ShanghaiTech University

# CS240 Algorithm Design and Analysis

## Spring 2024

## Problem Set 4

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Due: 23:59, May 31, 2024

1. Submit your solutions to the course Gradescope.
2. If you want to submit a handwritten version, scan it clearly.
3. Late homeworks submitted within 24 hours of the due date will be marked down 25%. Homeworks submitted more than 24 hours after the due date will not be accepted unless there is a valid reason, such as a medical or family emergency.
4. You are required to follow ShanghaiTech's academic honesty policies. You are allowed to discuss problems with other students, but you must write up your solutions by yourselves. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious penalties.

## Problem 1:

Given a set $C$, a collection of subsets of $C$ and an integer $k \geq 1$, the Set-Packing problem asks if there are $k$ subsets from the collection which are pairwise disjoint (i.e. no two sets share an element). Show that the Set-Packing problem is NP-complete.

**Solution**:

## Problem 2:

Given a Boolean CNF (conjunctive normal form) formula $\phi$ and an integer $k \geq 1$, the Stingy-SAT problem asks whether the formula has a satisfying assignment in which at most $k$ variables are set to true. Prove that Stingy-SAT is NP-complete.

**Solution**:

## Problem 3:

Given a set $C$, a collection of subsets of $C$ and an integer $k \geq 1$, the Set-Cover problem asks whether there are at most $k$ subsets from the collection which cover $C$, i.e. whose union includes all of $C$. Show that Set-Cover is NP-complete. Do not use a reduction from a problem which is very similar to Set-Cover.

**Solution**:

## Problem 4:

Consider a list with $n$ positive integers, and suppose we iterate through the numbers one by one in a random order. As we do this, we maintain a variable $b$ equal to the largest number we have seen so far. Initially $b$ is set to 0 . Compute the expected number of times $b$ is updated.

For example, if the input list is $[4, 7, 5]$, and we iterate through the list in the order $3, 1, 2$, then $b$ is updated twice, when we see 4 and 7 .

**Solution**: