

I. Multiple choice, choose single answer

Select ONLY ONE option that corresponds to each question below, and then write all the answers in the answer sheet on page 6.

1. (2 points) Which of the following scenarios under perspective projection may contain aliasing problems that cannot be efficiently solved by super-sampling with random uniform samples per pixel?
 - A. Line edges
 - B. Object boundaries
 - C. Texture patterns
 - D. Specular lighting
2. (2 points) In projection-based rendering pipeline, which of the following coordinate spaces may not have a unique coordinate system?
 - A. World coordinate space
 - B. View coordinate space
 - C. Clip coordinate space
 - D. Object coordinate space
3. (2 points) In order to realistically render a translucent glass with rough surfaces and caustics, which of the following statements is correct?
 - A. The rendering using bidirectional path tracing can produce clear result efficiently within a very short time.
 - B. The caustics can be easily captured by photon mapping algorithm.
 - C. The rough surface of the glass may exhibit scattering which is difficult to render efficiently using path tracing algorithm.
 - D. If the glass is thin, it cannot be well rendered by photon mapping either.
- B 4. (2 points) Given a 3D vertex $p = [2, 5, 8]^T$, which of the following homogeneous representation of vertex p indicates the same vertex in 3D space?
 - A. $[3, 7, 16, 2]^T$
 - B. $[2, 5, 8, 0]^T$
 - C. $[6, 15, 24, 3]^T$
 - D. $[2, 5, 8, -1]^T$
5. (2 points) Which of the following statements about the BVH acceleration structures for ray-geometry intersection is true?
 - A. BVH can create a tree structure for primitive searching without any spatial overlap.
 - B. With a BVH structure, you can perform primitive searching always with a constant time.
 - C. For each node of a BVH structure, there is no need to compute the overlapping between a geometry primitive and a bounding box.
 - D. BVH can be constructed with the similar process as in the KD tree construction, but with overlapping regions for each tree node.
6. (2 points) Which of the following statements about texture mapping is correct?
 - A. The role of texture mapping to enrich the surface details can usually be also achieved by more complex geometric modeling with specified vertex color.
 - B. Very sophisticated lighting effects can be achieved by texture mapping when the whole geometry and light source are static.
 - C. Antialiasing in texture mapping is more complicated and can be reduced efficiently by using mipmapping technique where texture with matched resolution is used adaptively.
 - D. A perfect surface parameterization to determine texture coordinates for each vertex can always be done fully automatically for any geometry given any 2D texture without any distortion.

7. (2 points) Marching cubes are useful for generating iso-surfaces from a discrete 3D scalar field. Which of the following statements about marching cubes algorithm is correct?
- A. The algorithm can always produce a triangle mesh that is a Denaulay triangulation.
 - B. A large memory size may be needed if the mesh resolution is fine.
 - C. The algorithm is only valid for discrete volumetric data.
 - D. Vertex normal of the isosurface mesh cannot be estimated from the volume data.
8. (2 points) Which of the following statements about comparison between normal mapping and displacement mapping is correct?
- A. Normal mapping and displacement mapping can simulate bumps on a smooth surface with the same visual effect.
 - B. In displacement mapping, there is no need to compute normal during lighting procedure.
 - C. Displacement mapping fakes the surface variation by altering the per-pixel normal only.
 - D. The surface contour with displacement mapping is usually non-smooth when projected compared to a normal mapping counterpart.

II. Multiple choice, choose multiple answers

Select ALL options that correspond to each of the questions below, and then write all the answers in the answer sheet on page 6.

9. (2 points) Which of the following statement(s) about surface modeling is/are true?
- A. Spline interpolation can be used to create curve/surface modeling with arbitrary number of points and with very cheap computation.
 - B. Bézier surface can be obtained by evaluating two Bézier curves along the independent dimension of the parameter space, with accurate normal computation.
 - C. Mesh subdivision from a given coarse triangle mesh into a very smooth and fine triangle mesh is possible, which requires mesh construction with neighboring information.
 - D. A B-spline surface can be considered as a stitching of multiple Bézier surfaces with different smoothness constraints.
10. (2 points) Suppose we are doing the ray-triangle intersection and we have already computed the intersection point p with barycentric coordinates $(b_1, b_2, 1 - b_1 - b_2)$. Which of the following conditions indicate that p is inside the triangle?
- A. $b_1 = 0, b_2 = 0$
 - B. $b_1 > 0, b_2 = 0$
 - C. $b_1 > 0, b_2 > 0$
 - D. $b_1 < 0, b_2 < 0$
11. (2 points) Which of the following statement(s) about direct lighting with finite area light source is/are true?
- A. There will be no aliasing error for area light source during rendering.
 - B. Soft shadow will appear for area light source.
 - C. For rendering diffuse surface, we can sample over the surface of the area light source and sum together all the sample contributions.
 - D. Bi-directional path tracing should be used for global illumination in case of small area light source.
12. (2 points) Which of the statement(s) about projection is/are true?

