CS 181 Artificial Intelligence (Fall 2022), Midterm Exam

Instructions

- Time: 10:15–11:55am (100 minutes)
- This exam is closed-book, but you may bring one A4-size cheat sheet. Put all the study materials and electronic devices into your bag and put your bag in the front, back, or sides of the classroom.
- You can write your answers in either English or Chinese.
- Two blank pieces of paper are attached, which you can use as scratch paper. Raise your hand if you need more paper.

1 Multiple choice (10 pt)

Each question has one or more correct answers. Select all the correct answer(s). For each question, you get 0 point if you select one or more wrong answers, but you get 0.5 point if you select a non-empty proper subset of the correct answers. Fill your answers in the table below.

1	2	3	4	5

6	7	8	9	10

1. The heuristic path algorithm (Pohl, 1977) is a greedy search algorithm in which the evaluation function is $f(n) = (1-w) \cdot g(n) + w \cdot h(n)$, where $w \in [0,1]$, $g(n)$ is the path cost (backward cost) and $h(n)$ is the heuristic function (forward cost). The costs are always positive. Suppose we only consider tree search. Which of the following statements is/are true?

   A. If h is admissible, then the heuristic path algorithm is always optimal.

   B. If $w = 0$, then the heuristic path algorithm is always optimal.

   C. If $w = 1$, then the heuristic path algorithm is always optimal.

   D. If h is admissible and $w \leq 0.5$, then the heuristic path algorithm is always optimal.

   E. If h is admissible and $w = 0.5$, then the heuristic path algorithm will always return the same path as the DFS algorithm.

2. In class we've talked about k-consistency. About k-consistency, which ONE of the following statements is true?

   A. A CSP is k-consistent if for each k nodes, any consistent assignment to $k-1$ can be extended to the kth node.

   B. A CSP is k-consistent if for each k constraints, any assignments that satisfy $k-1$ of them also satisfy the kth constraint.

   C. A CSP is k-consistent if it has k nodes.

   D. A CSP is k-consistent if it has k nodes and it satisfies all the constraints.

3. Which of the following statements about adversarial search is/are correct?

   A. If the opponent sometimes does not act rationally in the game, then we prefer expectimax algorithm rather than minimax algorithm.

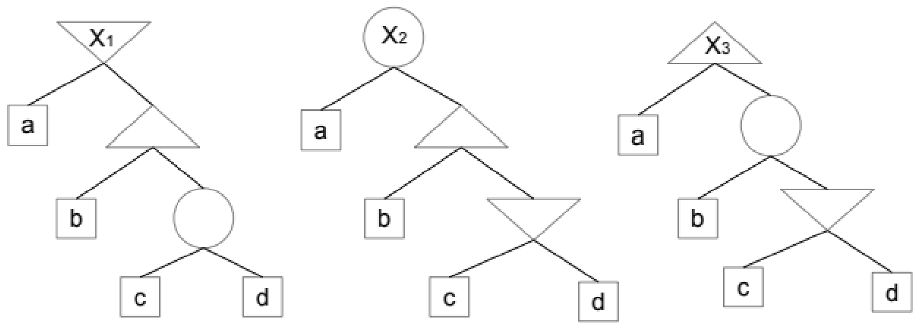
   B. In alpha-beta pruning, the order in which child nodes are evaluated may have a big effect on the effectiveness of pruning.

   C. Both the time complexity and space complexity of minimax algorithm are $O(bm)$, where the maximum depth of the tree is m and there are b legal moves at each point.

   D. If a depth-limit minimax is used as the main algorithm, the extracted policy may not be optimal.

   E. None of the above.

4. Consider three expectimax trees below. All three game trees use the same values at the leaves, represented by a, b, c, and d. The distribution over actions at each chance node (represented by a circle) can be arbitrary. The distributions at chance nodes can also vary between the trees. Which of the following relations can be determined, no matter what values a, b, c and d take?



