Computer Graphics I

Lecture 13: Global illumination 1

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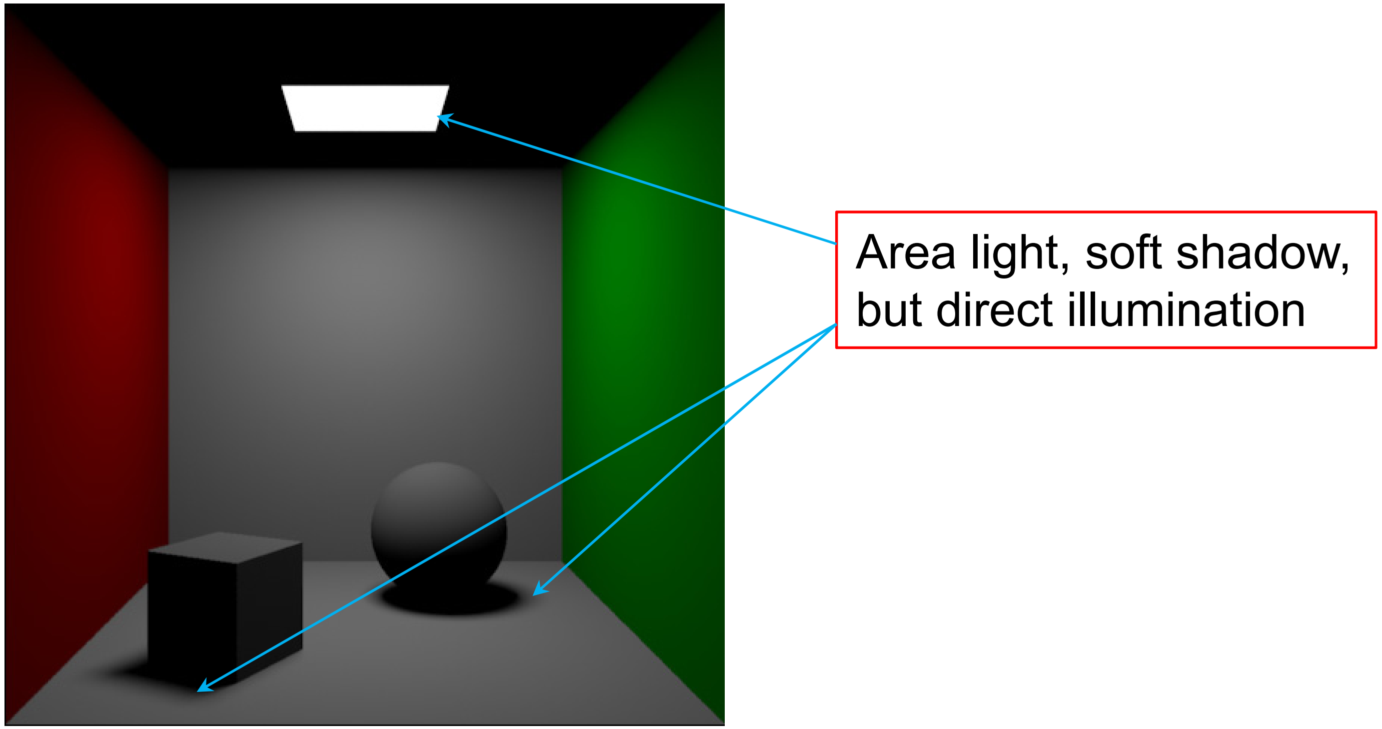
Illumination in a scene

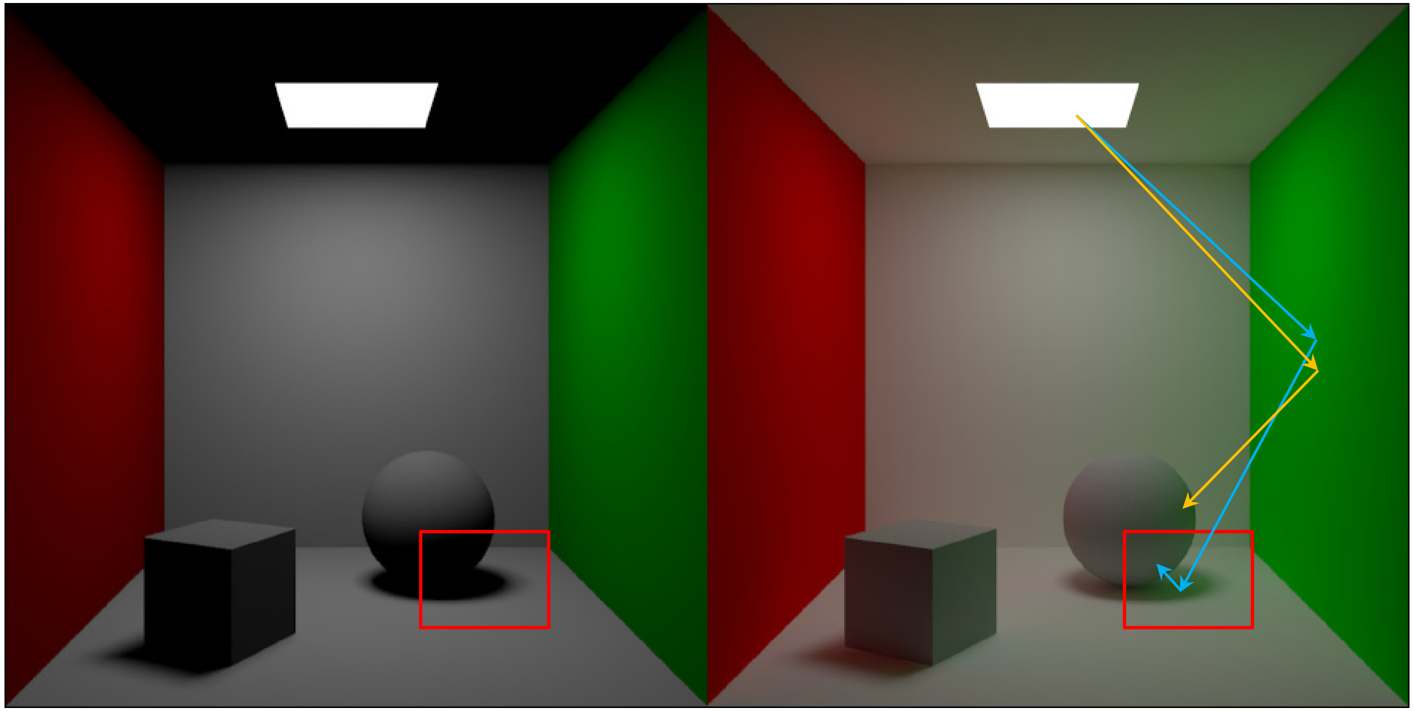
- **Direct illumination**
  - Illumination cast on objects directly from light sources



Illumination in a scene

- **Global illumination**
  - Illumination cast on objects from both light sources and surface inter-reflections
  - Direct illumination + indirect illumination

间接光源反射直接光源产生的光



1. Direct lighting

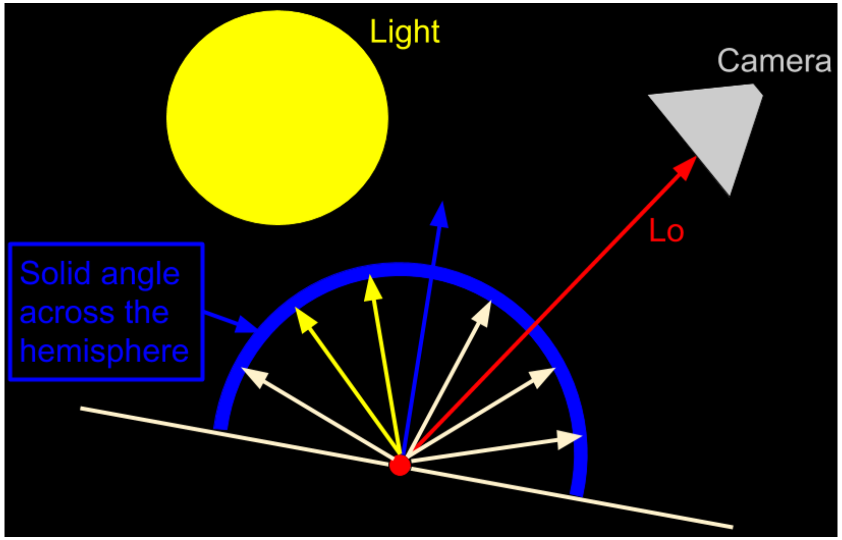
Direct lighting

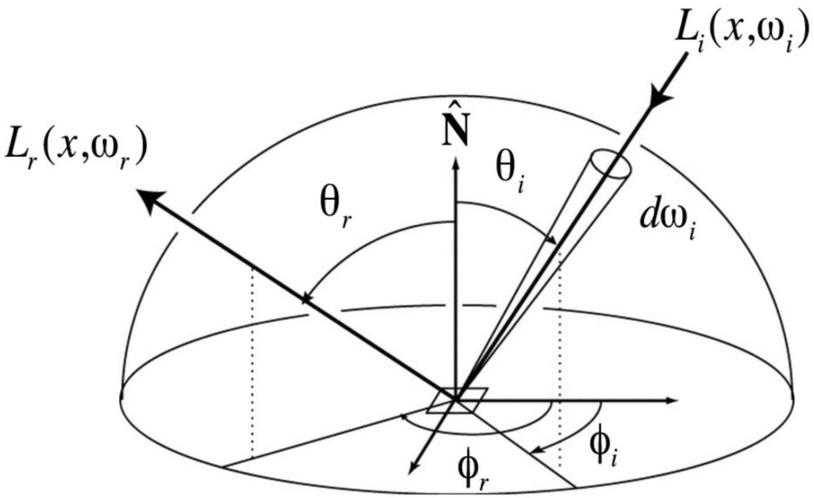
- **How to determine the radiance?**
  - Recall the rendering equation

Incoming light distribution

$$L_o(\mathrm{p}, \omega_o) = \int_{\mathcal{S}^2} f(\mathrm{p}, \omega_o, \omega_i)\, L_d(\mathrm{p}, \omega_i)\, |\cos \theta_i|\, d\omega_i$$

  - We need to consider direct light sources over the hemisphere





Direct lighting

- **Multiple importance sampling**
  - Assumption: proper distribution functions for f and L_d, but not for the whole product
  - Adopt a weighting scheme (reduce overall variance)

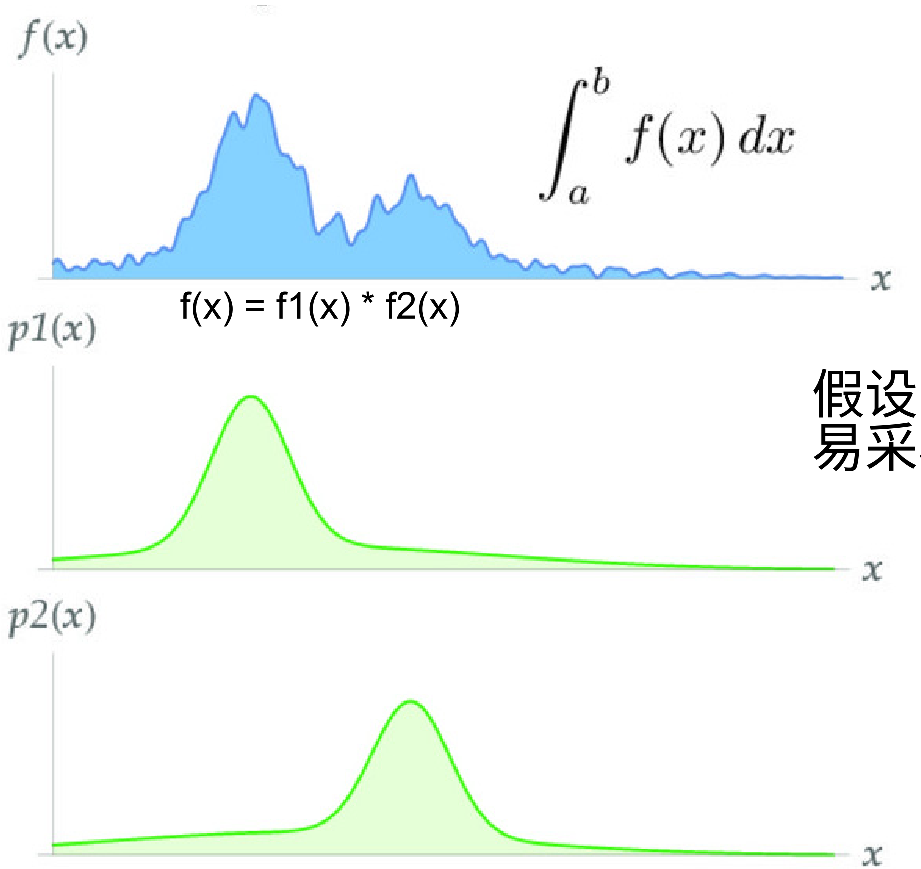
$$\frac{1}{n_f}\sum_{i=1}^{n_f}\frac{f(X_i)g(X_i)w_f(X_i)}{p_f(X_i)}+\frac{1}{n_g}\sum_{j=1}^{n_g}\frac{f(Y_j)g(Y_j)w_g(Y_j)}{p_g(Y_j)}$$

  - Choice for weight
    - Balance heuristic

$$w_s(x)=\frac{n_s p_s(x)}{\sum_i n_i p_i(x)}$$

Direct lighting

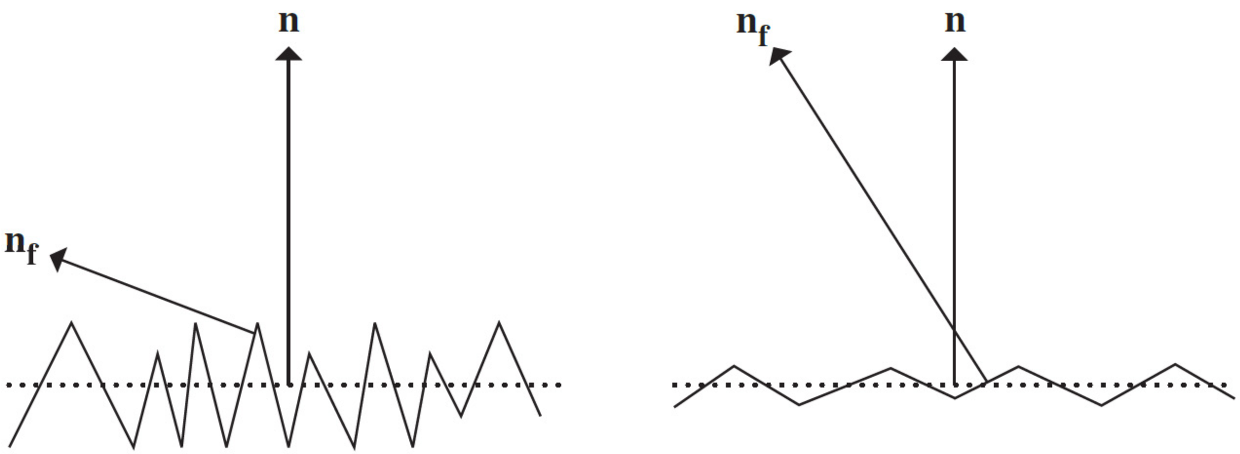
- **Multiple importance sampling**
  - Illustration



假设p1(x),p2(x)比较容易采样

Microfacet models

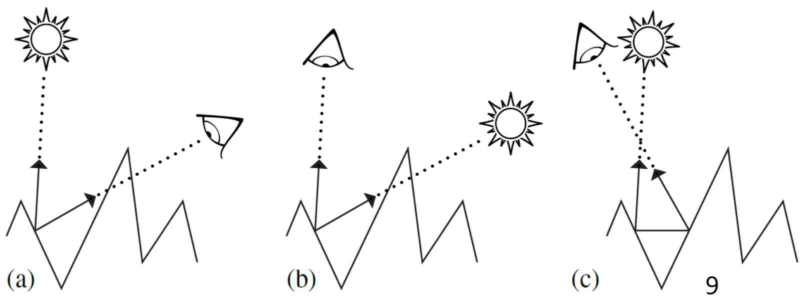
- **Microfacets**
  - Rough surfaces can be modeled as a collection of small microfacets
  - Essentially a heightfield
    - The distribution of facets is described statistically



Microfacet surface models are often described by a function that gives the distribution of microfacet normals $\mathbf{n}_f$ with respect to the surface normal $\mathbf{n}$

Microfacet models

- **Microfacet-based BRDF**
  - Statistically modeling the scattering of light from a large collection of microfacets
  - Assumption
    - Differential area is large compared to the size of microfacets
    - Their aggregate behavior determines the scattering
  - Three effects to consider
    - Occlusion, shadow, inter-reflection
  - Simplification
    - Assume V-shaped for each
    - Ignore most of inter-reflections