- A. Perspective projection is a linear transformation in homogeneous coordinates.
 - B. Clipping of objects is performed before projection.
 - C. With perspective projection, farther object will become smaller.
 - D. The perspective projection usually has a wider field-of-view than orthogonal projection.
13. (2 points) Delaunay triangulation has been widely used in creating triangle meshes. Which of the following statement(s) is/are its advantage(s)?
- A. It avoids singular triangles during triangulation.
 - B. Flipping can be used in progressive Delaunay triangulation.
 - C. Delaunay triangulation can only be used to construct convex triangulation.
 - D. Voronoi diagram and Delaunay triangulation are dual with each other.
14. (2 points) Which of the following statement(s) about image reconstruction is/are true?
- A. Box filter can produce aliasing-free image reconstruction result.
 - B. Gaussian filter is the best image reconstruction filter for ray tracing.
 - C. Mitchell filter is usually better than Gaussian since negative values enhance sharpness.
 - D. Image reconstruction based on sampled rays is usually a biased estimation.
15. (2 points) Which algorithm(s) is/are effective for anti-aliasing in texture mapping?
- A. Screen-space super-resolution
 - B. Isotropic mip-mapping
 - C. Anisotropic mip-mapping
 - D. High-order filtering
16. (2 points) A vector graphic image has pros and cons. What the following state(s) about vector graphic image is/are true?
- A. Font can be created using spline curves as a vector graphic image.
 - B. Vector graphic images are naturally aliasing-free.
 - C. Any natural image can be converted to a vector graphic image.
 - D. Zoom-in of a vector graphic image will blur the edges.
17. (2 points) Which of the following statement(s) about importance sampling for rendering with global illumination is/are correct?
- A. Importance sampling for a cosine-weighted hemi-sphere can be easily achieved by an inversion method.
 - B. Metropolis sampling can be a general method for importance sampling used in ray tracing.
 - C. Photon mapping can naturally achieve importance sampling property.
 - D. Importance sampling generally has faster convergence of radiance estimation.

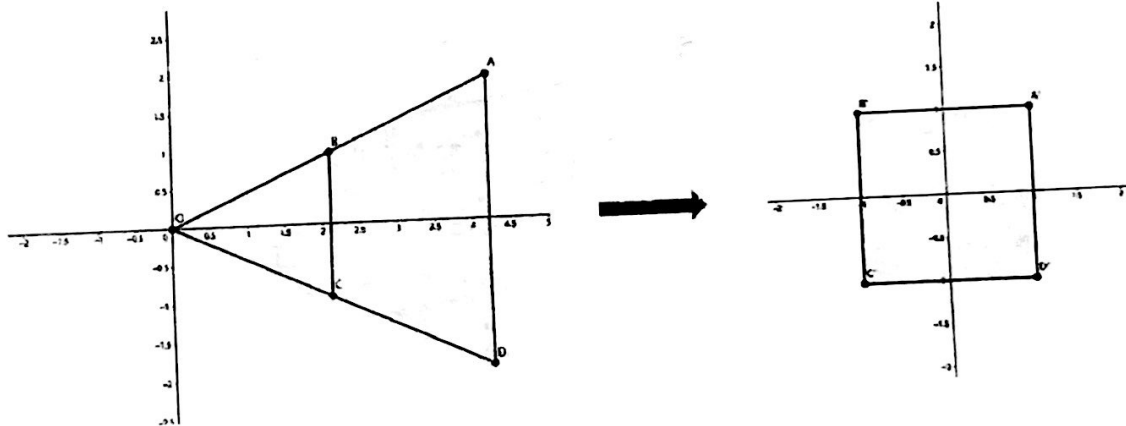
III. Short-answer and calculation questions

Write your answers in the blank space below the corresponding question. You should elaborate the specific calculation steps and clarify your notations.

