CS270 Digital Image Processing Final Project

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***Abstract*—We have reproduced a novel digital image processing technique aimed at improving the temporal resolution of echocardiographic imaging by incorporating a two-dimensional (2D) spatiotemporal meshless interpolation method using radial basis functions (RBFs). We developed an approach that initially downsamples echocardiography data from 1000 Hz to various lower rates (100 Hz, 250 Hz, and 500 Hz), then applies a 2D RBF interpolation for upsampling, effectively increasing the frame rate. The novel interpolation strategy, termed Intensity Variation Time Surface (IVTS), integrates both spatial and temporal information to reconstruct the frame sequences with enhanced temporal resolution.**

***Index Terms*—Echocardiography, Temporal Super-Resolution, Radial Basis Functions, Digital Image Processing, Spatio-Temporal Interpolation, Meshless Interpolation**

I. Introduction

Echocardiography is a crucial diagnostic tool in cardiology, providing real-time imaging of the heart to assess its structure and function. The quality of echocardiographic imaging is highly dependent on its temporal resolution, which is determined by the frame rate of the imaging system. Higher frame rates are essential for capturing fast-moving cardiac events but are often limited by the physical and technical constraints of ultrasound systems [1]. Traditional methods to enhance frame rates involve hardware modifications or complex software algorithms that can degrade image quality or increase computational costs.

Ultrasound localization microscopy (ULM) [2] breaks the diffraction limit resolution, improving the spatial resolution by 10 times in ultrasound imaging. However, the images must be acquired at a very high frame rate imposing high requirements on the ultrasound hardware. In this project, we will implement an interpolation method to ease this requirement on a high frame rate.

Recent advancements in digital image processing have introduced the possibility of enhancing image resolution through numerical methods, which offer a promising alternative to these traditional approaches. This project focuses on a novel 2D spatiotemporal interpolation technique [3] using radial basis functions (RBFs) [4] to increase the frame rate of echocardiographic data. By downsampling the original data to lower frequencies (100 Hz, 250 Hz, and 500 Hz) and then applying a meshless interpolation method, we aim to reconstruct the high-frequency data with enhanced temporal resolution [5].

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The innovative aspect of this approach lies in the use of an Intensity Variation Time Surface (IVTS), which allows for the integration of spatial and temporal information in the interpolation process. This meshless method provides flexibility in handling large datasets and facilitates a more straightforward mathematical implementation. Our results indicate that this method not only improves the temporal resolution [6] but also maintains or enhances the image quality compared to traditional frame rate enhancement techniques. This introduction sets the stage for a detailed presentation of our methodology, experimental setup, and a thorough analysis of our results, which collectively demonstrate the efficacy of our approach in real-world diagnostic applications.

We implemented the radial basis function 2D interpolation to upsample the acquired ultrasound dataset for establishing the Time Efficient Ultrasound Localization Microscopy method.

II. Related Work

The development of spatio-temporal interpolation methods for ultrasound imaging has been pivotal in addressing the limitations posed by high frame rate requirements. Tuccio et al. [4] introduced Radial Basis Function (RBF) 2D interpolation to enhance ultrasound localization microscopy (ULM), enabling the capture of super-resolved images at significantly reduced frame rates. Their method demonstrated that detailed vascular structures could be accurately reconstructed even when data was acquired at sub-100Hz levels.

Similarly, Jalilian et al. [7] applied RBF-based interpolation to echocardiography, allowing for high-resolution imaging from lower frame rates, which is crucial for detailed cardiac function analysis without the need for high-speed hardware [8].

Our project builds on these technologies by employing a combination of radial basis functions for both spatial and temporal enhancement across varied downsampling scenarios (100Hz, 250Hz, and 500Hz) from a high-frequency dataset (1000Hz). This approach is aimed at maintaining the clarity and detail necessary for accurate medical diagnosis, particularly in dynamic imaging processes like cardiac monitoring. Through the integration of robust interpolation techniques, this may be used for seeking to extend the practical applications of ULM in clinical settings later, where rapid and detailed imaging is essential.

