

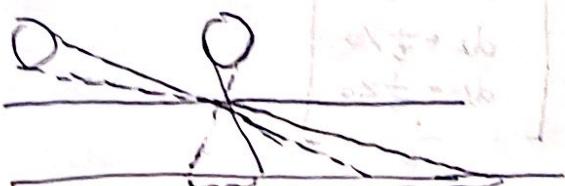
透視 projection

丢失：深度、角度、长度

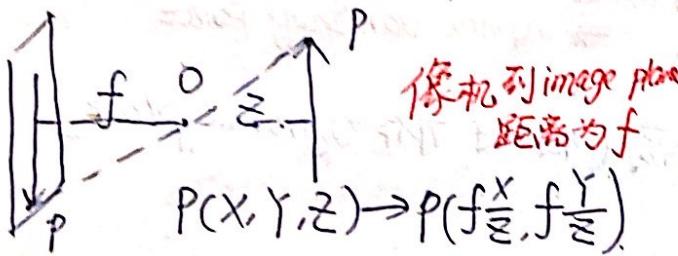
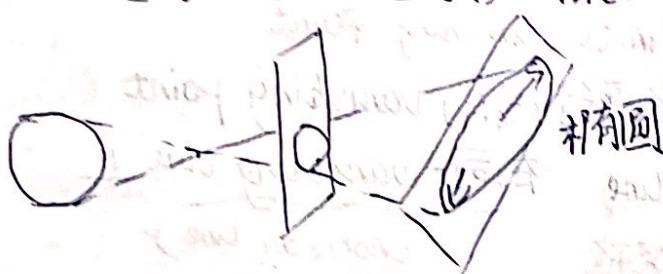
直线不会变曲线

平行线 vanishing line point

平行面 vanishing plane



越向两边，像越失真物体越矮



像机到image plane 距离为 f

$$P(x, y, z) \rightarrow p(f\frac{x}{z}, f\frac{y}{z})$$

$$B^R t \rightarrow A^A P = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}^B P$$

$$R \stackrel{\text{def}}{=} \begin{pmatrix} A_i^A, j_B^A, k_B^A \end{pmatrix} = \begin{pmatrix} i_A \cdot i_B & j_A \cdot j_B & k_A \cdot k_B \\ j_A \cdot i_B & j_A \cdot j_B & j_A \cdot k_B \\ k_A \cdot i_B & k_A \cdot j_B & k_A \cdot k_B \end{pmatrix}$$

物理世界 单位: m , pixel world 单位: (pixel)

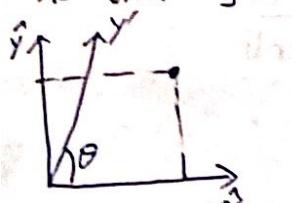
Pixel width (dx) m , height (dy) m .

$$\alpha = \frac{1}{dx} f, \beta = \frac{1}{dy} f$$

物理坐标、 (x, y)

normalized image plane 读取 (\hat{x}, \hat{y})

若考虑畸变的实际坐标、 (x', y')



$$\hat{x} = x' + y' \cos \theta$$

$$\hat{y} = y' \sin \theta$$

$$\begin{aligned} y' &= \frac{\hat{y}}{\sin \theta} \\ x' &= \hat{x} - \frac{\hat{y}}{\sin \theta} \cot \theta \end{aligned}$$

$$P = k \hat{P} \rightarrow \text{normalized image plane}$$

$$\hat{P} = \begin{pmatrix} \frac{1}{dx} & s & x_0 \\ 0 & \frac{1}{dy} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} [R \ t] P$$

world \rightarrow camera
 \downarrow
camera space
 \rightarrow image plane

$$\alpha = \frac{1}{dx} f, \beta = \frac{1}{dy} f$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} fx \\ fy \\ z \end{pmatrix}$$

