Digital Image Processing, 2024 Spring

Homework 2

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Due 23:59 (CST), May. 3, 2024

Problem 1: Image Sharping

(a) Let $\boldsymbol{a} = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T$, and $\boldsymbol{b} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T$.

Sobel operator among x direction: $S_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = \boldsymbol{a}\boldsymbol{b}^T$

Sobel operator among y direction: $S_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} = \boldsymbol{b}\boldsymbol{a}^T$

So both the Sobel operators can be represented as the outer product of two vectors, i.e. its separable.
From what we have learned, if a separable filter can be represented as $\boldsymbol{w} = \boldsymbol{w}_1\boldsymbol{w}_2^T$, then the convolution of the filter with an image can be computed as $\boldsymbol{I} * \boldsymbol{w} = (\boldsymbol{I} * \boldsymbol{w}_1) * \boldsymbol{w}_2^T$, where '$*$' represents convolution.
So we can implement the separated kernels sequencially to the origin image to get the results.
The S_x_a represents the convolution of origin image and $\boldsymbol{a}$, S_x_ab represents convolution of S_x_a and $\boldsymbol{b}^T$, which also means that the convolution of origin image and the S_x operator.
And S_y_b represents the convolution of origin image and $\boldsymbol{b}$, S_y_ba represents convolution of S_y_b and $\boldsymbol{a}^T$, which also means that the convolution of origin image and the S_y operator.
Specifically, for the convenience of checking, the negative values are turned to 0, and then the image pixel values are normalized to $[0, 255]$ after filtering.
And the results are shown in Figure 1.

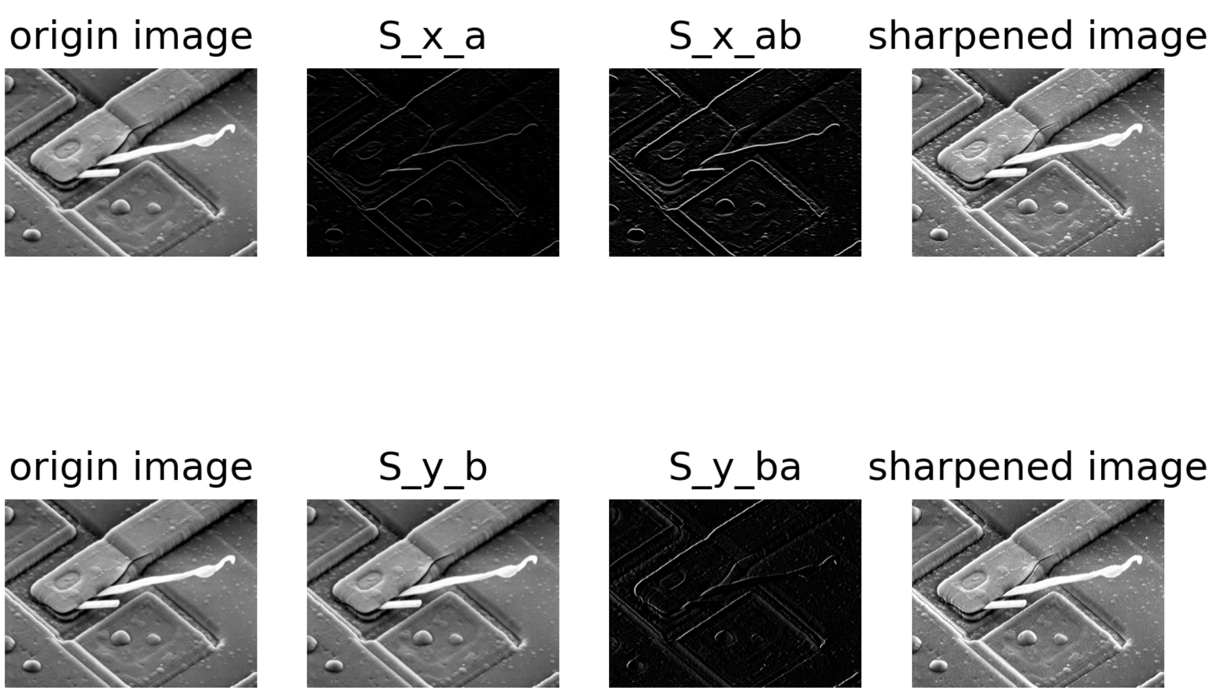

Figure 1. Processed images by Sobel operators among x and y directions.