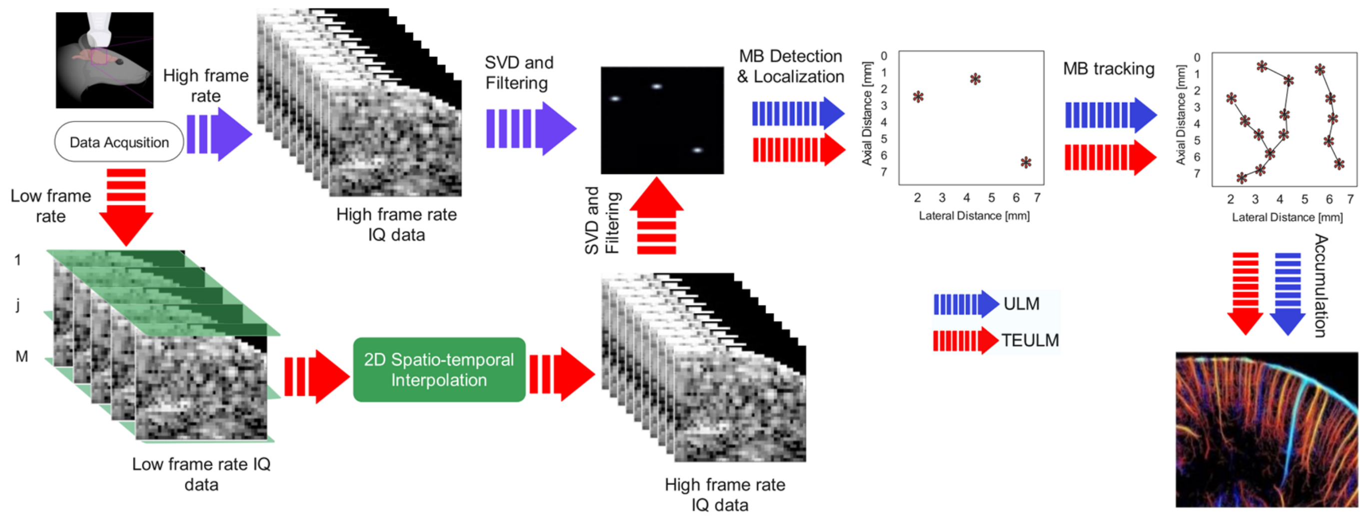

Fig. 1. Pipeline of the difference between Ultrasound Localization Microscopy (ULM) and Time Efficient Ultrasound Localization Microscopy (TEULM) methods.

III. Methods

Unlike traditional one-dimensional interpolation methods that only focus on time, the method we use takes into account both spatial and temporal dimensions of information, which is a two-dimensional interpolation method. Traditional methods only target one pixel, while our method can directly process the entire row and effectively balance spatial information. We will provide a detailed introduction to the specific methods of 2D Spatial-temporal Interpolation in Fig. 1

A. Data Preparation

Due to the high requirements of equipment and storage space for high sampling frequencies, our project aims to use data obtained at lower sampling frequencies to restore the same results as data sampled at the original frequency. When the down sample rate(DS) > 1, we need to change the maximum linking distance and minimum track length to adjust the new DS. If the new DS=k:

$$\text{new\_max\_linking\_dis} = k * \text{old\_max\_linking\_dis}$$

$$\text{new\_min\_track\_len} = d\frac{1}{k} * \text{old\_min\_track\_len}$$

These adjustments are made to maintain fairness when comparing downsampled data with the original data. The number of data frames after downsampling decreases. If parameters are not adjusted, it may lead to imbalanced or misleading results in comparison. Therefore, by adjusting the maximum link distance and minimum track length, it is possible to compare data at different downsampling levels more fairly.

B. Radial Basis Function

We have selected several commonly used RBFs:

- Gaussian (GA)
  Gaussian functions are infinitely smooth, making them suitable for interpolation problems that require high smoothness. Due to its fast decay property, Gaussian RBF has high interpolation accuracy when processing high-dimensional data. But at the same time, it consumes a large amount of computing resources.

$$\text{GA}(r) = e^{-cr^2}$$

- Multiquadric (MQ)
  Although it is not as smooth as Gaussian functions, MQ still has good smoothness. MQ has good applicability to different types of datasets, especially for non-uniform data distributions.

$$\text{MQ}(r) = \sqrt{1 + (cr)^2}$$

- Inverse multiquadric (IMQ)
  The smoothness of IMQ lies between Gaussian functions and multiple binomials. In some cases, IMQ may have better numerical stability than GA.

$$\text{IMQ}(r) = \frac{1}{\sqrt{1 + (cr)^2}}$$

C. Interpolation Formula

The first step is that we need to build the $\mathbf{\Phi}$ matrix, and we must ensure that the phi matrix is invertible. In our code, we calculate the Euclidean distance between two data points, so that we can build the r matrix.The element $r_{i,j}$ means the distances of X_i and X_j.

