CS 101 Fall 2024 - Quiz 3

October, 21, 2024 - 25 Minutes **Name:** **Student ID:**

HINT: $\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$

1. (3 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz. ◯ True ◯ False

2. (5 points) True or False

Determine whether the following statements are true or false.

(a) (1') Quick-sort is an in-place sorting algorithm, while merge-sort is not since it requires $\omega(n)$ extra space. ◯ True ◯ False

(b) (1') Randomized Quick-sort is unstable, while Quick-sort with deterministic method (choosing pivots) is stable. ◯ True ◯ False

(c) (1') There exists an comparison-based sort algorithm that needs $O(1)$ extra space and takes $o(n \log n)$ time. ◯ True ◯ False

(d) (1') For an array $\{a_n\}$ with distinct elements, for fixed i, j, if $\forall a_k \neq a_i, a_k \neq a_j, (a_k - a_i)(a_k - a_j) > 0$, then a_i and a_j will be compared in any case when using randomized quick-sort to make $\{a_n\}$ sorted. ◯ True ◯ False

(e) (1') When we use divide and conquer to solve a problem, we should divide the problem into one or more subproblems with the exact same scale, then recursively do them and merge their answers at last. ◯ True ◯ False

3. (5 points) Randomized quick-sort

(a) (1') If we use randomized quick-sort(i.e. randomly choosing pivots) to sort the array $[3, 4, 6, 2, 1, 5, 8, 0]$, the probability of 1 and 6 are compared is ______ .

(b) (1') Use the same method as above to sort an array with n distinct elements, the probability of i-th largest and j-th largest element $(i \neq j)$ are compared is ________ .

(c) (3') Prove that the expectation times of comparisons in the randomized quick-sort is $\Theta(n \log n)$. **Hint: The total expectation times can be obtained from the sum of the expectation of each comparison.**

4. (12 points) Solving Recursion

Solve the recursion relation with $T(1) = 1, T(0) = 0$:

(a) (3') $T(n) = T(n-1) + \Theta(c^n)$ ($c > 1$ is a constant).

(b) (4') $T(n) = T(\frac{n}{2}) + \Theta(\log n)$

(c) (5') $T(n) = \Theta(n) + \frac{1}{n}\sum_{i=0}^{n-1}(T(i) + T(n-i))$, you can write $\Theta(n)$ as cn for your convenience.