18. (7 points) Transformations

We have learned projection from 3D space to 2D image in class. In this question let's consider projection from 2D to 1D.

Now we have a camera placed at the origin point of x - y plane, looking at the positive direction of x -axis. The distance from camera to the near plane is n and distance from the camera to far plane is f . The field of view is 2θ .



1. For a point with coordinate $(x, y, 1)$, please write down the 3×3 projection matrix, with regard to n , f and θ .
2. If $x = 2$, $y = 1$, $\tan \theta = \frac{1}{2}$, $n = 1$ and $f = 3$, please calculate the coordinate of point in 1D screen space after projection. Suppose the resolution of 1D screen is 1000. (The screen in 1D is a line segment. We define the lower end of the line segment as the origin point of 1D screen space.)

Answer: Q1:

The projection matrix is:

$$\begin{bmatrix} \frac{2}{f-n} & 0 & \frac{n+f}{n-f} \\ 0 & \frac{1}{n \tan \theta} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n+f & 0 & -nf \\ 0 & n & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{f+n}{f-n} & 0 & -\frac{2nf}{f-n} \\ 0 & \frac{1}{\tan \theta} & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

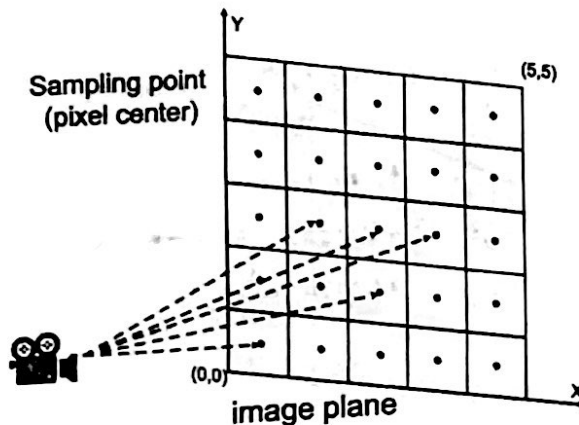
Q2:

The coordinate after projection is $(0.5, 1)$, So the 1D coordinate in screen space is 750.

19. (6 points) Ray tracing Given a camera's position $O = (1, 1, 1)$, look at $(1, 3, 1)$, reference up is $u = (0, 0, 1)$, focal length is 1 and fov on y is 90° . There is a triangle in the world space, given each vertex's position and normal as follows:

	position	normal
A	(4, 5, 2)	(0.60, 0.80, 0.00)
B	(1, 8, 2)	(0.00, 0.28, 0.96)
C	(3, 6, 4)	(0.42, 0.56, 0.70)

Then we generate a ray on an image of 25×25 at the pixel (17, 17). Is it possible for the ray to hit the triangle? If possible, please calculate the hit point and it's normal according to the given vertices.



Hint:

1. Sample point locates at the pixel center.
2. We assume the pixel index starts from (0, 0) and the image plane example is shown in the fig.

solution:

$$Pix_{NDC} = \frac{pix + (0.5, 0.5)}{(width, height)} = \frac{(17.5, 17.5)}{(25, 25)} = (0.7, 0.7)$$

$$Pix_{Screen} = 2 \times Pix_{NDC} - 1 = (0.4, 0.4) \dots (1pt)$$

$$h = \tan\left(\frac{fov}{2}\right) \cdot focal_len = 1$$

$$dir = (0, 1, 0) + (0.4, 0, 0.4) = (0.4, 1, 0.4)$$

$$O + t \cdot Dir = (1, 1, 1) + t \cdot (0.4, 1, 0.4) \dots (1pt)$$

$$O + t \cdot Dir = (1 - b_1 - b_2)P_0 + b_1P_1 + b_2P_2$$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{S_1 \cdot E_1} \begin{bmatrix} S_2 \cdot E_2 \\ S_1 \cdot S \\ S_2 \cdot Dir \end{bmatrix}$$

