CS 181 Artificial Intelligence (Fall 2021), Final Exam

Instructions

- Time: 10:30 – 12:10 (100 minutes)
- This exam is closed-book, but you may bring one A4-size cheat sheet. Put all the study materials and electronic devices (except a calculator) into your bag and put your bag in the front, back, or sides of the classroom.
- Two blank pieces of paper are attached, which you can use as scratch paper. Raise your hand if you need more paper.

1 Multiple choices (10 pt)

Each question has one or more correct answers. Select all the correct answers. For each question, you get 1 point if your select all the correct answers and nothing else, 0 point if you select one or more wrong answers, and 0.5 point if you select a non-empty proper subset of the correct answers.

1	2	3	4	5

6	7	8	9	10

1. Which of the following statements about Markov models, hidden Markov models (HMM), and Dynamic Bayes Nets (DBN) is/are correct?
   A. For most Markov chains, the stationary distribution is independent of the initial distribution.
   B. For HMMs, the forward algorithm and the Viterbi algorithm have the same time complexity.
   C. Every HMM is a DBN, but not every discrete DBN can be represented as a HMM.
   D. Particle filtering is preferable to exact HMM inference when the state space is large and speed is more important than accuracy.
   E. None of the above.

2. Which of the following statements are true for MDPs?
   A. MDPs are non-deterministic search problems.
   B. When solving MDPs, the value of γ (discount factor) can change the speed of learning, but the final policy that would be learnt remains the same.
   C. Value Iteration is guaranteed to converge to unique optimal values.
   D. When running Value Iteration, the values often converge before the policy.
   E. None of the above.

3. Which of the following is/are not given in Online Learning compared with Offline Planning?

   A. Set of states S.
   B. Set of actions A.
   C. Transition model for states and actions.
   D. Reward function for every allowed transition between states.
   E. All of the above.

4. One could simplify the particle filtering algorithm by getting rid of the resampling step and instead keeping weighted particles all the time, with the weight of a particle being the product of all observation probabilities $P(E_i|X_i)$ up to and including the current timestep. Which of the following statements is/are true?

   A. This will always work as well as standard particle filtering.
   B. This will generally work less well than standard particle filtering because all the particles will cluster in the most likely part of the state space.
   C. This will generally work less well than standard particle filtering because most particles will end up in low-likelihood parts of the state space.
   D. This will generally work less well than standard particle filtering because the number of particles you have will decrease over time.
   E. None of the above.

5. Which of the following statements about Reinforcement Learning are true?

   A. Q-Learning is a passive reinforcement learning method.
   B. Direct Evaluation follows a policy π and makes use of the information about states connections. It is slow because it requires enough samples in order to converge to the true values of states.
   C. When running Q-Learning with sub-optimal actions, it may still converge to the optimal policy.
   D. When running Q-Learning, using exploration functions often leads to lower regret than using random exploration.
   E. None of the above.

6. Which of the following statements about overfitting and underfitting is/are correct?

   A. Overfitting is more likely to occur when training examples are not representative of the true data distribution.
   B. Limiting the model expressiveness is an approach to avoiding overfitting.
   C. Underfitting is more likely to occur when there are insufficient training examples.
   D. The characteristic of underfitting is that it fits the training data very closely, but does not generalize well.
   E. None of the above.

7. Which of the following statements about the Naive Bayes classification model is/are correct? There are n feature variables and one label variable. Both feature and label variables are discrete and have a constant domain size.

   A. The total number of parameters in the Naive Bayes model is linear in n.
   B. The total number of parameters in the Naive Bayes model is exponential in n.
   C. The goal of inference in Naive Bayes is to compute the posterior distribution over the label variable.
   D. An important assumption in Naive Bayes is that the features are independent given the label variable.
   E. None of the above.

8. Which of the following statements about K-means is/are correct?

   A. The result of K-means is not influenced by the initial center points.

   B. K-means converges when there is no longer any point changing the center it is assigned to.

   C. K-means is an unsupervised learning algorithm.

   D. K-means is guaranteed to converge to a global optimum.

   E. None of the above.

9. Which of the following statements about sequence labeling is/are **incorrect**?

   A. "我骑车差点摔倒，好在我一把把把把住了" This example demonstrates the importance of contextual information in sequence labeling.

   B. Using a hidden Markov model (HMM) for sequence labeling is better than simply predicting the most frequent label for each word because HMM does not consider relations between adjacent labels.

