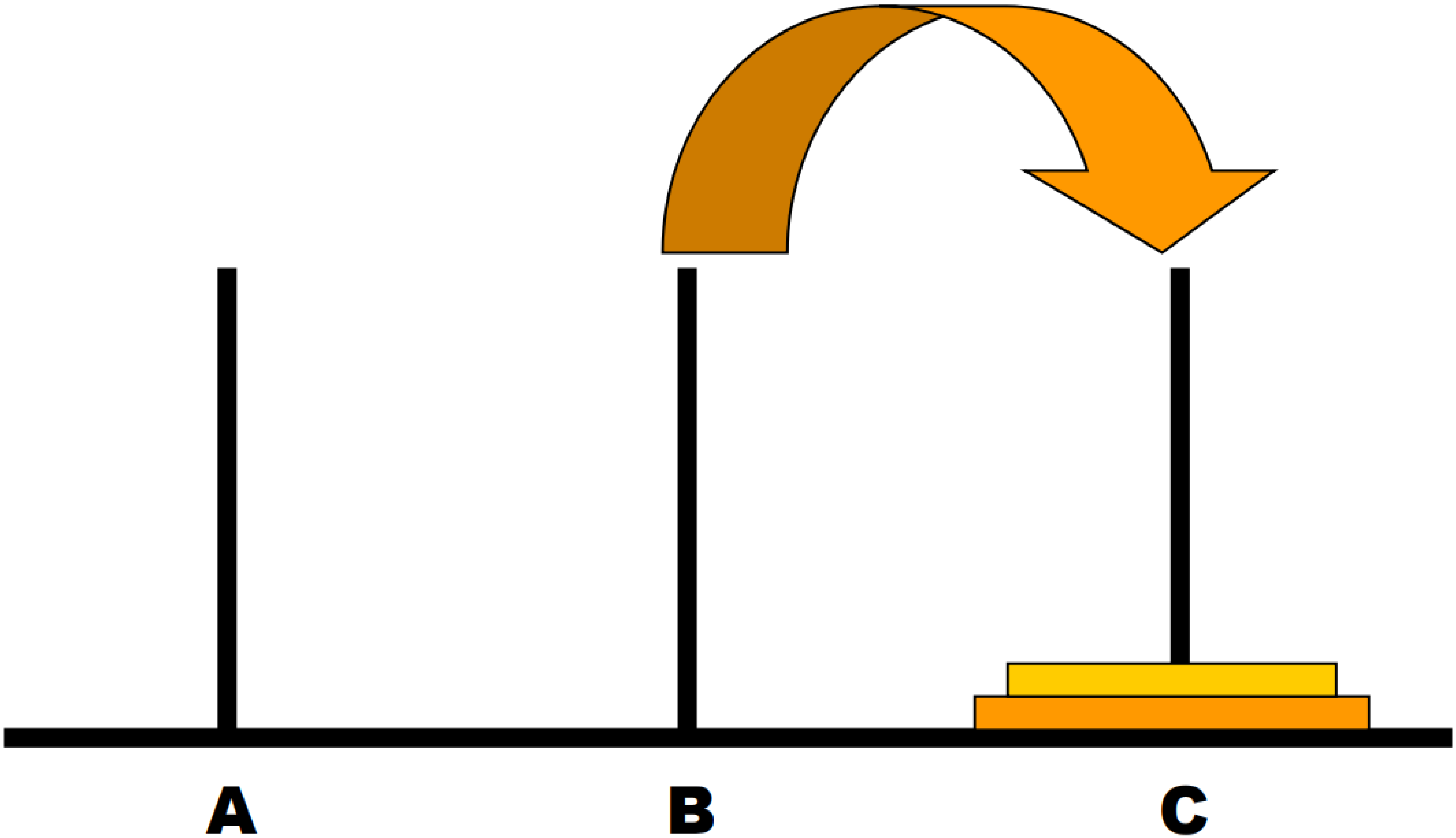
Move 1



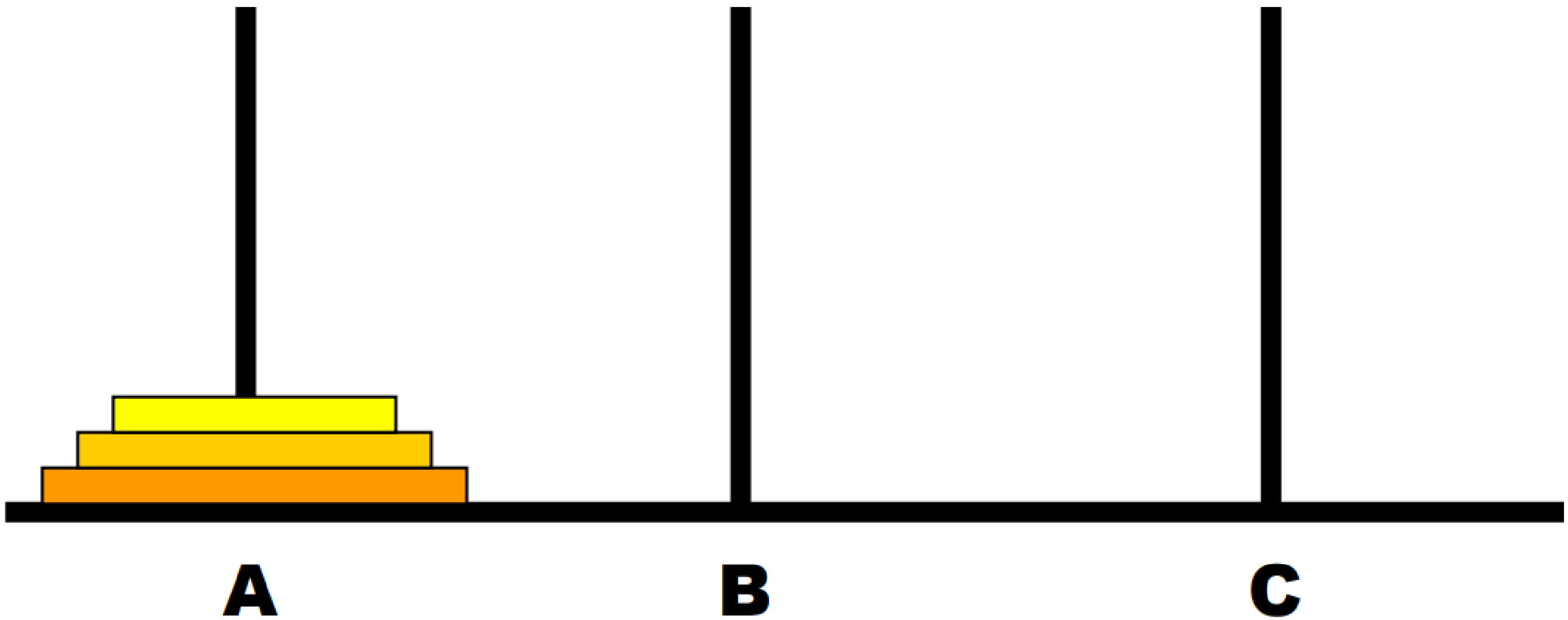
Move 2



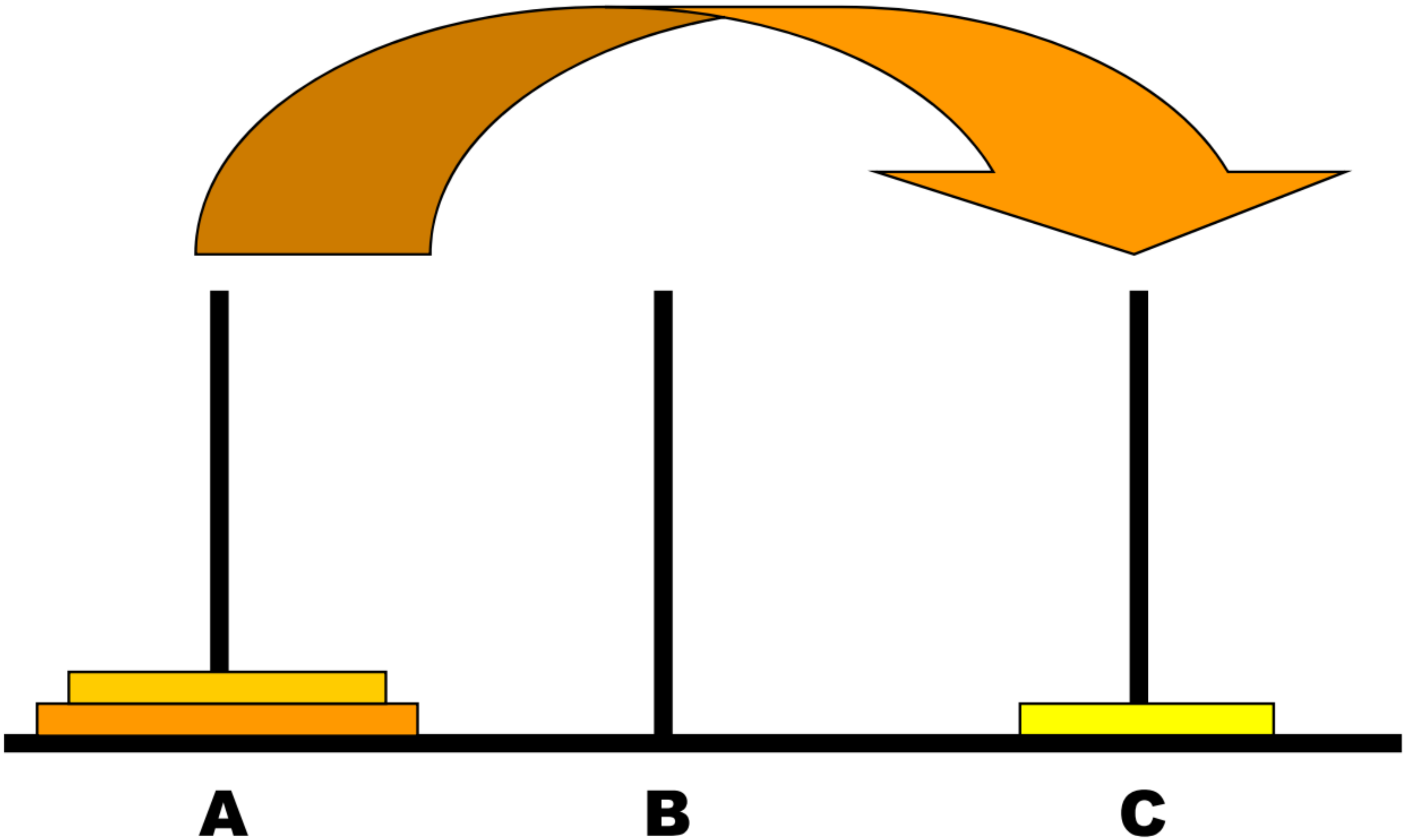
Move 3



