CS 101 Fall 2024 - Quiz 9
December 2, 2024 - 20 Minutes **Name:** **Student ID:**

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1. (1 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz. ◯ True ◯ False

2. (7 points) True or False

Determine whether the following statements are true or false.

(a) (1') Given a directed acyclic graph $\mathbf{G}$. If u appears before v in a topological sort, there may exist a path from v to u in $\mathbf{G}$. ◯ True ◯ False

(b) (1') Given a directed acyclic graph $\mathbf{G}$. If there doesn't exist a path from u to v in $\mathbf{G}$, then u will always appear before v in a topological sort. ◯ True ◯ False

(c) (1') Given a directed acyclic graph $\mathbf{G}$. If u always appears before v in all topological sortings of $\mathbf{G}$, then there exists a path from u to v in $\mathbf{G}$. ◯ True ◯ False

(d) (1') Running topological sort in DAG takes $\Theta(|V|\log|V|)$ time. ◯ True ◯ False

(e) (1') A subgraph of a DAG may not be a DAG. ◯ True ◯ False

(f) (1') Topological sort can be applied to any connected graph. ◯ True ◯ False

(g) (1') Given an arbitrary coin system, a greedy algorithm of picking the largest denomination of coin which is not greater than the remaining amount to be made will always produce the optimal result. ◯ True ◯ False

3. (5 points) Match Sequence

Given a long sequence S of n characters, find an efficient way to detect whether it contains a subsequence S' with m characters. Characters in S' may not be consecutive in S, but they must follow the same order. For example,

$$A, B, C, A$$

is in

$$C, A, B, Q, D, C, A, A$$

(a) (3') Give an efficient algorithm (less than $O(n^2)$) for this problem. **Use natural language** to show your answer.

1. Start from beginning of both sequence S and subsequence S'.
2. Iterate through the sequence S, for each character,
   - If the current character in S matches the current character in S',

     _____
   - Else,

     _____
3. If you have matched all characters in S', return true.
4. If you finish checking all characters in S and have not matched all characters in S', return false.

(b) (2') What is the time complexity of your algorithm? _____

4. (4 points) Does Greedy Work?

There are n pieces of wood, the i-th of which has length l_i. At each time, one can choose two pieces of wood with length a and b respectively, and use $\max\{a, b\}$ units of glue to glue them into one. The new piece of wood is of length $a + b$. Xiao Wang wants to find minimum amount of glue needed to glue all the pieces of wood into one. He came up with a greedy algorithm: Take two pieces of wood with minimal length each time and glue them into one, and repeat that until there is only one piece of wood left.

Do you think this algorithm can always give the optimal solution? If so, give a proof. If not, provide a counterexample. A counterexample should contain the input, the solution given by the greedy algorithm, and the optimal solution.