$$\text{Where } E_1 = P_1 - P_0$$

$$E_2 = P_2 - P_0$$

$$S = O - P_0$$

$$S_1 = Dir \times E_2$$

$$S_2 = S \times E_1$$

$$P_0 = (4, 5, 2)$$

$$P_1 = (1, 8, 2)$$

$$P_2 = (3, 6, 4)$$

$$E_1 = (-3, 3, 0)$$

$$E_2 = (-1, 1, 2)$$

$$S = (-3, -4, -1)$$

$$S_1 = (1.6, -1.2, 1.4)$$

$$S_2 = (3, 3, -21)$$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{1}{6} \\ \frac{1}{2} \end{bmatrix} \dots\dots\dots (4\text{pts for all values above})$$

$$\therefore P = O + t \cdot Dir = (3, 6, 3) \dots\dots\dots (1\text{pts})$$

$$b_0 = \frac{1}{3}$$

$$n = b_0 \cdot n_0 + b_1 \cdot n_1 + b_2 \cdot n_2$$

$$= \frac{1}{3}(0.6, 0.8, 0) + \frac{1}{6}(0, 0.28, 0.96) + \frac{1}{2}(0.42, 0.56, 0.70)$$

$$= (0.41, 0.59, 0.51) = \left(\frac{41}{100}, \frac{178}{300}, \frac{51}{100}\right) \dots\dots\dots (1\text{pts})$$

20. (8 points) Lighting Calculation on a Bézier Surface
Given a 3D Bézier surface's control points

P_{ij}	$i = 0$	$i = 1$	$i = 2$
$j = 0$	(0.0, 0.0, 0.0)	(3.0, 4.0, 4.0)	(6.0, -2.0, 8.0)
$j = 1$	(-1.0, 0.0, -0.5)	(2.0, 4.0, 3.5)	(5.0, -2.0, 7.5)

1. Evaluate the position and normal of point P_m where $(u, v) = (0.4, 0.4)$.
2. We use Phong lighting model to calculate the illumination of the point P_m . Supposing the ambient, diffuse and specular components of the object are

$$k_a = (0.1, 0.1, 0.1)$$

$$k_d = (0.6, 0.7, 0.4)$$

$$k_s = (1, 1, 1)$$

respectively and the shininess α is 16. There is a point light located at $P_l = (5.0, 4.6, 3.0)$ in the scene with ambient i_a , diffuse i_d and specular i_s radiance as $i_a = i_d = i_s = (1, 1, 1)$. The camera position is located at $(2.0, 2.6, 3)$. Please calculate the radiance of the reflected light using Phong lighting model.

Hint: Could be degraded to calculation on a Bézier Curve. solution:

1. It is apparent that $P_{i1} = P_{i0} + (-1, 0, -0.5)$. So to compute the position and tangent of $B(0.4, 0.4)$, we can just compute the position and tangent $B_0(0.4)$ of Bézier curve P_{00}, P_{10}, P_{20} and move the point by $0.4 \cdot (-1, 0, -0.5) = (-0.4, 0, -0.2)$.

$$B_0(0.4) = (1-t)^2 P_{00} + 2(1-t)t P_{10} + t^2 P_{20} = (2.4, 1.6, 3.2)$$

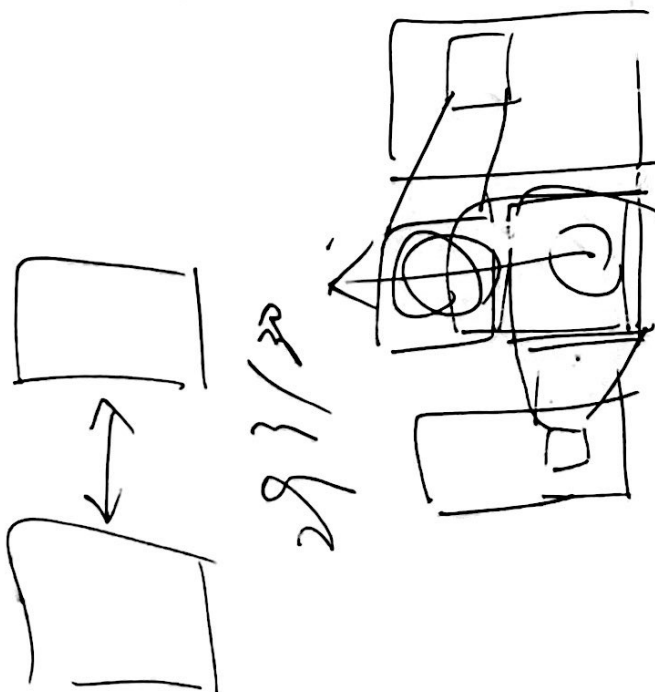
$$B(0.4, 0.4) = B_0(0.4) + (-0.4, 0, -0.2) = (2, 1.6, 3)$$