Microfacet models

- **Torrance-Sparrow model (1967)**
  - One of the first microfacet models for computer graphics
  - Used for modeling metallic surfaces
  - A collection of smooth mirrored microfacets
  - A $D(\omega_h)$ that gives the probability
    - A microfacet has orientation ω_h

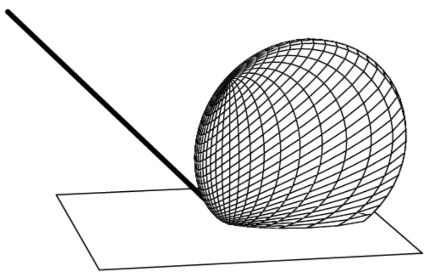
$$f_r(\mathrm{p}, \omega_o, \omega_i) = \frac{D(\omega_h)\, G(\omega_o, \omega_i)\, F_r(\omega_o)}{4\, \cos\theta_o\, \cos\theta_i}$$

$$G(\omega_o, \omega_i) = \min\left(1, \min\left(\frac{2(\mathbf{n}\cdot\omega_h)(\mathbf{n}\cdot\omega_o)}{\omega_o\cdot\omega_h}, \frac{2(\mathbf{n}\cdot\omega_h)(\mathbf{n}\cdot\omega_i)}{\omega_o\cdot\omega_h}\right)\right)$$

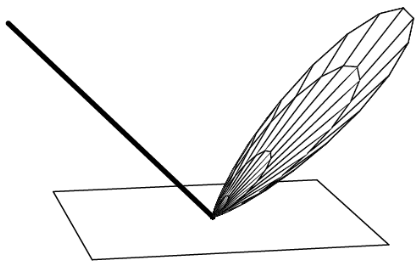
Microfacet models

- **Blinn microfacet distrubution**
  - Blinn (1977) proposed a model for the distribution of microfacets
    - The normal is approximated with exponential fall-off
    - Normalized Blinn microfacet distribution is:

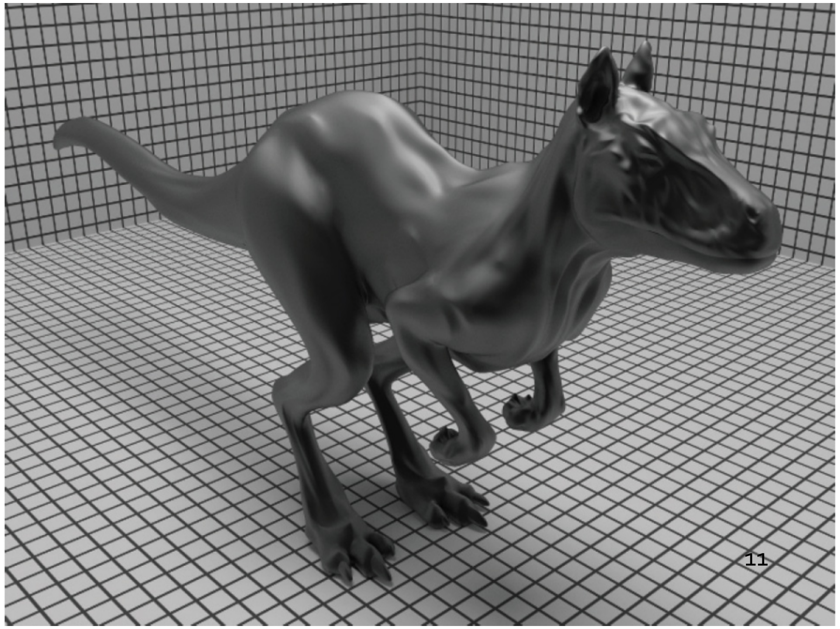
$$D(\omega_h) = \frac{e+2}{2\pi}(\omega_h \cdot \mathbf{n})^e$$



(a)



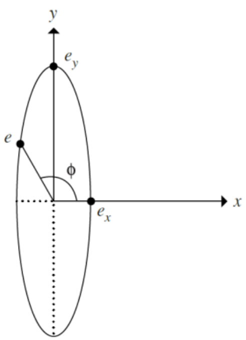
(b)

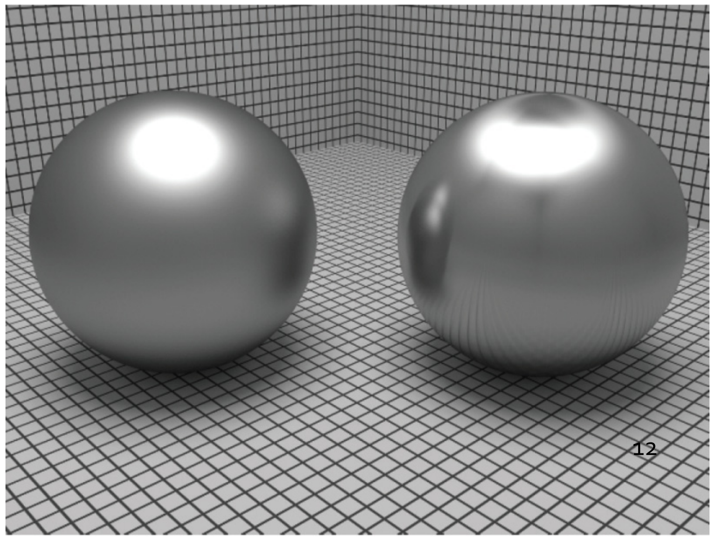


Microfacet models

- **Anisotropic microfacet model**
  - Ashikhmin and Shirley (2000,2002) developed a microfacet distribution function
    - To model the appearance of anisotropic surfaces
    - An anisotropic variant of Blinn's exponential fall-off microfacet distribution
    - Physically-based, with intuitive parameters

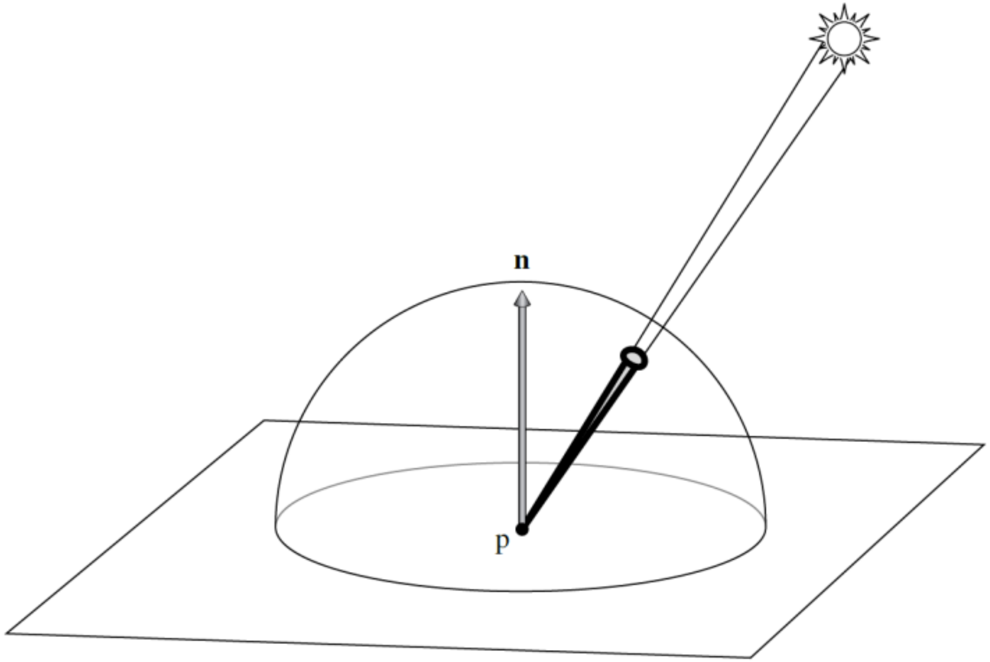
$$D(\omega_h) = \frac{\sqrt{(e_x + 2)(e_y + 2)}}{2\pi} (\omega_h \cdot \mathbf{n})^{e_x \cos^2 \phi + e_y \sin^2 \phi}$$





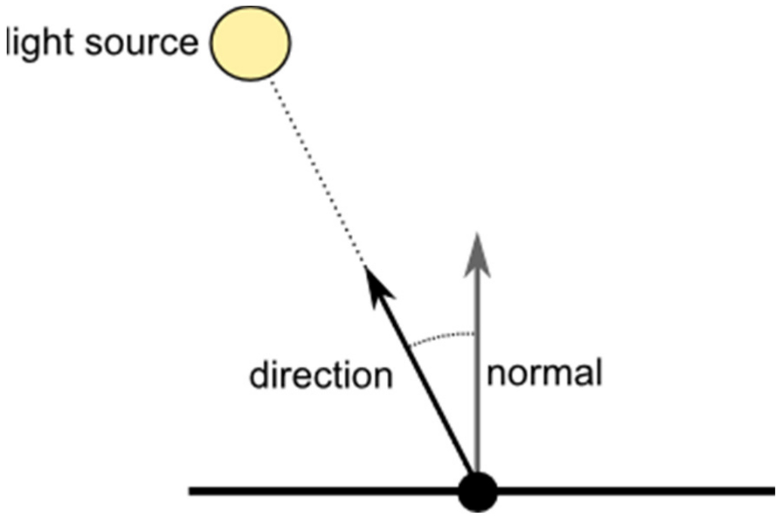
Sampling light sources

- **Lights with singularities**
  – Lights coming from extremely small ranges of solid angles
  – Point lights, directional lights etc.



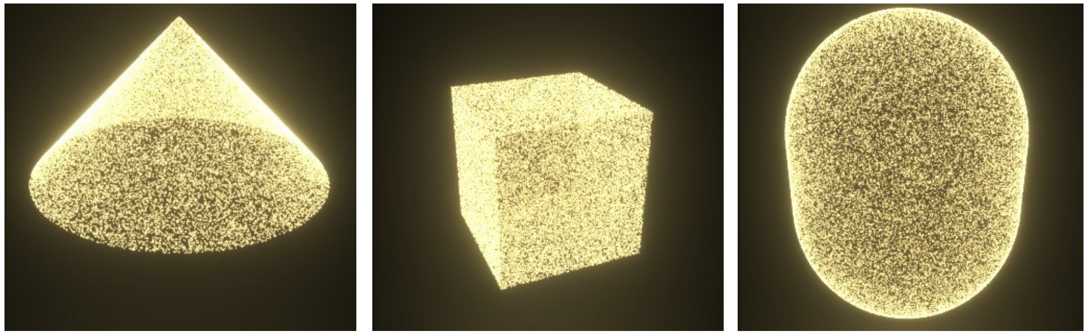
Sampling light sources

- **Point lights and directional lights**
  - The light distribution L_d is a delta function
  - Return samples along that specific direction



Sampling light sources

- **Area lights**
  - An emission profile attached to a shape
  - Sampling shapes
    - Sample uniformly in terms of area



  - Area light sampling
    - Combine shape distribution and uniform distribution

2. Light transport equation

Light transport equation

- **Governing equation**
  - Describe the equilibrium distribution of radiance in a scene
  - Give the total reflected radiance at a point on a surface
  - In terms of
    - Emission from the surface
    - Surface BSDF(BRDF/BSSRDF)
    - Distribution of incident illumination arriving at the point
  - Numerical computation for a solution of light transport equation (LTE)

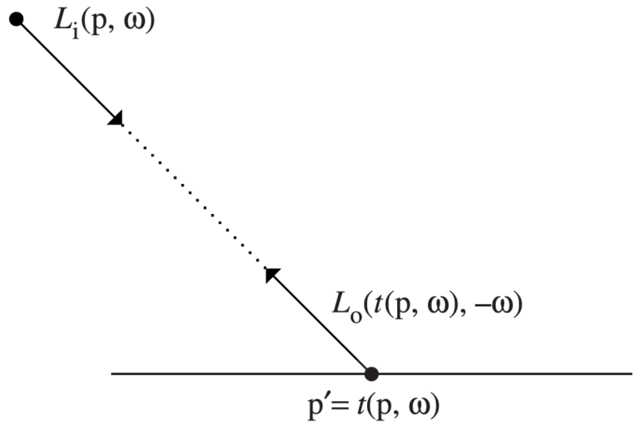
Light transport equation

- **Basic derivation**
  - Energy balance
    - Exitant radiance must be equal to emitted radiance + fraction of incident radiance scattered:

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{S^2} f(p, \omega_o, \omega_i)\, L_i(p, \omega_i)\, |\cos\theta_i|\, d\omega_i$$

Le:打到的点本身发光

    - Assume now: no participating media
      - Radiance is constant along rays through the scene
    - We can relate incident radiance at p to outgoing radiance from another point p'



$$L_i(p, \omega) = L_o(t(p, \omega), -\omega)$$

Light transport equation

- **Basic derivation**
  - Consider the entire scene as a light field
    - We can describe the field by L(p,ω)
  - Dropping the subscript, we obtain the LTE equation

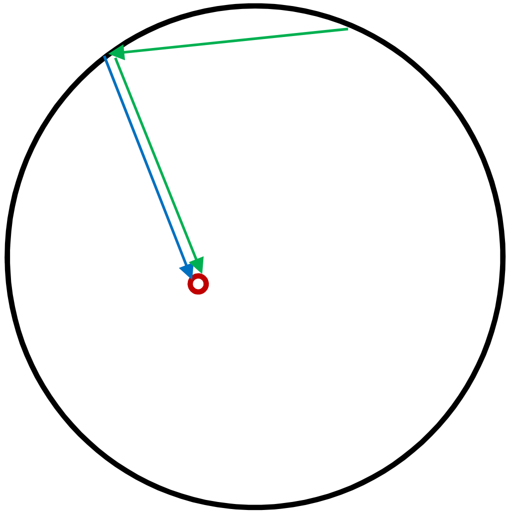
$$\boxed{L(\mathrm{p}, \omega_\mathrm{o})} = L_\mathrm{e}(\mathrm{p}, \omega_\mathrm{o}) + \int_{\mathcal{S}^2} f(\mathrm{p}, \omega_\mathrm{o}, \omega_\mathrm{i}) \boxed{L(t(\mathrm{p}, \omega_\mathrm{i}), -\omega_\mathrm{i})} \left|\cos\theta_\mathrm{i}\right| \mathrm{d}\omega_\mathrm{i}$$

直到t(p,wi)为直接光源则停止递归

Same light field at different positions and solid angles

Light transport equation

- **Analytical solutions**
  - Impossible to solve in general
  - Difficulties
    - BSDF models
    - Arbitrary scene geometry
    - Intricate visibility relationships
  - Possible for extremely simple settings
    - Consider the interior of a sphere with Lambertian surface



$$f(\mathrm{p}, \omega_\mathrm{o}, \omega_\mathrm{i}) = c$$

    - And emit constant amount of radiance in all directions

$$L(\mathrm{p}, \omega_\mathrm{o}) = L_\mathrm{e} + c \int_{\mathcal{H}^2(\mathbf{n})} L(t(\mathrm{p}, \omega_\mathrm{i}), -\omega_\mathrm{i}) \, |\cos\theta_\mathrm{i}| \, \mathrm{d}\omega_\mathrm{i}$$

Light transport equation

- **Analytical solutions**
  - Integrate out

$$L(\mathrm{p}, \omega_\mathrm{o}) = L_\mathrm{e} + c \int_{\mathcal{H}^2(\mathbf{n})} L(t(\mathrm{p}, \omega_\mathrm{i}), -\omega_\mathrm{i})\, |\cos\theta_\mathrm{i}|\, \mathrm{d}\omega_\mathrm{i} \quad \Longrightarrow \quad L = L_\mathrm{e} + c\pi L$$

  - Replace cπ with ρ_{hh} as Lambertian surface reflectance, consider successive substitution

$$L = L_\mathrm{e} + \rho_\mathrm{hh}(L_\mathrm{e} + \rho_\mathrm{hh}(L_\mathrm{e} + \cdots = \sum_{i=0}^{\infty} L_\mathrm{e}\rho_\mathrm{hh}^i$$

phh<1 -> 收斂

  - Explanation of the series
    - Exitant radiance = emitted radiance + light scatted by a BSDF once + light scattered twice + ...
    - Convergence: $L = \sum_{i=0}^{\infty} L_\mathrm{e}\rho_\mathrm{hh}^i = \frac{L_\mathrm{e}}{1-\rho_\mathrm{hh}}$

Light transport equation

- **Analytical solutions**
  - Similar idea of successive substitution on

$$L(\mathrm{p}, \omega_{\mathrm{o}}) = L_{\mathrm{e}}(\mathrm{p}, \omega_{\mathrm{o}}) + \int_{\mathcal{S}^2} f(\mathrm{p}, \omega_{\mathrm{o}}, \omega_{\mathrm{i}})\, L_{\mathrm{d}}\, |\cos\theta_{\mathrm{i}}|\, \mathrm{d}\omega_{\mathrm{i}}$$

where

$$L_{\mathrm{d}} = L_{\mathrm{e}}(t(\mathrm{p}, \omega_{\mathrm{i}}), -\omega_{\mathrm{i}}) + \int_{\mathcal{S}^2} f(t(\mathrm{p}, \omega_{\mathrm{i}}), \omega') L(t(t(\mathrm{p}, \omega_{\mathrm{i}}), \omega'), -\omega') |\cos\theta'|\, \mathrm{d}\omega'$$