\downarrow
(normalized) image plane \rightarrow pixel

x_0, y_0 : principal offset point

θ : skew angle

$\alpha(f_x), \beta(f_y)$ focal length

\hat{P} 单位: m P : 单位 pixel

$$\alpha = kf, \beta = lf \quad \text{unit aspect ratio}$$

k, l : 拉伸的系数

K : 自由度 5

$[R \ t]$ 自由度 $3+3=6$

$$\lambda x_i = \underbrace{k[R \ t] X_i}_{= P X_i} \quad \parallel$$

$$\Rightarrow \underbrace{x_i \times P X_i}_{\text{提供2个方程}} = 0$$

提供2个方程

at least 6 pairs $\Rightarrow P$

$$x = \alpha \hat{x} + x_0$$

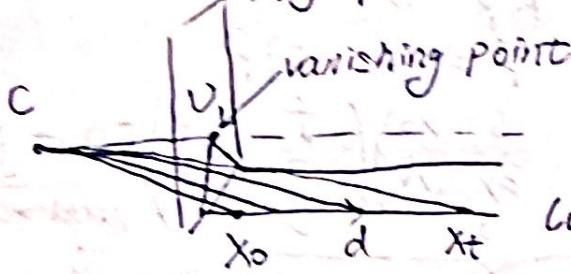
$$y = \beta \hat{y} + y_0$$

$$x = \alpha \hat{x} - \alpha \hat{y} \cot \theta + x_0$$

$$\Rightarrow y = \beta \frac{\hat{y}}{\sin \theta} + y_0$$

\Downarrow

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha \cot \theta & x_0 \\ 0 & \frac{\beta}{\sin \theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \\ 1 \end{pmatrix}$$



same direction \Rightarrow same vanishing point

$$x_t = x_0 + t \cdot d = \begin{bmatrix} x_0 + td_1 \\ y_0 + td_2 \\ z_0 + td_3 \end{bmatrix} \leftrightarrow \begin{bmatrix} d_1 + \frac{t}{t} x_0 \\ d_2 + \frac{t}{t} y_0 \\ d_3 + \frac{t}{t} z_0 \end{bmatrix}$$

$$t \rightarrow \infty \quad x_\infty = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \end{pmatrix}$$

$$\boxed{\vec{cv} \parallel d}$$

$$v = P x_\infty = k[R \ t] x_\infty$$

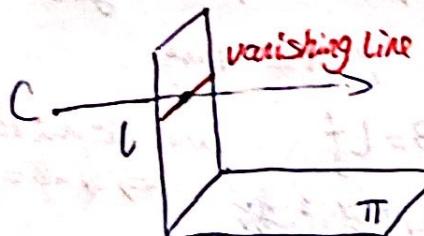
finite vanishing point

同一平面，各平行线的 vanishing point

不平行的 plane \Rightarrow 不同的 vanishing line 在同一 vanishing line 上

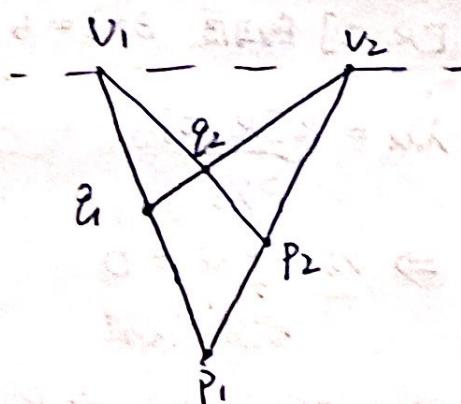
若 $d \parallel$ 侧面 (Chorizon line)

\Rightarrow infinite vanishing point



$$l \parallel \pi$$

C 和 l 等高 (相对于 pi 作为 ground plane)



$$\overleftrightarrow{P_1Q_1}$$

$$P_1Q_1 \text{ 直线: } P_1 \times Q_1 \Rightarrow l_1$$

$$P_2Q_2 \text{ 直线: } P_2 \times Q_2 \Rightarrow l_2$$

$$\hookrightarrow \text{直线交点 } l_1 \times l_2 = V$$

$$P_i = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad \text{2D 的齐次坐标}$$

$$l = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow ax + by + c = 0$$

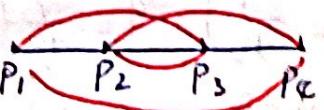
$$P_i = \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$\frac{||P_1 - P_3|| \cdot ||P_2 - R||}{||P_2 - P_3|| \cdot ||P_1 - R||}$$

measuring height

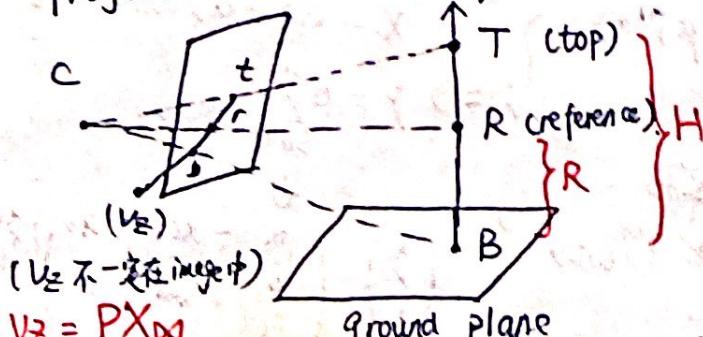
projective invariant 投影不变量

cross ratio



$$\text{ratio} = \frac{||\infty - R|| \cdot ||T - B||}{||\infty - B|| \cdot ||T - R||} = \frac{H}{H - R}$$