For tangent on y direction:

$$\tan(t) = \text{norm}(((1-t)P_{10} + tP_{20}) - ((1-t)P_{00} + tP_{10}))$$

$$\tan(0.4) = \text{norm}((3, 0, 4)) = (0.6, 0, 0.8)$$

For tangent on x direction is always, $\text{norm}((-1, 0, -0.5)) = \text{norm}((-2, 0, -1))$ we can normalize later. So normal of the point P_m is $\text{norm}(\text{cross}(\tan_x, \tan_y))$, obviously $n = (0, 1, 0)$ for $P_m = (2, 1.6, 3)$.



2.

$$\mathbf{n} = (0, 1, 0)$$

$$\mathbf{L} = \text{norm}((P_l - P_m)) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\mathbf{R} = 2(\mathbf{L} \cdot \mathbf{n})\mathbf{n} - \mathbf{L} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\mathbf{V} = \text{norm}((P_{cam} - P_m)) = (0, 1, 0)$$

$$\mathbf{L} \cdot \mathbf{n} = \frac{1}{\sqrt{2}}$$

$$\mathbf{R} \cdot \mathbf{V} = \frac{1}{\sqrt{2}}$$

$$I_p = k_a i_a + k_d (\mathbf{L} \cdot \mathbf{N}) i_d + k_s (\mathbf{R} \cdot \mathbf{V})^2 i_s$$

$$= (0.1, 0.1, 0.1) + \frac{1}{\sqrt{2}} (0.6, 0.7, 0.4) + \frac{1}{\sqrt{2}}^{10} (1, 1, 1)$$

$$\approx (0.55551407, 0.62622475, 0.41409271)$$

21. (11 points) Texture Mapping

Given the following mapping between the screen space and the texture space as shown in the graph: $p_{00} \rightarrow p'_{00}(0.3, 0.3)$, $p_{10} \rightarrow p'_{10}(0.7, 0.2)$, $p_{01} \rightarrow p'_{01}(0.4, 0.5)$, where p_{00} , p_{10} , and p_{01} are points in the screen space and p'_{00} , p'_{10} , and p'_{01} are points in the texture space. The texture coordinates are normalized and the given 2D texture image resolution is 128×128 .

Suppose T_{00} is the original 128×128 texture, T_{10} is the 64×128 texture, T_{01} is the 128×64 texture, T_{11} is the 64×64 texture, \dots .

Suppose p'_{00} is evaluated to colors $c_{00}, c_{10}, c_{01}, c_{11}, \dots$ for $T_{00}, T_{10}, T_{01}, T_{11}, \dots$ respectively.

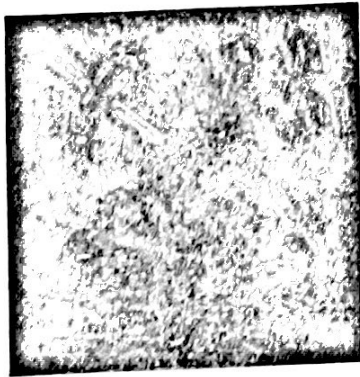
Please compute the ripmap color evaluated at p_{00} , which is an expression of $c_{00}, c_{10}, c_{01}, c_{11}, \dots$.

Hint:

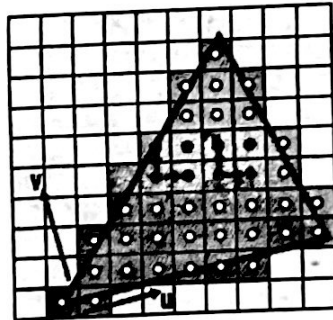
1. Ripmap, not mipmap.
2. You can use finite difference approximation to calculate the derivatives.
3. You can use the forward difference for the finite difference approximation:

$$\frac{du}{dx} \approx \frac{u(x + \Delta x) - u(x)}{\Delta x},$$

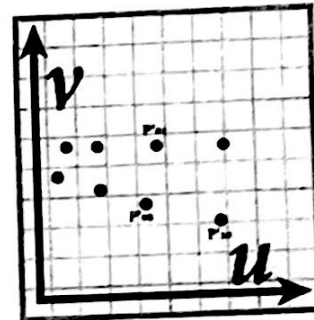
where in screen space, Δx is always 1.