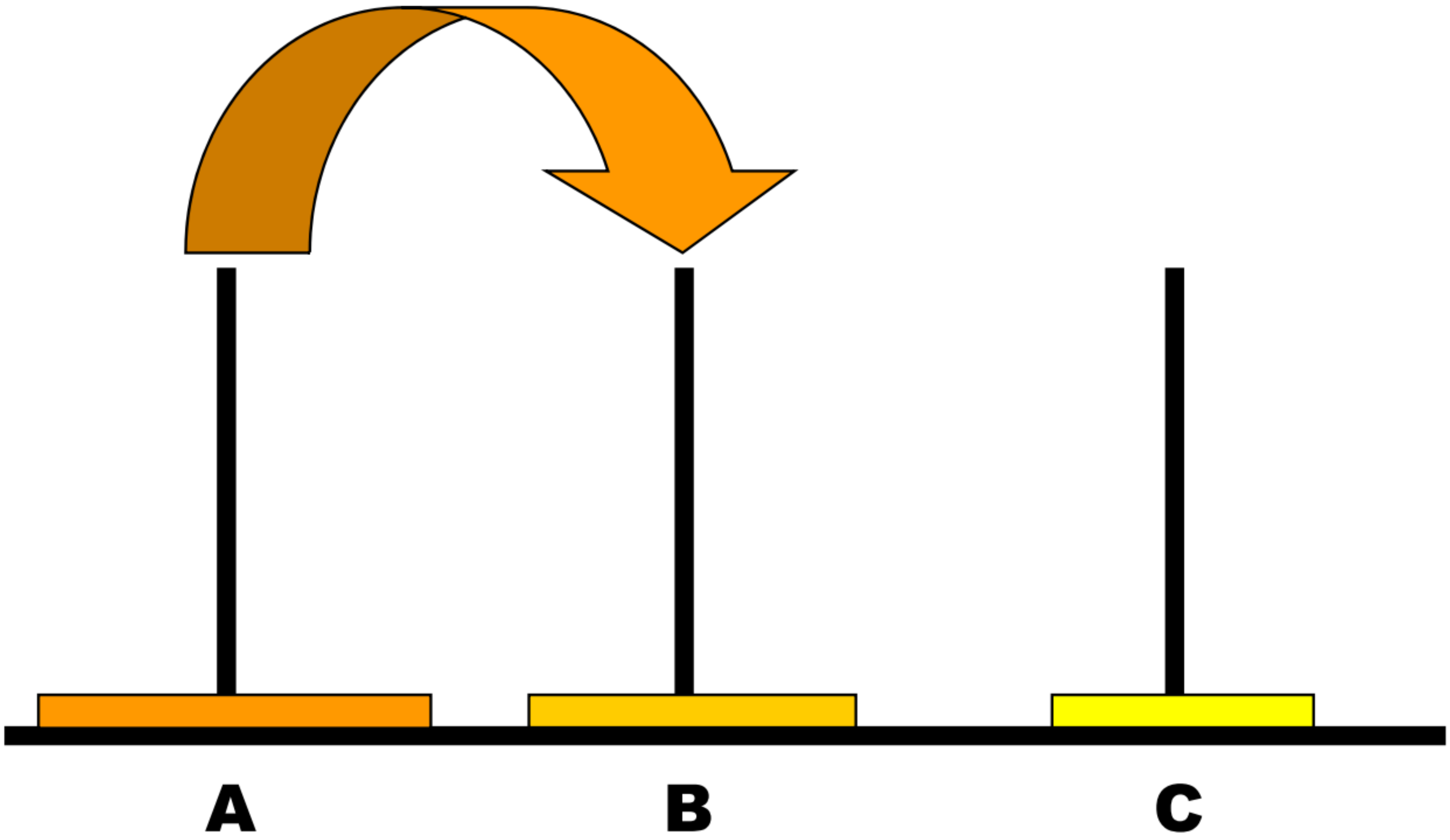
A 3-Disk Tower



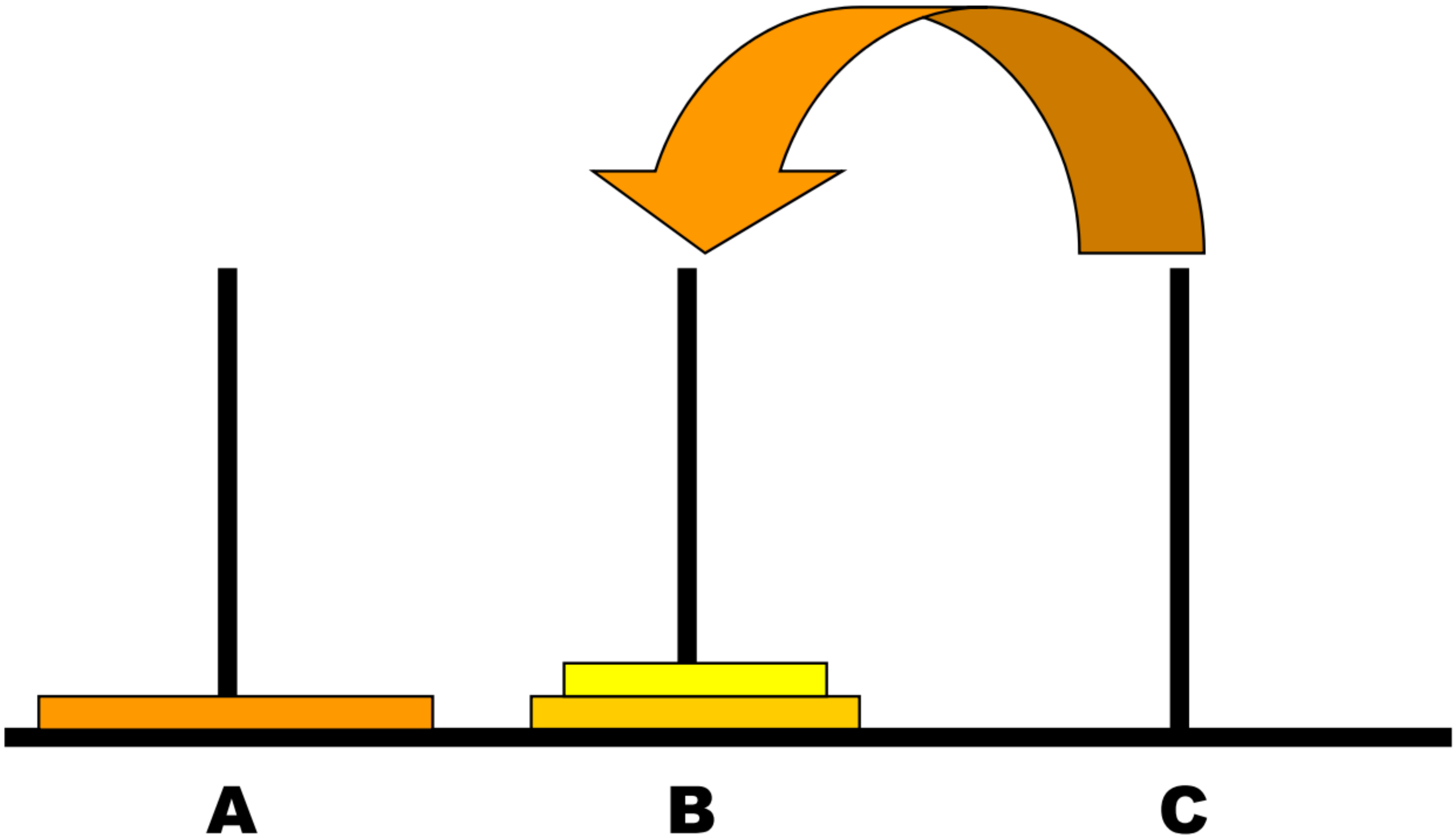
Move 1



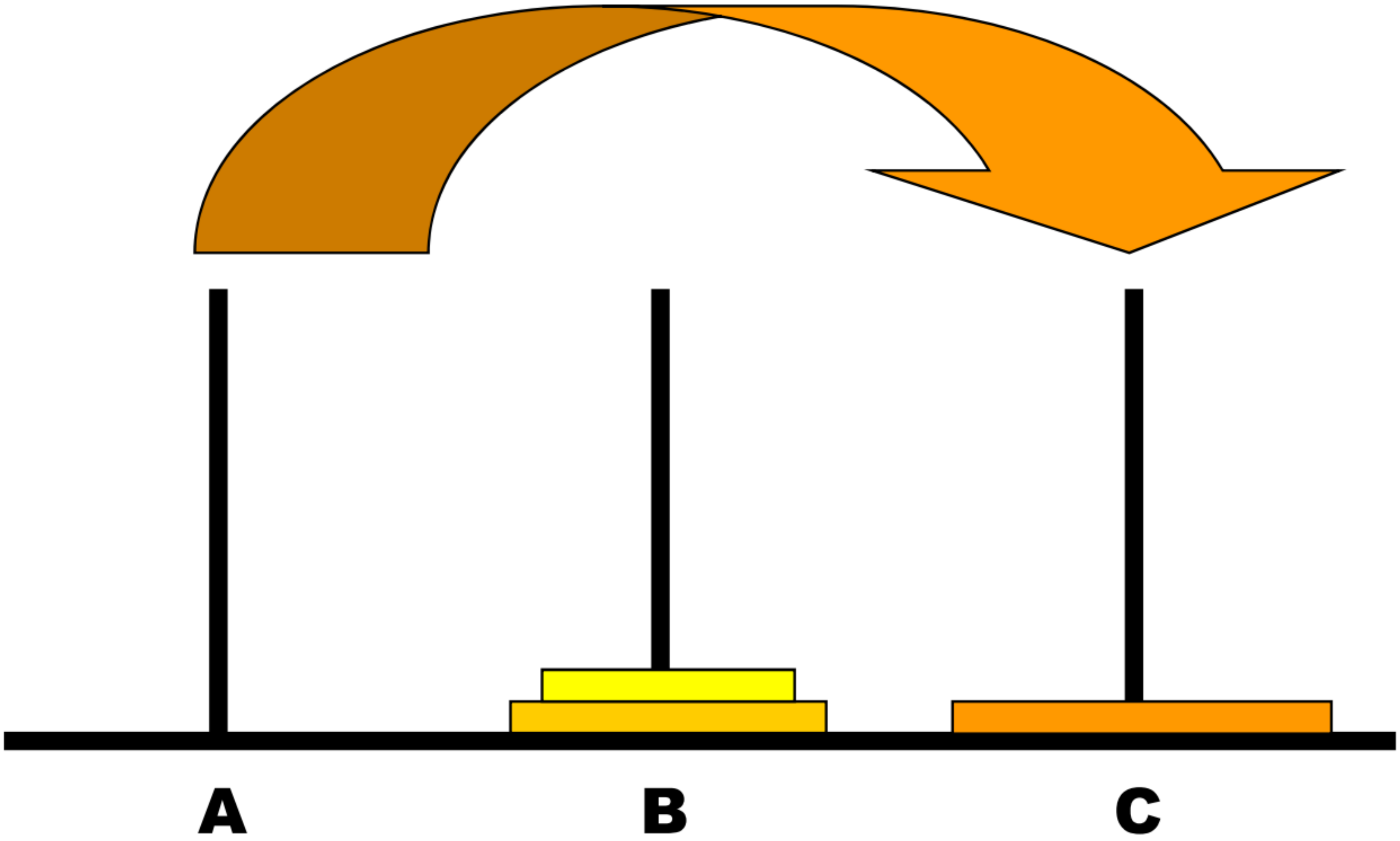
Move 2