  - This is the mathematical base for developing rendering algorithms

Light transport equation

- **The surface form**
  - Light transport equation on a surface
  - Definition
    - Exitant radiance from a point p′ to a point p

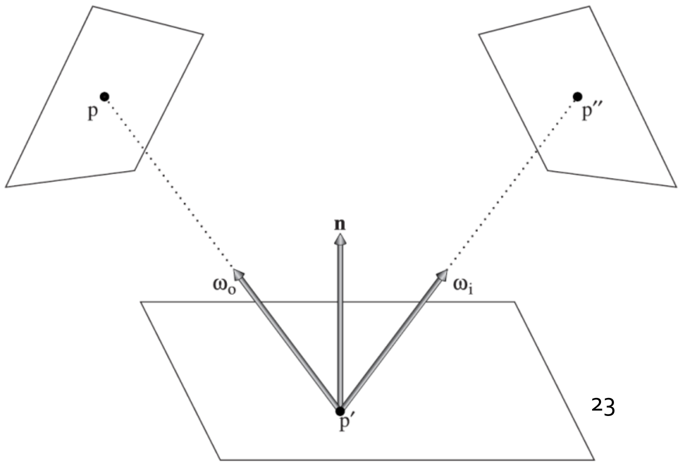
$$L(\mathrm{p}' \to \mathrm{p}) = L(\mathrm{p}', \omega)$$

- If p′ and p are mutually visible, and $\omega = \widehat{\mathrm{p} - \mathrm{p}'}$
  - BSDF at p′ is

$$f(\mathrm{p}'' \to \mathrm{p}' \to \mathrm{p}) = f(\mathrm{p}', \omega_\mathrm{o}, \omega_\mathrm{i})$$

where

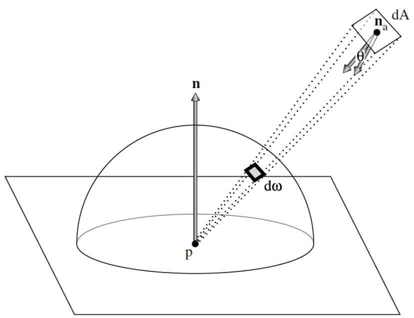
$$\omega_\mathrm{i} = \widehat{\mathrm{p}'' - \mathrm{p}'} \qquad \omega_\mathrm{o} = \widehat{\mathrm{p} - \mathrm{p}'}$$



Light transport equation

- **The surface form**
  - Jacobian relating solid angle to surface area: $|\cos\theta'|/r^2$
  - Geometric term
    - Jacobian term + original $|\cos\theta|$ term + binary visibility function V

$$G(\mathrm{p} \leftrightarrow \mathrm{p}') = V(\mathrm{p} \leftrightarrow \mathrm{p}') \frac{|\cos\theta|\,|\cos\theta'|}{\| \mathrm{p} - \mathrm{p}' \|^2}$$



  - Substituting into the light transport equation
    - Convert to surface integral

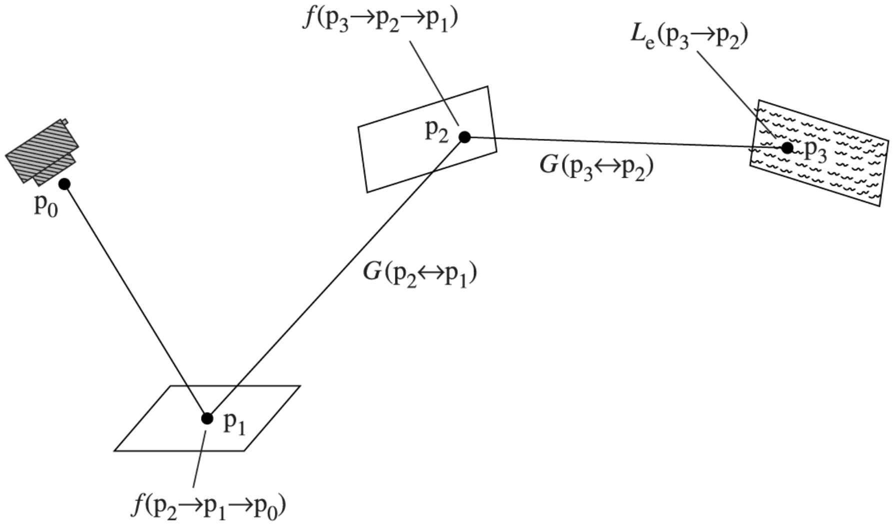
$$L(\mathrm{p}' \to \mathrm{p}) = L_e(\mathrm{p}' \to \mathrm{p}) + \int_A f(\mathrm{p}'' \to \mathrm{p}' \to \mathrm{p}) L(\mathrm{p}'' \to \mathrm{p}') G(\mathrm{p}'' \leftrightarrow \mathrm{p}')\, \mathrm{d}A(\mathrm{p}'')$$

All surfaces in the scene

Light transport equation

- **Integral over paths**
  - Expand the three-point light transport equation

$$
\begin{aligned}
L(\mathrm{p}_1 \to \mathrm{p}_0) = \boxed{\begin{aligned}
& L_e(\mathrm{p}_1 \to \mathrm{p}_0) \\
& + \int_A L_e(\mathrm{p}_2 \to \mathrm{p}_1) f(\mathrm{p}_2 \to \mathrm{p}_1 \to \mathrm{p}_0) G(\mathrm{p}_2 \leftrightarrow \mathrm{p}_1) \mathrm{d}A(\mathrm{p}_2) \\
& + \int_A \int_A L_e(\mathrm{p}_3 \to \mathrm{p}_2) f(\mathrm{p}_3 \to \mathrm{p}_2 \to \mathrm{p}_1) G(\mathrm{p}_3 \leftrightarrow \mathrm{p}_2) \\
& \qquad \times f(\mathrm{p}_2 \to \mathrm{p}_1 \to \mathrm{p}_0) G(\mathrm{p}_2 \leftrightarrow \mathrm{p}_1)\, \mathrm{d}A(\mathrm{p}_3)\, \mathrm{d}A(\mathrm{p}_2) + \cdots
\end{aligned}}
\end{aligned}
$$



Each term represents
A path of increasing length

Light transport equation

- **Integral over paths**
  - The infinite sum can be written compactly as

$$L(\mathrm{p}_1 \to \mathrm{p}_0) = \sum_{n=1}^{\infty} P(\bar{\mathrm{p}}_n) \qquad \bar{\mathrm{p}}_n = \mathrm{p}_0, \mathrm{p}_1, \ldots, \mathrm{p}_n$$

Ray scatter path

- *P* is on the camera plane and p_n is on a light source

$$P(\bar{\mathrm{p}}_n) = \underbrace{\int_A \int_A \cdots \int_A}_{n-1} L_{\mathrm{e}}(\mathrm{p}_n \to \mathrm{p}_{n-1})$$

$$\times \left(\prod_{i=1}^{n-1} f(\mathrm{p}_{i+1} \to \mathrm{p}_i \to \mathrm{p}_{i-1}) G(\mathrm{p}_{i+1} \leftrightarrow \mathrm{p}_i) \right) \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)$$

Light transport equation

- **Integral over paths**
  - Throughput of the path
    - The product of a path's BSDF and geometry terms

$$T(\bar{\mathrm{p}}_n) = \prod_{i=1}^{n-1} f(\mathrm{p}_{i+1} \to \mathrm{p}_i \to \mathrm{p}_{i-1}) G(\mathrm{p}_{i+1} \leftrightarrow \mathrm{p}_i)$$

$$\Downarrow$$

$$P(\bar{\mathrm{p}}_n) = \underbrace{\int_A \int_A \cdots \int_A}_{n-1} L_\mathrm{e}(\mathrm{p}_n \to \mathrm{p}_{n-1}) T(\bar{\mathrm{p}}_n)\, \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)$$

Light transport equation

- **Delta distributions in the integrand**
  - The integrand will generally be integrated out
  - Reduce dimensionality

- **For example**
  - A point light source

$$P(\bar{\mathrm{p}}_2) = \int_A L_\mathrm{e}(\mathrm{p}_2 \to \mathrm{p}_1)\, f(\mathrm{p}_2 \to \mathrm{p}_1 \to \mathrm{p}_0)\, G(\mathrm{p}_2 \leftrightarrow \mathrm{p}_1)\, \mathrm{d}A(\mathrm{p}_2)$$

$$= \frac{\delta(\mathrm{p}_\mathrm{light} - \mathrm{p}_2) L_\mathrm{e}(\mathrm{p}_\mathrm{light} \to \mathrm{p}_1)}{p(\mathrm{p}_\mathrm{light})} f(\mathrm{p}_2 \to \mathrm{p}_1 \to \mathrm{p}_0) G(\mathrm{p}_2 \leftrightarrow \mathrm{p}_1)$$

Light transport equation

- **Partitioning the integrand**
  - We can decompose the path integral into three components

$$L(\mathrm{p}_1 \to \mathrm{p}_0) = P(\bar{\mathrm{p}}_1) + P(\bar{\mathrm{p}}_2) + \sum_{i=3}^{\infty} P(\bar{\mathrm{p}}_i)$$

- First term
  - Emitted radiance at p_1
- Second term
  - Solve with an accurate direct lighting solution
- Third term (indirect lighting)
  - Solve with faster but less accurate approach

Light transport equation

- **Partitioning the integrand**
  - For each term
    - Partition the light sources: small area light sources and large area light sources (sampled differently)

$$
\begin{aligned}
P(\bar{\mathrm{p}}_n) &= \int_{A^{n-1}} (L_{\mathrm{e,s}}(\mathrm{p}_n \to \mathrm{p}_{n-1}) + L_{\mathrm{e,l}}(\mathrm{p}_n \to \mathrm{p}_{n-1}))\, T(\bar{\mathrm{p}}_n)\, \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n) \\
&= \boxed{\int_{A^n} L_{\mathrm{e,s}}(\mathrm{p}_n \to \mathrm{p}_{n-1})\, T(\bar{\mathrm{p}}_n)\, \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)} \\
&\quad + \boxed{\int_{A^n} L_{\mathrm{e,l}}(\mathrm{p}_n \to \mathrm{p}_{n-1})\, T(\bar{\mathrm{p}}_n)\, \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)}.
\end{aligned}
$$

    - Partition the BSDF: delta and non-delta distribution

$$
\begin{aligned}
P(\bar{\mathrm{p}}_n) = \int_{A^{n-1}} & L_{\mathrm{e}}(\mathrm{p}_n \to \mathrm{p}_{n-1}) \\
& \times \prod_{i=1}^{n-1} \left(\boxed{f_{\Delta}(\mathrm{p}_{i+1} \to \mathrm{p}_i \to \mathrm{p}_{i-1})} + \boxed{f_{\neg\Delta}(\mathrm{p}_{i+1} \to \mathrm{p}_i \to \mathrm{p}_{i-1})} \right) \\
& \times G(\mathrm{p}_{i+1} \leftrightarrow \mathrm{p}_i) \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)
\end{aligned}
$$

3. Path tracing for global lighting

Path tracing

- **Path tracing**
  - The first general-purpose unbiased Monte-Carlo light transport algorithm (by Kajiya 1986)
  - Incrementally generate paths of scattering
    - Starting from the camera
    - Ending at light sources

- **Overview**
  - Starting from the path integral form of LTE

$$L(\mathrm{p}_1 \to \mathrm{p}_0) = \sum_{i=1}^{\infty} P(\bar{\mathrm{p}}_i)$$

Path tracing

- **Two problems to solve**
  - How to turn infinite sum to finite sum?
  - Given a particular term, how to generate one or more paths to compute integral?

- **Physical fact**
  - Paths with more vertices scatter less light (conservation of energy)
  - We will always estimate the first few terms, then start to apply Russian roulette
  - Stop sampling after a finite number of terms

Path tracing

- **Computing with Russian roulette sampling**
  - Three term estimates as

$$P(\bar{\mathrm{p}}_1) + P(\bar{\mathrm{p}}_2) + P(\bar{\mathrm{p}}_3) + \frac{1}{1-q}\sum_{i=4}^{\infty} P(\bar{\mathrm{p}}_i)$$

  - Recursively apply Russian roulette sampling

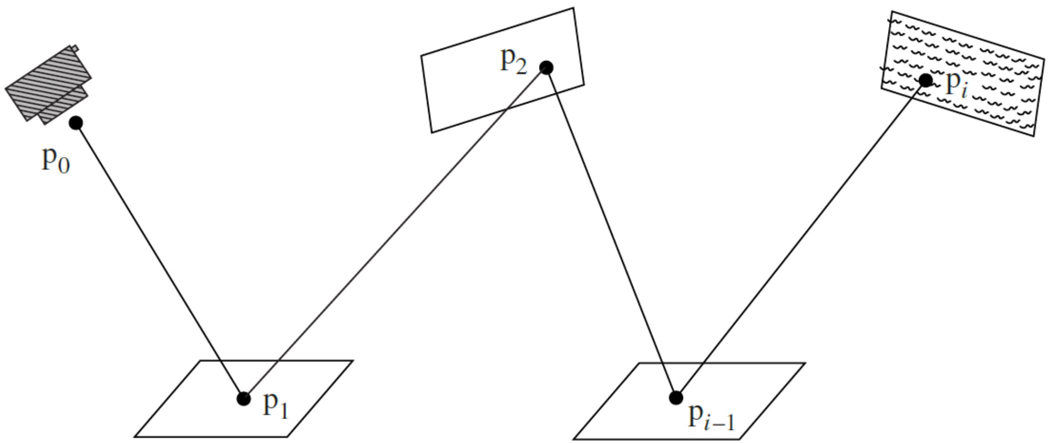
$$\frac{1}{1-q_1}\left(P(\bar{\mathrm{p}}_1) + \frac{1}{1-q_2}\left(P(\bar{\mathrm{p}}_2) + \frac{1}{1-q_3}\left(P(\bar{\mathrm{p}}_3) + \cdots\right.\right.\right.$$

Path tracing

- **Path sampling**
  - How to evaluate each P term?
    - Look again at the form of P term

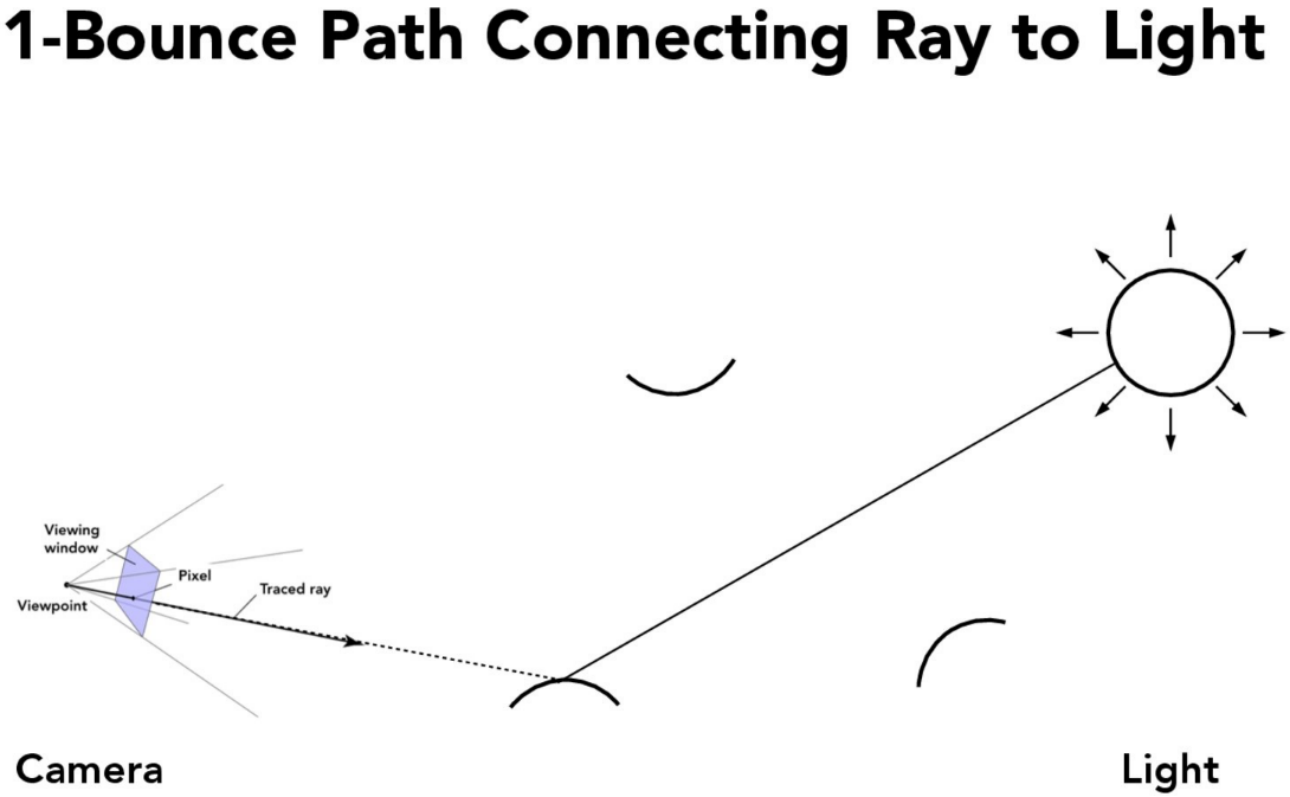
$$P(\bar{\mathrm{p}}_n) = \underbrace{\int_A \int_A \cdots \int_A}_{n-1} L_e(\mathrm{p}_n \to \mathrm{p}_{n-1})$$
$$\times \left(\prod_{i=1}^{n-1} f(\mathrm{p}_{i+1} \to \mathrm{p}_i \to \mathrm{p}_{i-1}) G(\mathrm{p}_{i+1} \leftrightarrow \mathrm{p}_i) \right) \mathrm{d}A(\mathrm{p}_2) \cdots \mathrm{d}A(\mathrm{p}_n)$$