We also tried to use the gradient of Sobel kernels to sharpen the image.
i.e.

$$G = \sqrt{(S\_x\_ab)^2 + (S\_y\_ba)^2}$$

The gradient of Sobel kernels and its sharpened image are shown in Figure 2.

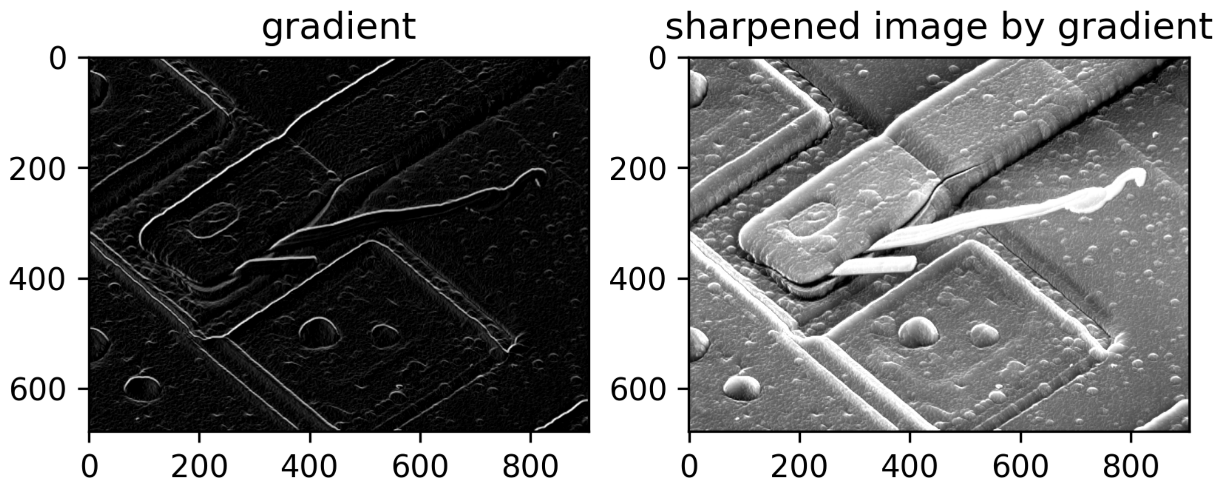

Figure 2. Sharpened by the gradient of Sobel kernels.

(b) Gaussian Highpass Filter:
The Gaussian Highpass Filter is:

$$H(u,v) = 1 - \exp\left(-\frac{D(u,v)^2}{2D_0^2}\right)$$

Where

$$D(u,v) = \left[\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2\right]^{\frac{1}{2}}$$

In order to do FFT, we need to pad the image to the size of $2^m * 2^n$, where m and n are integers.
So the images size varies: $678*906 \Rightarrow 1024*1024$. And aftering filtering, the image is cropped to the original size.

The Gaussian high pass filter is shown in Figure 3.

To have a better effect of visualization on the frequency domain, the image is shown by applying the log transformation on the magnitude of the Fourier transform of the image.
i.e. $I_{\log} = 20\log(1 + |I|)$, where I is the spectrum generated by Fourier transform of the image.
The spectrum of the origin image and its filtered spectrum by the Gaussian highpass filter is shown in Figure 4.
The results of filtering the image with the Gaussian high pass filter and its sharpened image is shown in Figure 5.
The filtered image is generated by using the inverse Fourier transform to the frequency domain results in the real domain.