The interpolation basis function is defined as:

$$\phi(r_j) \equiv \phi(\|\mathbf{X} - \mathbf{X}_j\|) = \text{RBF}(\|\mathbf{X} - \mathbf{X}_j\|)$$

After we get $\phi(r)$ and the exact RBF that we have chosen, we can now construct linear combinations. Given data points $(\mathbf{X}_j, \mathbf{Y}_j)$, where $\mathbf{X}_j$ is the spatial coordinate, $\mathbf{Y}_j$ is the corresponding strength value, and our goal is to find a continuous

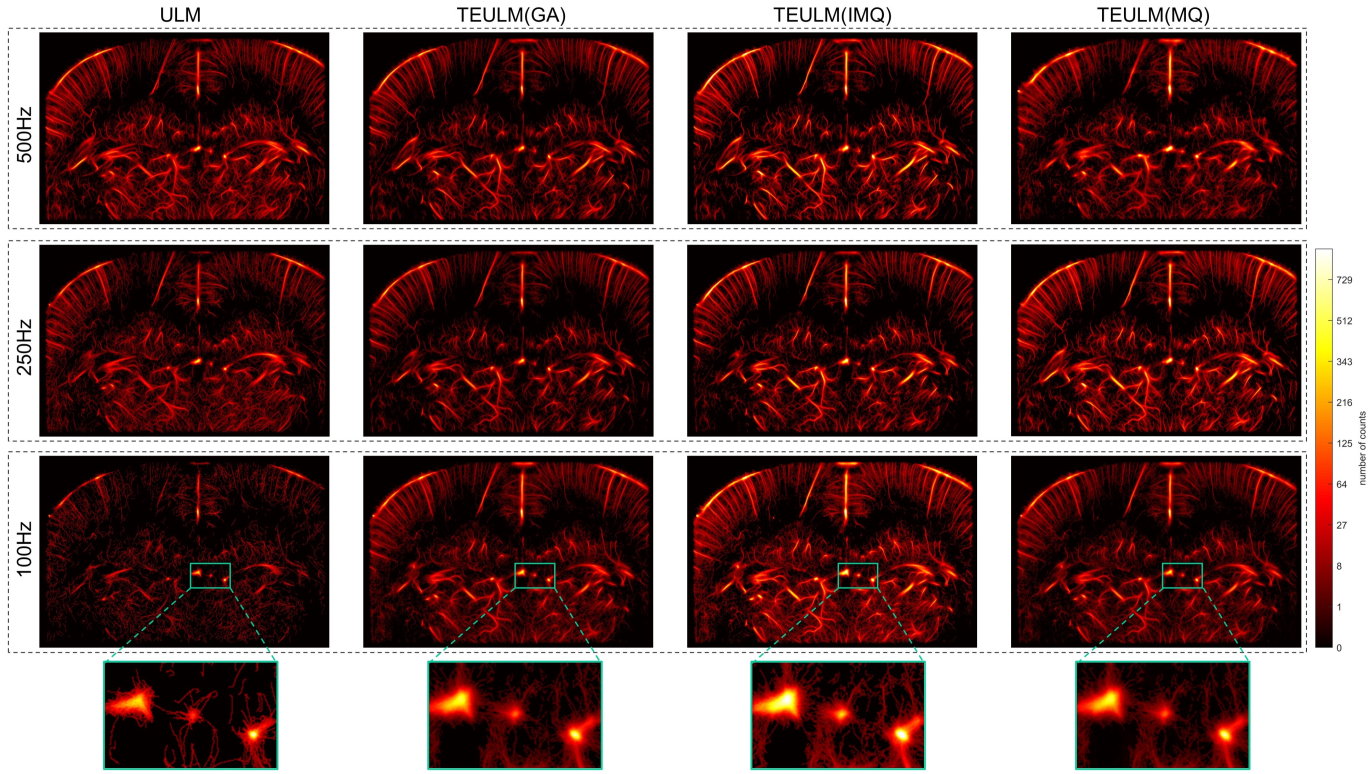

Fig. 2. Comparison of ULM and TEULM under different kernel functions

function $S(X)$, such that $S(X_j) = Y_j$ is satisfied for all data points. So we have:

$$S(\mathbf{X}) = \sum_{j=1}^{N} \beta_j \phi(\|\mathbf{X} - \mathbf{X}_j^c\|_2)$$

where β_j is the interpolation coefficient we need.To find β_j,we use the following interpolation condition:

$$S(\mathbf{X}_j) = f(\mathbf{X}_j), j = 1, 2, \ldots, N$$

By solving a system of linear equations:

$$\mathbf{\Phi}_{N\times N}\mathbf{B}_{N\times 1} = \mathbf{F}_{N\times 1}$$

where $\mathbf{\Phi}, \mathbf{B}, \mathbf{F}$ can be expanded as follows:

$$\Phi = \begin{bmatrix} \phi(r_{1,1}) & \phi(r_{1,2}) & \cdots & \phi(r_{1,N}) \\ \phi(r_{2,1}) & \phi(r_{2,2}) & \cdots & \phi(r_{2,N}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(r_{N-1,1}) & \phi(r_{N-1,2}) & \cdots & \phi(r_{N-1,N}) \\ \phi(r_{N,1}) & \phi(r_{N,2}) & \cdots & \phi(r_{N,N}) \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} f(X_1) \\ f(X_2) \\ \vdots \\ \vdots \\ f(X_N) \end{bmatrix}$$

So the main computational workload is focused on how to calculate the inverse of $\mathbf{\Phi}$.

