Name:
ID:
E-mail:

Digital Image Processing
Quiz 1

Problem 1: Pixel Adjacency (20 pts)

Consider the image segment shown.

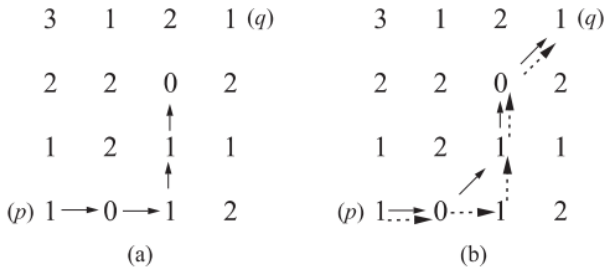
(a) Let $V = \{0, 1\}$, show the shortes 4-, 8- and m-path between p and q and compute its lengths. If a particular path does not exist between these two points, explain why. (11 pts)

(b) Calculate the Euclidean distance, City-block distance and Chessboard distance between p and q respectively. (9 pts)

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

Solution:

(a) When $V = \{0, 1\}$, 4-path does not exist between p and q (1 pt) because it is impossible to get from p to q by traveling along points that are both 4-adjacent and also have values from V(2 pt). Fig (a) shows this condition (2 pt), it is not possible to get to q. The shortest 8-path is shown in Fig (b) (1pt); its length is 4 (2 pt). The length of the shortest m- path (shown dashed (1 pt)) is 5 (2 pt).



(b) Euclidean distance: $3\sqrt{2}$ (3 pts) City-block distance: 6 (3 pts) Chessboard distance: 3 (3 pts)

Problem 2: Convolution and Correlation (20 pts)

(a) If there is an image of size $M \times N$ and a filter of size 5×5, what is the size of the full convolution result of them? (3 pts)

(b) Compute cropped correlation and cropped convolution of A and B (Use B as the filter), respectively (14 pts):

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 3 & -2 & 2 \\ -3 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 & -2 \\ -2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix}$$

(c) Under what circumstances does convolution have the same result as correlation (3 pts)

Solution:

(a) For an image of size $M \times N$ (assuming $M = N$ for simplicity) and a filter of size 5×5, the size of the full convolution result is $(M + 4) \times (N + 4)$. (3 pts)

(b)

$$\text{Cropped Correlation Result} = \begin{bmatrix} 35 & -17 & -10 \\ -20 & 2 & 30 \\ 5 & 27 & 2 \end{bmatrix} (7pts)$$

$$\text{Cropped Convolution Result} = \begin{bmatrix} -7 & 20 & 16 \\ 20 & 26 & -23 \\ -1 & -31 & 24 \end{bmatrix} (7pts)$$

(c) Specifically, convolution and correlation produce the same output when one of the input signals is symmetric about its midpoint and when the other input signal is unchanged. (3 pts)

Problem 3 : Histogram Processing (40 pts)

(a) Fill the histogram equalization table.(32 pts)

r_k	n_k	$p_r(r_k)$	s_k	s_k	$p_s(s_k)$
0	2				
1	7				
2	1				
3	2				
4	4				
5	8				
6	2				
7	1				

(b) Describe how adaptive histogram equalization is implemented and what people do to speed it up.(8 pts)

Solution:

(a)

(b) Traverse every pixel with a $W \times W$ patch, process histogram equalization within each patch and update the center pixel.(4 pts) For faster processing AHE, it is proposed to update a center patch of size $M \times M$ instead of just the center pixel in each HE in each within the $W \times W$ patch HE.(4 pts)

r_k	n_k	$p_r(r_k)$	s_k	s_k	$p_s(s_k)$
0	2	0.07	0.49	0	0.07
1	7	0.26	2.31	2	0
2	1	0.04	2.59	3	0.26
3	2	0.07	3.08	3	0.11
4	4	0.15	4.13	4	0.15
5	8	0.30	6.23	6	0
6	2	0.07	6.72	7	0.30
7	1	0.04	7	7	0.11

Problem 4: Spatial filtering (20 pts)

(a) Choose one of the given kernel that fits the description (12 pts)

1. Suitable for sharpening edges and enhancing high-frequency components in an image. Can be prone to noise amplification, often used with noise reduction method.
2. Suitable for emphasizing edges in vertical direction
3. Effective in detecting edges with a 45-degree orientation.

$$\text{A: } \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{B: } \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{C: } \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad \text{D: } \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) How to get the mask from original image?

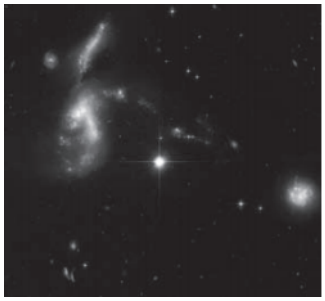


Figure 1: original image

Figure 2: mask

Solution:

(a) 1. B (4 pts) 2. A (4 pts) 3.C (4 pts)

(b) First, use lowpass filtering like a Gaussian kernel to smooth the image. (4 pts) Then, threshold the filtered image to get the mask. (4 pts)