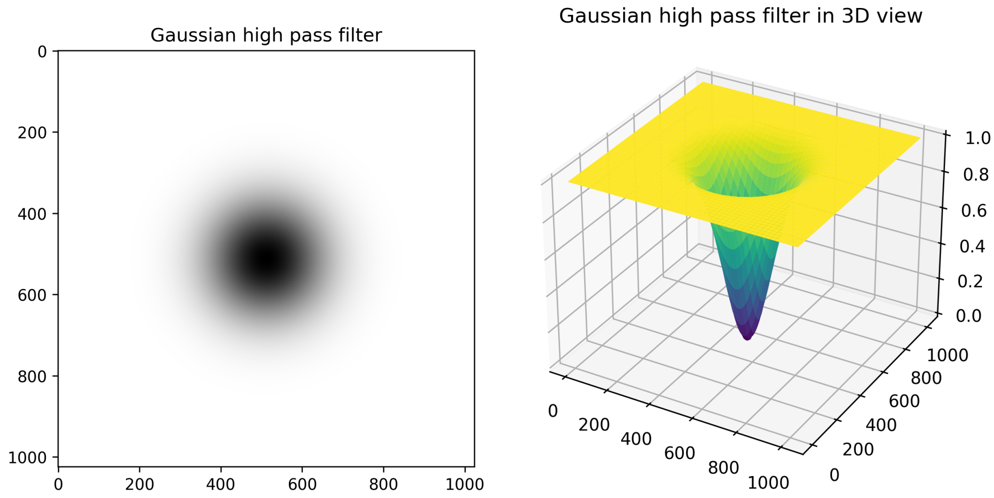

Figure 3. Gaussian high pass filter

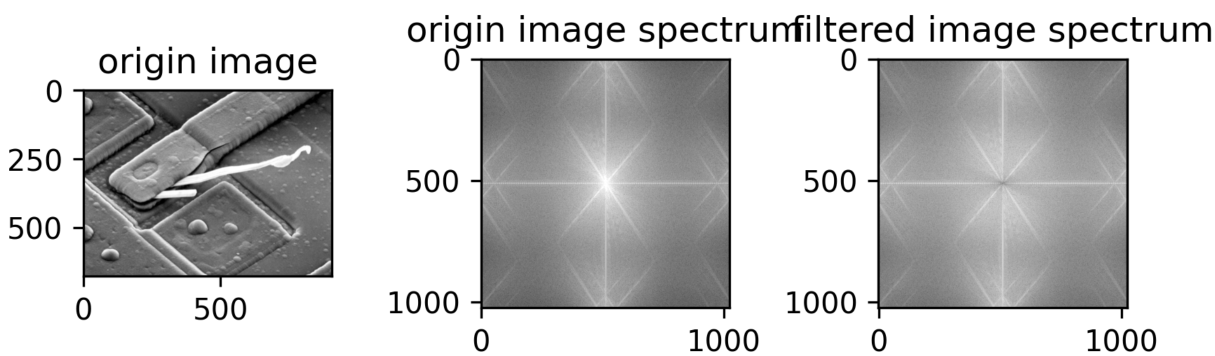

Figure 4. Spectrum of Gaussian Highpass Filtered iamge.

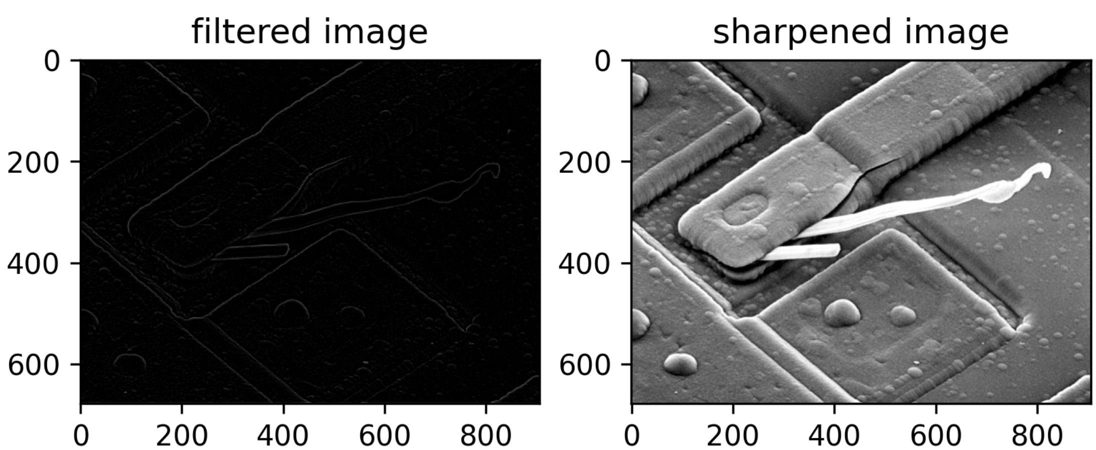

Figure 5. Results of filtering the image with the Gaussian high pass filter.

