Name:
ID:
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Digital Image Processing

Quiz 3

Problem 1: Thresholding (27 pts)

(a) Thresholding is a straightforward and efficient method for segmenting images into distinct regions based on their intensity values. Among the following options, which may affect the accuracy of thresholding ? Select all options that match the description. (6 pts)

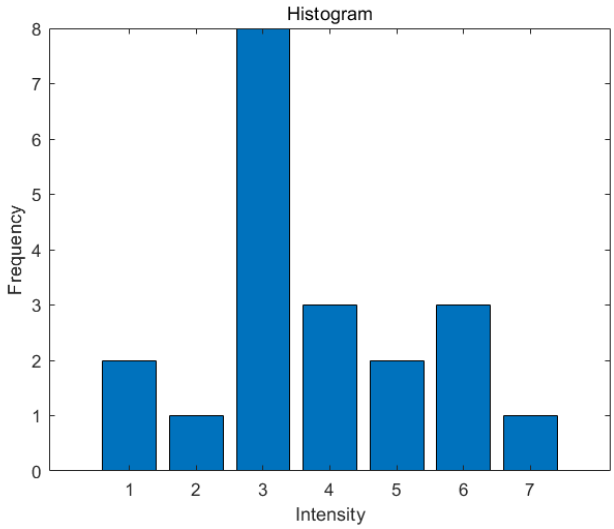
A. The uniformity of the illumination source

B. The noise content in the image

C. Histogram distribution of images

D. The relative sizes of objects and background

(b) Given the histogram of an image, apply Otsu's method to determine the optimal threshold and subsequently calculate the between-class variance, to reduce the amount of calculation, only the intensity from 3 to 6 (included) should be considered as threshold. Ensure that all calculations are precise to two decimal places. (21 pts)



Solution:

(a) A.B.C.D. (6 pts)

(b) The optimal threshold is 4, corresponding between class variance is 1.86 (1 pts).

$$p_1 = \sum_{i=1}^{k} p_i$$

$$p_2 = \sum_{i=k+1}^{L-1} p_i$$

$$m_1 = \frac{1}{p_1} \sum_{i=1}^{k} i p_i$$

$$m_2 = \frac{1}{p_2} \sum_{i=k+1}^{L-1} i p_i$$

$$\sigma_B^2 = p_1 p_2 (m1 - m2)^2$$

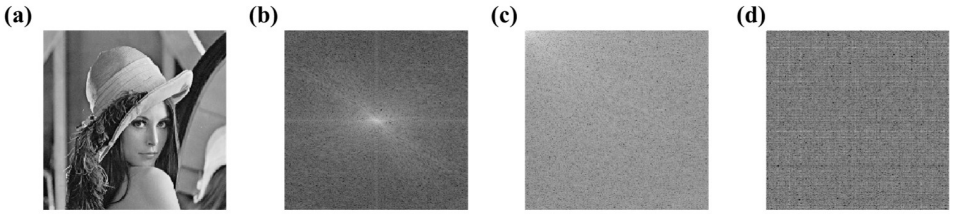
k = 3, $p_1 = 0.55$, $p_2 = 0.45$, $m_1 = 2.55$, $m_2 = 5.22$, $\sigma_B = 1.77$ (5 pts)
k = 4, $p_1 = 0.70$, $p_2 = 0.30$, $m_1 = 2.86$, $m_2 = 5.83$, $\sigma_B = 1.85$ (5 pts)

k = 5, $p_1 = 0.80$, $p_2 = 0.20$, $m_1 = 3.13$, $m_2 = 6.25$, $\sigma_B = 1.56$ (5 pts)

k = 6, $p_1 = 0.95$, $p_2 = 0.05$, $m_1 = 3.58$, $m_2 = 7.00$, $\sigma_B = 0.56$ (5 pts)

Problem 2: Image transforms (25 pts)

(a) The images (b-d) are obtained by applying different types of transformations to the original image (a). Please provide the name of each transformation corresponding to the images (b-d), and describe where the average brightness (direct component) of the transformed image is concentrated. (9 pts)



(b) Given an eight-point signal $x(n) = [3, 8, 8, 7, 6, 3, 2, 1]$, calculate the Haar wavelet transform with three-level operation. The answer format is: [c_3, d_3, $d_2(1)$, $d_2(2)$, $d(1)$, $d(2)$, $d(3)$, $d(4)$]. (16 pts)

Solution:

(a) (b) DFT (1 pts), (c) DCT (1 pts) (d) Hadamard (1 pts)

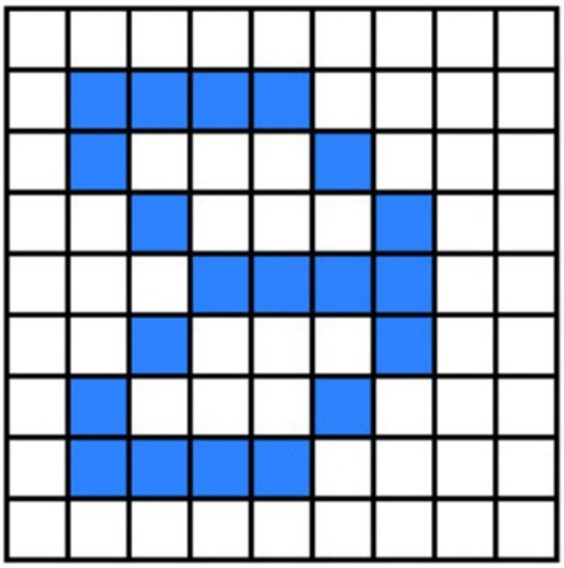
(b) Center of spectrum (2 pts), (c) the top left corner (2 pts), (d) first pixels (on the top left corner). (2 pts)