   A. X1 $\leq$ X3.

   B. X1 $\geq$ X2.

   C. X2 $\geq$ X3.

   D. The relation between X2 and X3 is uncertain.

   E. None of the above.

5. Which of the following statements about propositional logic is/are correct?

   A. If a sentence $\neg S$ is satisfiable, then S is unsatisfiable.

   B. Compared with backward chaining, forward chaining is goal driven and more appropriate for problem-solving.

   C. $A \wedge (B \vee C)$ is equivalent to $(A \wedge B) \vee (A \wedge C)$.

   D. A complete inference means everything that is entailed can be proved.

   E. None of the above.

6. Which of the following statements of equivalence is/are correct?

   A. $A \wedge (B \vee (A \wedge B)) \equiv (A \wedge B)$.

   B. $(A \Rightarrow B) \vee (B \Rightarrow A) \equiv A \vee \neg A$.

   C. $((A \vee B) \wedge (\neg C \vee \neg D \vee E) \Rightarrow (A \vee B) \wedge (\neg D \vee E)) \equiv False$.

   D. $(A \wedge \neg B) \vee (B \wedge \neg A) \equiv (A \vee B) \wedge (A \vee \neg B)$.

   E. None of the above.

7. Which of the following statements about first-order logic is/are correct?

A. "A sibling is another child of one's parents" can be translated as the following FOL expression:
$\forall x, y\ Sibling(x, y) \iff (\neg(x = y) \wedge \exists p\ Parent(p, x) \wedge Parent(p, y))$.

B. $\exists x Likes(x, Beef) \equiv \neg\forall x \neg Likes(x, Beef)$.

C. $\exists x \forall y$ MakeFriend$(x, y) \equiv \forall y \exists x$ MakeFriend(x, y).

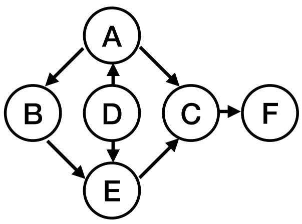
D. "Richard's brothers are John and Geoffrey" can be translated as the following FOL expression:
$\forall x Brother(x, Richard) \Rightarrow (x = John \vee x = Geoffrey)$.

E. None of the above.

8. Given $P(A|B) = P(A|B, C)$ for random variables A, B and C, which of the following expressions hold?

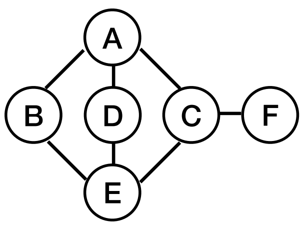
    A. $P(C|B) = P(C|B, A)$.
    B. $P(A, C|B) = \sum_a P(a, C|B) \cdot \sum_c P(A, c|B)$.
    C. $P(A, B, C) = P(A|B)P(C|B)P(B)$.
    D. $P(A, C|B) = P(A|C)P(B|C)$.

9. Consider the Bayes Network below, which of the following statements is/are correct?



    A. $A \perp\!\!\!\perp E\ |B, D$.
    B. $A \perp\!\!\!\perp E\ |B, F, D$.
    C. $A \perp\!\!\!\perp E\ |B, C, D, F$.
    D. $B \perp\!\!\!\perp C\ |A, E$.
    E. $B \perp\!\!\!\perp D\ |A, E$.

10. Consider the Markov Network below, which of the following statements is/are correct?



    A. $A \perp\!\!\!\perp E\ |B, D$.
    B. $A \perp\!\!\!\perp E\ |B, C, D, F$.
    C. This Markov Network encodes the same conditional independence as the Bayesian Network in the previous question.
    D. The maximum clique in this Markov Network is of size 2.
    E. For any Bayesian Network, we can construct a Markov Network that encodes exactly the same conditional independence.

Solution:

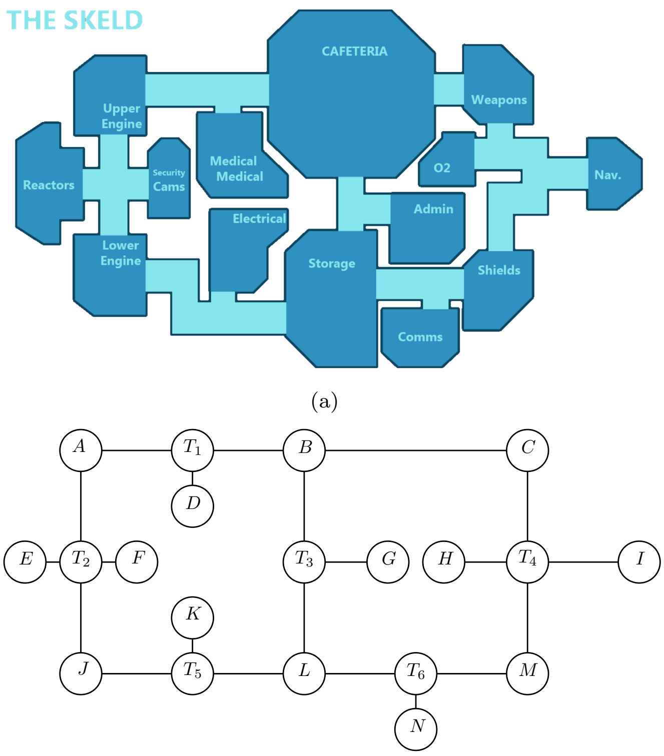
1. BD
2. A
3. ABD
4. AD
5. CD
6. AB
7. ABD or BD
8. ABC
9. AD
10. BD