Problem 2: Homomorphic Filtering

The Homomorphic filter is:

$$H(u,v) = (\gamma_H - \gamma_L) * \left[1 - \exp\left(-c\left[\frac{D(u,v)}{D_0}\right]^2\right)\right] + \gamma_L$$

Where

$$\gamma_L < 1, \gamma_H \geq 1, D(u,v) = \left[\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2\right]^{\frac{1}{2}}$$

In this specific situation, we take $\gamma_L = 0.5, \gamma_H = 2.0, D_0 = 80, c = 1$.

In order to do FFT, we need to pad the image to the size of $2^m * 2^n$, where m and n are integers.

So the images size varies: $1162 * 746 \Rightarrow 2048 * 1024$.

And aftering filtering, the image is cropped to the original size.

The Homomorphic filter is shown in 6.

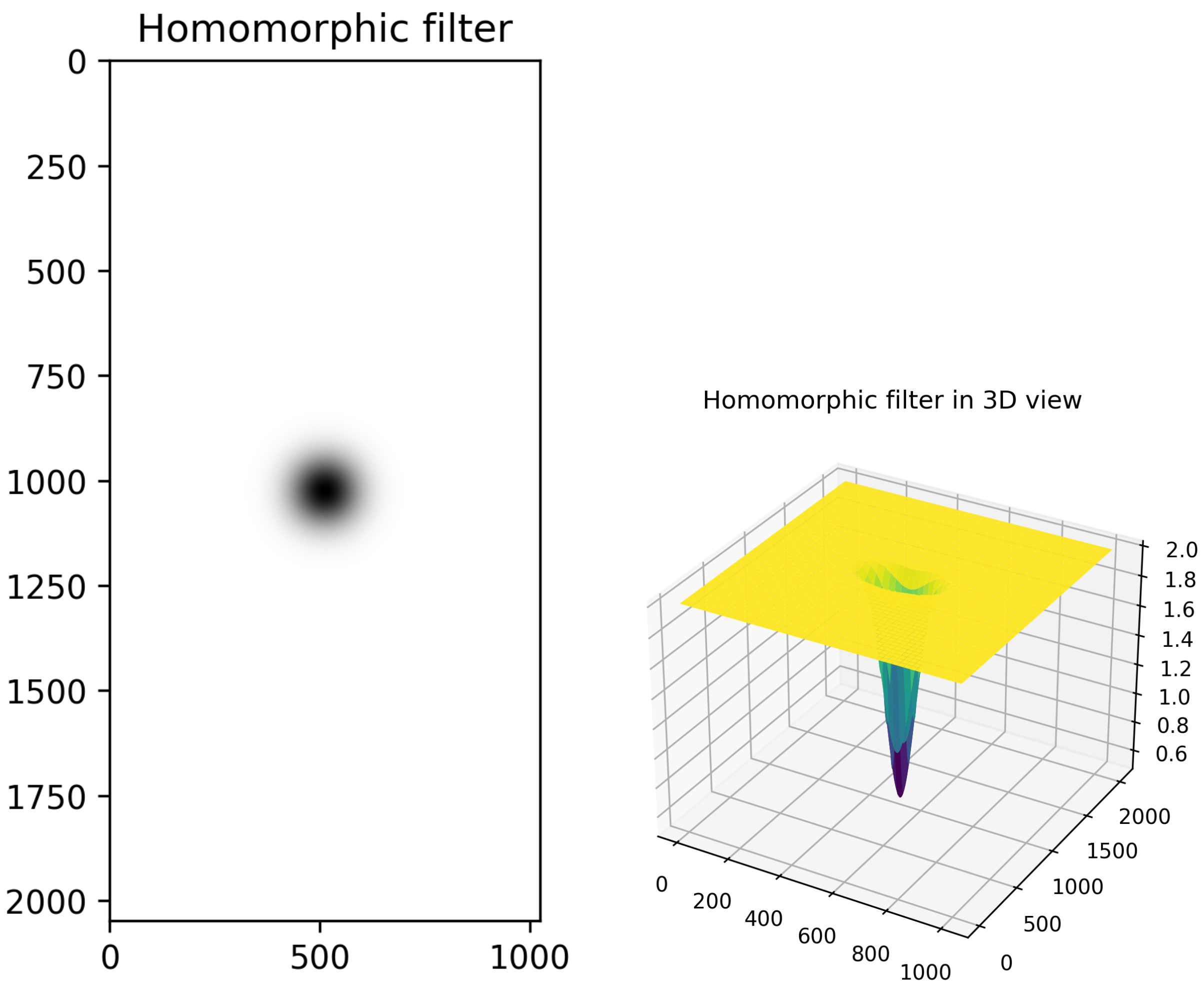

Figure 6. Homomorphic filter

The results of filtering the image with the Homomorphic filter is shown in Figure 7.

The variance of pixel values with in the white box $[500, 1100] \times [90, 180]$ is 0.0017427355, which exceeds 3×10^{-4}.

