CS 101 Fall 2024 - Quiz 8

November 24, 2024 - 20 Minutes **Name:** **Student ID:**

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1. (2 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz. ◯ True ◯ False

2. (7 points) True or False

(a) (1') If we use BFS(breadth-first search) to find a path from u to v on an unweighted graph, this path will be the shortest path between u and v. ◯ True ◯ False

(b) (1') If we consider the disjoint set time complexity, the time complexity of Kruskal's algorithm can reach to $\Theta\left(|E|\alpha(|V|)\log|V|\right)$. ◯ True ◯ False

(c) (1') The maximum weight edge in a graph will not be in any MST. ◯ True ◯ False

(d) (1') Let G be a weighted undirected graph with positive weights where edge e has weight w_e for all $e \in E$. And G' is a copy of G except that every edge e has weight $w_e + \frac{2}{w_e}$. Then any MST in G is also a MST in G' (for every pair of (G, G')). ◯ True ◯ False

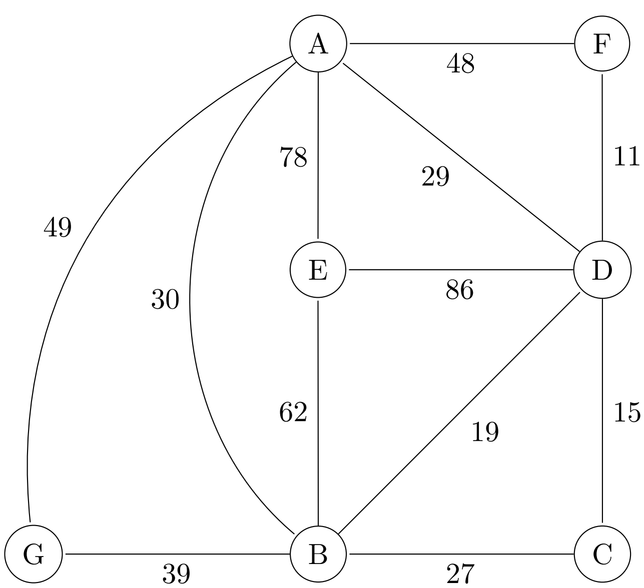
(e) (1') If T is a MST of G, then $\forall u, v \in G$ the path on the tree T connecting u and v is the shortest path from u to v in G. ◯ True ◯ False

(f) (1') Given a graph $G = (V, E)$, $w : E \to \mathbb{R}$ assigns the weight of every edge. $\forall C \subset E$ which is a cycle, if $\exists e$ satisfies $\forall e' \in C, w(e) > w(e')$, then e won't be in any MST of G. ◯ True ◯ False

(g) (1') Let $G = (V, E)$ be a connected undirected graph. If $e_0 \subset E$ is an edge such that $w(e_0) = \min\{w(e) | e \in E\}$, then e_0 belongs to every MST of G. ◯ True ◯ False

3. (6 points) Select the MST

In this problem, we want you to find the MST of the given graph. For every edge in the graph, check whether it is in the MST. **Select those edges which are in the MST in the table.**



Edge	Whether In MST
AB	◯ Selected
AD	◯ Selected
AE	◯ Selected
AF	◯ Selected
AG	◯ Selected
BC	◯ Selected
BD	◯ Selected
BE	◯ Selected
BG	◯ Selected
CD	◯ Selected
DE	◯ Selected
DF	◯ Selected

4. (10 points) MST with special edge

Given a weighted undirected graph $G = (V, E \cup \{\mathbf{e_0}\})$. For each edge in E, it can be represented as a triple: $e_i = (u_i, v_i, w_i)$. u_i and v_i mean the indices of the vertices connected by the edge, and w_i means the weight of the edge. There exists a special edge $\mathbf{e_0}$ which can be represented by $\mathbf{e}_0 = (u_0, v_0, w_0)$. We want you to find the minimal spanning tree containing the special edge $\mathbf{e}_0$ based on **Kruskal's algorithm**.

Please write the answers in the blanks below. Write the most precise and simplified form.

(You may use disjoint sets with union-by-rank and path-compression optimization in the problem. You can use 'Make-set', 'Union', 'Find-Set' to refer to the functions of those in the disjoint set. Take $\alpha(n)$ as a constant.)

Usage of three functions:

Make-Set **Require:** v vertex in V.
MAKE-SET(v) {Make a set in the disjoint sets.}

Find-Set **Require:** v vertex in V.
FIND-SET(v) {Find and return the corresponding set which v belongs to.}

Union **Require:** S_1, S_2 sets in the disjoint sets.
UNION(S_1, S_2) {Union S_1 and S_2 into a set in the disjoint sets.}

Algorithm 1 Minimum Spanning Tree with Special Edge

Require: V vertex set, E edge set, $\mathbf{e}$ special edge
Ensure: Minimum Spanning Tree containing $\mathbf{e}_0$

$A \leftarrow \varnothing$
for each vertex v in set V **do**
  MAKE-SET(v)
end for
$(u_0, v_0, w_0) \leftarrow \mathbf{e_0}$
________ (A) ________
________ (B) ________
Sort(E , ascending) {Sort the edge in ascending weight}
for each edge e in set E **do**
  $(u, v, w) \leftarrow e$
  if ____ (C) ____ **then**
    $A \leftarrow A \cup \{e\}$
    ________ (D) ________
  end if
end for
return A

(A) ________________________________

(B) ________________________________

(C) ________________________________

(D) ________________________________

- What's the time complexity of the algorithm? (Using $|E|$ and $|V|$) $\Theta($________________$)$