$$= \frac{||v_2 - r|| \cdot ||t - b||}{||v_2 - b|| \cdot ||t - r||}$$



$$v_2 = P X_\infty$$

$$P = K[R \ t] = [P_1 \ P_2 \ P_3 \ P_4]$$

$$P_1 = P(1, 0, 0, 0)^T, P_2 = P(0, 1, 0, 0)^T, P_3 = P(0, 0, 1, 0)^T$$

$e_i: x, y, z$ direction 的 vanishing point

$$P_4 = P(0, 0, 0, 1)^T \Rightarrow \text{原点在 image 上的位置.}$$

$$\lambda_i v_i = K[R \ t] \begin{bmatrix} e_i \\ 0 \end{bmatrix} = K R e_i \Rightarrow e_i = \lambda_i R^T K^{-1} v_i$$

$$e_i^T e_j = 0 \Rightarrow v_i^T K^{-T} \underline{R R^T} K^{-1} v_j = 0$$

$$\Rightarrow v_i^T K^{-T} K^{-1} v_j = 0$$

假设 $\alpha = \beta, \theta = 90^\circ$, 则 K 共有 f, x_0, v_0 3 个未知数 $\Rightarrow 3$ vanishing point $\Rightarrow K$

1° 3 finite vanishing point \Rightarrow 求出 K . (f, u_0, v_0)

2° 2 finite vanishing point \Rightarrow

3° 1 finite vanishing point \Rightarrow f 无法求出,

(u_0, v_0) 为该 vanishing point

$$K \text{ 得知后可求 } R = [r_1 \ r_2 \ r_3]$$

$$\lambda_i v_i = K[R \ t] \begin{bmatrix} e_i \\ 0 \end{bmatrix} \Rightarrow \lambda_i K^{-1} v_i = r_i, \|r_i\| = 1$$

stereo

structure from motion (SFM)

motion: given set of correspondence point \Rightarrow Camera params
epipolar geometry

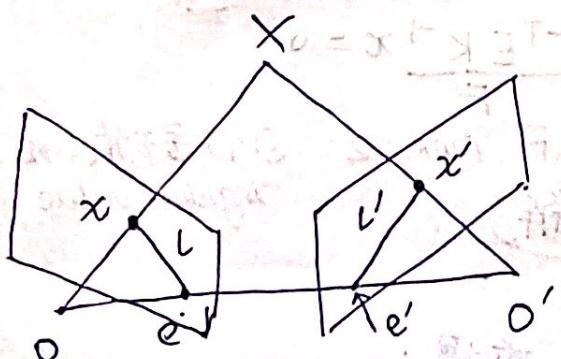
OO' : base line

epipolar plane: 过 base line 的 所有平面

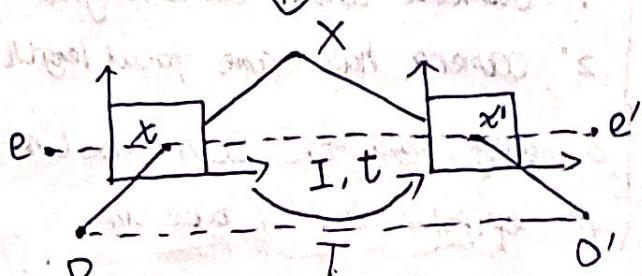
epipolar: e, e'

epipolar line: l, l'

x 的 对应点 必 在 l' 上 : 面上找 \Rightarrow 线上找



↓ 2个成像平面平行



无 epipolar (epipole 在无穷远)

x, x' 等高

$$E = [t_x] R = [t_x] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$\vec{T} = (T, 0, 0)$$

$$x'^T E x = 0$$

$$(u' \ v' \ 1) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = 0 \Rightarrow u = v'$$

$$x' = Rx + t$$

$$tx \cdot x' = t \cdot Rx + t \cdot t = 0$$

$$x' \cdot (t \times x') = 0$$

$$x' \cdot (t \times Rx) = 0$$

$$x'^T [tx] Rx = 0$$

设 $X = (x, 1)^T \Rightarrow x = [I \ 0]X = x$

$$x' = [R \ t] X = Rx + t$$

$$\Rightarrow x'^T [tx] Rx = 0$$

$$x^T E x = 0 \quad \text{rank}(E) = 2 \Rightarrow \text{将1个归一化: } 5 \text{个自由元}$$

\therefore 已知 $x \Rightarrow l' = [tx] Rx = Ex$

已知 $x' \Rightarrow x^T (E^T x') = 0 \Rightarrow l = E^T x' \quad \text{null}(E) = e$

l, l' : epipolar line $\Rightarrow Ee = 0, E^T e = 0 \Rightarrow \text{rank}(E) = 2$
epipolar 无 epipolar line.