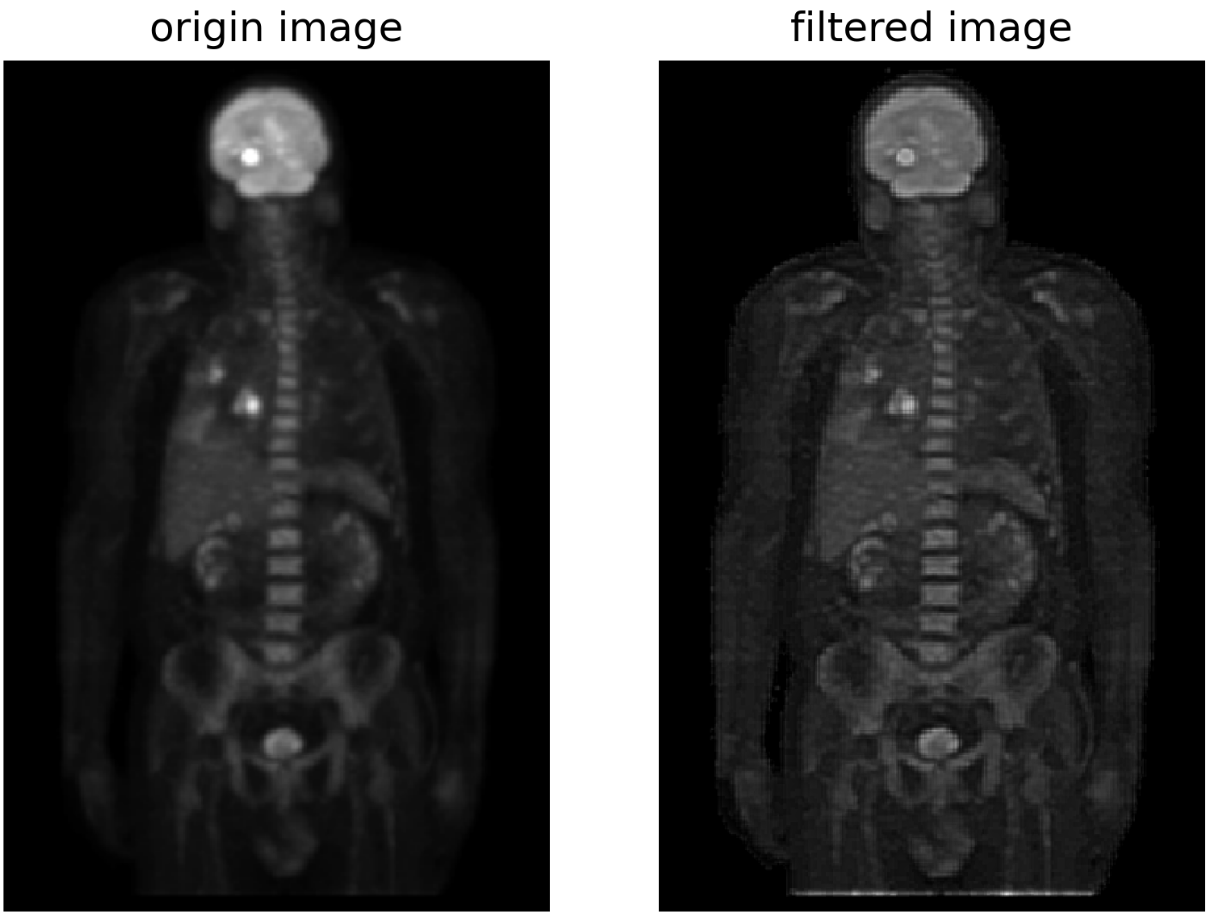

Figure 7. Result after Homomorphic filtering

Problem 3: Color space conversion

1. Convert RGB image to HSI image.

The formula to convert RGB to HSI is as follows:

$$\theta = \arccos\left\{\frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{\frac{1}{2}}}\right\}$$

$$H = \begin{cases}\theta & \text{if } B \le G \\ 360-\theta & \text{if } B > G\end{cases}$$

$$S = 1 - \frac{3}{R+G+B}\min\{R,G,B\}$$

$$I = \frac{R+G+B}{3}$$

And the HSI channels should be normalized to $[0,1]$.
Since H represents the hue, which is a circular value, and its range is $[0,360]$.
S represents the saturation, its range is $[0,1]$.
I represents the intensity, its range is $[0,255]$.
So to normalize the HSI channels, we can use the following formula:

$$H \leftarrow \frac{H}{360}, S \leftarrow \frac{S}{1}, I \leftarrow \frac{I}{255}$$

For details, since we need to avoid the situation that the denominator is 0, so we can add $\epsilon = 10^{-9}$ on the denominator when calulating θ and S.
Also, in order to fit the domain of the function arccos, we need to clip the values into $[-1,1]$ when calulating θ.