2 Among Us (10 pt)

Prepare for departure but beware the impostor! You are one of the 5 crewmembers on the spaceship, The Skeld. All crewmates are working together to complete tasks and maintain the spaceship. However, there is one impostor among us bent on killing everyone!

2.1 Maintain The Spaceship (5 pt)

Here is the map of the spaceship, The Skeld. Figure 1(b) is a graph representation of the map. Each node represents a room or a crossing point. The edges represent the corridors.



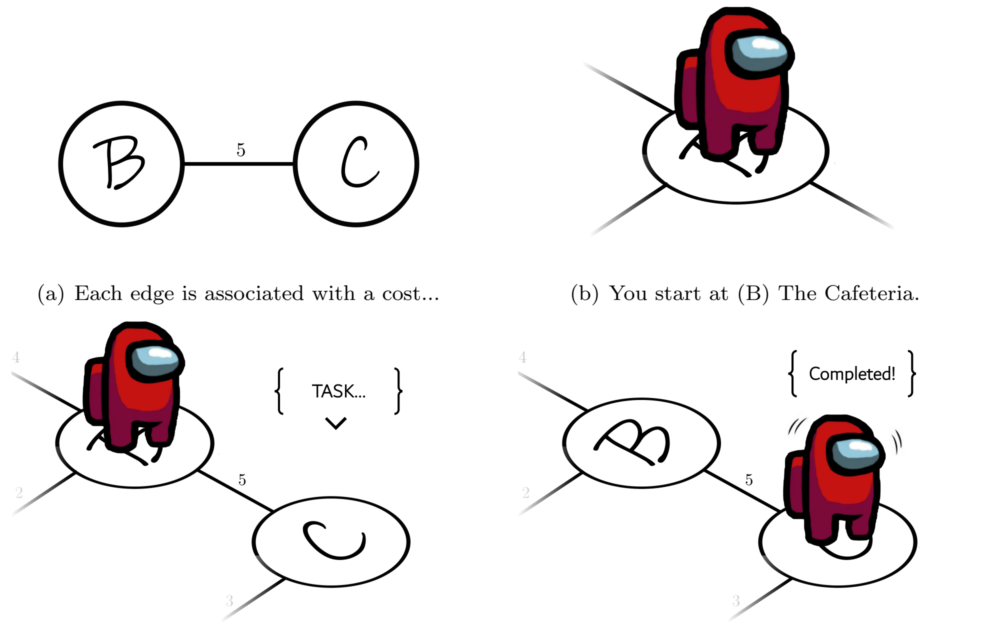
(a)

(b)

Figure 1: Map of The Skeld

To ensure the spaceship working properly, you are required to complete several tasks. These tasks are located at (C) The Weapon Room, (D) The Medical Bay and (K) The Electrical Room. Starting from (B) The Cafeteria, you want to find a way to complete these tasks with the lowest time cost.

Assume that completing each task takes no time. That is, you are only required to visit (C), (D) and (K), respectively. Each edge is associated with a cost representing the time to pass through this edge. Different edges may have different costs. You cannot stay on the edges. If you take an action, you move from one node to another.



(a) Each edge is associated with a cost...

(b) You start at (B) The Cafeteria.

(c) Find a way to complete all the tasks with the lowest cost.

(d) Visit the nodes to complete the tasks. The cost only comes from the edges.