$\hat{x}^T E \hat{x} = 0$ (真正写法有1)

若 C_1, C_2 内参已知:

$$(x, y, 1)^T F \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

$$x' = K' \hat{x}' \quad x = K \hat{x}$$

$$Ax = 0 \quad (\text{只有0解})$$

$$\Rightarrow \hat{x}' = K'^{-1} x' \quad \hat{x} = K^{-1} x$$

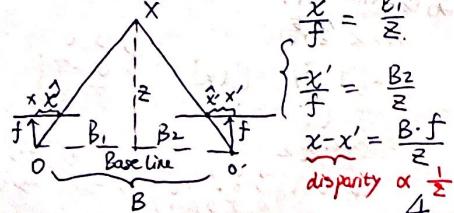
$$\Rightarrow \text{rank}(A) = 9 \Rightarrow 8 \text{点法}$$

$$\Rightarrow (K'^{-1} x')^T E (K^{-1} x) = 0$$

$$F = K'^{-T} E K^{-1} \quad \text{保证 } E, F \text{ rank}=2: \text{SVD 舍去最小的 singular value.}$$

将1个归一化: 8个自由元 \Rightarrow 8点法求解

像机到 image plane 距离为 f



$$\left\{ \begin{array}{l} \frac{x}{f} = \frac{B_1}{z} \\ \frac{x'}{f} = \frac{B_2}{z} \\ x - x' = \frac{B \cdot f}{z} \\ \text{disparity } \propto \frac{1}{z} \end{array} \right.$$

前提:

- 1° Camera center same height
- 2° camera has same focal length
- 3° image plane 平行, 且 // base line
- 4° epipolar line // base line.

homography 单应性

\Rightarrow 多视图重建 multi-view stereo MVS \Rightarrow depth \Rightarrow 点云

2D 情况

1.



$$R: \theta \quad 1$$

$$t: (x, y) \quad 2$$

2. affine transform 平行关系



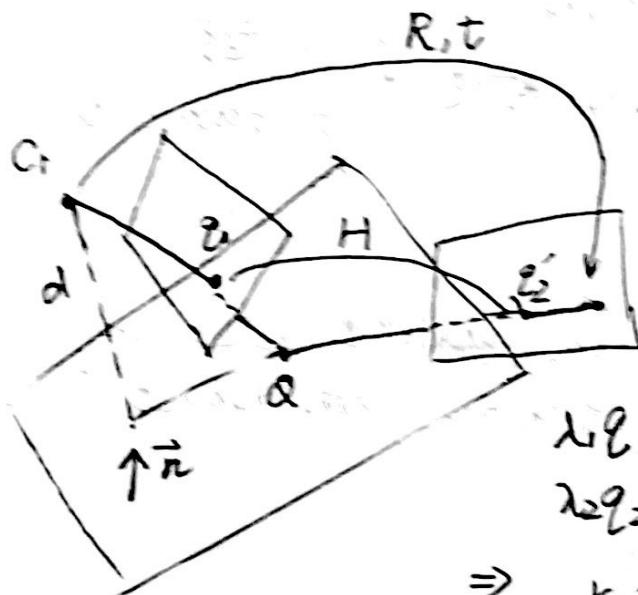
3 点确定关系 6

$$x' = Mx + t$$

3. projective homography



4 点确定关系 (8个自由度)



$$q_1 \in C_1 \text{ 且 } q_1 \in Q_1$$

$$C_1 \subset Q_1$$

$$Q_2 = RQ_1 + t$$

$$\vec{n} \cdot \vec{q}_1 + d = 0 \Rightarrow 1 = -\frac{\vec{n}^T \vec{q}_1}{d}$$

$$Q_2 = RQ_1 + t \cdot 1 = (R - t \frac{\vec{n}^T}{d})Q_1$$

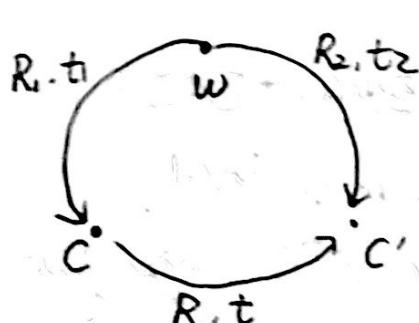
$$\lambda_1 q_1 = k_1 [I \ 0] \begin{bmatrix} q_1 \\ 1 \end{bmatrix} = k_1 q_1$$