    - Sample surface areas



Path tracing

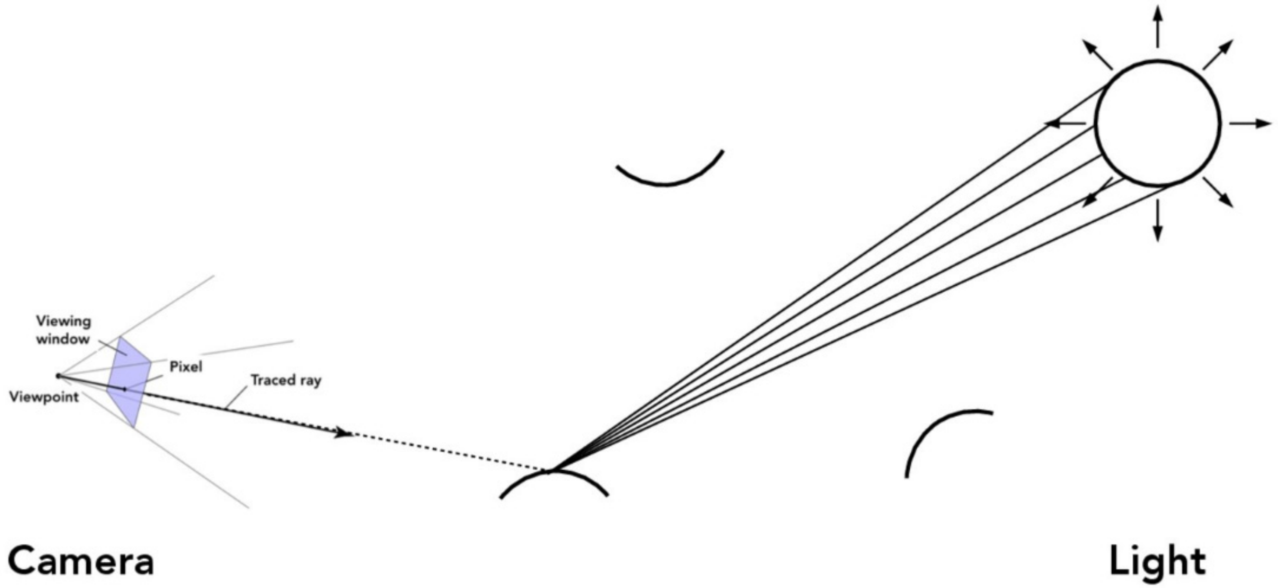
- **Implementation**
  - Light path



Path tracing

- **Implementation**
  - Light path

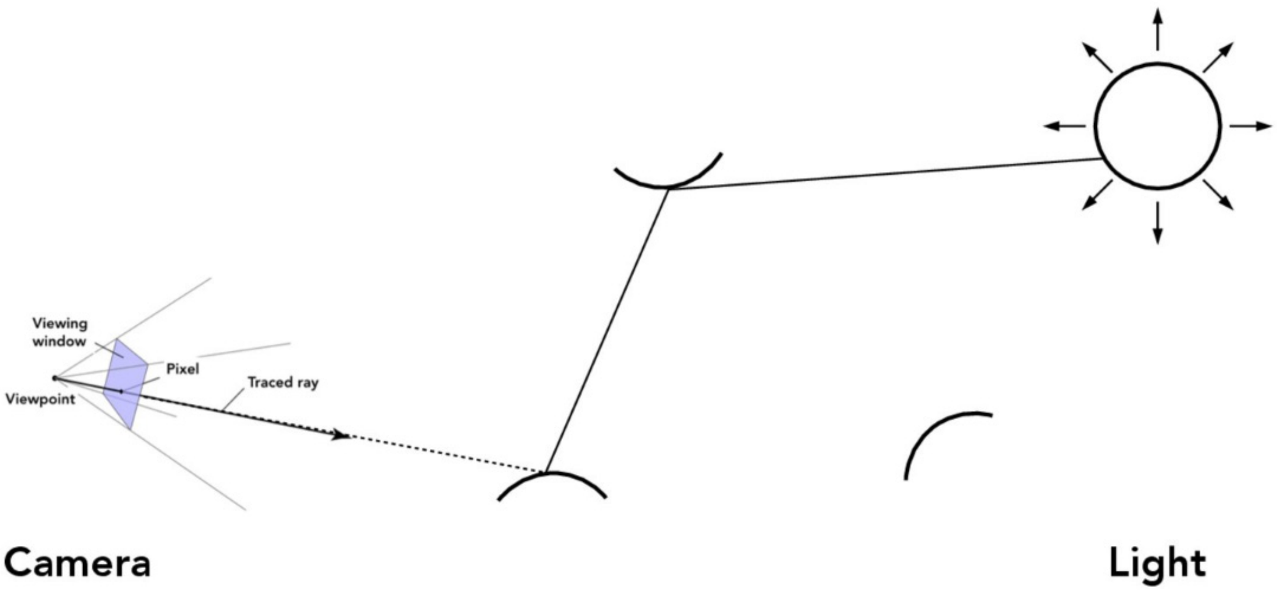
1-Bounce Paths Connecting Ray to Light



Path tracing

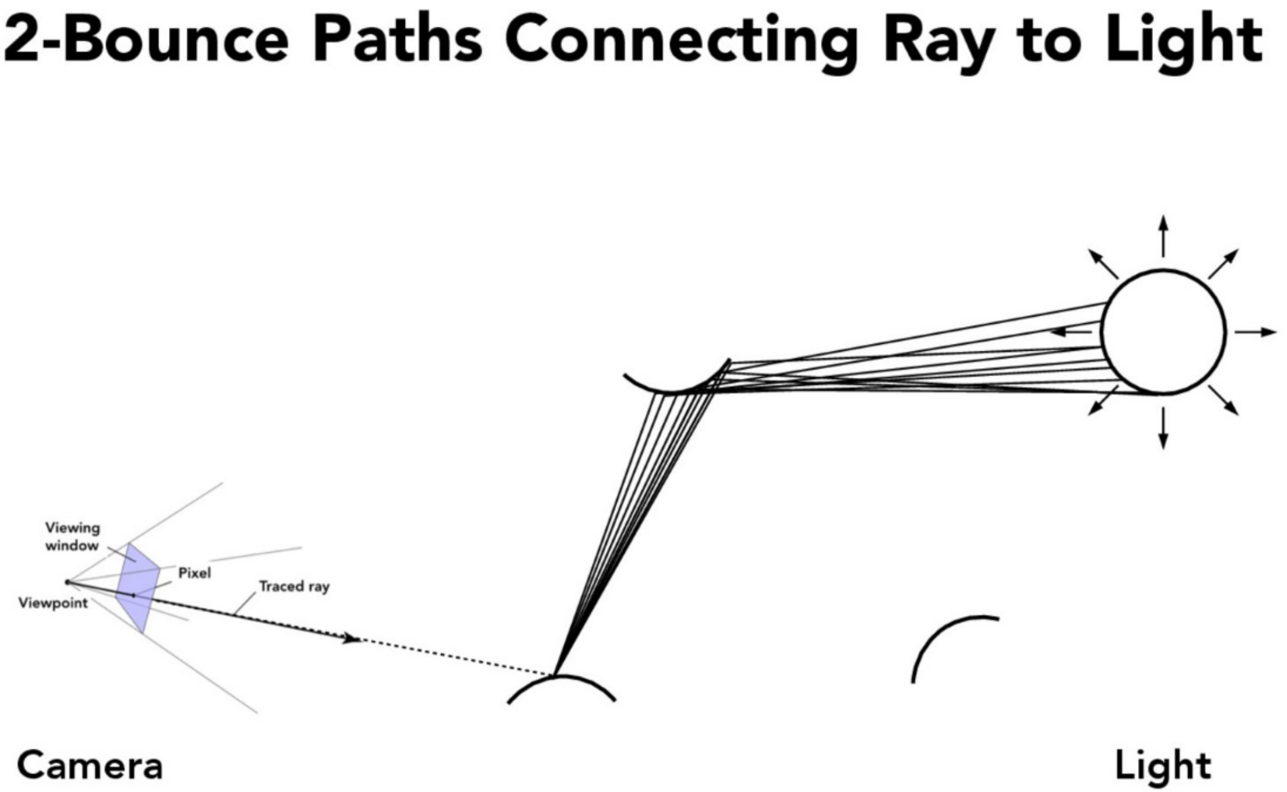
- **Implementation**
  - Light path

2-Bounce Path Connecting Ray to Light



Path tracing

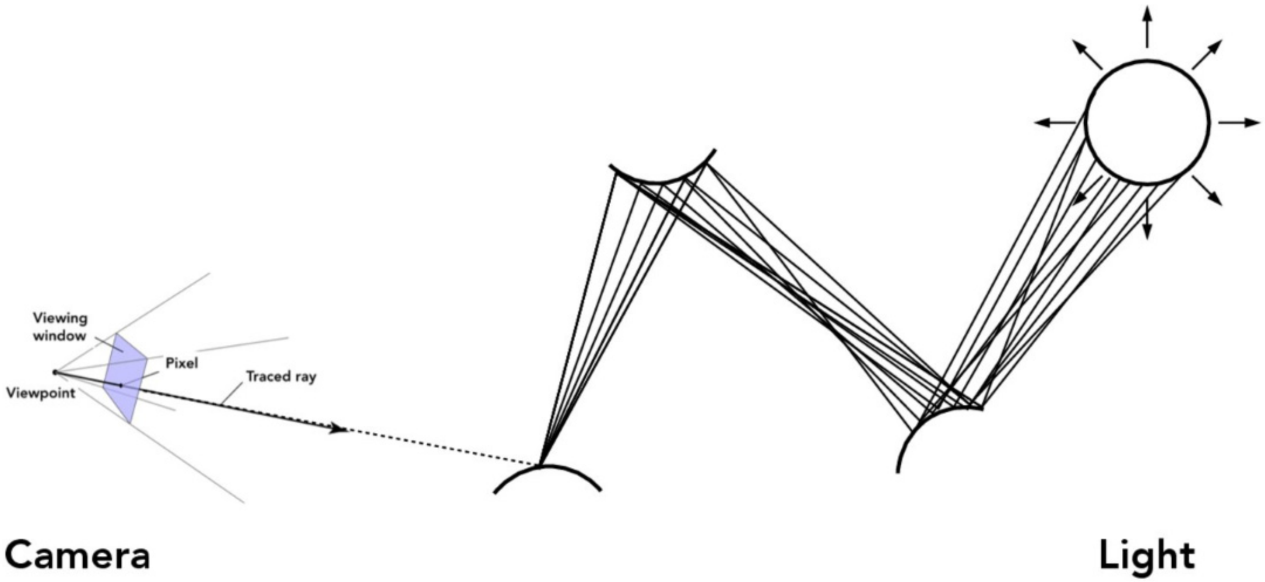
- **Implementation**
  - Light path



Path tracing

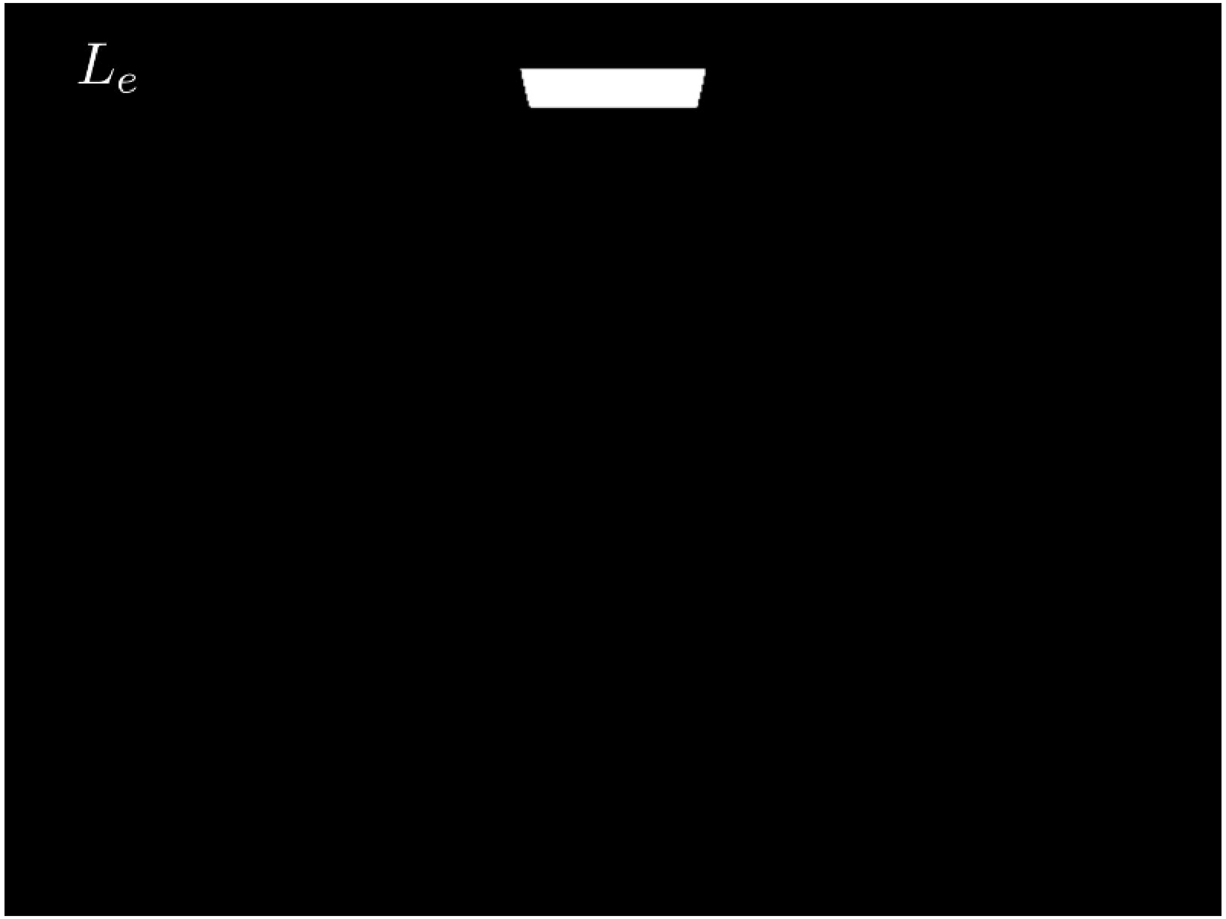
- **Implementation**
  - Light path

3-Bounce Paths Connecting Ray to Light