Figure 2: Rules in comics

Formulate this problem as a search problem, then answer the following questions:

2.1.1 The State Space (1pt)

There are 20 nodes in this graph. You are going to complete 3 tasks located at (C), (D) and (K). What is the size of the (search) state space in this question? Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ 20 ○ 60 ○ 160 ○ 8000 ○ 3^{20}

Solution:
160

2.1.2 The Branching Factor (1pt)

What is the branching factor in this question? Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ 1 ○ 2 ○ 3 ○ 4 ○ 8

Solution:
4

2.1.3 A* Search (2pt)

Suppose you plan to solve this problem using the A* search algorithm. Consider the following heuristic function:

$$h(s) = \begin{cases} \max_{i \in r(s)} \text{distance}(i, p(s)) & |r(s)| \geq 1 \\ 0 & |r(s)| = 0 \end{cases}$$

where s is the state, $p(s)$ is your position under state s, $r(s)$ is the set of remaining task nodes under state s, distance(i, j) is the minimal cost from node i to node j.

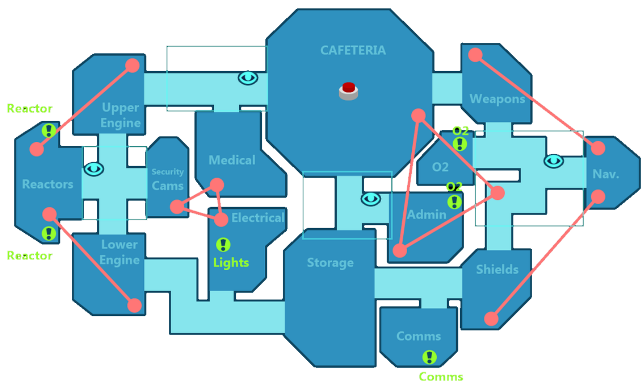
Is this heuristic function admissible? Is it consistent? Choose the correct answer by filling up the square like ■. Ambiguous answers will receive no points.

- ☐ h is admissible.
- ☐ h is not admissible.
- ☐ h is consistent.
- ☐ h is not consistent.

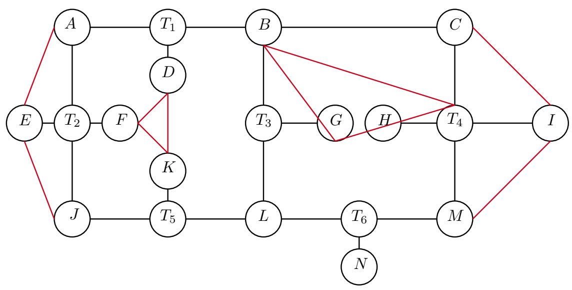
Solution:
AC

2.1.4 The Impostor (1 pt)

Life is easier for the impostor. He may sneak through the vents (secret paths) to quickly move around. Moving through vents has no costs. In Figure 3(a), the red dots show the positions of the vents. The vents are connected by red lines. In Figure 3(b), we connect the nodes that have connected vents with red lines.



(a)



(b)

Figure 3: Map of The Skeld with Vents

The impostor needs to pretend that he is one of the crewmembers, so he will do the same task as you do. The only difference is that he can move through the vents at no cost. Nancy argues that for the impostor, at least one previously consistent heuristic can become inconsistent. Is Nancy right?

Note: we define heuristics for this problem as being a function of the previously defined states: They cannot incorporate anything that does not exist in the crewmember version of the task that we are comparing to.

Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ Yes, Nancy is right.

○ No, Nancy is wrong.

Solution:

Yes, Nancy is right.

2.2 The Emergency Meeting (5 pt)

One crewmember's body is found in (D) The Medical Bay! He was killed by the impostor. All 4 survivors (including the impostor) decide to hold an emergency meeting, attempting to find out who is the impostor.

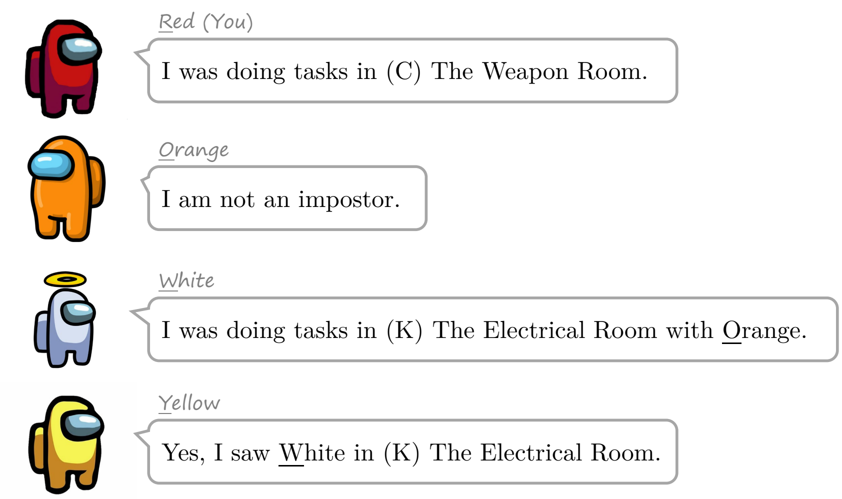


Figure 4: The Emergency Meeting

We already know that:

1. There's one and only one impostor.
2. The impostor's location must be (D) The Medical Bay.
3. Red (you) is not an impostor.