   C. Max Entropy Markov Models (MEMM) suffer from the label bias problem.

   D. Conditional Random Fields (CRF) are undirected graphical models.

   E. None of the above.

10. Recall that in sequence labeling, HMM inference uses the Viterbi algorithm to find the most likely label sequence of the input sentence. What if we use the Forward algorithm? What does it output? Only ONE answer is correct.

    A. It outputs the most likely label sequence of the input sentence.

    B. It outputs the most unlikely label sequence of the input sentence.

    C. It outputs the most likely word sequence of the label sequence.

    D. It outputs the posterior distribution over the last label given the input sentence.

    E. The output of the Forward algorithm does not make sense at all.

Solution:

1. ABD
2. AC
3. CD
4. C
5. CD
6. AB
7. ACD
8. BC
9. B
10. D

2 Hidden Markov Models (10 pt)

Consider a Markov model with a binary state X (i.e., X_t is either 0 or 1). $P(X_0 = 0) = 0.4$ and $P(X_0 = 1) = 0.6$. The transition probabilities are given as follows:

X_t	X_{t+1}	$P(X_{t+1}\|X_t)$
0	0	0.6
0	1	0.4
1	0	0.3
1	1	0.7

Note: keep three decimal places in your calculation.

(a) Calculate $P(X_1)$. (2pt)

(b) What is the stationary distribution, i.e., $P(X_\infty)$? (2pt)

Now we incorporate binary sensor readings E_t (note that we are starting from E_0). The emission probabilities are given as follows:

X_t	E_t	$P(E_t\|X_t)$
0	0	0.6
0	1	0.4
1	0	0.7
1	1	0.3

(c) If $E_0 = 1, E_1 = 1$, calculate $P(X_1|E_0, E_1)$. Write down all the steps to get full points. (3pt)

(d) If $E_0 = 1, E_1 = 1, E_2 = 0$, what is the most likely sequence X_0, X_1, X_2? Write down all the steps to get full points. (3pt)

Solution:

(a) $P(X_1 = 1) = \sum_{X_0} P(X_0)P(X_1 = 1|X_0) = 0.4 * 0.4 + 0.6 * 0.7 = 0.58$

$P(X_1 = 0) = 1 - 0.58 = 0.42$

(b) $P(X_\infty = 1) = 0.3 * P(X_\infty = 1) + 0.4 * P(X_\infty = 0) \rightarrow P(X_\infty = 1) = \frac{4}{7}P(X_\infty = 0)$

$P(X_\infty = 0) + P(X_\infty = 1) = 1$

hence, $\frac{11}{7}P(X_\infty = 0) = 1 \rightarrow P(X_\infty = 0) = \frac{7}{11}, P(X_\infty = 1) = \frac{4}{11}$

(c)

First step (1pt):

$P(X_0 = 1|E_0 = 1) \propto P(X_0 = 1)P(E_0 = 1|X_0 = 1) = 0.6 * 0.3 = 0.18\alpha$

$P(X_0 = 0|E_0 = 1) \propto P(X_0 = 0)P(E_0 = 1|X_0 = 0) = 0.4 * 0.4 = 0.16\alpha$

The second step (1pt):

$P(X_t|E_{0:t}) \propto P(E_t|X_t) \sum_{X_{t-1}} P(X_{t-1}|E_{0:t-1})P(X_t|X_{t-1})$

then, $P(X_1 = 1|E_{0:1}) \propto 0.3 * (0.16 * 0.4 + 0.18 * 0.7) = 0.057\alpha$

$P(X_1 = 0|E_{0:1}) \propto 0.4 * (0.16 * 0.6 + 0.18 * 0.3) = 0.06\alpha$

Normalization (1pt):

Hence $P(X_2 = 1|E_{0:2}) = 0.057/(0.057 + 0.06) = 0.48717948717949,$

$P(X_2 = 0|E_{0:2}) = 0.51282051282051$

(d)

step(0) $m(X_0 = 1|E_0 = 1) \propto P(X_0 = 1)P(E_0 = 1|X_0 = 1) = 0.6 * 0.3 = 0.18\alpha$

$m(X_0 = 0|E_0 = 1) \propto P(X_0 = 0)P(E_0 = 1|X_0 = 0) = 0.4 * 0.4 = 0.16\alpha$

step1: (1pt)

$m(X_1 = 1|E_{0:1}) = 0.3 * \max(0.16 * 0.4, 0.18 * 0.7) = 0.0378\alpha$, $(\text{argmax } X_0 = 1)$