D. Get New Coefficients Matrix

Once we get $\mathbf{\Phi}^{-1}$,then we can get $\mathbf{B}$,where :

$$\mathbf{B} = \mathbf{\Phi}^{-1}\mathbf{F}$$

Then we need to build the new coefficients matrix, the matrix entries must be constructed as follows:

$$h_{ij} = \phi(\|\mathbf{X}_i - \mathbf{X}_j^c\|_2), \forall j \in 1, \ldots, N.$$

After we use the above formula to calculate all the data points, we put all the h_{ij} together to make a new matrix $\mathbf{H}$, then we calculate the interpolated data.

$$\mathbf{F}_{\text{est}} = \mathbf{HB} = \mathbf{H\Phi^{-1}F}$$

where $\mathbf{F}_{\text{est}}$ represents the interpolated data we finally need.

IV. Experiments

We have explored recovering the Ultrasound images with the method Time Efficient Ultrasound Localization Microscopy (TEULM) from different downsample rates (2 times, 4 times, and 10 times, representing the correspondence lower frequencies 500Hz, 250Hz, and 100Hz). The comparison was done between the downsampled data and reconstruction data, as shown in Fig.2. We also tried different Radial basis functions (RBFs) with suitable parameters.

In the actual experiments, we utilized PyTorch to offload the interpolation operations to the GPU for accelerated computation. During the calculation of the phi matrix and its inverse, due to the large size of the matrices, we employed a combination of CPU and GPU to complete this part.

A. Metrics

- Root Mean Square Error (RMSE): The similarity between the original image, reconstructed at 1000Hz (i.e. the ground truth), and the images interpolated at different DS rates, with $DS = 2, 4, 10$, is assessed pixel-by-pixel using the root mean square error:

$$RMSE = \sqrt{\sum_{i=1}^{N_{pix}} \frac{(|I_{DS} - I_1|)^2}{N_{pix}}} \tag{1}$$

where N_{pix} denotes the total number of pixels in the reconstructed image.
- DICE score: The $DICE$ score evaluates the similarity between two images by measuring the overlapping area of two images over the area of their union [9]:

$$DICE = \frac{2 \times |I_{DS} \cap I_1|}{|I_{DS}| \cup |I_1|} \tag{2}$$

The $DICE$ score quantitatively assesses the resemblance of the reconstructed binary images generated by ULM and TEULM across various downsample rates. To generate the binary images from the result, we convert the RGB image into a gray level image (with range [0,255]) and set all pixels with intensity > 1 to be 1, and intensity ≤ 1 to be 0, which is a binary image. Then we use the converted binary image to compute the DICE score.

We applied interpolation with different radial basis functions mentioned in III-B. And compared them with the ground truth (constructed result without being downsampled). The reconstruction results use metrics RMSE and DICE score mentioned in IV-A to show the quantities. The DICE scores are shown in table I, and the RMSE are shown in table II.

B. Ablation study

The data we used contains two parts: real and imaginary. In implementation, we found that interpolation can be performed on the real and imaginary parts together or separately. Therefore, we conducted experiments on the interpolation method using the RBF basis function as GA. Here, we present the results and metrics at 500Hz as an example.

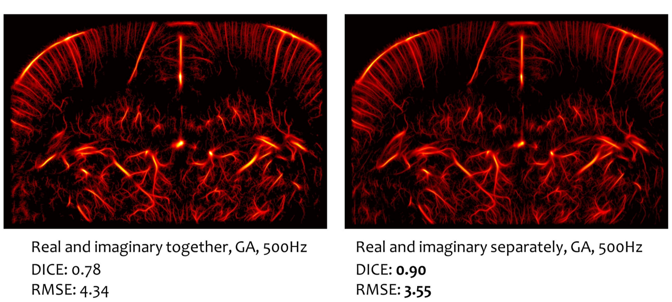

Fig. 3. Comparison of different interpolation method

We found that under the same conditions, separating the real and imaginary parts for interpolation has a better performance.

