CS101 Fall 2024 - Quiz 13
December 30, 2024 - 12 Minutes **Name:** **Student ID:**

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1. (2 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz. ◯ True ◯ False

2. (18 points) True or False

(a) (1') A problem in NP is NP-Complete if any other problems in NP can be reduced to it in polynomial time. ◯ True ◯ False

(b) (1') A problem in NP is NP-Complete if it can be reduced to another problem in NP in polynomial time. ◯ True ◯ False

(c) (1') A problem in NP is NP-Complete if it can be reduced to the 3-SAT problem in polynomial time. ◯ True ◯ False

(d) (1') A problem in NP is NP-Complete if the 3-SAT problem can be reduced to it in polynomial time. ◯ True ◯ False

(e) (1') A problem in NP is NP-Complete if some problems in NP can be reduced to it in polynomial time. ◯ True ◯ False

(f) (1') A problem in NP is NP-Complete if there exists another NP-Complete problem which can be reduced to it in polynomial time. ◯ True ◯ False

(g) (1') According to the Cook-Levin Theorem, any problem in NP can be reduced to SAT in polynomial time. ◯ True ◯ False

(h) (1') If a problem is in NP-Complete, any NP-Complete problem can reduce to it in polynomial time. ◯ True ◯ False

(i) (1') Any problem in P is in NP. ◯ True ◯ False

(j) (1') If $L_1 \leq_p L_2$ and L_1 is in NP, then so is L_2. ◯ True ◯ False

(k) (1') If $L_1 \leq_p L_2$ and L_1 is in NP-Complete, then so is L_2. ◯ True ◯ False

(l) (1') If $L_1 \leq_p L_2$ and $L_1 \in \mathsf{P}$, then $L_2 \in \mathsf{P}$. ◯ True ◯ False

(m) (1') If $L_1 \leq_p L_2$ and $L_1 \notin \mathsf{P}$, then $L_2 \notin \mathsf{P}$. ◯ True ◯ False

(n) (1') For any positive integer $k \geq 2$, k-SAT $\in$ NP. ◯ True ◯ False

(o) (1') For any positive integer $k \geq 2$, k-SAT $\in$ NP-Complete. ◯ True ◯ False

(p) (1') For any problem $L \in \mathsf{NP}, L \leq_p$ 3-SAT. ◯ True ◯ False

(q) (1') 2-COLOR $\leq_p$ 3-COLOR $\leq_p$ 3-SAT. ◯ True ◯ False

(r) (1') If CLIQUE can be solved in polynomial time, then any problem in NP can be solved in polynomial time. ◯ True ◯ False

3. (10 points) P and NP worlds

Give out statements, please select their correctness from the four below.

A. True if and only if P = NP B. True regardless of whether P = NP or not
C. False if and only if P = NP D. False regardless of whether P = NP or not

(a) (2') There are problems in NP that cannot be solved in exponential time. _____

(b) (2') If problem $L_1 \in \mathsf{NP}$, then for any problem $L_2 \in \mathsf{P}$, L_2 can be reduced to L_1 in polynomial time. _____

(c) (2') Minimum Spanning Tree Problem is in NP. _____

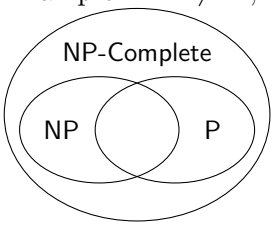
(d) (2') Minimum Spanning Tree Problem is in NP-Complete. _____

(e) (2') 3-SAT $\leq_p$ 2-SAT. _____

4. (10 points) Venn diagrams for P and NP

Draw Venn diagrams which represent the relationship among P, NP, and NP-Complete under 2 circumstances.

Example: If $A \neq B, A \cap B \neq \emptyset, A \subset C, B \subset C$, then the venn diagrams can be drawn like:



(a) (4') If P = NP, draw Venn diagrams which represent the relationship among P, NP, and NP-Complete under 2 circumstances.

(b) (6') If P $\neq$ NP, draw Venn diagrams which represent the relationship among P, NP, and NP-Complete under 2 circumstances.