$$\lambda_2 q_2 = k_2 [I \ 0] \begin{bmatrix} q_2 \\ 1 \end{bmatrix} = k_2 q_2$$

$$\Rightarrow k_2^{-1} q_2 = \lambda (R - t \frac{\vec{n}^T}{d}) k_1^{-1} q_1$$

$$q_2 = \lambda \underbrace{k_2 (R - t \frac{\vec{n}^T}{d}) k_1^{-1}}_H q_1$$

考虑外参:



$$\underbrace{\begin{bmatrix} R_2 & t_2 \\ 0 & 1 \end{bmatrix}}_{C' \rightarrow W} \underbrace{\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}}_{C \rightarrow C'} \underbrace{\begin{bmatrix} R_1 & t_1 \\ 0 & 1 \end{bmatrix}}_{W \rightarrow P} P = p$$

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_2 & t_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} R_2 & t_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^{-1} & -R_1^{-1}t_1 \\ 0 & 1 \end{bmatrix}$$

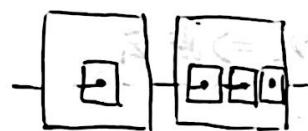
$$R = R_2 R_1^{-1}$$

$$t = -R_2 R_1^{-1} t_1 + t_2$$

$$\Rightarrow H = k_2 (R_2 R_1^{-1} - (-R_2 R_1^{-1} t_1 + t_2) \frac{\vec{n}^T}{d}) k_1^{-1}$$

$$= k_2 R_2 (I - \frac{1}{d} (R_1^{-1} t_1 + R_2^{-1} t_2) A^T R_1) R_1^{-1} k_1^{-1} = \begin{bmatrix} R_2 R_1^{-1} & -R_2 R_1^{-1} t_1 + t_2 \\ 0 & 1 \end{bmatrix}$$

stereo matching



图像特征

1° Pixel → local feature → global feature

信息不足

感受野 reception feature

failure



⇒ 各向异性

1° 无纹理 / 弱纹理
textureless

2° 重复纹理 repetition

3° 动态 dynamic

4° 遮挡 occlusion

马氏距离

Mahalanobis distance

$$d^2(x, y) = \sum \alpha_i (x_i - y_i)^2 = \sum M_{ii} (x_i - y_i)^2$$

↳ 对角矩阵

$$x^T M y = \sum M_{ij} x_i^T y_j \geq 0 \Rightarrow M \text{ 半正定}$$

M_{ij} 的相关性 ⇒ attention

distance F, L_1 , L_2 , 马氏

↳ 对比直方图, 限制 sparse 的个数: L_1 正则化

VQ - VE vector quantization 量化 ⇒ more robust

Fitting a homography

$$\lambda x'_i = H x_i \Rightarrow x'_i \times H x_i = 0$$

$$H = \begin{pmatrix} h_1^T \\ h_2^T \\ h_3^T \end{pmatrix} \quad Hx = \begin{pmatrix} h_1^T x_i \\ h_2^T x_i \\ h_3^T x_i \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \times \begin{pmatrix} h_1^T x_i \\ h_2^T x_i \\ h_3^T x_i \end{pmatrix} = 0$$

(H)h: 自由度 $\delta \Rightarrow$ 至少 4 对匹配点

$$\begin{pmatrix} 0 & -x_i^T & y' x_i^T \\ x_i^T & 0 & -x' x_i^T \\ -y' x_i^T & x' x_i^T & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = 0$$

$$Ah = 0$$

find $h \Rightarrow \min \|Ah\|$

let $h: \min A^T A$'s eigenvalue.

$A^T A h = \lambda_{\min} h$.

$$h^T A^T A h = \lambda_{\min} \|h\|^2 = \lambda_{\min}$$

$$\|Ah\| = \lambda_{\min}$$

MVS Net

视锥



对深度做 slice.

匹配: basic: pixel 层面

deep learning \rightarrow feature 层面

MVS Net: 不同视角的图像, 相同网络提取 feature

核心: 计算 volume.

Cost volume

假设每个 view depth 已知 \Rightarrow warp 到 reference view T

n -view \Rightarrow n-feature volume V_i (匹配特征)

找深度 \Rightarrow s.t. V_i 匹配到 reference

cost volume: 特征 $\xrightarrow{\text{卷积}}$ 回归 g.t. 深度.

1. 离散化深度
2. depth 求正

MVS Net.

先得到 depth \Rightarrow 点云

一个 Network: 唯一的场景
(CSDF)

10 个场景 train \Rightarrow 可用于第 11 个

Combine: MVS 得到 depth, 作为 prior 为 NERT

limitation: cost volume 3D

high resolution \Rightarrow Memory

Ransac. 随机采样一致. Random Sample Consensus

e.g. Homography

1. random select a group of matches
2. compute H of selected matches
3. Find **inlier** to the ~~H~~ H
4. 若 inlier 数目超 threshold
对所有 inlier 用 least square.

Pros: 1. simple, general.

2. work well

Cons: 1. params

2. low inlier rate
 \Rightarrow work X

3. initialization X.

MRF.

$T(x)$: density prob: hit at x

$T(x)$ transmittance prob: $0 \rightarrow t$ not hit any particle

$$T(t + \delta t) = T(t)(1 - \sigma_{ct} \delta t)$$

$$\frac{T(t + \delta t) - T(t)}{\delta t} = -T(t)\sigma_{ct}$$

$$\delta t \rightarrow 0 \quad \frac{T'(t)}{T(t)} = -\sigma_{ct}$$

$$\int_a^b \frac{T'(t)}{T(t)} dt = - \int_a^b \sigma_{ct} dt$$

$$(\log T(t)) \Big|_a^b = - \int_a^b \sigma_{ct} dt \quad T(a \rightarrow c) = e^{- \int_a^b \sigma_{ct} dt - \int_b^c \sigma_{ct} dt}$$

$$T(a \rightarrow b) = e^{- \int_a^b \sigma_{ct} dt} \Rightarrow = T(a \rightarrow b) \cdot T(b \rightarrow c)$$

$$C(a \rightarrow b) = \int_a^b T(a \rightarrow t) \sigma_{ct} C(ct) dt + \boxed{T(b)} C_bg$$

a $\rightarrow t$ not hit, 在 t hit 到 c or $C(ct)$ 的 particle

Piecewise constant 逐段一致.

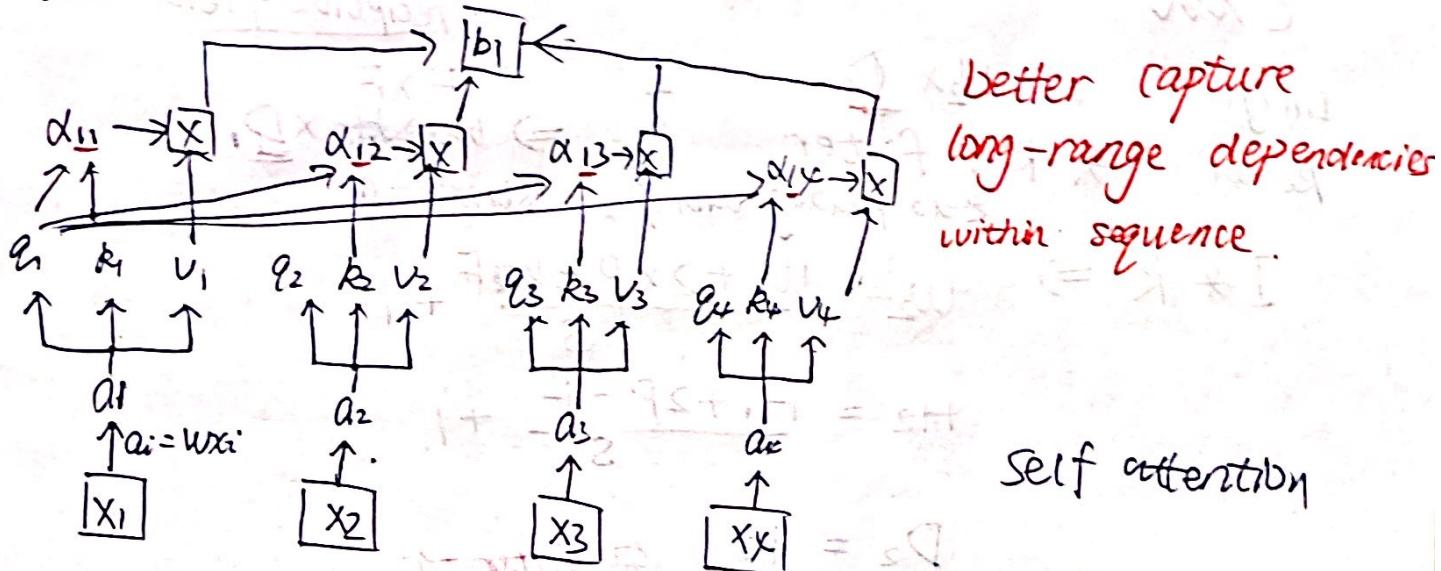
$$T(t) = T(0 \rightarrow t) = T(0 \rightarrow t_n) \cdot T(t_n \rightarrow t)$$