TABLE I
DICE SCORE OF DIFFERENT DOWNSAMPLE RATES WITH ULM METHOD AND TEULM WITH DIFFERENT RADIAL BASIS FUNCTIONS. WHICH IS COMPUTED WITH THE FORMULA 2.

Methods	RBFs	DS **rates**		
		2	4	10
ULM	-	0.81	0.71	0.40
TEULM	GA	**0.90**	0.82	0.77
TEULM	IMQ	0.88	0.78	0.79
TEULM	MQ	0.89	0.87	0.78

TABLE II
RMSE OF DIFFERENT DOWNSAMPLE RATES WITH ULM METHOD AND TEULM WITH DIFFERENT RADIAL BASIS FUNCTIONS. WHICH IS COMPUTED WITH THE FORMULA 1.

Methods	RBFs	DS **rates**		
		2	4	10
ULM	-	4.21	4.46	4.69
TEULM	GA	**3.55**	4.54	4.79
TEULM	IMQ	4.20	4.30	5.12
TEULM	MQ	3.65	4.21	4.52

V. CONCLUSION

In this project, we employed spatio-temporal interpolation to preprocess and reconstruct microbubble data, effectively restoring the original sampling frequency. This algorithm reduces the sampling frequency, thereby lowering the demands on equipment and storage space in practical applications. We conducted experiments at 100Hz, 250Hz, and 500Hz, comparing the GA, IMQ, and MQ basis functions. The results showed that the GA basis function at 500Hz achieved the best performance. During the experiments, we discovered that the impact of hyperparameters on the kernel function might be a significant factor influencing the results of spatio-temporal interpolation.

Additionally, we identified potential issues with matrices during the actual interpolation process, such as precision errors in the inversion of large, ill-conditioned matrices, leading to them being deemed non-invertible. Therefore, we also experimented with a series of methods, such as introducing hyperparameters and adding a regularization term, to make them invertible.

REFERENCES

[1] Botelho CMA, Pena JLB, Passos BR, Fortes PRL, Moreira MCV. Chronic Obstructive Pulmonary Disease: The Role of Myocardial Deformation Indices and Right Ventricle Three-Dimensional Echocardiography. Int. J. Cardiovasc. Sci. 2022;35(3):329-3.

[2] B. Heiles et al. "Performance benchmarking of microbubble-localization algorithms for ultrasound localization microscopy." Nature Biomedical Engineering (2022): 1-12.

[3] D. P. Perrin, N. V. Vasilyev, G. R. Marx, and P. J. del Nido, "Temporal enhancement of 3d echocardiography by frame reordering," *JACC: Cardiovascular Imaging*, vol. 5, no. 3, pp. 300–304, 2012.

[4] G. Tuccio, S. Afrakhteh, G. Iacca and L. Demi, "Time Efficient Ultrasound Localization Microscopy Based on A Novel Radial Basis Function 2D Interpolation," in IEEE Transactions on Medical Imaging, vol. 43, no. 5, pp. 1690-1701, May 2024, doi: 10.1109/TMI.2023.3347261. keywords: Interpolation;Standards;Location awareness;Ultrasonic imaging;Spatial resolution;Rats;Pipelines;Frame rate;super resolution;TEULM;ULM;2D interpolation;RBFs,

[5] H. Jalilian, S. Afrakhteh, G. Iacca, and L. Demi, “Increasing frame rate of echocardiography based on a novel 2d spatio-temporal meshless interpolation,” *Ultrasonics*, vol. 131, p. 106953, 2023.
[6] Andersen SB, Taghavi I, Hoyos CAV, et al. Super-Resolution Imaging with Ultrasound for Visualization of the Renal Microvasculature in Rats Before and After Renal Ischemia: A Pilot Study. Diagnostics (Basel). 2020;10(11):862. Published 2020 Oct 22. doi:10.3390/diagnostics10110862
[7] S. Afrakhteh, H. Jalilian, G. Iacca, and L. Demi, “Increasing the frame rate of echocardiography images based on a novel interpolation technique,” in *2022 IEEE International Ultrasonics Symposium (IUS)*, 2022, pp. 1–4.
[8] “Temporal super-resolution of echocardiography using a novel high-precision non-polynomial interpolation,” *Biomedical Signal Processing and Control*, vol. 78, p. 104003, 2022.
[9] Y.-H. Nai, B. W. Teo, N. L. Tan, S. O’Doherty, M. C. Stephenson, Y. L. Thian, E. Chiong, and A. Reilhac, “Comparison of metrics for the evaluation of medical segmentations using prostate mri dataset,”Computers in Biology and Medicine, vol. 134, p. 104497, 2021.