CS 101 Fall 2024 - Quiz 11
December 16, 2024 - 20 Minutes **Name:** **Student ID:**

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1. (2 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz. ◯ True ◯ False

2. (4 points) True or False

Determine whether the following statements are true or false.

(a) (1') The Floyd-Warshall algorithm can return the shortest path between all pairs of nodes in a connected graph with n nodes in $O(|V|^3)$, while it only needs $O(|V|^2)$ to find the shortest path between a given pair of nodes in the graph(for worst case). ◯ True ◯ False

(b) (1') We can modify the Floyd-Warshall algorithm to detect whether there exists a negative cycle or not in a directed graph. ◯ True ◯ False

(c) (1') Since the solutions of DP problems depend on other subproblems, one needs to at least give out the solutions of some subproblems to solve the problem. ◯ True ◯ False

(d) (1') We could always use recursion in DP and hence do not need to allocate extra space to store solutions of subproblems, which could save the memory use. ◯ True ◯ False

3. (4 points) Floyd-Warshall's Algorithm

Given the Floyd-Warshall's algorithm, Please write the fill in the blanks below. Write the most precise and simplified form.

Hint: *Mind the order of loop!*

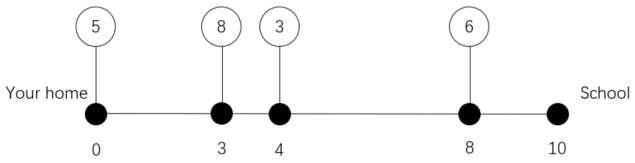
Algorithm 1 Floyd-Warshall's algorithm

```
1:  procedure FLOYD-WARSHALL(V, E)
2:      let dist be a |V| × |V| array of minimum distances initialized to ∞ (infinity)
3:      for each edge (u, v) ∈ E do
4:          dist[u][v] ← w(u, v)                    ▷ The weight of the edge (u, v)
5:      end for
6:      for each vertex v ∈ V do
7:          dist[v][v] ← 0
8:      end for
9:      for i from 1 to |V| do
10:         for j from 1 to |V| do
11:             for k from 1 to |V| do
12:                 if _____ then
13:                     _____
14:                 end if
15:             end for
16:         end for
17:     end for
18:     return dist
19: end procedure
```

4. (10 points) Remove the road sign

There is a road from your home to school. The length of the road is L kilometers, and your home is located at coordinate 0, and the school is located at coordinate L. There are n signs along the road, where the i 'th sign has a value a_i, indicating that you must travel at a speed of exactly a_i minutes per kilometer until you reach the next sign. There is also a sign at coordinate 0 which sets the initial speed. We can use the signs to calculate the time to travel from home to school. For example, in Fig. 1, the total time is $3 \times 5 + 1 \times 8 + 4 \times 3 + 2 \times 6 = 47$ minutes.

We now want to remove at most k signs in a way that minimizes the time it takes to travel from home to school. You cannot remove the sign at coordinate 0. We want to design a dynamic programming algorithm.



(a) (6') Define $OPT(i, j)$ be the minimum time needed to spend if we already walk to sign i, and choose j signs (i.e remove $i - j$ signs). Give your Bellman equation and explanation to solve the subproblems. (Write the Bellman Equation will receive 4 points, while the explanation will receive 2 points)

(b) (2') What is the answer to this question in terms of your subproblems?

(c) (2') What is the runtime complexity of your algorithm?