So with above formulas, we can convert the RGB image to HSI image.

2. Convert HSI image to RGB image.

The formula to convert HSI to RGB is as follows:

$$0^\circ \le H < 120^\circ$$

$$B = I(1-S), \quad R = I\left[1+\frac{S\cos(H)}{\cos(60^\circ-H)}\right], \quad G = 3I-(R+B)$$

$$> 120^\circ \le H < 240^\circ$$

$$R = I(1-S), \quad G = I\left[1+\frac{S\cos(H-120^\circ)}{\cos(180^\circ-H)}\right], \quad B = 3I-(R+G)$$

$$> 240^\circ \le H < 360^\circ$$

$$G = I(1-S), \quad B = I\left[1+\frac{S\cos(H-240^\circ)}{\cos(300^\circ-H)}\right], \quad R = 3I-(G+B)$$

And the RGB channels should be normalized to $[0,255]$.
So to normalize the RGB channels, we need to recover the HSI by using the following formula:

$$H \leftarrow H*360, S \leftarrow S*1, I \leftarrow I*255$$

For specific details, all values of H are using the radians, so the numbers of the degrees above should be converted to radians while implementing.

With these formulas, we can convert the HSI image to RGB image.

With the analysis above, we can convert the RGB image to HSI image, and recover the RGB image from the HSI image.

The Figure 8 is the result of the conversions.