The crewmembers must tell the truth. The impostor may tell the truth or tell lies, so we won't consider his words.

To formulate this problem as a CSP problem, let's define the following 8 variables:

- $I_i \in \{0, 1\}, \forall i \in \{\text{R, O, W, Y}\}$, indicates whether i is an impostor. $I_i = 1$ if and only if i is an impostor.
- $L_i \in \{\text{C, D, K}\}, \forall i \in \{\text{R, O, W, Y}\}$, indicates the location of crewmember i.

Then we can formulate the constraints above:

1. If $I_\text{R} = 0$, then $L_\text{R} = \text{C}$.
2. If $I_\text{W} = 0$, then $L_\text{W} = L_\text{O} = \text{K}$.

3. If $I_{\mathrm{Y}} = 0$, then $L_{\mathrm{W}} = \mathrm{K}$.
4. $\sum_{i \in \{\mathrm{R,\ O,\ W,\ Y}\}} I_i = 1$.
5. $\forall i \in \{\mathrm{R, O, W, Y}\}$, If $I_i = 1$, then $L_i = \mathrm{D}$.
6. $I_{\mathrm{R}} = 0$.

2.2.1 Warm Up (1 pt)

How many unary constraints are there? Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ 0 ○ 1 ○ 2 ○ 3 ○ 4

Solution:
1

2.2.2 Least Constraining Value (1 pt)

Assume that we have assigned values: $I_{\mathrm{R}} = 0$ and $L_{\mathrm{R}} = \mathrm{C}$. Then we decide to apply the Least Constraining Value (LCV) strategy. What should we do? Suppose we are using forward checking as our filtering strategy. Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ We should choose the next variable among I_i, $\forall i \in \{\mathrm{O, W, Y}\}$.

○ We should choose the next variable among L_i, $\forall i \in \{\mathrm{O, W, Y}\}$.

○ If we choose I_{O} as the next variable, then we should assign 0 to it.

○ If we choose I_{O} as the next variable, then we should assign 1 to it.

Solution:
C

2.2.3 Forward Checking & Arc Consistency (2 pt)

Assume that we have assigned values: $I_{\mathrm{R}} = 0$ and $L_{\mathrm{R}} = \mathrm{C}$ and we are running backtracking search. Then we assign 1 to variable I_{W}. Which of the following statements is/are correct? Choose the correct answer(s) by filling up the square like ■. Ambiguous answers will receive no points.

☐ If we apply forward checking as the filtering strategy, we will backtrack before we make the next variable assignment.

☐ If we apply arc consistency as the filtering strategy, we will backtrack before we make the next variable assignment.

☐ None of the above.

Solution:
B

2.2.4 The Impostor (1 pt)

Who is the impostor? Remember that the impostor may also tell the truth! Choose the correct answer by filling up the circle like ●. Ambiguous answers will receive no points.