Path tracing

- **Path integral of LTE (K=P)**



Path tracing

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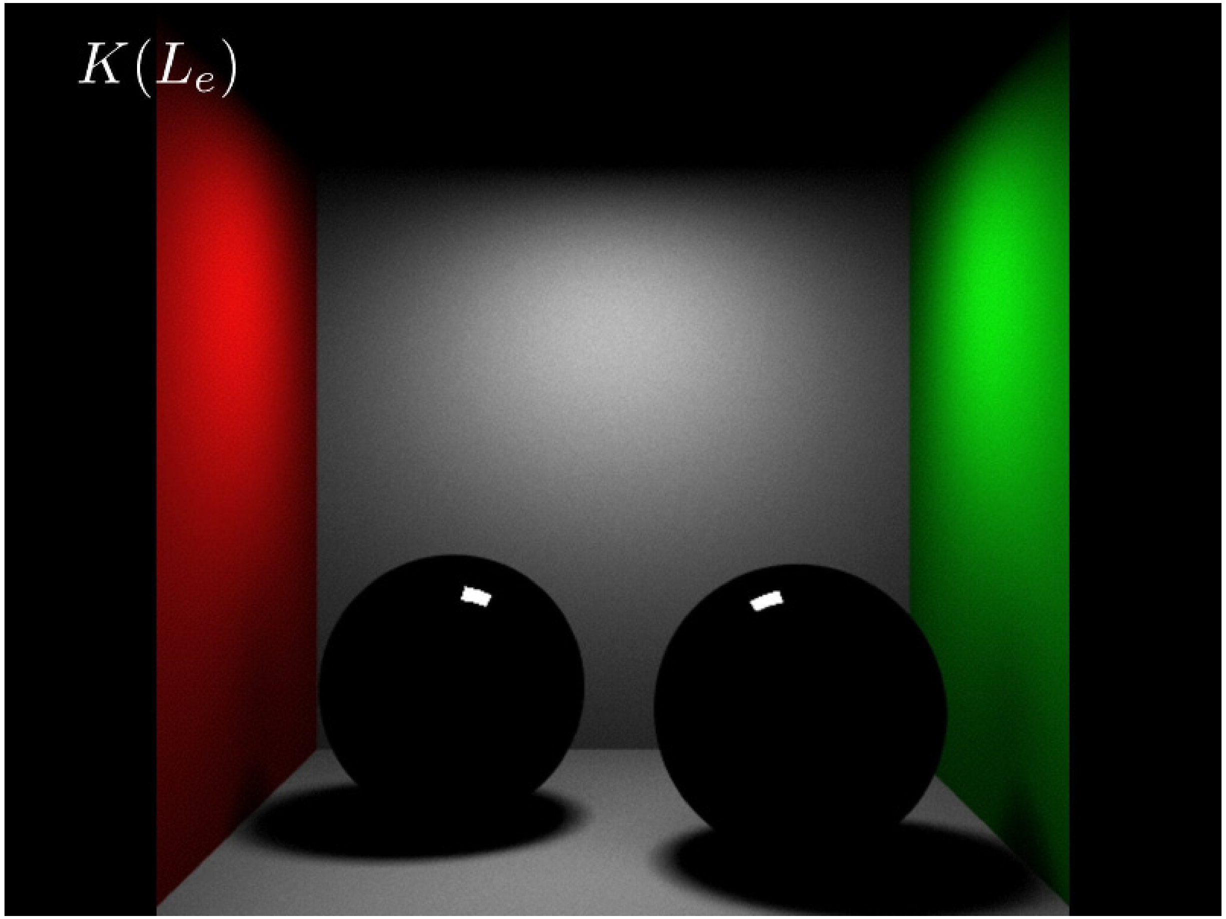
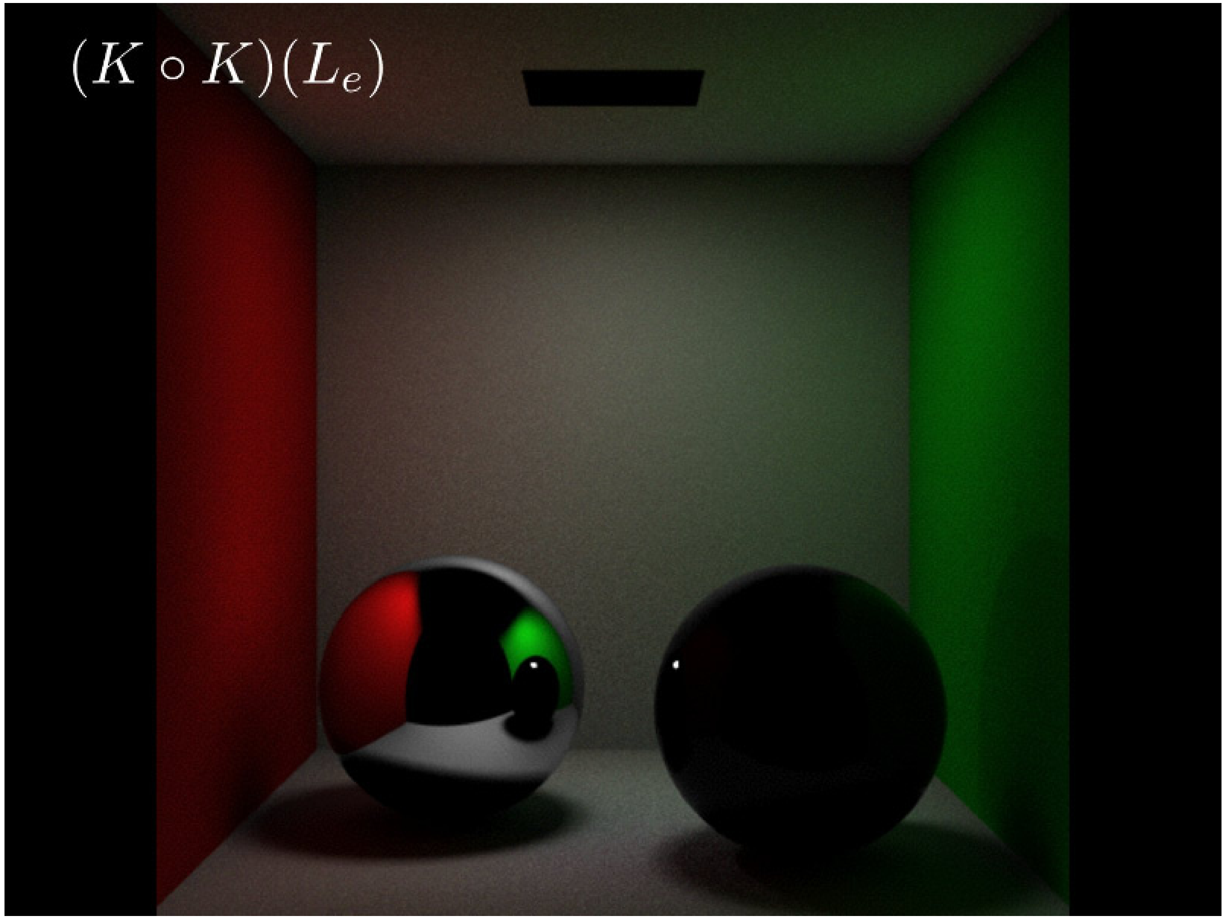


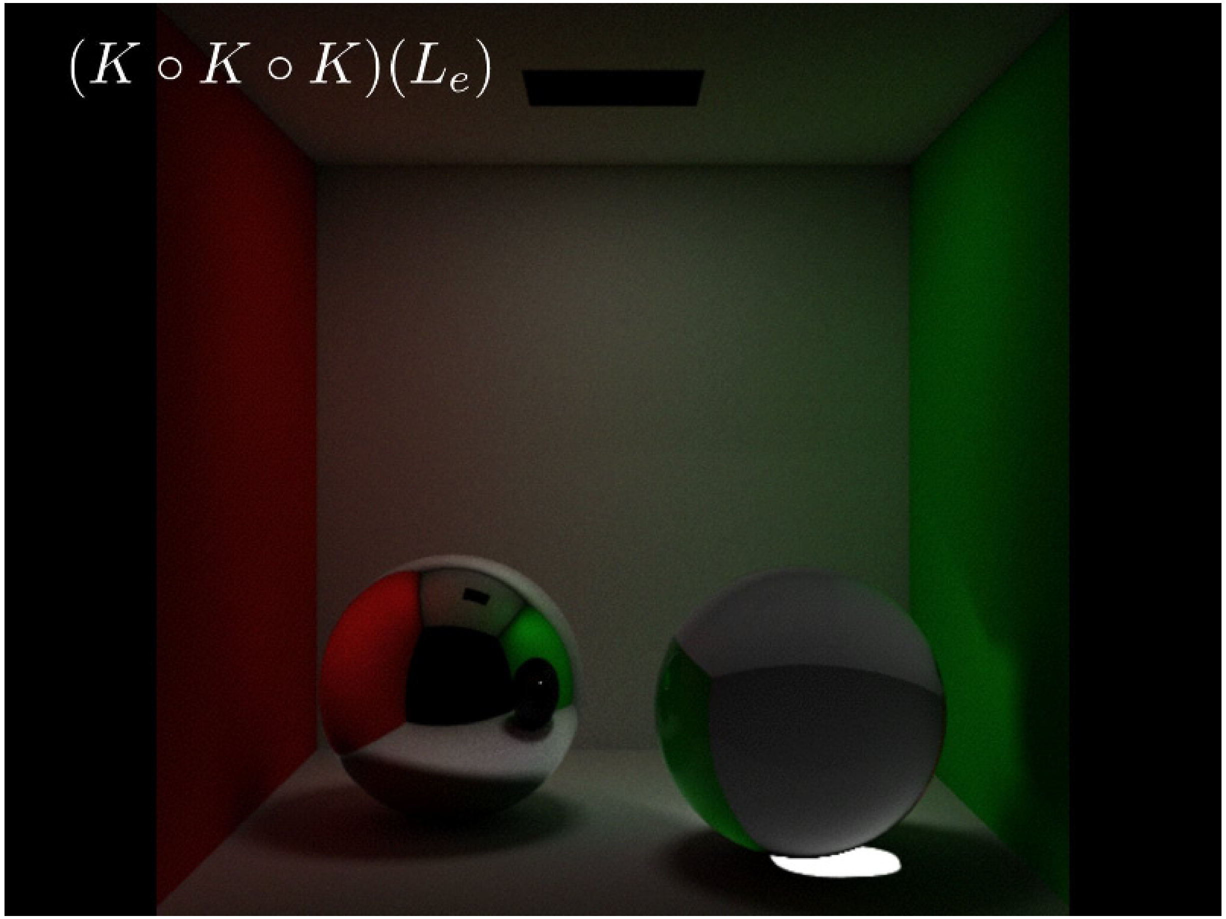
Path tracing

- **Path integral of LTE (K=P)**



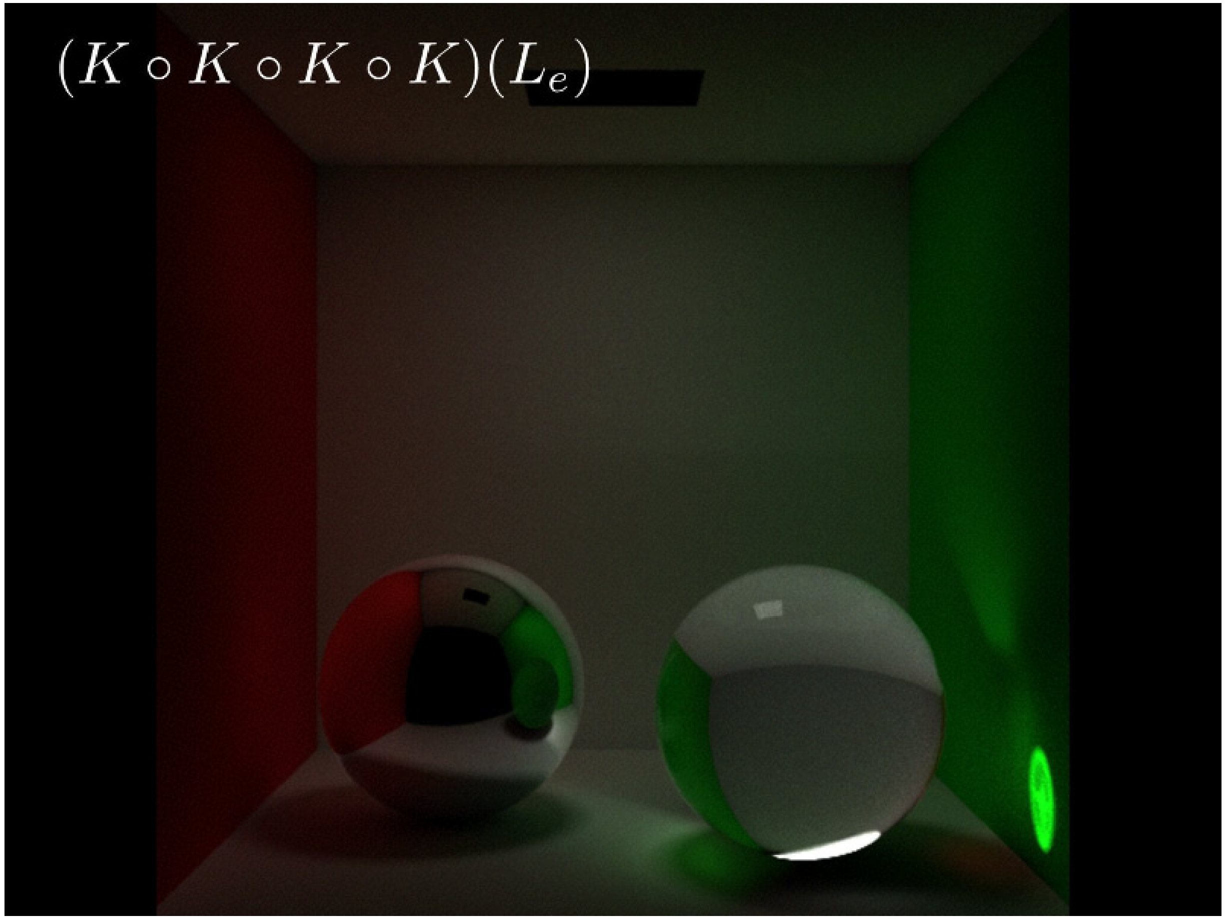
Path tracing

- **Path integral of LTE (K=P)**



Path tracing

- **Path integral of LTE (K=P)**



Path tracing

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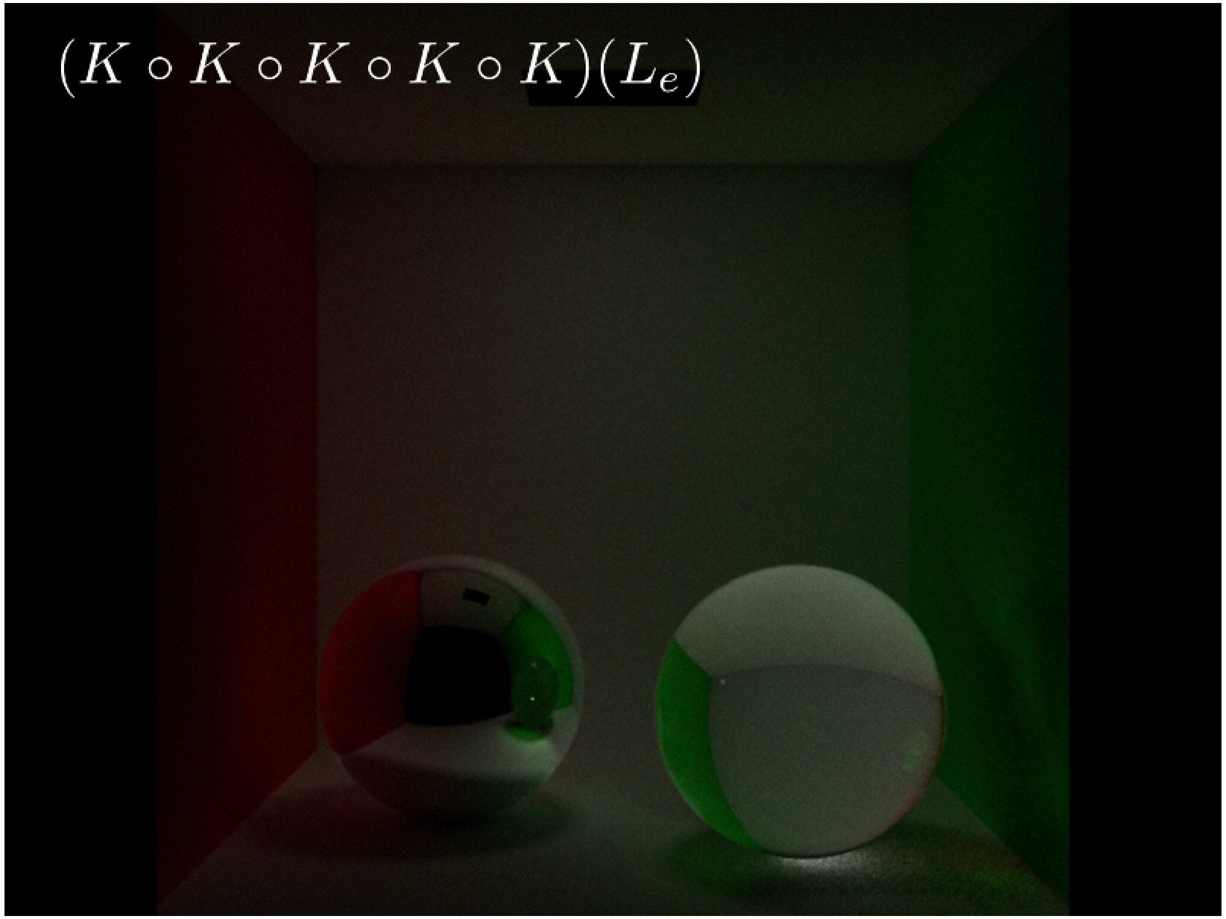
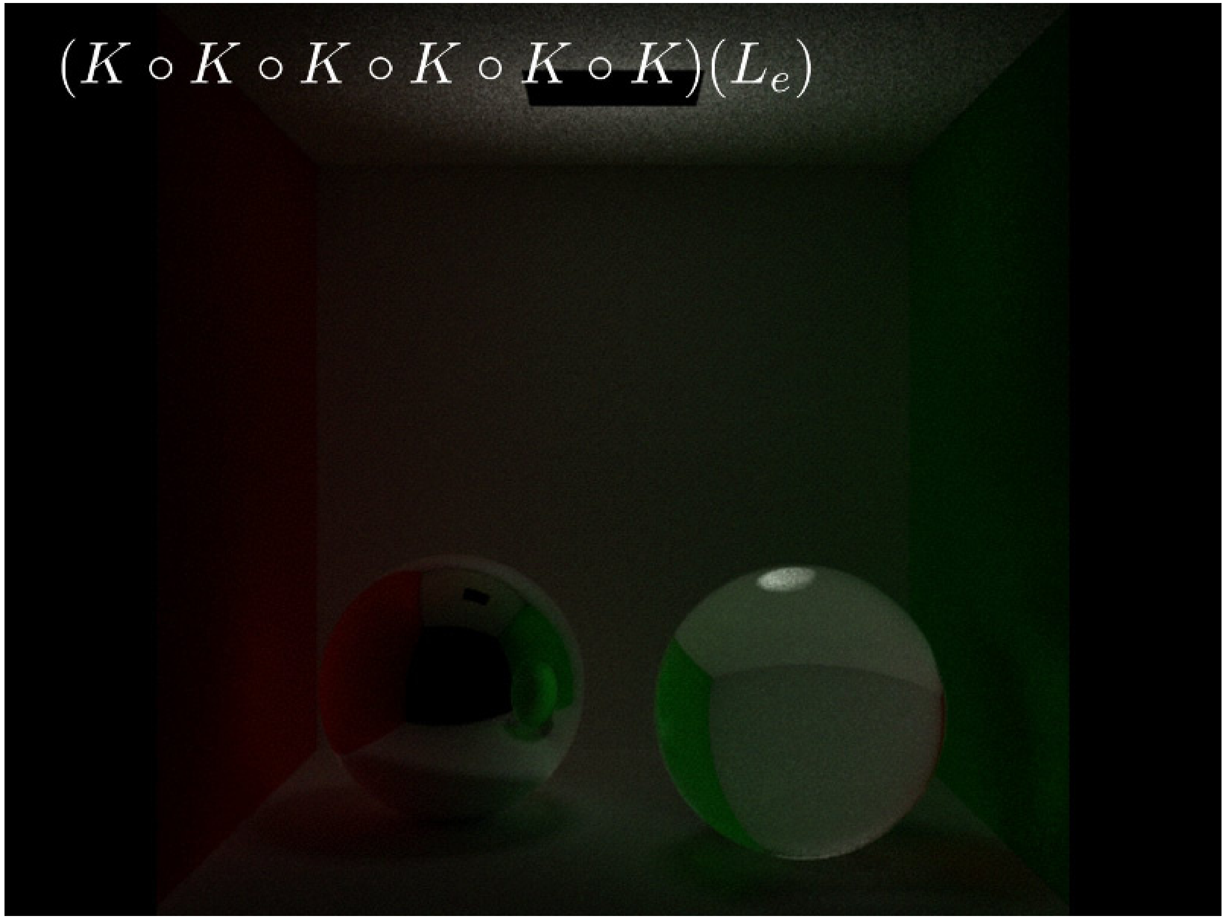