$$\begin{aligned}
 C(t_{n+1}) &= \sum_{n=1}^{N-1} \int_{t_n}^{t_{n+1}} T(t) \sigma_{ct} C(ct) dt \\
 &= \sum_{n=1}^{N-1} \int_{t_n}^{t_{n+1}} T(t_n) T(t_n \rightarrow t) \sigma_n C_n dt \\
 &= \sum_{n=1}^{N-1} T(t_n) \underbrace{\int_{t_n}^{t_{n+1}} T(t_n \rightarrow t) \sigma_n C_n dt}_{\text{constant}} \\
 &\quad \hookrightarrow = \sigma_n C_n \int_{t_n}^{t_{n+1}} \exp\left(-\int_{t_n}^t \sigma_{ct} d\tau\right) dt \\
 &= \sum_{n=1}^{N-1} T(t_n) (1 - \exp(-\sigma_n(t_{n+1} - t_n)) C_n) \\
 &\quad \quad \quad \text{constant} = \int_{t_n}^{t_{n+1}} \exp\left(-\int_{t_n}^t \sigma_{ct} d\tau\right) dt \\
 &= \int_{t_n}^{t_{n+1}} \exp(-\sigma_n(t_n)(t - n)) dt \\
 &= \frac{1}{\sigma_n} \cdot (1 - \exp(-\sigma_n(t_{n+1} - t_n)))
 \end{aligned}$$

SDF. signed distance field.

$$SDF(p) = \text{sign}(p) \cdot \min_{q \in S} \|p - q\|.$$

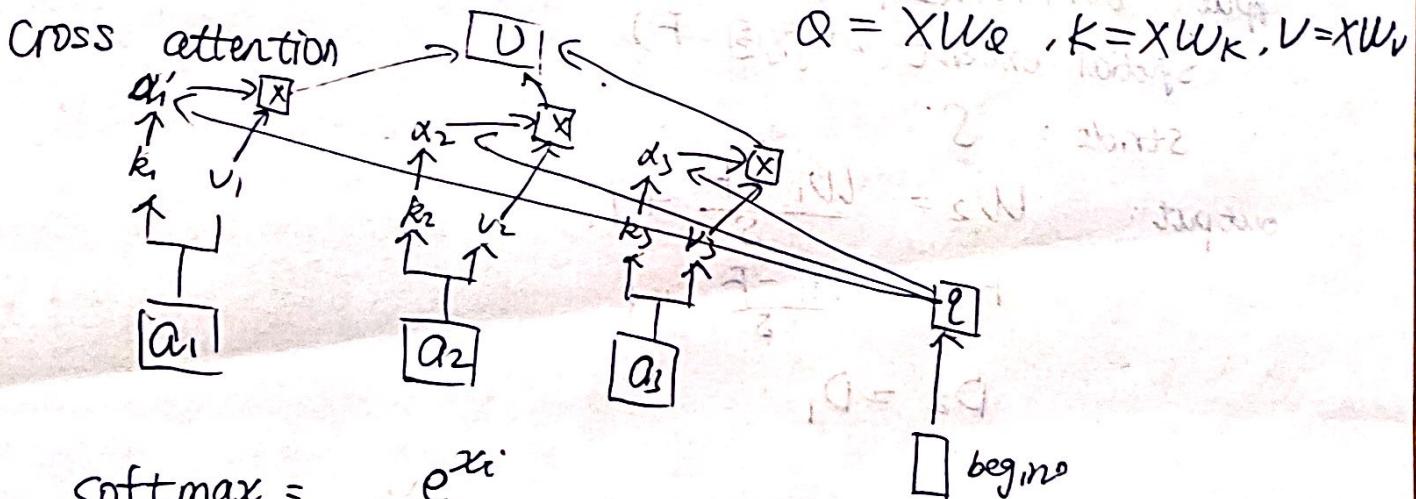
attention



$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{dk}}\right)V$$

output: weighted sum of V

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



adjust focus dynamically

transformer 可起到降低的作用 (局部压缩)

CNN

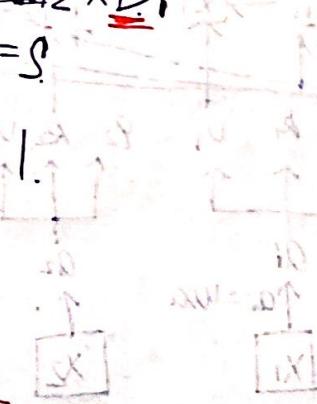
image: $W_1 \times H_1 \times D_1$
 kernel: 共 K 个 filter
 zero padding with P , stride = S