○ <u>O</u>range ○ <u>W</u>hite ○ <u>Y</u>ellow ○ Cannot be determined

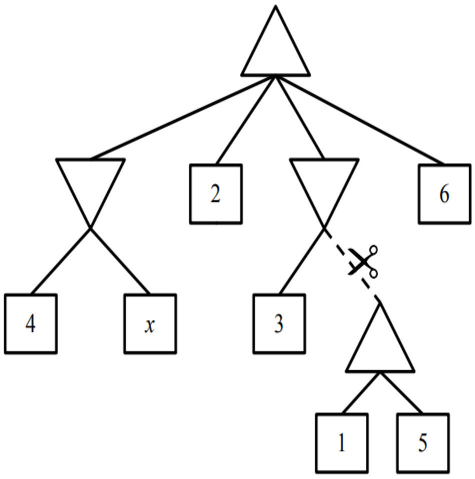
Solution:
<u>Y</u>ellow

3 Tree Pruning! (10 pt)

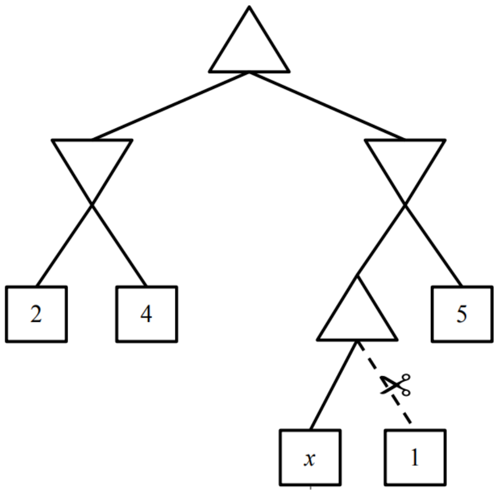
3.1 Alpha-Beta Pruning (3 pt)

For each of the game-trees shown below, please determine the range of values of x so that the branch with a scissor will be pruned. If the pruning will not happen for any value of x, write “none”. If pruning will happen for all values of x, write “all”. Assuming that nodes are evaluated from left to right.

(a) Answer: ______________________________



(b) Answer: ______________________________

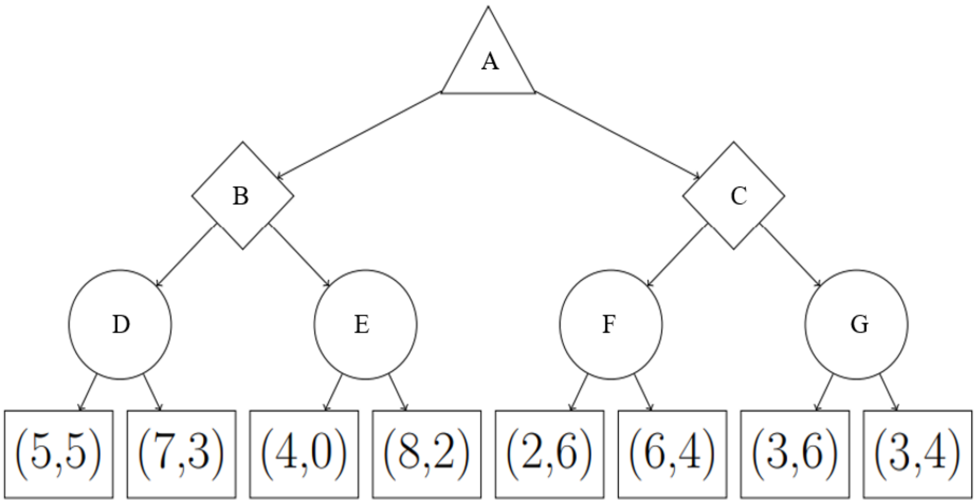


Solution:

(a)x≥3 (b)None

3.2 Pruning in a 2-player game (7 pt)

Consider the following game tree for a two-player game. Each leaf node assigns a score to each player (e.g., the leaf node (7, 3) corresponds to player 1 getting 7 points and player 2 getting 3 points). The values in leaf nodes can take any positive integers. While player 1's goal is to maximize her own score, player 2's goal is to maximize his relative score (i.e., maximizing player 2's score minus player 1's score). An upward-facing triangle represents a node for player 1, a diamond represents a node for player 2, and a circle represents a chance node (where either branch could be chosen with equal probability). Assume that branches are explored left-to-right.



(a) What are the values of all the internal nodes? Hint: The value of each node should be a tuple (e.g., (0,1)) in this question. (3 pt)

A	B	C	D	E	F	G

(b) Does there exist any branch that can be pruned? If so, write down all the branches that can be pruned (e.g., A→B). If not, write "No pruning possible". Hint: Pruning will not occur on a branch from a chance node to a leaf node. (1 pt)

(c) Now we modify the rules a little. We declare that no pair of scores at a leaf node can have a sum greater than 10, and both players know this. Under the modified rules, does there exist any branch that can be pruned? If so, write down all the branches that can be pruned (e.g., A→B). If not, write "No pruning possible". Briefly explain your reason. (3 pt)

Solution:

(a)

A	B	C	D	E	F	G
(6,4)	(6,4)	(3,5)	(6,4)	(6,1)	(4,5)	(3,5)

(b) No pruning possible.

(c) $C \rightarrow G$ can be pruned.