$m(X_1 = 0|E_{0:1}) = 0.4 * \max(0.16 * 0.6, 0.18 * 0.3) = 0.0384\alpha$, $(\text{argmax } X_0 = 0)$

step2: (1pt)

$m(X_2 = 1|E_{0:2}) = 0.7 * \max(0.0384 * 0.4, 0.0378 * 0.7) = 0.018522\alpha$, $(\text{argmax } X_1 = 1)$

$m(X_2 = 0|E_{0:2}) = 0.6 * \max(0.0384 * 0.6, 0.0378 * 0.3) = 0.013824\alpha$, $(\text{argmax } X_1 = 0)$

backtracking (1pt):

$m(X_2 = 1|E_{0:2}) > m(X_2 = 0|E_{0:2})$, hence, $X_2 = 1$

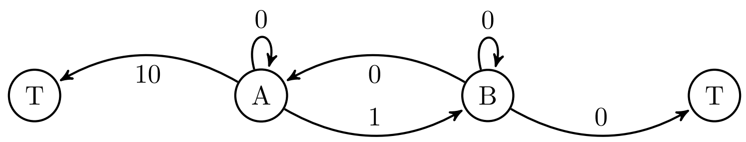
then, $X_1 = 1, X_0 = 1$.

Answer: 1, 1, 1

3 MDP and RL: Left or Right (10 pt)

3.1 MDP (6 pt)

Consider the following MDP.



The state space $\mathcal{S}$ and action space $\mathcal{A}$ are:

$$\mathcal{S} = \{A, B, T\}$$
$$\mathcal{A} = \{left, right\}$$

T denotes the terminal state (the two T states in the figure are the same). When in a terminal state, the agent has no more action to take and gets no more reward. In a non-terminal state (A or B), the agent can only go left or right, but its action only succeeds (going in the intended direction) with probability p_1. With probability p_2, it goes in the opposite direction. With probability $1 - p_1 - p_2$, it does not move. In the figure, the numbers on the arrows denote the reward associated with going from one state to another (regardless of the action taken).

For example, if the agent is at state A and takes action $left$:

- with probability p_1, the next state will be T and the agent will get a reward of 10. The episode is then terminated.
- with probability p_2, the next state will be B and the reward will be 1.
- with probability $1 - p_1 - p_2$, the next state will still be A and the reward will be 0.

For this problem, the discount factor γ is 1. Let π_p^* be the optimal policy, which may or may not depend on the value of p_1 and p_2. Let $Q^{\pi_p^*}$ and $V^{\pi_p^*}$ be the corresponding Q and V functions of π_p^*.

(a) If $p_1 = 1, p_2 = 0$, what is π_p^*?

- ☐ $\pi_p^*(A) = left$, $\pi_p^*(B) = left$
- ☐ $\pi_p^*(A) = left$, $\pi_p^*(B) = right$
- ☐ $\pi_p^*(A) = right$, $\pi_p^*(B) = left$
- ☐ $\pi_p^*(A) = right$, $\pi_p^*(B) = right$

(b) If $p_1 = 0, p_2 = 1$, what is π_p^*?

- ☐ $\pi_p^*(A) = left$, $\pi_p^*(B) = left$
- ☐ $\pi_p^*(A) = left$, $\pi_p^*(B) = right$
- ☐ $\pi_p^*(A) = right$, $\pi_p^*(B) = left$
- ☐ $\pi_p^*(A) = right$, $\pi_p^*(B) = right$

(c) Suppose $\pi_p^*(A) = right$. Which of the following statements must be true? (Select all that apply)
Hint: If $x = y$, then $x \geq y$ and $x \leq y$.

- ☐ $Q^{\pi_p^*}(A, left) \leq Q^{\pi_p^*}(A, right)$
- ☐ $Q^{\pi_p^*}(A, left) \geq Q^{\pi_p^*}(A, right)$

- ☐ $Q^{\pi_p^*}(A, left) = Q^{\pi_p^*}(A, right)$
- ☐ $V^{\pi_p^*}(A) \leq Q^{\pi_p^*}(A, left)$
- ☐ $V^{\pi_p^*}(A) \geq Q^{\pi_p^*}(A, left)$
- ☐ $V^{\pi_p^*}(A) = Q^{\pi_p^*}(A, left)$
- ☐ $V^{\pi_p^*}(A) \leq Q^{\pi_p^*}(A, right)$
- ☐ $V^{\pi_p^*}(A) \geq Q^{\pi_p^*}(A, right)$
- ☐ $V^{\pi_p^*}(A) = Q^{\pi_p^*}(A, right)$

(d) Assume $p_1 + p_2 = 1, p_1 \geq 0.5$ below:

(i) $V^{\pi_p^*}(B) = \alpha V^{\pi_p^*}(A) + \beta$. Find α and β in terms of p_1.