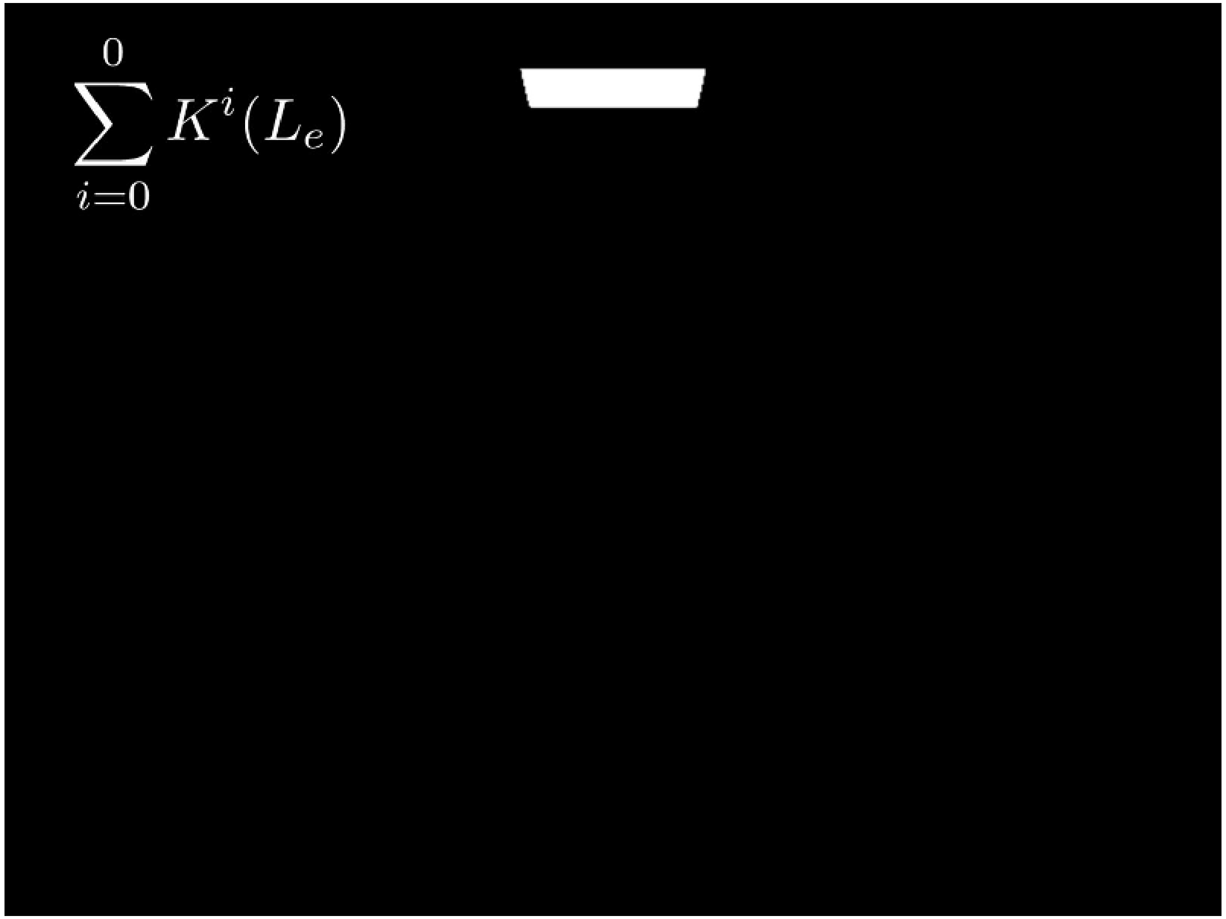
Path tracing

- **Path integral of LTE (K=P)**



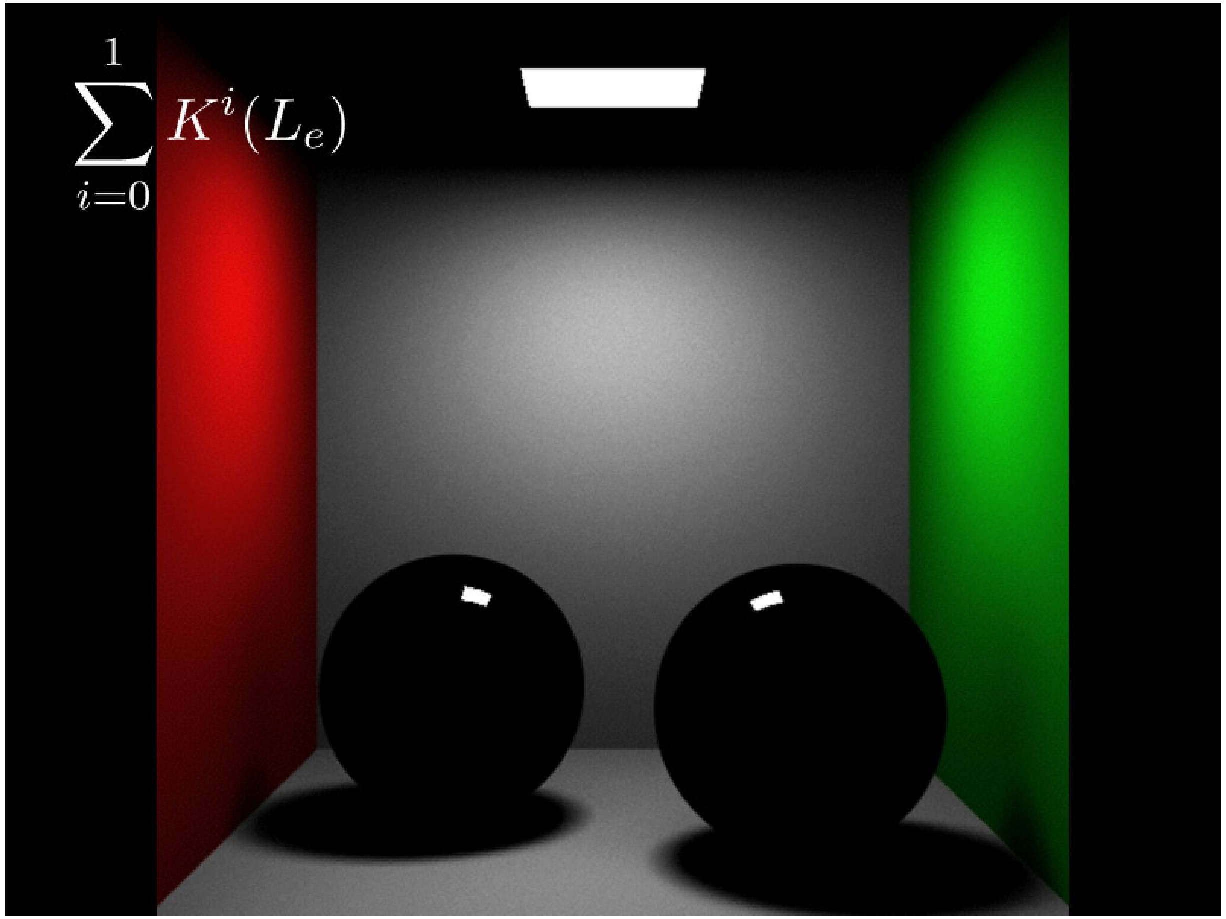
Path tracing

- **Path integral of LTE (K=P)**



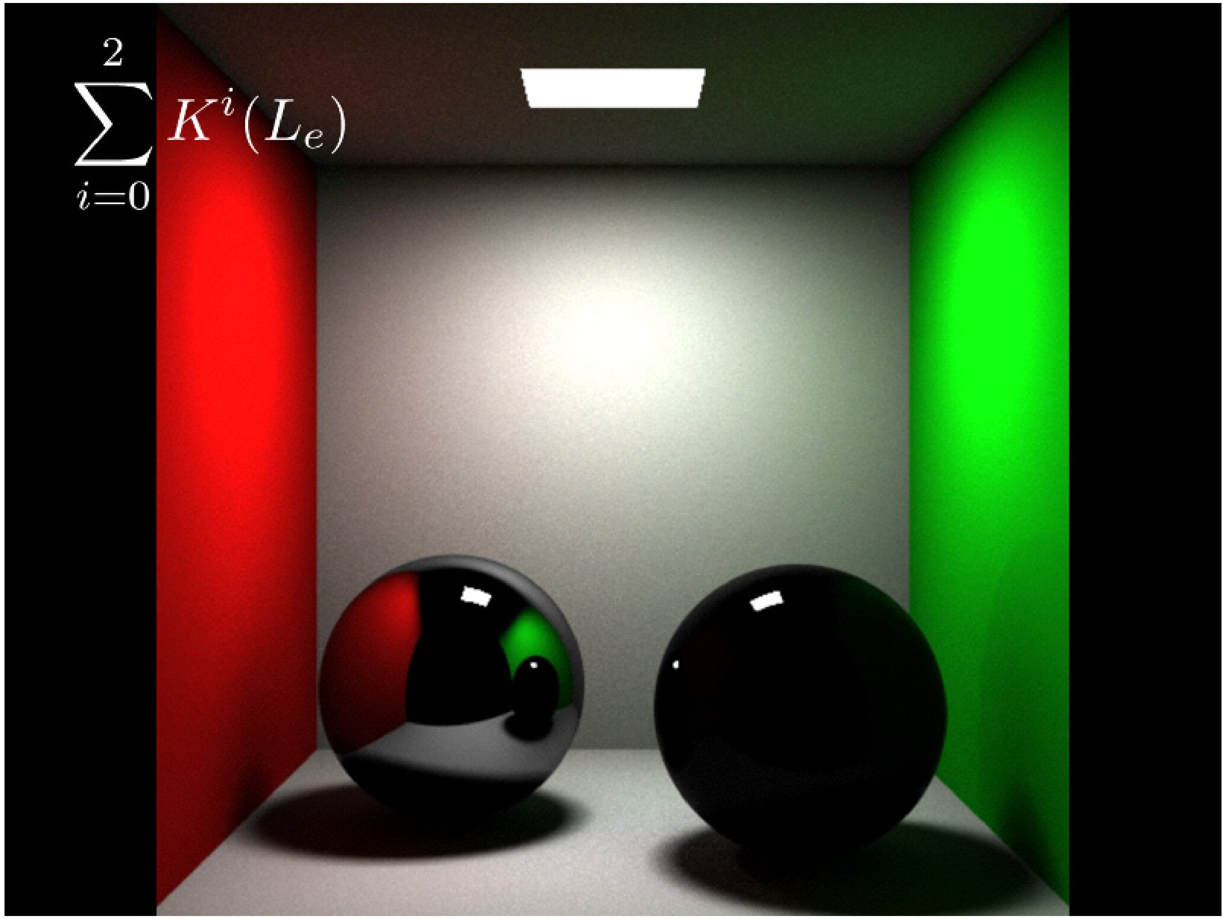
Path tracing

- **Path integral of LTE (K=P)**



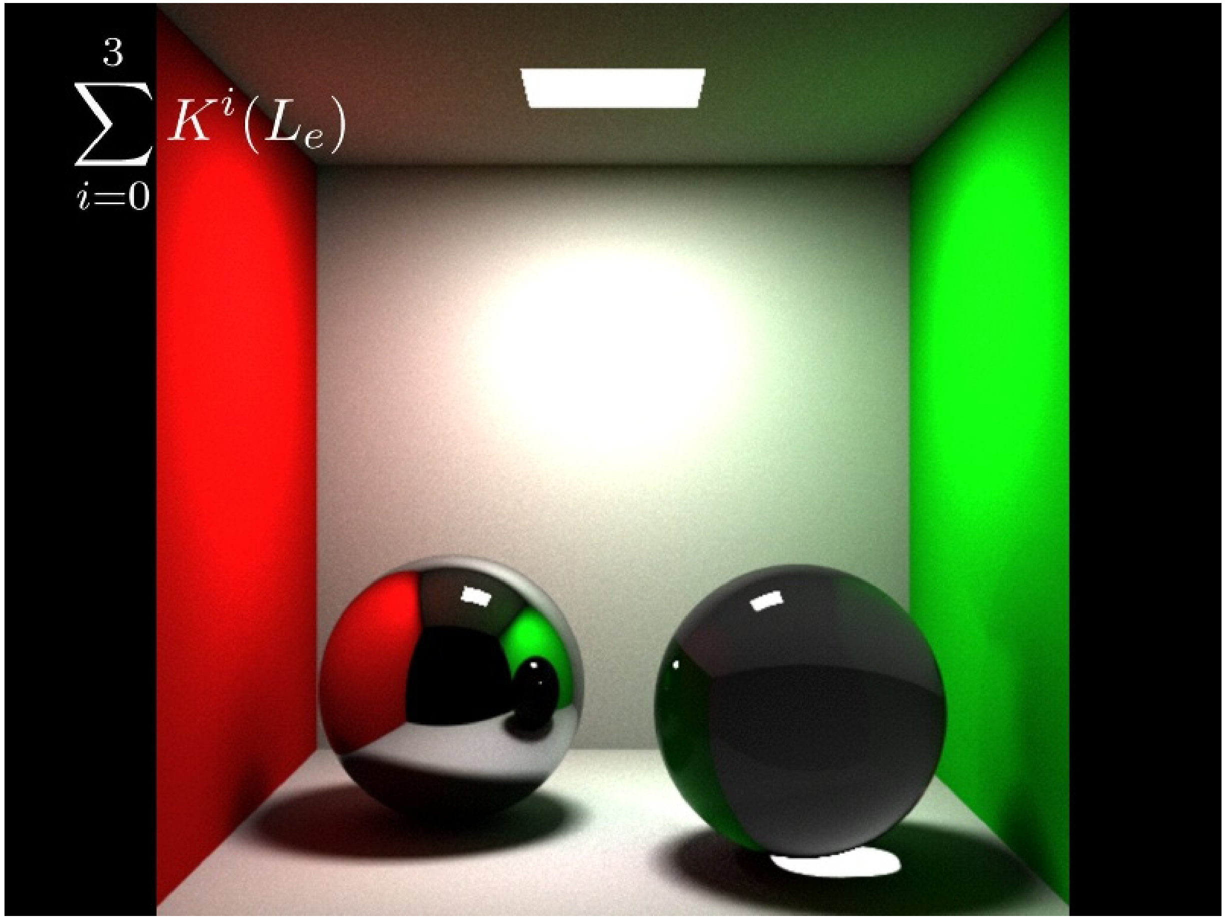
Path tracing

- **Path integral of LTE (K=P)**



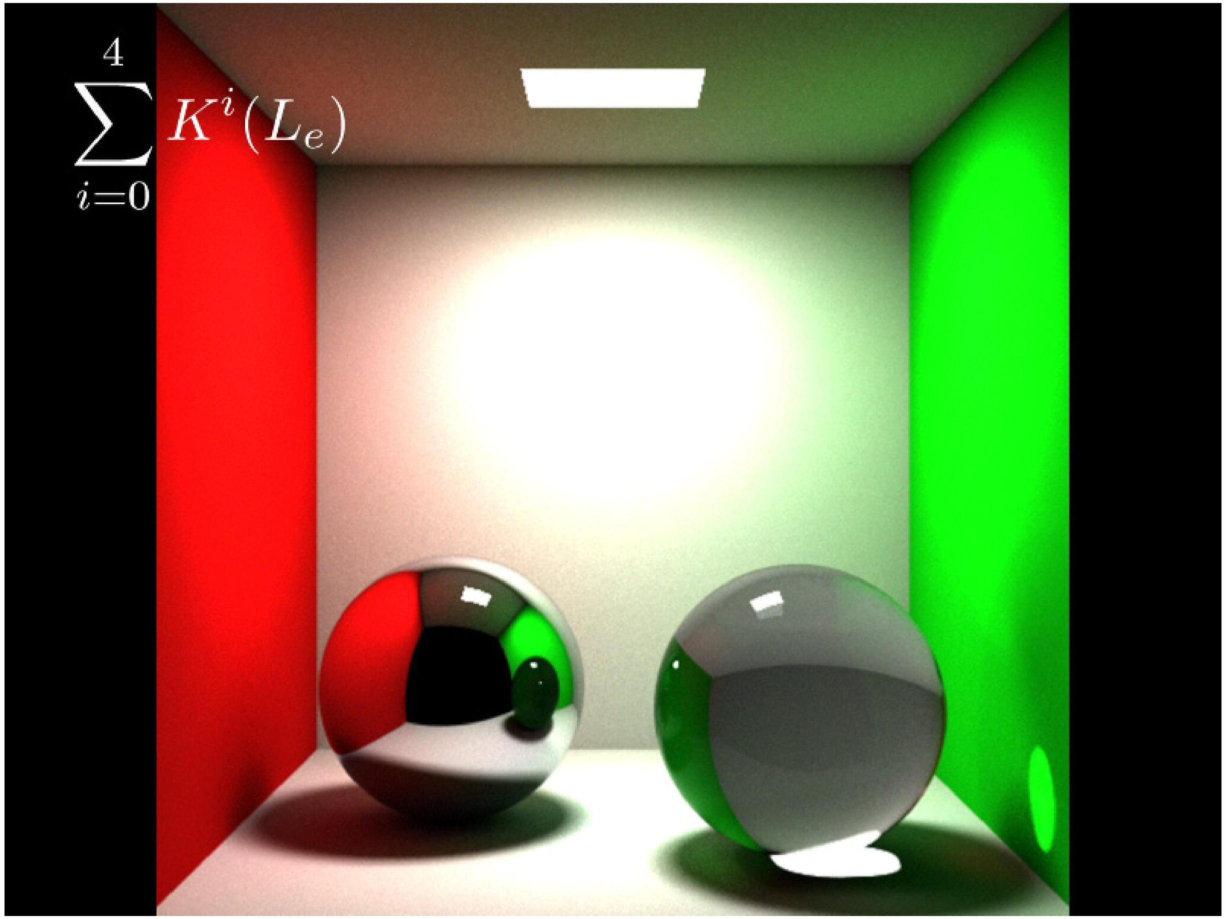
Path tracing

- **Path integral of LTE (K=P)**



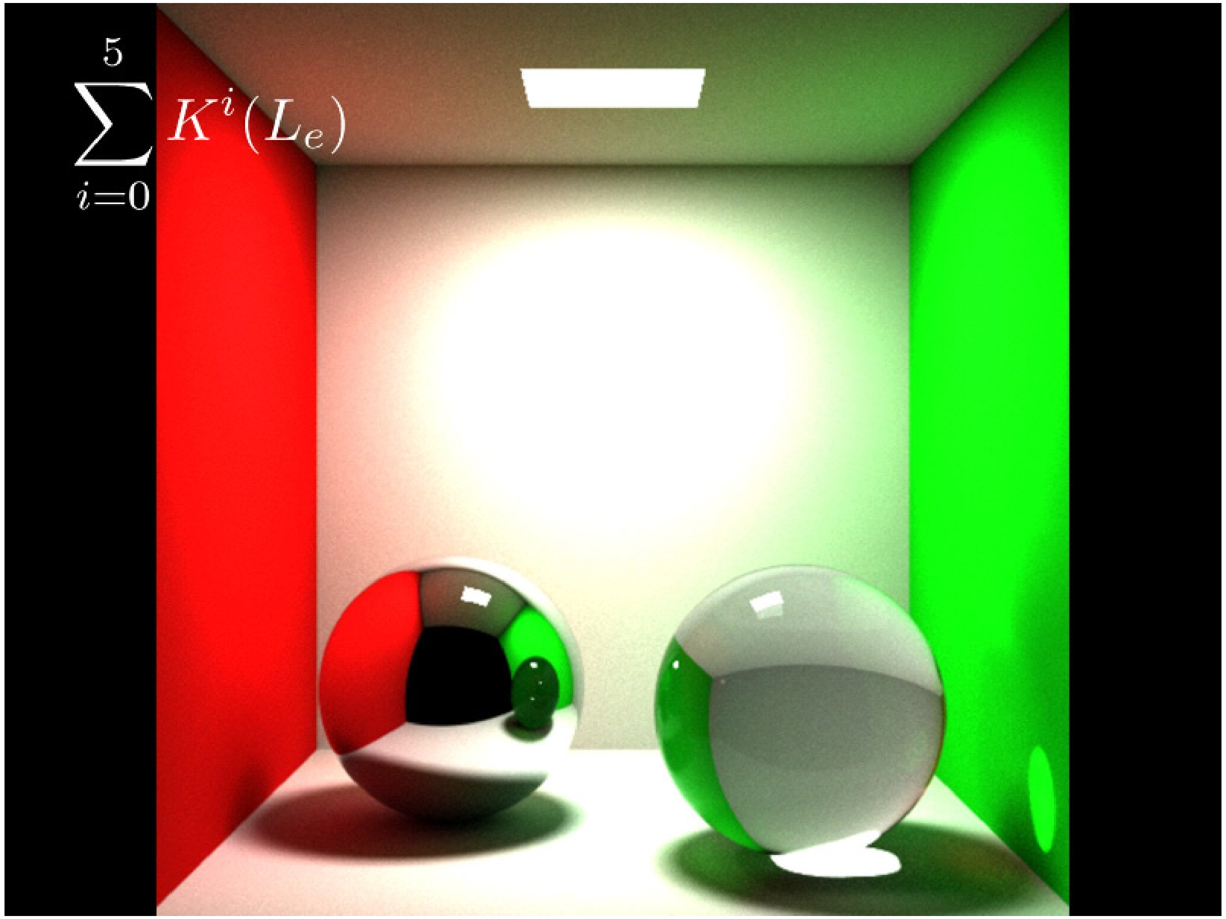