(a) Ripmap explanation



Screen space



Texture space

(b) Ripmap coordinates

Answer:

$$L_x = \sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2} = \sqrt{(0.4 \times 128)^2 + (0.1 \times 128)^2} = \frac{64}{5} \sqrt{17}$$

$$L_y = \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} = \sqrt{(0.1 \times 128)^2 + (0.2 \times 128)^2} = \frac{64}{5} \sqrt{5}$$

$$D_x = \log_2 L_x \approx 5.722$$

$$D_y = \log_2 L_y \approx 4.839$$

So we should sample from $T_{54}, T_{64}, T_{55}, T_{65}$.

$$\begin{aligned} c &= (1 - 0.839) ((1 - 0.722)c_{54} + 0.722c_{64}) \\ &\quad + (0.839) ((1 - 0.722)c_{55} + 0.722c_{65}) \\ &= 0.161 \cdot 0.278c_{54} + 0.161 \cdot 0.722c_{64} \\ &\quad + 0.839 \cdot 0.278c_{55} + 0.839 \cdot 0.722c_{65} \\ &= 0.044758c_{54} + 0.116242c_{64} + 0.233242c_{55} + 0.605758c_{65} \end{aligned}$$

22. (13 points) Sampling

Suppose we have one random variable ξ that is sampled from the standard uniform distribution $\xi \sim U(0,1)$. A curve in 2D space is represented as $r = \sin \theta + \cos \theta$, $\theta \in [0, \frac{\pi}{2}]$ in polar coordinate system. Now we wish to generate samples that are uniformly distributed over the curve. (i.e. $p(x, y) = c$) Please write down the Cartesian coordinate (x, y) of the sampled point with regard to ξ .

Answer:

Solution 1:

let $p(\theta) = rc = c(\sin \theta + \cos \theta)$.

$$\int_0^{\frac{\pi}{2}} c(\sin \theta + \cos \theta) d\theta = 1 \Rightarrow c = \frac{1}{2}$$

$$P(\theta) = \int_0^{\theta} \frac{1}{2}(\sin \theta' + \cos \theta') d\theta' = \frac{1}{2}(\sin \theta - \cos \theta + 1)$$

Let $P(\theta) = \xi$

$$\theta = \arcsin(\sqrt{2}\xi - \frac{\sqrt{2}}{2}) + \frac{\pi}{4}$$

Given $x = r \cos \theta = \cos^2 \theta + \sin \theta \cos \theta$, and $y = r \sin \theta = \sin^2 \theta + \sin \theta \cos \theta$

$$\cos \theta + \sin \theta = 2\xi^2 - 2\xi$$

$$\sin^2 \theta = \left[\xi - \frac{1}{2} + \sqrt{\frac{1}{4} + \xi - \xi^2} \right]^2$$

$$\cos^2 \theta = \left[\frac{1}{2} - \xi + \sqrt{\frac{1}{4} + \xi - \xi^2} \right]^2$$

So

$$\begin{cases} x = \frac{1}{2} + 2\xi - 2\xi^2 - (2\xi - 1)\sqrt{\frac{1}{4} + \xi - \xi^2} \\ y = \frac{1}{2} + 2\xi - 2\xi^2 + (2\xi - 1)\sqrt{\frac{1}{4} + \xi - \xi^2} \end{cases}$$

Solution 2:

The curve is part of the circle $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$. The center of the circle is $(0.5, 0.5)$ First we uniformly sample the unit circle for $\phi \in [-\frac{\pi}{4}, \frac{3\pi}{4}]$

$$x = \cos(\pi\xi - \frac{\pi}{4})$$

$$y = \sin(\pi\xi - \frac{\pi}{4})$$

Then we transform it to target curve:

$$\begin{cases} x = \frac{\sqrt{2}}{2} \cos(\pi\xi - \frac{\pi}{4}) + 0.5 = \frac{1}{2} \cos \pi\xi + \frac{1}{2} \sin \pi\xi + \frac{1}{2} \\ y = \frac{\sqrt{2}}{2} \sin(\pi\xi - \frac{\pi}{4}) + 0.5 = \frac{1}{2} \sin \pi\xi - \frac{1}{2} \cos \pi\xi + \frac{1}{2} \end{cases}$$