- $\alpha =$ ____________
- $\beta =$ ____________

(ii) $Q^{\pi_p^*}(A, left) = \alpha V^{\pi_p^*}(B) + \beta$. Find α and β in terms of p_1.

- $\alpha =$ ____________
- $\beta =$ ____________

(iii) $Q^{\pi_p^*}(A, right) = \alpha V^{\pi_p^*}(B) + \beta$. Find α and β in terms of p_1.

- $\alpha =$ ____________
- $\beta =$ ____________

Solution:

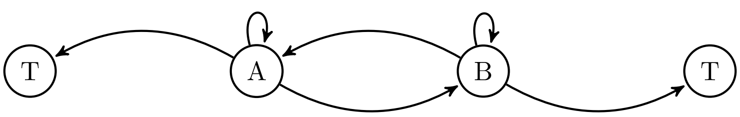
(a) C

(b) B

(c) AEGHI

(d) $\alpha = p_1$, $\beta = 0$ $\quad$ $\alpha = 1 - p_1$, $\beta = 9p_1 + 1$ $\quad$ $\alpha = p_1$, $\beta = 10 - 9p_1$

3.2 Reinforcement Learning (4 pt)



In this part, we are exploring an MDP model with the same structure as above and unknown transition and reward functions. So we will learn the underlying MDP model from experience. Suppose the agent chooses actions based on some policy π and generates the following samples.

t	s_t	a_t	s_{t+1}	r_t
1	B	$left$	B	-1
2	B	$left$	A	2
3	A	$right$	B	1
4	B	$left$	A	2
5	A	$right$	T	5

Assume a discount factor $\gamma = 0.5$ and a learning rate $\alpha = 0.2$.

3.2.1 Model-Based Learning

In model based learning, we first need to estimate the transition model and reward function. Based on the samples above, estimate the following parameters. (Note: p_1 and p_2 have the same definition as in 3.1)

$$p_1 = \_\_\_\_\_\_\_\_\_\_ \qquad p_2 = \_\_\_\_\_\_\_\_\_\_$$

3.2.2 Model-Free: Q-learning

Assume that all the Q-values are initialized to 0. Apply Q-learning we learnt in class. What are the Q-values learned by running Q-learning with all the transitions shown above? Note that we update Q-values immediately after we receive a sample.

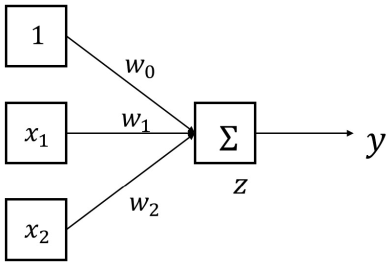
$$Q(B, left) = \_\_\_\_\_\_\_\_\_\_ \qquad Q(A, right) = \_\_\_\_\_\_\_\_\_\_$$

Solution:

$p_1 = 0.6, p_2 = 0.2$ $\qquad$ $Q(B, left) = 0.6144, Q(A, right) = 1.1792$

4 Machine Learning - Perceptron (10 pt)

Consider the following perceptron, for which the inputs are the always-one feature and two binary features $x_1 \in \{0,1\}$ and $x_2 \in \{0,1\}$. The output is $y \in \{0,1\}$.



$$y = \begin{cases} 1 & \text{if } (w_0 + w_1 \cdot x_1 + w_2 \cdot x_2) > 0 \\ 0 & \text{otherwise} \end{cases}$$

4.1 Forward & Backward Propagation (2 × 2pt)

(i) Calculate the output if the weight vector $[w_0\ w_1\ w_2] = [1\ -1\ 2]$ and input vector $[x_1\ x_2] = [1\ 2]$

Your answer: __________

(i) Say z is an intermediate variable after calculating summation: $z = w_0 + w_1 \cdot x_1 + w_2 \cdot x_2$. What is the derivative of z with respect to x_1 and x_2 respectively?

Your answer: __________; __________

4.2 Weights (2 × 3pt)

(i) Which of the following choices for the weight vector $[w_0\ w_1\ w_2]$ can classify y as $y = (x_1$ OR $x_2)$? Here OR refers to the logical OR operation.