Path tracing

- **Path integral of LTE (K=P)**



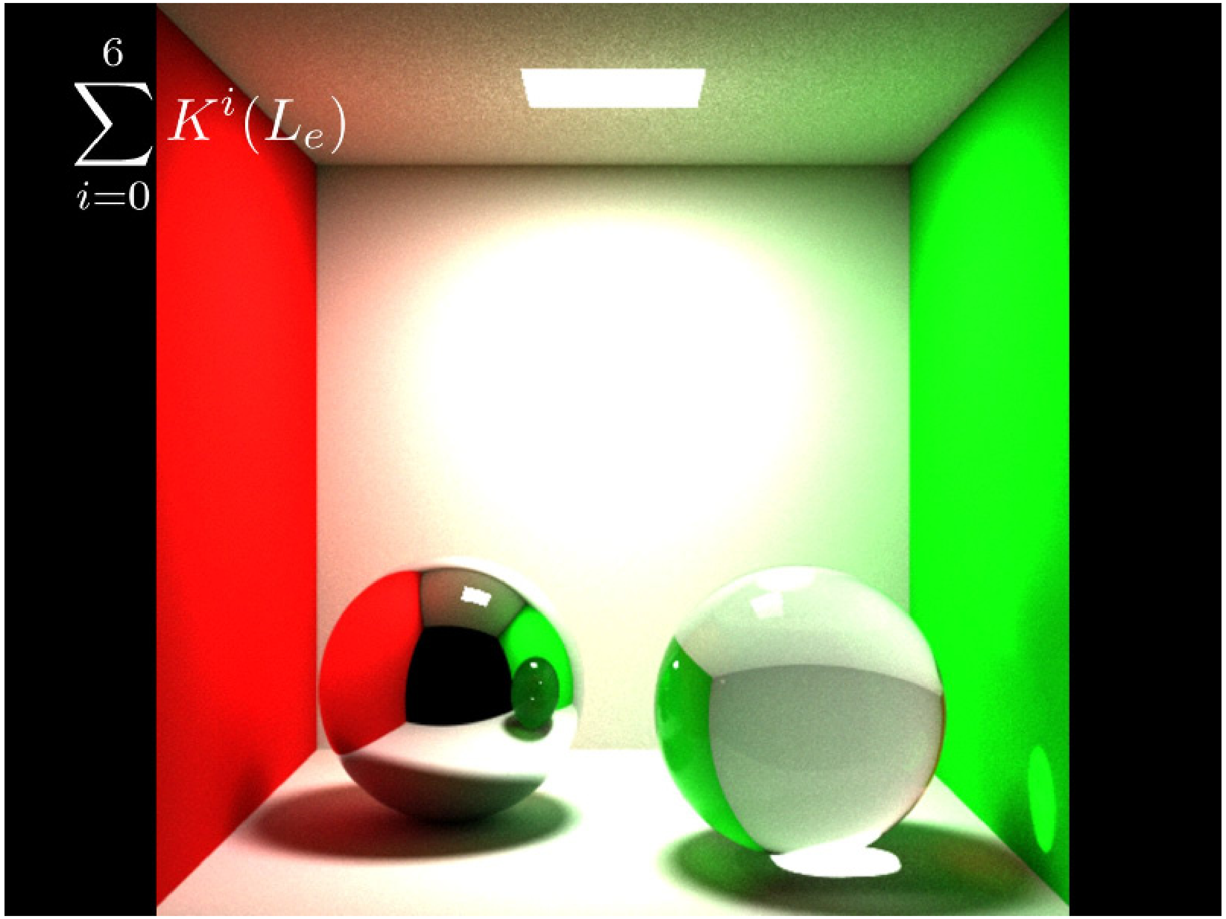
Path tracing

- **Path integral of LTE (K=P)**



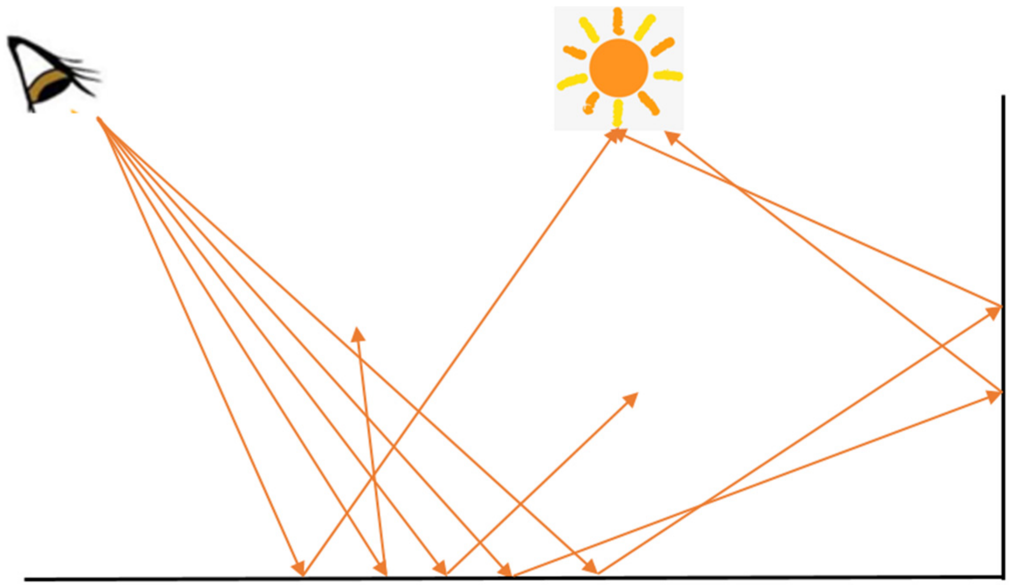
Path tracing

- **Path integral of LTE (K=P)**



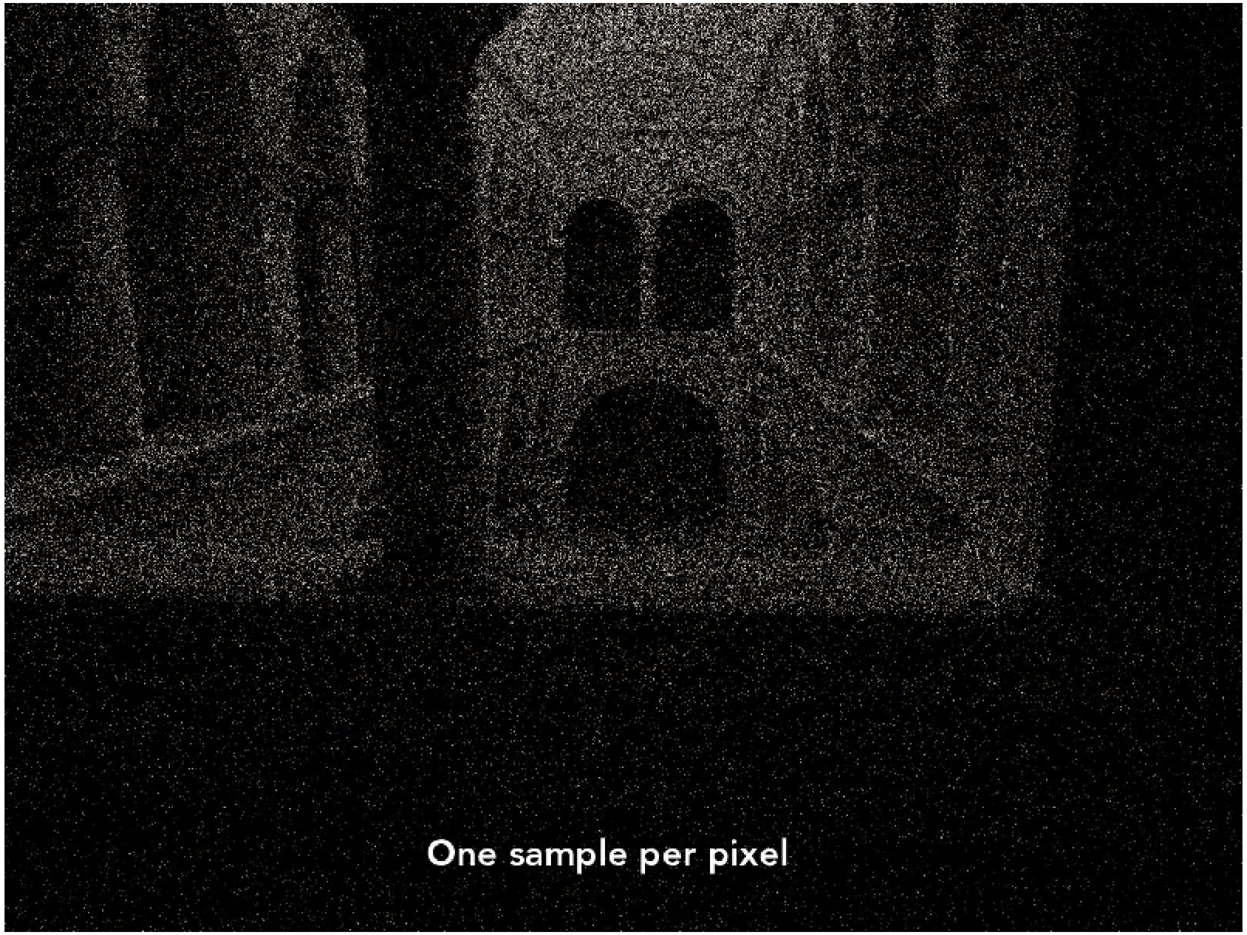
Path tracing

- **Approximation**
  - Instead of shooting multiple rays per intersection, we shoot only one ray
  - Instead of shooting only a few rays per pixel, we should large amount of rays per pixel