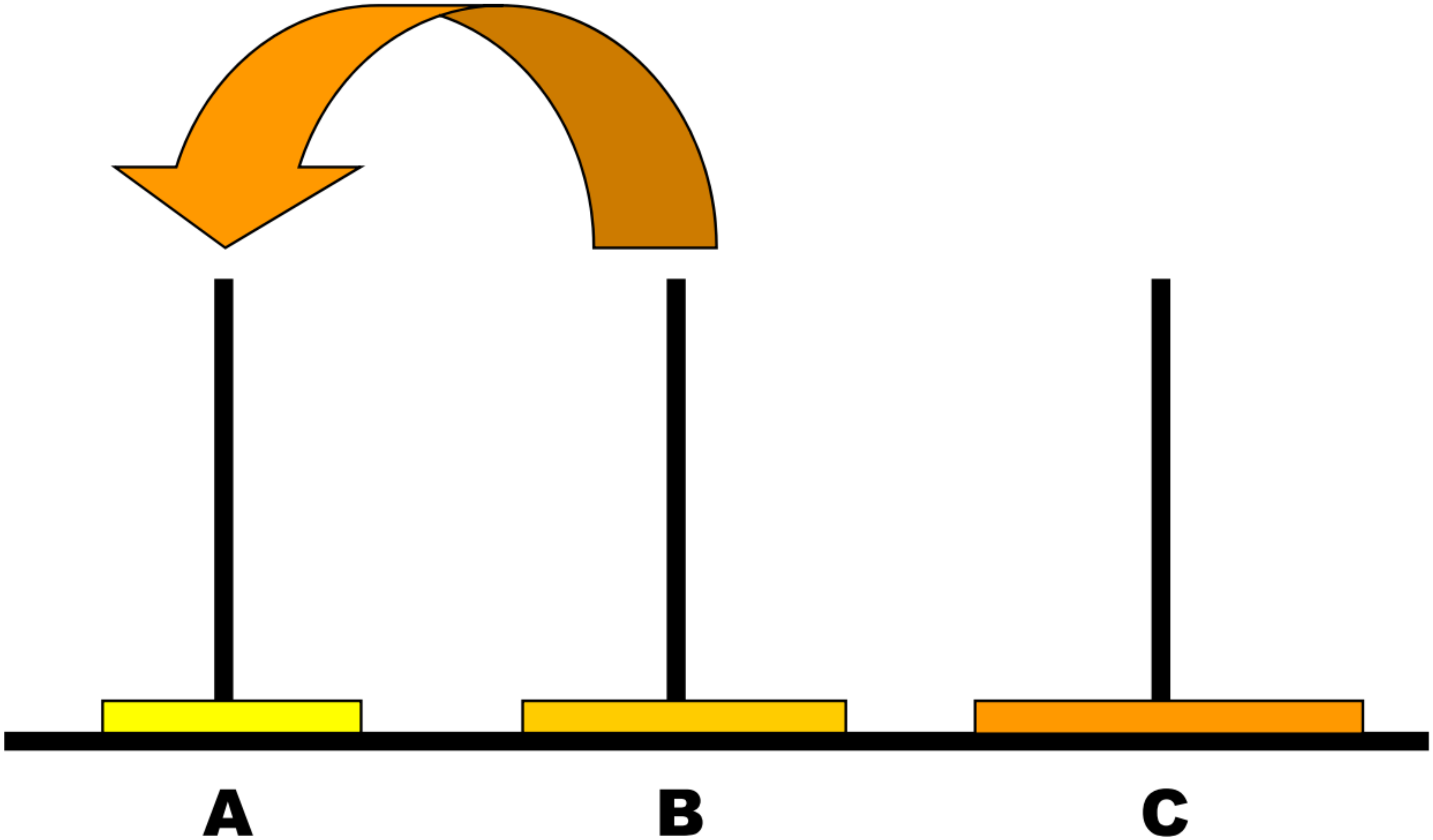
Move 3



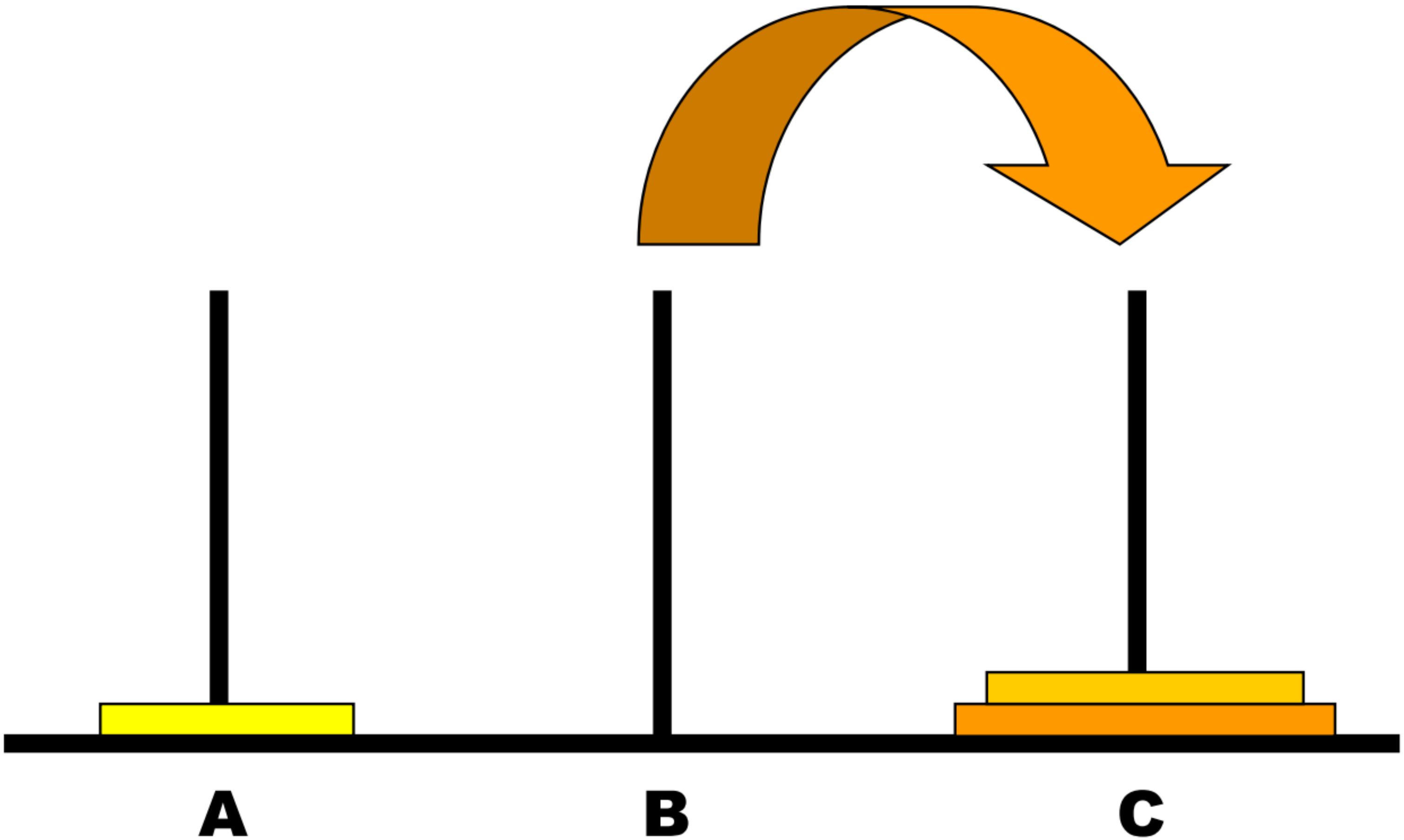
Move 4



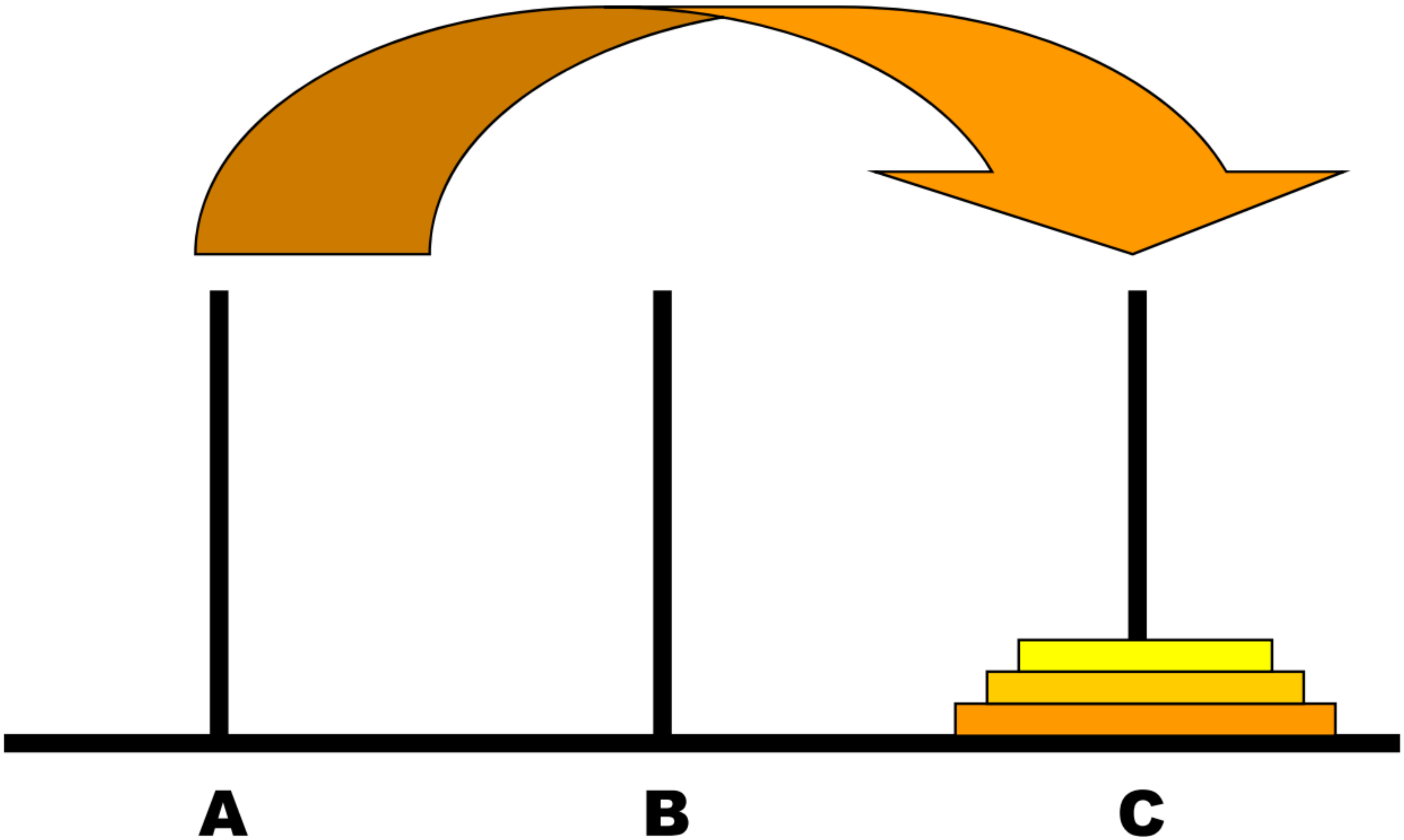
Move 5



Move 6



Move 7



The Algorithm

- **void move(int count, int start, int finish, int temp)**
- Precondition
  - There are at least **count** disks on the tower start
- Postcondition
  - The top **count** disks on start have been moved to tower finish;
  - Tower temp (used for temporary storage) has been returned to its starting position