Example reason: From B (6,4) we know the score of player 1 is at least 6. When we reach node C and after visiting F, only when G satisfy score_2-$\text{score}_1 \geq 1$ will we choose it. However, let G=(x, y), to satisfy $x + y \leq 10$, we cannot come up with a value combination to make $x \geq 6$ and $y - x \geq 1$.

4 Logic (10 pt)

4.1 Propositional Logic (4pt)

After learning logic in CS 181, Eric wrote some logic sentences with his friends. However, after two weeks, now they totally forget the meaning of all the propositional symbols that appear in the logic sentences. Although they still remember the corresponding English sentences, they do not know the correspondence between the English and logic sentences (i.e., the English and logic sentences appearing in the same row below may not have the same meaning). Your goal is to figure out each propositional symbol's meaning.

English	Propositional Logic
I do not win the championship	$A \vee \neg B \vee D$
If I am not sick and I drink tea, then I win the championship	$\neg C \vee A$
I drink beer or tea but not both	$\neg D$
If I drink beer, then I am sick	$(B \vee C) \wedge (\neg B \vee \neg C)$

i) A:

ii) B:

iii) C:

iv) D:

Solution:

A: I am sick

B: I drink tea

C: I drink beer

D: I win the championship

4.2 First Order Logic (2pt)

We define the following symbols.

- Job(x,y): Predicate representing person x has job y.
- Friend(x,y): Predicate representing person x is a friend of y.
- Parent(x,y): Predicate representing person x is a parent of y.
- Eric, Mike: Constants representing people's names
- Nurse, Doctor: Constants representing people's jobs

Please translate the following sentences to first order logic.

i) Mike has a friend called Eric who is a doctor.

ii) Any nurse is not a parent of Mike.

Solution:

$Job(Eric, Doctor) \wedge Friend(Eric, Mike)$

$\forall x\ Job(x, Nurse) \Rightarrow \neg Parent(x, Mike) or \neg \exists x Job(x, Nurse) \wedge Parent(x, Mike)$

4.3 Forward and Backward Chaining (2pt)

Suppose we have the following clauses in a knowledge base:

- $Student(x) \wedge Fail(x, z) \wedge Friend(x, y) \wedge Pass(y, z) \Rightarrow Cry(x)$
- $Sober(x) \wedge Exam(y) \Rightarrow Pass(x, y)$
- $Sleepy(x) \wedge Exam(y) \Rightarrow Fail(x, y)$
- $Student(x) \wedge Play(x, y) \Rightarrow Sleepy(x)$
- $Exam(CS181)$
- $Student(Eric)$
- $Play(Eric, GameX)$
- $Friend(Eric, Mike)$
- $Sober(Mike)$

Suppose we want to prove $Cry(Eric)$.

i) How many new facts (e.g., $Sleepy(Eric)$) need to be added into KB during forward chaining.

ii) How many goals or subgoals (e.g., $Student(x)$) need to be created during backward chaining.

Solution:

4

10

4.4 CNF Conversion (2pt)

Convert the following sentence into CNF:

$\forall x\ [\forall y\ A(y) \vee B(x, y)] \Rightarrow [\exists y\ C(y, x) \wedge B(x, y)]$.

Note: When needed, use $F(\cdots), G(\cdots), \cdots$ to represent Skolem functions during existential instantiation.

Solution:

$(\neg A(F(x)) \vee C(G(x), x))$

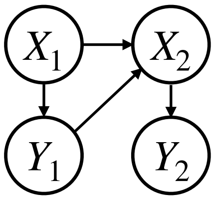
$\wedge(\neg B(x, F(x)) \vee C(G(x), x))$

$\wedge(\neg A(F(x)) \vee B(x, G(x)))$

$\wedge(\neg B(x, F(x)) \vee B(x, G(x)))$

5 Bayesian Network (10 pt)

Consider the following Bayesian network associated with binary random variables X_1, X_2, Y_1, Y_2. Answer the following questions. (Note: $+x_1$ represents $X_1 =$ true and $-x_1$ represents $X_1 =$ false)



$P(X_1)$	
$+x_1$	0.5
$-x_1$	0.5

$P(Y_i \mid X_i)$, $i = 1, 2$		
$+y_i$	$+x_i$	0.4
$-y_i$	$+x_i$	0.6
$+y_i$	$-x_i$	0.2
$-y_i$	$-x_i$	0.8

$P(X_2 \mid X_1, Y_1)$			
$+x_2$	$+x_1$	$+y_1$	0.5
$-x_2$	$+x_1$	$+y_1$	0.5
$+x_2$	$+x_1$	$-y_1$	0.4
$-x_2$	$+x_1$	$-y_1$	0.6
$+x_2$	$-x_1$	$+y_1$	0.4
$-x_2$	$-x_1$	$+y_1$	0.6
$+x_2$	$-x_1$	$-y_1$	1.0
$-x_2$	$-x_1$	$-y_1$	0

5.1 Markov network (1pt)

Convert the given Bayesian network to a Markov network and draw it below.