Path tracing

- **Different sample per pixel ray**



Path tracing

- **Different sample per pixel ray**

Path tracing

- **Different sample per pixel ray**

Path tracing

- **Different path length (bounce number)**

Path tracing

- **Different path length (bounce number)**

Path tracing

- **Different path length (bounce number)**

Path tracing

- **Different path length (bounce number)**

Path tracing

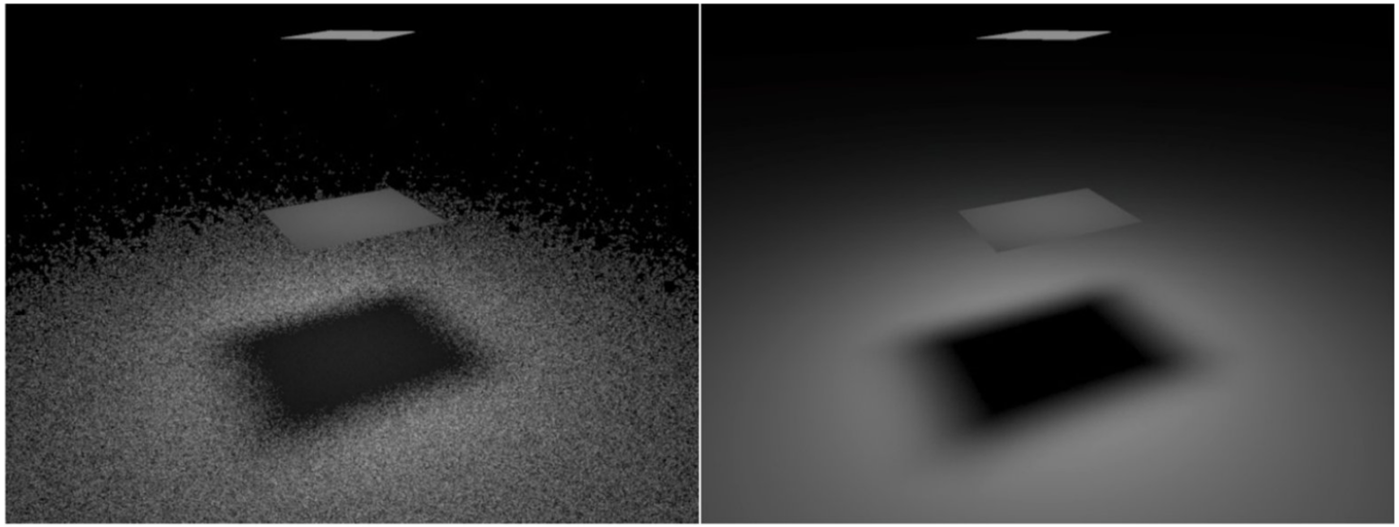
- **Different path length (bounce number)**

Path tracing

- **Different path length (bounce number)**

Path tracing

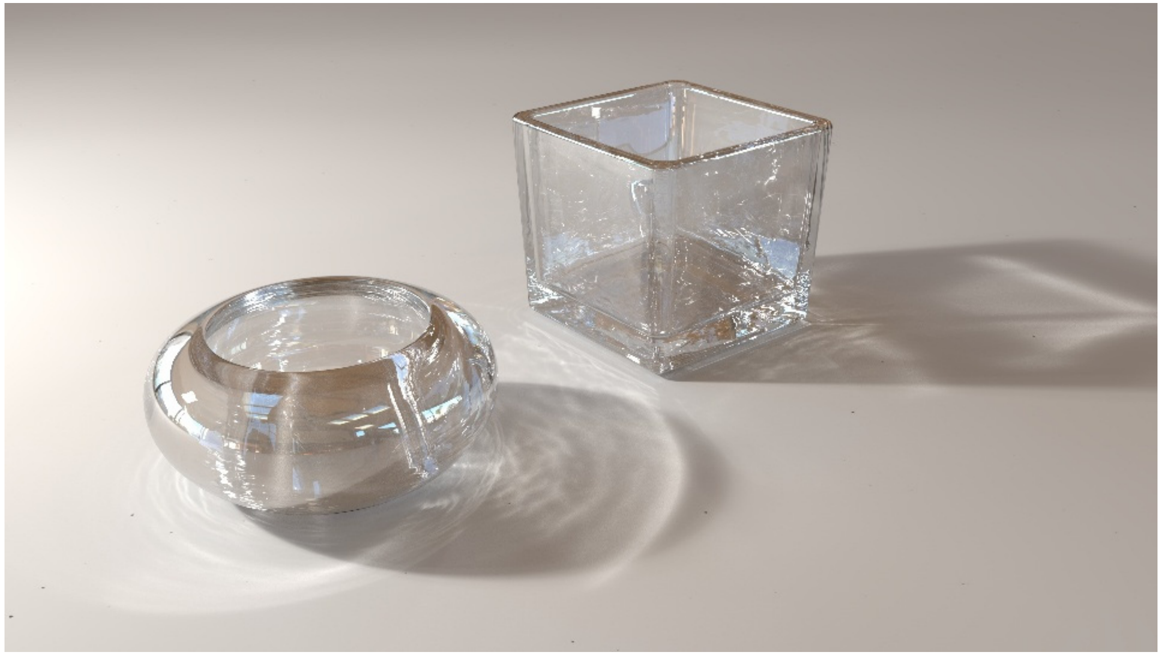
- **Solid angle v.s. light area sampling**



Solid angle sampling | Light area sampling

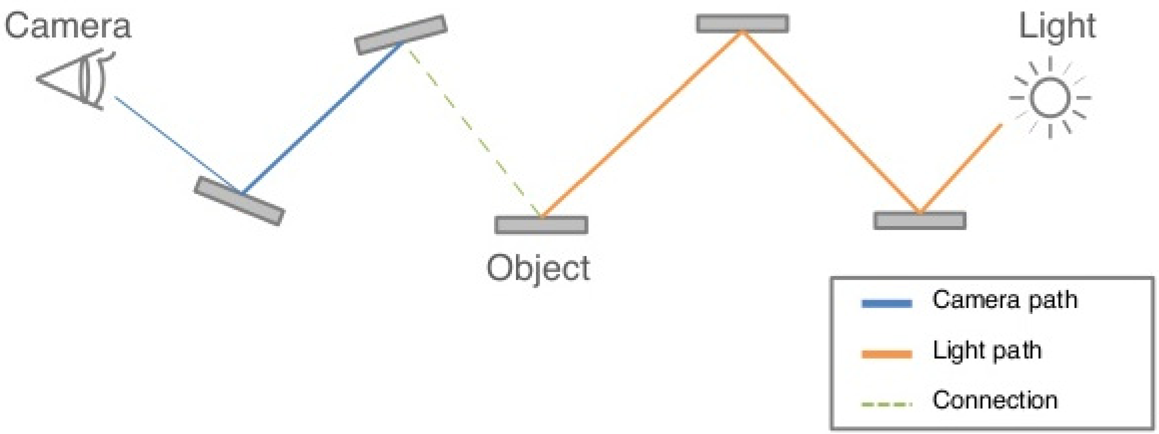
Bidirectional path tracing

- **Problem for path tracing**
  - Exhibit high variance for particular lighting conditions
  - For light with limited area but with strong intensity
  - For example, caustics



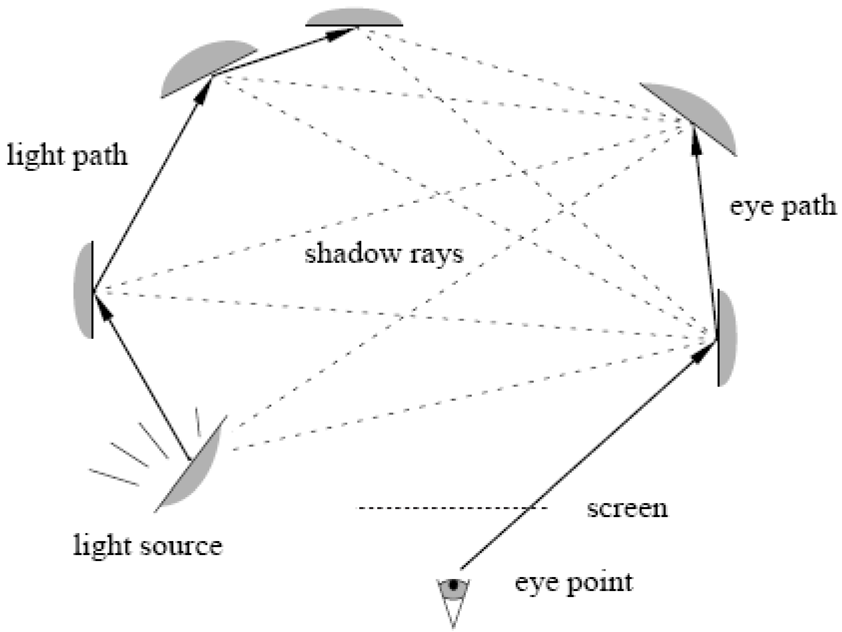
Bidirectional path tracing

- **Basic principle**
  - Constructing paths
    - From both camera and light sources
    - Two paths are connected in the middle with visibility ray



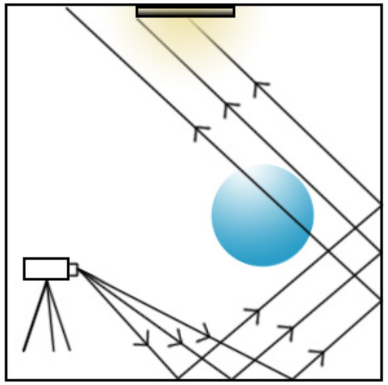
Bidirectional path tracing

- **For each vertex on camera path**
  - Check visibility for each vertex on light path
  - Render each sampled path with BSDF

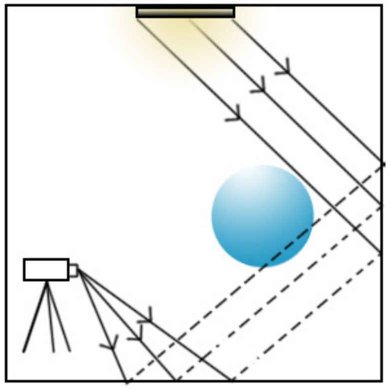


Bidirectional path tracing

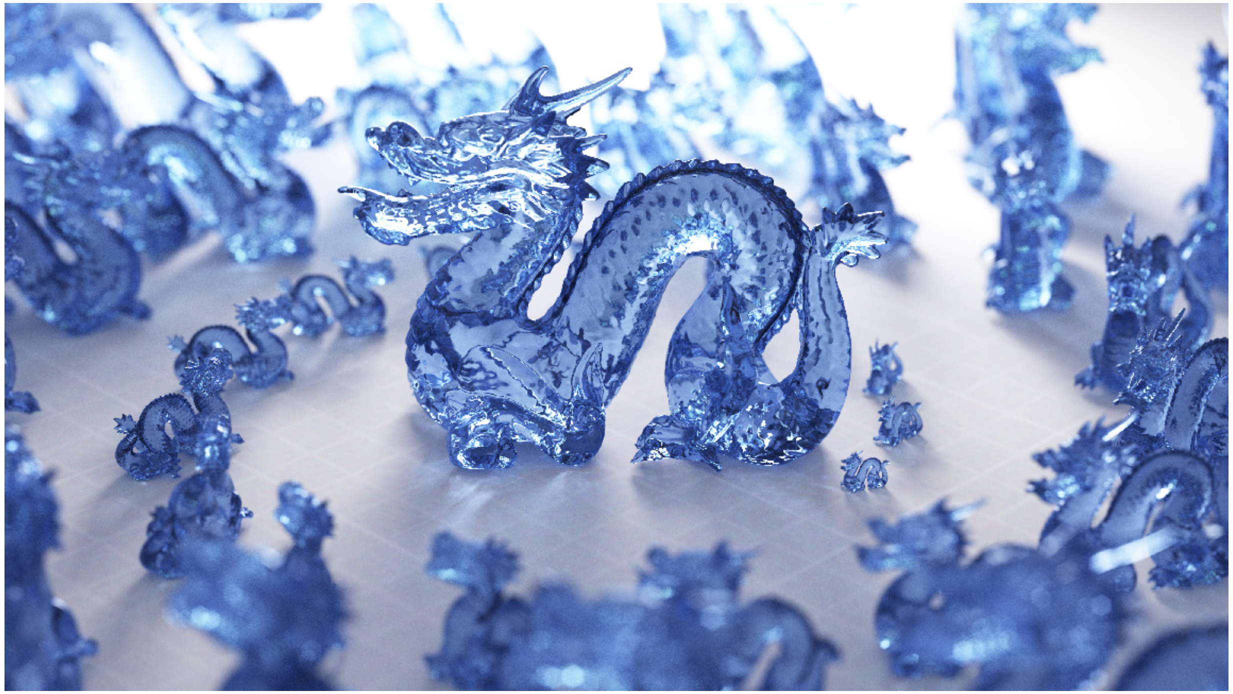
- **Rendering**



Normal Pathtracing



Bidirectional pathtracing



4. Instant global illumination

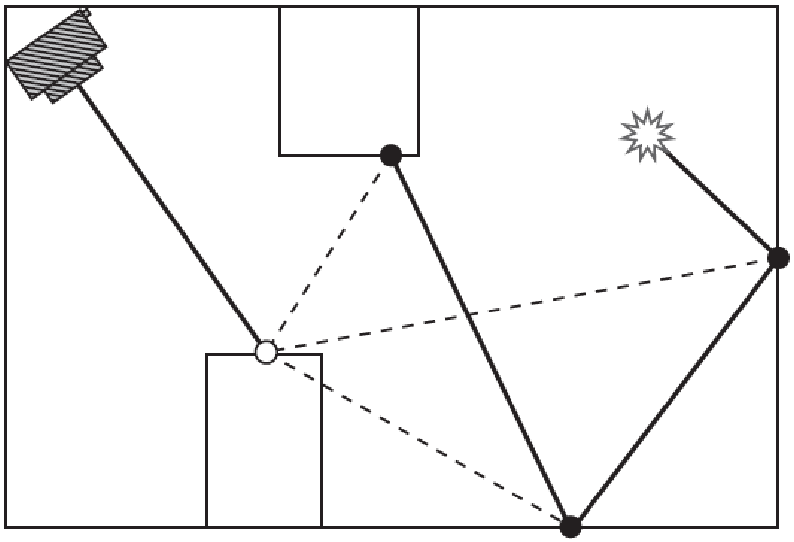
Instant global illumination

- **Problem for (bidirectional) path tracing**
  - A cost requiring tens to thousands of samples in each pixel for non-noisy image
  - Undesirable computational cost

- **Basic idea**
  - Follow a small number of light-carrying paths starting from the light source
  - Construct point light source (virtual light sources) when the paths intersect surfaces in the scene
  - These point sources approximate the indirect radiance distribution

Instant global illumination

- **Virtual light sources**
  - Point light source by light path intersection with surface
  - Indirection illumination
  - Connection to bi-directional path tracing
    - Camera path: one segment
    - Light path: multiple segments



Instant global illumination

- **Handling close point**
  - G term may become very large

$$G(\mathrm{p} \leftrightarrow \mathrm{p}') = V(\mathrm{p} \leftrightarrow \mathrm{p}') \frac{|\cos\theta|\,|\cos\theta'|}{\|\,\mathrm{p} - \mathrm{p}'\,\|^2}$$

    - Bright splotches in the image

Instant global illumination

- **Handling close point**
  - Reply on identity

$$a = \min(a, b) + \max(a - b, 0)$$

  - Rewrite reflectance integral

$$\int_A f(\mathrm{p}, \mathrm{p}')\, G(\mathrm{p} \leftrightarrow \mathrm{p}')\, \mathrm{d}A$$

$$\Downarrow$$

$$\int_A f(\mathrm{p}, \mathrm{p}') \min(G(\mathrm{p_i} \leftrightarrow \mathrm{p_1}), G_{\mathrm{limit}})\, \mathrm{d}A$$
$$+ \int_A f(\mathrm{p}, \mathrm{p}') \max(G(\mathrm{p_i} \leftrightarrow \mathrm{p_1}) - G_{\mathrm{limit}}, 0)\, \mathrm{d}A$$

Instant global illumination

- **Rendering with virtual light sources**

Direct illumination only

Using 4 virtual lights

Using 64 virtual lights

Next lecture: Global illumination 2