The Algorithm

```
void move(int count, int start, int finish, int temp);
int main() {
   int disks = 3;
   printf("Disks: %d\n", disks);
   move(disks, 1, 3, 2);
   return 0;
}
void move(int count, int start, int finish, int temp) {
   if (count == 1) {
      printf("%s %d %s %d %s %d\n", "move disk", count,
               "from tower", start, "to tower", finish);
      return;
   }
   move(count – 1, start, temp, finish);
   printf("%s %d %s %d %s %d\n", "move disk", count,
            "from tower", start, "to tower", finish);
   move(count – 1, temp, finish, start);
}
```

The Algorithm

Output:

Disks: 3

Move disk 1 from tower 1 to tower 3

Move disk 2 from tower 1 to tower 2

Move disk 1 from tower 3 to tower 2

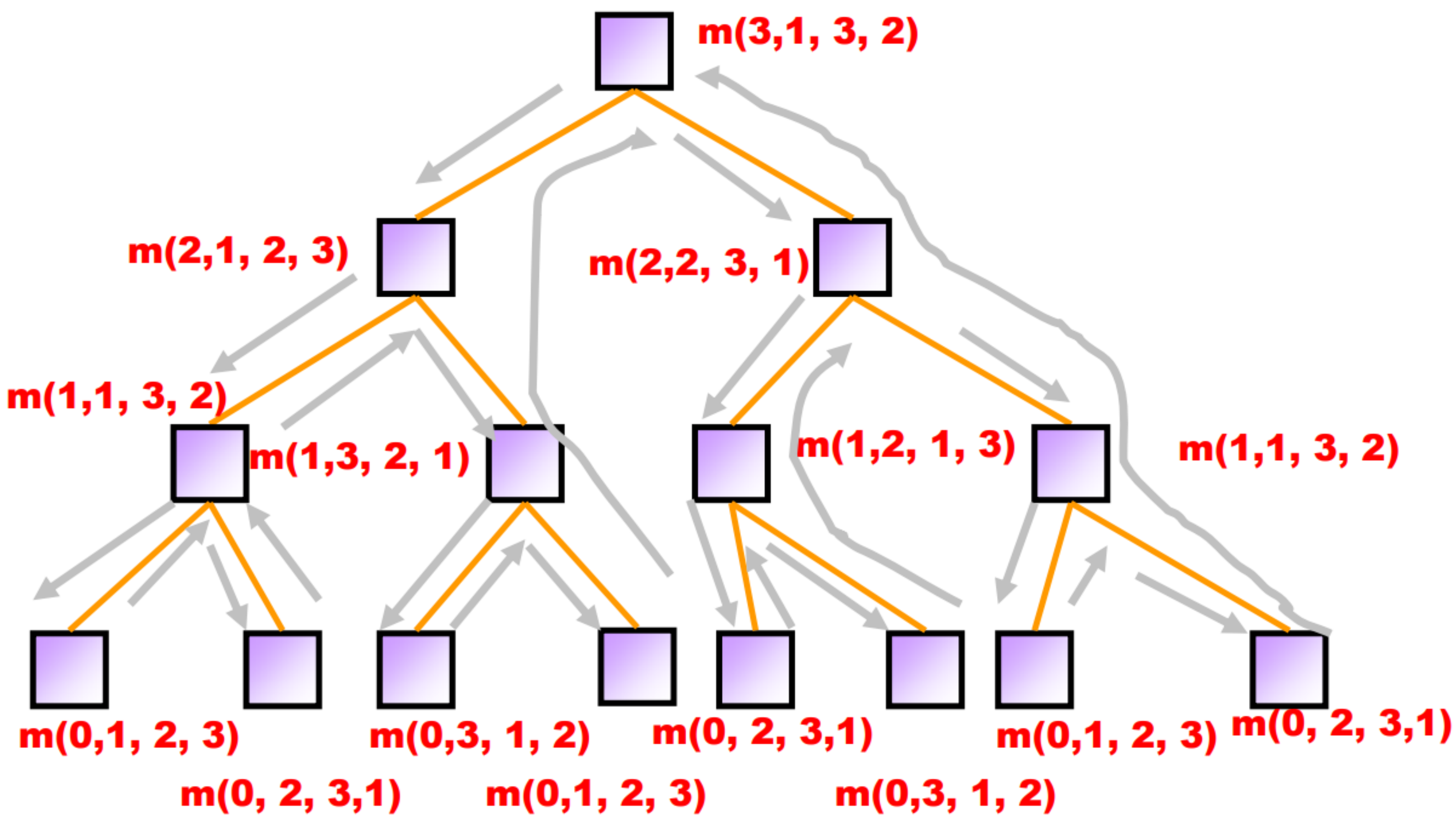
Move disk 3 from tower 1 to tower 3

Move disk 1 from tower 2 to tower 1

Move disk 2 from tower 2 to tower 3

Move disk 1 from tower 1 to tower 3

The Recursion Tree



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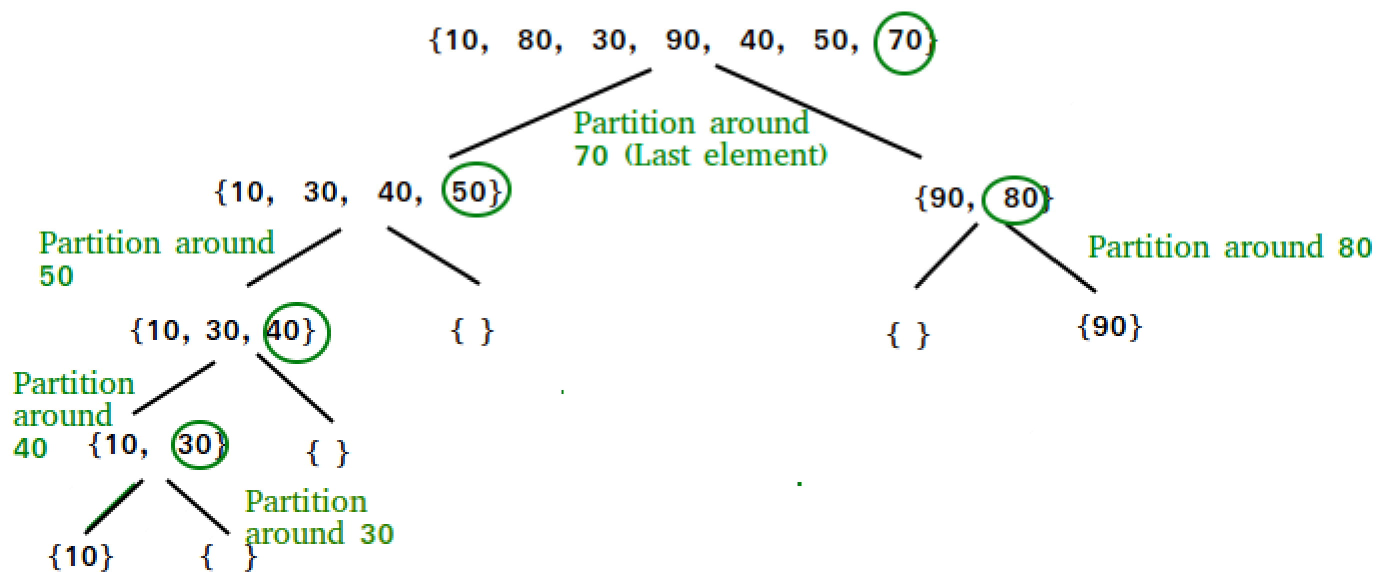
Sorting a Sequence of Data

- **Quick sort**
  - a divide-and-conquer algorithm
  - pick an element as pivot; partition the data sequence into two sequences around the pivot
    - The left sequence contains element smaller than pivot
    - The right sequence contains element larger than pivot
  - This partition is recursively applied to the sequence until no partition can be done

Sorting a Sequence of Data

- **An example of quick sort**



Recursive Quick Sort

- **The algorithm**
  - Key part: partition of the data sequence

```
partition(arr[], low, high)
{
    // pivot (Element to be placed at right position)
    pivot = arr[high];

    i = (low - 1)  // Index of smaller element

    for (j = low; j <= high - 1; j++)
    {
        // If current element is smaller than the pivot
        if (arr[j] < pivot)
        {
            i++;    // increment index of smaller element
            if(i != j) swap arr[i] and arr[j]
        }
    }
    swap arr[i + 1] and arr[high])
    return (i + 1)
}
```

Recursive Quick Sort

- **The algorithm**
  - Key part: partition of the data sequence

```
arr[] = {10, 80, 30, 90, 40, 50, 70}
Indexes:  0   1   2   3   4   5   6


low = 0, high =  6, pivot = arr[h] = 70
Initialize index of smaller element, i = -1

Traverse elements from j = low to high-1
j = 0 : Since arr[j] <= pivot, do i++
i = 0
arr[] = {10, 80, 30, 90, 40, 50, 70} // No change as i and j
                                     // are same

j = 1 : Since arr[j] > pivot, do nothing
// No change in i and arr[]
```

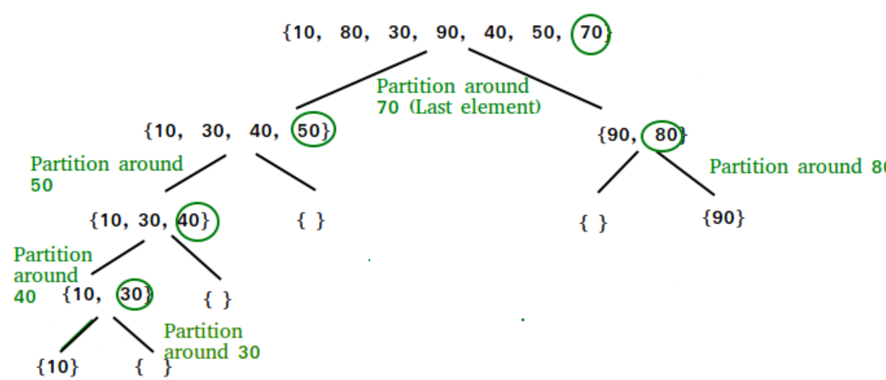
Recursive Quick Sort

- **The algorithm**
  - Key part: partition of the data sequence

```
j = 2 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])
i = 1