23. (10 points) Ray Marching Based Volume Visualization

Suppose we have the following 4 samples at p_1, p_2, p_3, p_4 ($\|p_2 - p_1\| = \|p_3 - p_2\| = \|p_4 - p_3\|$) along a camera ray. p_1, p_2, p_3 fall inside a participating media with radiance and transparency as:

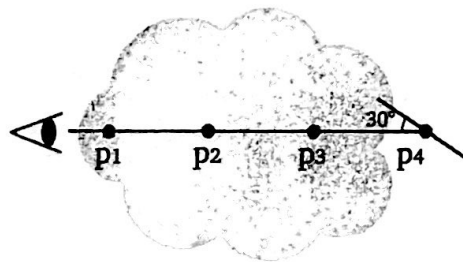
$$p_1 : (c_1 = 0.1, T_1 = 0.3), \quad p_2 : (c_2 = 0.8, T_2 = 0.4), \quad p_3 : (c_3 = 0.3, T_3 = 0.4).$$

p_4 falls on a uniform emitter outside of the volume with radiance $L_e = 1$.

Q1: Determine the radiance c got by the camera ray using these 4 samples with a front-to-back composition order from p_1 to p_4 .

Suppose early ray termination is applied when opacity $\alpha > 0.95$.

Q2: Determine the radiance c got by the camera ray using these 4 samples with a front-to-back composition order from p_1 to p_4 .



Answer:

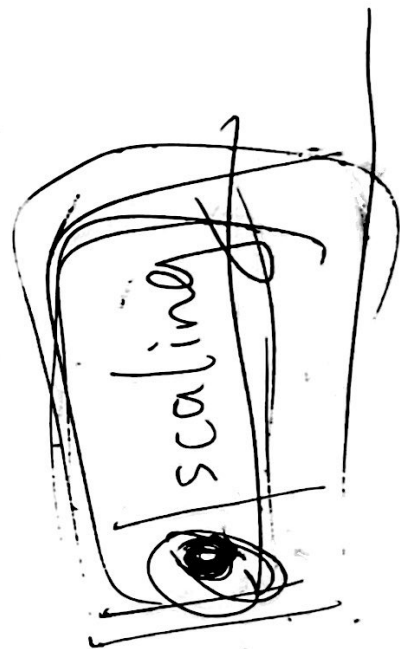
Q1:

$$\begin{aligned}
 c_{dst}^{(0)} &= 0 \\
 \alpha_{dst}^{(0)} &= 0 \\
 c_{dst}^{(1)} &= c_{dst}^{(0)} + (1 - \alpha_{dst}^{(0)}) c_1 = 0.1 \\
 \alpha_{dst}^{(1)} &= \alpha_{dst}^{(0)} + (1 - \alpha_{dst}^{(0)}) \alpha_1 = 0.7 \\
 c_{dst}^{(2)} &= c_{dst}^{(1)} + (1 - \alpha_{dst}^{(1)}) c_2 = 0.34 \\
 \alpha_{dst}^{(2)} &= \alpha_{dst}^{(1)} + (1 - \alpha_{dst}^{(1)}) \alpha_2 = 0.88 \\
 c_{dst}^{(3)} &= c_{dst}^{(2)} + (1 - \alpha_{dst}^{(2)}) c_3 = 0.376 \\
 \alpha_{dst}^{(3)} &= \alpha_{dst}^{(2)} + (1 - \alpha_{dst}^{(2)}) \alpha_3 = 0.952 \\
 c &= c_{dst}^{(3)} + T_{dst}^{(4)} L_e \cos \theta \\
 &= c_{dst}^{(3)} + T_{dst}^{(3)} L_e \cos \theta \\
 &= c_{dst}^{(3)} + (1 - \alpha_{dst}^{(3)}) L_e \cos \theta \\
 &= c_{dst}^{(3)} + (1 - 0.952) L_e \cos \theta \\
 &= 0.376 + 0.048 \cdot 1 \cdot 0.5 \\
 &= 0.4
 \end{aligned}$$

Q2:

$$\alpha_{dst}^{(3)} = 0.952 > 0.95$$

$$c = c_{dst}^{(3)} = 0.376$$



zyh FVV.

动态 FVV H