Solution:
Directed arrows to undirected edges.

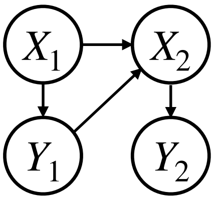
5.2 Exact inference (6pt)

5.2.1 Calculate $P(+y_2 \mid +x_2, +x_1, +y_1)$ (1pt)

Solution:
0.4 by conditional independence

5.2.2 Calculate $P(+y_2 \mid +x_1, +y_1)$ (2pt)

Solution:
two methods: 1. by enumeration $P(+y_2 \mid +x_1, +y_1) \propto \sum_{x_2} P(+y_2, +x_1, +y_1, x_2)$
2. $P(+y_2 \mid +x_1, +y_1) = \sum_{x_2} P(+y_2 \mid x_2) P(x_2 \mid +y_1, +x_1)$, simpler.
same results: 0.3



$P(X_1)$	
$+x_1$	0.5
$-x_1$	0.5

$P(Y_i \mid X_i)$, $i = 1, 2$		
$+y_i$	$+x_i$	0.4
$-y_i$	$+x_i$	0.6
$+y_i$	$-x_i$	0.2
$-y_i$	$-x_i$	0.8

$P(X_2 \mid X_1, Y_1)$			
$+x_2$	$+x_1$	$+y_1$	0.5
$-x_2$	$+x_1$	$+y_1$	0.5
$+x_2$	$+x_1$	$-y_1$	0.4
$-x_2$	$+x_1$	$-y_1$	0.6
$+x_2$	$-x_1$	$+y_1$	0.4
$-x_2$	$-x_1$	$+y_1$	0.6
$+x_2$	$-x_1$	$-y_1$	1.0
$-x_2$	$-x_1$	$-y_1$	0

5.2.3 Steps to calculate $P(+y_2)$ using variable elimination. (3pt)

You should first list the initial factors. For each variable elimination step, you need to show **the random variable that you pick and eliminate, and the formula (e.g., sums and products) of obtaining a new factor from related factors**.
Note: You do not need to really calculate the value of $P(+y_2)$. Just provide the correct steps and you will get full points.

Solution:

- Initial factors: $P(+y_2|X_2), P(Y_1|X_1), P(X_1), P(X_2|X_1, Y_1)$
- choose to eliminate hidden r.v. Y_1, $P(X_2|X_1) = \sum_{y_1} P(y_1|X_1)P(X_2|X_1, y_1)$
- resulting factors: $P(+y_2|X_2), P(X_2|X_1), P(X_1)$
- choose to eliminate hidden r.v. X_1, $P(X_2) = \sum_{x_1} P(X_2|x_1)P(x_1)$
- resulting factors: $P(+y_2|X_2), P(X_2)$
- choose to eliminate hidden r.v. X_2, $P(+y_2) = \sum_{x_2} P(+y_2|x_2)P(x_2)$

Note: it is OK to not write explicit distributions, e.g., writing $f(X_1, X_2)$ instead of $P(X_2|X_1)$.

5.3 Approximate Inference

5.3.1 Prior Sampling (1pt)

If we use prior sampling for X_1, X_2, Y_1, Y_2, what is the expected number of times we get $(+x_1, -x_2, +y_1, -y_2)$ if we produce 1000 samples?

Solution:
$P(+x_1, -x_2, +y_1, -y_2) * 1000 = 80$

5.3.2 Likelihood Weighting (2pt)

We want to use likelihood weighting to approximate $P(+x_1| + y_1, -y_2)$. Write down the weights of the following samples, and use them to estimate $P(+x_1| + y_1, -y_2)$.

$+x_1$	$+x_2$	$+y_1$	$-y_2$	______
$-x_1$	$+x_2$	$+y_1$	$-y_2$	______

The estimated $P(+x_1| + y_1, -y_2)$ is ______.

Solution:
$0.4 * 0.6 = 0.24$
$0.2 * 0.6 = 0.12$
$2/3$