arr[] = {10, 30, 80, 90, 40, 50, 70} // We swap 80 and 30

j = 3 : Since arr[j] > pivot, do nothing
// No change in i and arr[]

j = 4 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])
i = 2
arr[] = {10, 30, 40, 90, 80, 50, 70} // 80 and 40 Swapped
j = 5 : Since arr[j] <= pivot, do i++ and swap arr[i] with arr[j]
i = 3
arr[] = {10, 30, 40, 50, 80, 90, 70} // 90 and 50 Swapped

We come out of loop because j is now equal to high-1.
Finally we place pivot at correct position by swapping
arr[i+1] and arr[high] (or pivot)
arr[] = {10, 30, 40, 50, 70, 90, 80} // 80 and 70 Swapped
```

Recursive Quick Sort

- **The algorithm**
  – The recursive part




```
quickSort(arr[], low, high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[pi] is now
           at right place */
        pi = partition(arr, low, high);

        quickSort(arr, low, pi - 1);  // Before pi
        quickSort(arr, pi + 1, high); // After pi
    }
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Declaring dynamic array structure

```
struct ARRAY
{
    int size;
    int count;
    float* data;
};
```

Note that the size of the array (for storage) could be larger than the real number (count) of the array

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - What functions we need (by design)?

```
bool create_array(int, ARRAY*);
void destroy_array(ARRAY*);

bool array_add_element(ARRAY*, float);
bool array_remove_element(ARRAY*, int);

int get_array_count(const ARRAY*);
int get_array_size(const ARRAY*);

float& get_array_element(int, const ARRAY*);
void set_array_element(int, float, ARRAY*);

void print_array(const ARRAY*);
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Create the array

```
bool create_array(int size, ARRAY* p_array_out)
{
	if (size <= 0)
		return false;

	if (p_array_out != NULL)
	{
		p_array_out->data = (float*)malloc(sizeof(float) * size);
		if (p_array_out->data == NULL)
				return false;

		p_array_out->size = size;
		p_array_out->count = 0;

		return true;
	}
	else
		return true;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Destroy the array

```
void destroy_array(ARRAY* p_array)
{
    if (p_array != NULL)
    {
        if (p_array->data != NULL)
                free(p_array->data);

        p_array->size = 0;
        p_array->count = 0;
    }
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Add array element (dynamically)

```
bool array_add_element(ARRAY* p_array, float element_value)
{
	if (p_array == NULL || p_array->data == NULL
			|| p_array->count < 0 || p_array->size <= 0)
		return false;

	if (p_array->count + 1 <= p_array->size)
	{
		p_array->count++;
		p_array->data[p_array->count - 1] = element_value;
		return true;
	}
	else
	{
		...
	}

	return true;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Add array element (dynamically)