A. [1 1 0]
B. [-1.5 1 1]
C. [-2 1 1.5]
D. Any weights that satisfy $(-w_1 - w_2) < w_0 < \min(0, -w_1, -w_2)$.
E. None of the above.

Your answer: __________

(ii) Which of the following choices for the weight vector $[w_0\ w_1\ w_2]$ can classify y as $y = (x_1$ AND $x_2)$? Here AND refers to the logical AND operation.

A. [1 1 0]
B. [-1.5 1 1]
C. [-2 1 1.5]
D. Any weights that satisfy $(-w_1 - w_2) < w_0 < \min(0, -w_1, -w_2)$.
E. None of the above.

Your answer: __________

Solution:

Answer: 1
Answer w_1, w_2
Answer: E
Answer: BCD

5 The Unusual Hilichurl (10 pt)

Hey there, Travelers! The Mimi Tomo event is about to begin!

Hilichurls are a goblin-like family of monsters speaking the Hilichurlian language. A certain "Unusual Hilichurl" has been spotted gathering its fellow hilichurls out in the wild recently... Katheryne of the Adventurers' Guild is thus calling on Travelers to investigate this matter. Together with Hilichurlian expert Ella Musk and your companion Paimon, find hilichurls that are open to communication and ask them for more information on the whereabouts of the Unusual Hilichurl!

Ella Musk (Scholar of Hilichurlian Linguistics)

Good morning, traveler! Oh, you've found a hilichurl?

Paimon (Your Companion)

Yes! But we don't know how to talk to him... We'll be glad if you could help!

Ella Musk (Scholar of Hilichurlian Linguistics)

Ye muhe dala? Mimi mani ye.

Lonely Hilichurl

Ya tomo mani, mani yaya tome nye... Mi muhe du, du celi lata gusha.

You (The Traveler)

What does it say?

Ella Musk (Scholar of Hilichurlian Linguistics)

It appears to want... some sort of objects? But I'm not sure... Could you please check the "Handy Handbook of Hilichurlian" I wrote? I remember there's a simple Probabilistic Context-Free Grammar (PCFG) for Hilichurlian noun phrases (NP).

You (The Traveler)

Sure. Here it is!

0.6:	NP → CD AdjString Noun
0.4:	NP → CD Nominal
0.5:	AdjString → Adj AdjString
0.5:	AdjString → Λ
1.0:	Nominal → Adj NounNounCompound
1.0:	NounNounCompound → Noun Noun
0.8:	CD → *unu*
0.2:	CD → *du*
0.5:	Adj → *celi*
0.5:	Adj → *lata*
0.6:	Noun → *lata*
0.4:	Noun → *gusha*

Table 1: The PCFG in Musk's handbook. "Λ" denotes the empty string. For example, string "Adj Λ" is equivalent to string "Adj", and string "CD Λ Noun" is equivalent to string "CD Noun".

5.1 The Longest NP (2pt)

Paimon (Your Companion)

Wow, that seems interesting! I am wondering what is the longest NP that can be generated by this grammar?

You (The Traveler)

That's easy. It's ...

Instruction: Please select the proper response from the candidates below by filling up the circle like ●, not ✓:

- ○ three words.
- ○ four words.
- ○ infinitely many words.

Solution:

infinitely many words.

5.2 Convert to Chomsky Normal Form (2pt)

Ella Musk (Scholar of Hilichurlian Linguistics)

Good job. But in order to parse the Hilichurl's words, we first need to convert this PCFG into Chomsky Normal Form (CNF). Could you help me with that?

You (The Traveler)

Sure! Glad to help.

Instruction: Please complete the converted grammar by filling in the blanks. Please derive the answers based on the equivalence of the two grammars, instead of directly applying the conversion method taught in class.

_[1]_:	NP → CD X
_[2]_:	NP → CD Noun
0.4:	NP → _[3]_ _[4]_
0.5:	X → Adj Noun
0.5:	X → Y Noun
0.5:	Y → Adj Y
0.5:	Y → Adj Adj
1.0:	Nominal → Adj NounNounCompound
1.0:	NounNounCompound → Noun Noun
0.8:	CD → *unu*
0.2:	CD → *du*
0.5:	Adj → *celi*
0.5:	Adj → *lata*
0.6:	Noun → *lata*
0.4:	Noun → *gusha*

Table 2: Converted grammar in CNF

[1] ______ [2] ______

[3] ______ [4] ______

Solution:

0.3:	NP $\rightarrow$ CD X
0.3:	NP $\rightarrow$ CD Noun
0.4:	NP $\rightarrow$ CD Nominal
0.5:	X $\rightarrow$ Adj Noun
0.5:	X $\rightarrow$ Y Noun
0.5:	Y $\rightarrow$ Adj Y
0.5:	Y $\rightarrow$ Adj Adj
1.0:	Nominal $\rightarrow$ Adj NounNounCompound
1.0:	NounNounCompound $\rightarrow$ Noun Noun
0.8:	CD $\rightarrow$ *unu*
0.2:	CD $\rightarrow$ *du*
0.5:	Adj $\rightarrow$ *celi*
0.5:	Adj $\rightarrow$ *lata*
0.6:	Noun $\rightarrow$ *lata*
0.4:	Noun $\rightarrow$ *gusha*

Table 3: Converted grammar in CNF

5.3 The Parse Tree (2pt)

Ella Musk (Scholar of Hilichurlian Linguistics)

Now we can parse his words! Erm... What did he say?

Paimon (Your Companion)

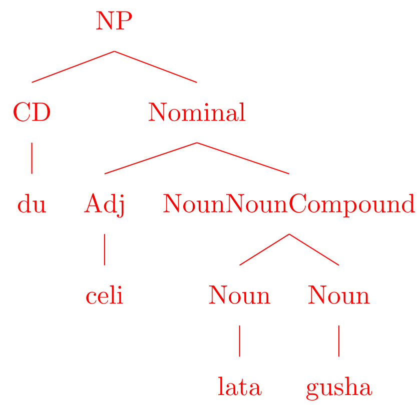
I think it's *du celi lata gusha*.

You (The Traveler)

Okay. I'll try to parse this phrase.

Instruction: Please draw a complete constituency parse tree based on the grammar you've just constructed. If multiple parse trees are possible, draw the one with the highest probability. Remember we are parsing a noun phrase, so the root of the parse tree is NP instead of S.

Solution:



5.4 The Probability (2pt)

Paimon (Your Companion)

Then, what is the probability of generating this phrase? I mean, *du celi lata gusha*.

You (The Traveler)

Let me see. It's...

Instruction: Please give a proper response to your companion's question:

Solution:

0.0096 + 0.0015 = 0.0111

Paimon (Your Companion)

Paimon would like to remind you that there might be multiple trees corresponding to this phrase...

You (The Traveler)

I know that!

5.5 Convert to a Dependency Tree (2pt)

Ella Musk (Scholar of Hilichurlian Linguistics)

Aha! I've found the constituency parse tree of this phrase in my handbook.

You & Paimon

Oh?!

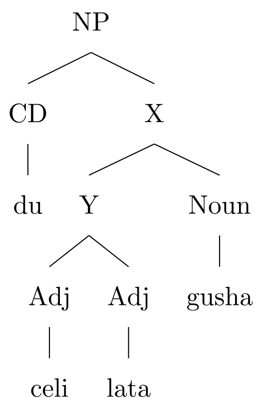


Figure 1: Constituency Parse Tree

Ella Musk (Scholar of Hilichurlian Linguistics)

I want to turn it into a dependency parse tree. Would you like to help me?

Paimon (Your Companion)

But Paimon thinks we cannot build a dependency parse tree for now... We may need more information.

You (The Traveler)

I think...

Instruction: Please select the proper response from the candidates below by filling up the circle like ●, not ✓:

○ Paimon is right. We need more information to convert this tree into a dependency parse tree.

○ we can build a dependency parse tree just based on this constituency parse tree. There exists one and only one dependency parse tree given this constituency parse tree, since dependency grammars are a subclass of CFGs.

Solution:

Paimon is right.

(End of this problem)

Ella Musk (Scholar of Hilichurlian Linguistics)

Fine. I get it. *celi lata gusha* refers to a plant called "Small Lamp Grass".

You (The Traveler)

I have that kind of plant in my package. Wait a minute... I'll share some with him.

Lonely Hilichurl

Yeye dada! Mosi mita! Odomu tomo zido mi!

Ella Musk (Scholar of Hilichurlian Linguistics)

Ya zido? Ye? Mi?

Lonely Hilichurl

Mi! Mi!

Ella Musk (Scholar of Hilichurlian Linguistics)

It seems to be happy. It shows us the way to find the Unusual Hilichurl.

You & Paimon

Great! Let's go!