(b) [13.44,4.95,-2,3,-3.54,0.70,2.12,0.71] or [4.75,1.75,-1,1.5,-2.5,0.5,1.5,0.5] (16 pts)

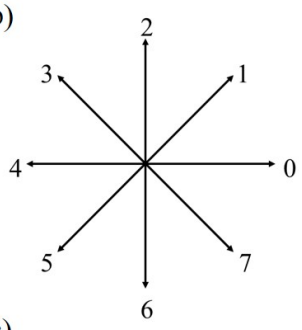
Problem 3: Chain code (24 pts)

Apply Moore boundary following algorithm on the image (a): mark the position of b_i and c_i on the grids of the image, where b_i is the boundary and c_i is the preceding pixel of b_i (16 pts). Write the Freeman chain code based on 8-connectivity (b) for the image (a) (8 pts). In freeman chain code, the direction of each line segment is indicated by a numerical number, such as an example shown in (c). Assume the origin of the image is at the left-up corner, the x-axis is oriented vertically downward and the y-axis is oriented horizontally to the right. The coordinate of the uppermost-leftmost nonzero point is (2, 2).

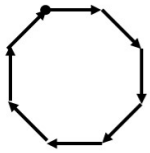
(a)



(b)



(c)

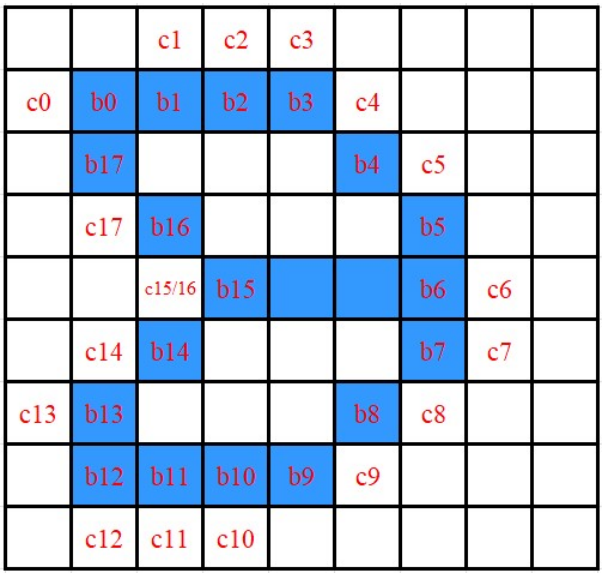


Freeman chain code is
0 7 6 5 4 3 2 1.

• start point

Solution:

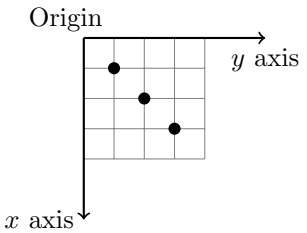
freeman code is 0 0 0 7 7 6 6 5 5 4 4 4 2 1 1 3 3 2. (8 pts) + (16 pts)



Problem 4: Hough transform (24 pts)

Suppose you have a 5×5 binary image (pixel values are only 0 or 1) containing the following pixel values:

0	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	0



Here, 1 represents edge points, and 0 represents the background. Use the Hough Transform to detect lines in the image (The four coordinates in the image are (1,1),(2,2),(3,3)). We parameterize the lines in polar coordinates $r = x\cos(\theta) + y\sin(\theta)$, where r is the distance from the origin to the line, and θ is the angle between the line and the counterclockwise direction of the X-axis.

(a) Manually calculate the Hough space mapping for each edge point for θ values of $0°$, $45°$ and $135°$. (Hint: $\cos(45°) = \sin(45°) \approx 0.707$, $\sin(135°) \approx 0.707$, $\cos(135°) \approx -0.707$) (15 pts)

(b) Determine which (r, θ) combinations receive the most votes. (9 pts)

Solution:

(a) **Point A**(1, 1):

$$\begin{aligned} \theta = 0^\circ: &\quad r = 1 \cdot 1 + 1 \cdot 0 = 1 \\ \theta = 45^\circ: &\quad r = 1 \cdot 0.707 + 1 \cdot 0.707 = 1.414 \\ \theta = 135^\circ: &\quad r = 1 \cdot (-0.707) + 1 \cdot 0.707 = 0 \end{aligned}$$

Point B(2, 2):

$$\begin{aligned} \theta = 0^\circ: &\quad r = 2 \\ \theta = 45^\circ: &\quad r = 2 \cdot 0.707 + 2 \cdot 0.707 = 2.828 \\ \theta = 135^\circ: &\quad r = 0 \end{aligned}$$

Point C(3, 3):

$$\begin{aligned} \theta = 0^\circ: &\quad r = 3 \\ \theta = 45^\circ: &\quad r = 3 \cdot 0.707 + 3 \cdot 0.707 = 4.242 \\ \theta = 135^\circ: &\quad r = 0 \end{aligned}$$

(b) We observe that for $\theta = 135^\circ$, all points have an r value of 0, indicating that these points all lie on the same line, and the angle of this line is 135°. Therefore, this line in the Hough space will receive the most votes, confirming the existence of a line from the top-left to the bottom-right in the image.