```
bool array_add_element(ARRAY* p_array, float element_value)
{
    ...
    else
    {
        float* data_prev = p_array->data;
        p_array->data = (float*)realloc(data_prev, p_array->size
                                                  + ARRAY_INCREMENT_SIZE);
        if (p_array->data == NULL)
            return false;

        p_array->size += ARRAY_INCREMENT_SIZE;

        p_array->count++;
        p_array->data[p_array->count - 1] = element_value;
        return true;
    }

    return true;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Remove array element

```
bool array_remove_element(ARRAY* p_array, int i)
{
     if (i < 0 || i >= p_array->count)
          return false;

     int rest_count = p_array->count - i - 1;
     int copy_size = sizeof(float) * rest_count;

     float* p_temp_data = (float*)malloc(copy_size);
     if (p_temp_data == NULL)
          return false;
     memcpy(p_temp_data, &p_array->data[i + 1], copy_size);

     memcpy(&p_array->data[i], p_temp_data, copy_size);
     p_array->data[p_array->count - 1] = 0.0f;

     p_array->count--;

     ...
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Remove array element

```
bool array_remove_element(ARRAY* p_array, int i)
{
     ...

     if (p_array->count < p_array->size - 2 * ARRAY_INCREMENT_SIZE)
     {
          float* p_temp_data = p_array->data;
          p_array->data = (float*)realloc(p_temp_data, p_array->size
                                                 - ARRAY_INCREMENT_SIZE);
          if (p_array->data == NULL)
                    return false;
     }

     return true;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Remove array element

```
bool array_remove_element2(ARRAY* p_array, int i)
{
	if (i < 0 || i >= p_array->count)
		return false;

	for (int j = i; j < p_array->count - 1; j++)
		p_array->data[j] = p_array->data[j + 1];
	p_array->data[p_array->count - 1] = 0.0f;

	p_array->count--;

	if (p_array->count < p_array->size - 2 * ARRAY_INCREMENT_SIZE)
	{
		float* p_temp_data = p_array->data;
		p_array->data = (float*)realloc(
				p_temp_data, p_array->size - ARRAY_INCREMENT_SIZE);
		if (p_array->data == NULL)
			return false;
	}
	return true;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Other array functions

```
int get_array_count(const ARRAY* p_array)
{
	return p_array->count;
}

int get_array_size(const ARRAY* p_array)
{
	return p_array->size;
}

float& get_array_element(int i, const ARRAY* p_array)
{
	return p_array->data[i];
}

void set_array_element(int i, float data_value, ARRAY* p_array)
{
	p_array->data[i] = data_value;
}
```

Recursive Quick Sort for Array

- **Constructing a dynamic array**
  - Print(display) the array on the screen

```
void print_array(const ARRAY* p_array)
{
    for (int i = 0; i < get_array_count(p_array) - 1; i++)
        printf("%f, ", get_array_element(i, p_array));

    printf("%f\n", get_array_element(
                        get_array_count(p_array) - 1, p_array));
}
```

Recursive Quick Sort for Array

- **Implementing quicksort with dynamic array**
  - Partition function

```
int quick_sort_array_partition
        (ARRAY*p_array , int low, int high)
{
    float pivot = get_array_element(high,p_array);
    int i = low-1;

    for (int j = low; j < high; j++)
    {
        if (get_array_element(j, p_array) < pivot)
        {
            i++;
            if (i != j)
                swap_data(&get_array_element(i, p_array),
                          &get_array_element(j, p_array));
        }
    }
    swap_data(&get_array_element(i+1, p_array),
         &get_array_element(high, p_array));

    return i+1;
}
```

```
void swap_data(float* a, float* b)
{
    float t = *a;
    *a = *b;
    *b = t;
}
```

```
partition(arr[], low, high)
{
    // pivot (Element to be placed at right position)
    pivot = arr[high];

    i = (low - 1)  // Index of smaller element

    for (j = low; j <= high - 1; j++)
    {
        // If current element is smaller than the pivot
        if (arr[j] < pivot)
        {
            i++;
            if(i != j) swap arr[i] and arr[j]
        }
    }
    swap arr[i + 1] and arr[high])
    return (i + 1)
}
```

Recursive Quick Sort for Array

- **Implementing quicksort with dynamic array**
  - Recursive function call for partitioning

```
bool quick_sort_array(ARRAY* p_array_in_out, int low, int high)
{
    if (low < high)
    {
        int partition_index =
                quick_sort_array_partition(p_array_in_out, low, high);

        quick_sort_array(p_array_in_out, low, partition_index - 1);
        quick_sort_array(p_array_in_out, partition_index + 1, high);

        return true;
    }
    else
        return false;
}
```

Recursive Quick Sort for Array

- **How to use the quicksort for dynamic array?**

```
void test_array_sort()
{
    ARRAY a;
    if (!create_array(20, &a))
    {
        printf("unable to create the array!\n");
        return;
    }

    printf("Please input the array elements:\n");

    while (1)
    {
        char str[32];
        scanf("%s", str);
        if (strcmp(str, "quit") == 0)
        break;
        array_add_element(&a, (float)atof(str));
    }
    ...
}
```

Recursive Quick Sort for Array

- **How to use the quicksort for dynamic array?**

```
void test_array_sort()
{
    ...

    printf("Your input ARRAY is:\n");
    print_array(&a);

    printf("\nSorting the whole array...\n");
    quick_sort_array(&a, 0, get_array_count(&a) - 1);
    printf("Your sorted array is:\n");
    print_array(&a);

    destroy_array(&a);//don't forget to destroy the array
}
```

Designing Recursive Algorithms

- Find the **key step**
  - How can this problem be divided into parts?
  - How will the key step in the middle be done?
- Find a **stopping rule**
  - Small, special case that is trivial or easy to handle without recursion
- Outline your algorithm
  - Combine the stopping rule and the key step, using an **if** statement to select between them
- Check termination
  - Verify that the **recursion always terminates**

Recursion or Iteration ?

- The main advantage of using recursive functions: When the problem is recursive in nature, a recursive function results in short, clear code.
- The main disadvantage of using recursive functions: recursion is more expensive than iteration.
- Any problem that can be solved recursively can also be solved iteratively (by using loops).
- A recursive approach is normally chosen in preference to an iterative approach when the recursive approach more naturally mirrors the problem and results in a program that is easier to understand and debug.

Summary

- Study several simple examples to see whether recursion should be used and how it will work.
- Attempt to formulate a method that will work more generally
  - How can this problem be divided into parts?
- Ask whether the remainder of the problem can be done in the same or simple way.
- Find a stopping rule.
- Algorithm must terminate.