$$I * K \Rightarrow$$

$$W_2 = \frac{W_1 + 2P - F}{S} + 1$$

$$H_2 = \frac{H_1 + 2P - F}{S} + 1$$

$$D_2 = K$$

每个 filter \rightarrow 

每个 kernel: $W_1 \times H_1 \times D_1 \rightarrow$ 单独的数

Pooling: 无参数, 不改变 通道数

input $W_1 \times H_1 \times D_1$

Spatial extent (空间范围) F

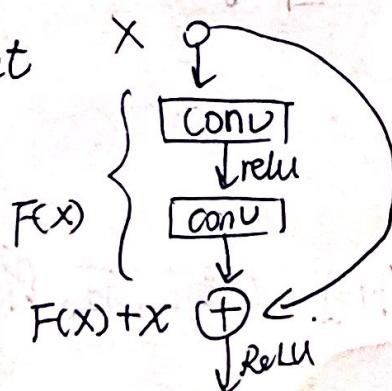
stride: S

output: $W_2 = \frac{W_1 - F}{S} + 1$

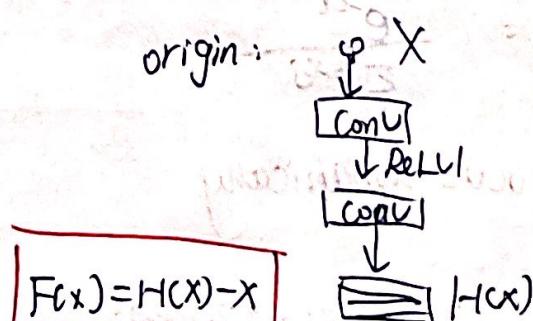
$H_2 = \frac{H_1 - F}{S} + 1$

$D_2 = D_1$

ResNet



origin:



skip connection

不同深度可提 feature

↓
ResNext

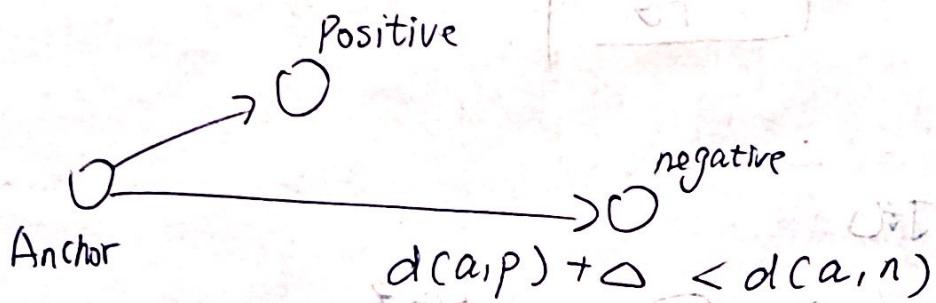
GoogLeNet

deepernet 扩大深度

\Rightarrow computational efficiency

triplet loss

$$L = \max(d(a,p) - d(a,n) + \Delta, 0)$$



Δ margin: 使 a 距 n 尽可能大

fine-grained 增加显著性

category level: 区分人、狗、猫

fine-grained image classification

细密区分: eg. 人脸识别

区分狗的品种

可引入 bias 作为先验.

先用 cross-entropy

后用 triplet loss

image sampling 不可导
 \Rightarrow bilinear interaction \Rightarrow 连续.



depth estimator

用 几何特征 Geometry feature 做约束

做 masked-auto-encoder

$10^6 \sim 10^7$

1. auto encoder.

图像信息 redundant \Rightarrow mask 75% 以上 feature

数据增强

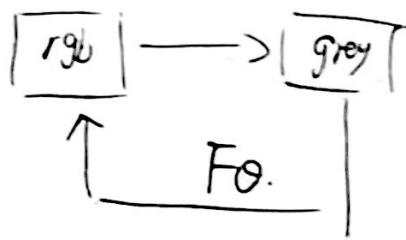
NLP { Bert
GPT

bidirectional encoder represent from transform

Mask 中间，周围推出

given 前，predict 后。

2° coloring



3° rotation

4° 图像切割、打孔



/ Given ①, predict 位置

5° contrastive learning 对比学习 (Self-supervised)



$$\|I_1 - I_2\| < \|I_1 - I_{2'}\|$$

6° supervised learning \Rightarrow knowledge

interaction \Leftrightarrow feedback

7° scale / flip vertically/horizontally

Monodepth \Rightarrow multi-task learning
 匹配 \Leftarrow 难点 独/无纹理、
 重复纹理、动态、遮挡
 occlusion.

简化MLP:

① random connection ② local + global.

③ remove connect, 减少 node, depth, local connection \Rightarrow convolution.

④ knowledge distillation 知识蒸馏

$$\left\| \mathbf{f}_{\text{teacher}}^{\text{student}} - \mathbf{f}_{\text{student}}^{\text{teacher}} \right\|$$