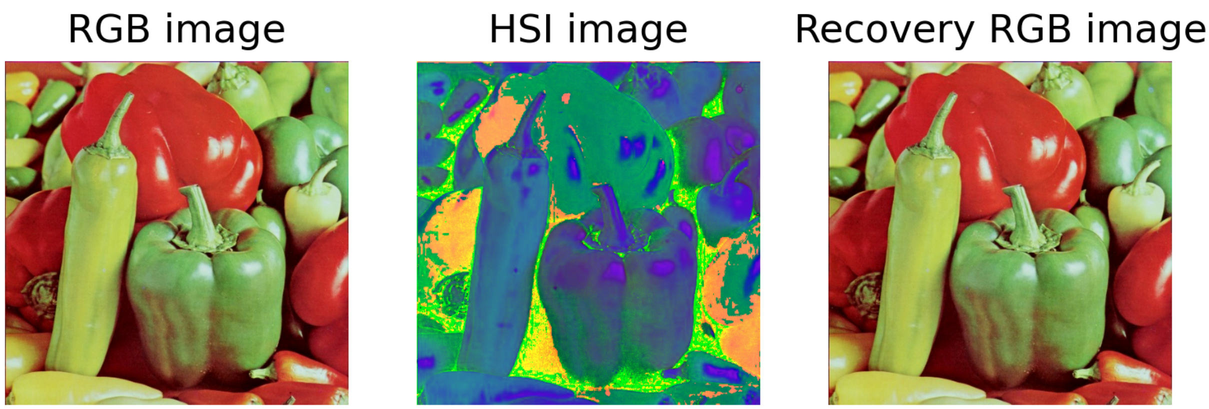

Figure 8. RGB to HSI conversion and the recovery

Problem 4: Image Restoration

(a) The spectrum of the origin image is generated through FFT.
To shift the zero frequency to the center of the spectrum, we times $(-1)^{u+v}$ for each pixel (u, v) on the origin image before applying FFT.

To have a better effect of visualization on the frequency domain, the image is shown by applying the log transformation on the magnitude of the Fourier transform of the image.
i.e. $I_{\log} = \log(1 + |I|)$, where I is the spectrum generated by Fourier transform of the image.
The spectrum of the origin image after fftshift is shown in Figure 9.

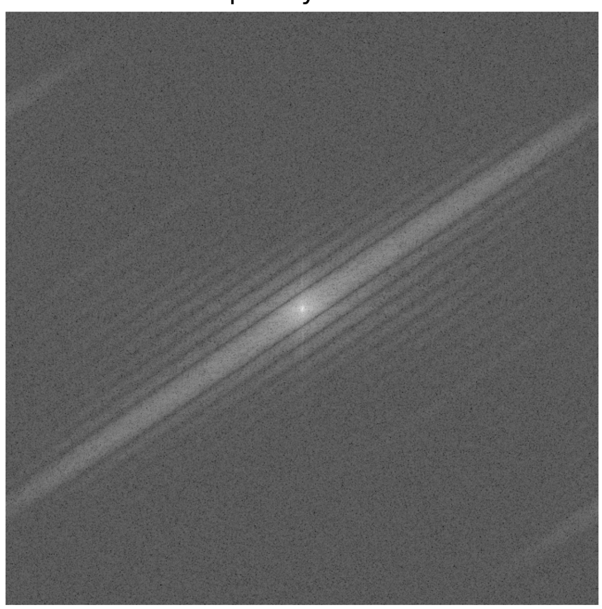

Figure 9. FFT shifted spectrum

(b) Then we can apply transformation on the spectrum to get $theta$ and d for the strip.
The Radon transform is:

$$g(\rho, \theta) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\delta(x \cos\theta + y \sin\theta - \rho)$$

where δ is the Dirac delta function.
$\rho_{\max} = \lceil \frac{\sqrt{2}}{2} N \rceil = 453$, so the vertical index is in range $[-\rho_{\max}, \rho_{\max}]$.

The image by applying Radon transform on the origin image is shown in Figure 10.
We can get the horizontal coordinate with the highest intensity in the Radon transformed image, which is $\theta = 124°$.

The we rotate the spectrum by $180° - \theta = 56°$, and do vertical projection to find the distance between two similar dark strips.
The rotated spectrum and the intensity of the vertical projection are shown in Figure 11.

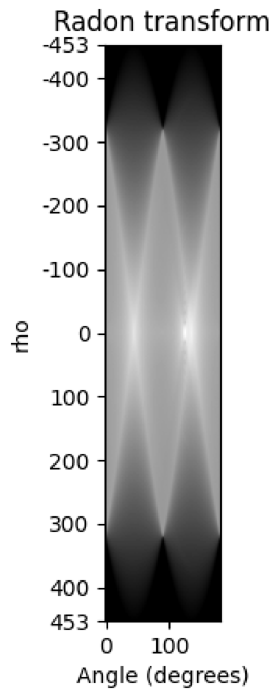

Figure 10. Radon Transformed image

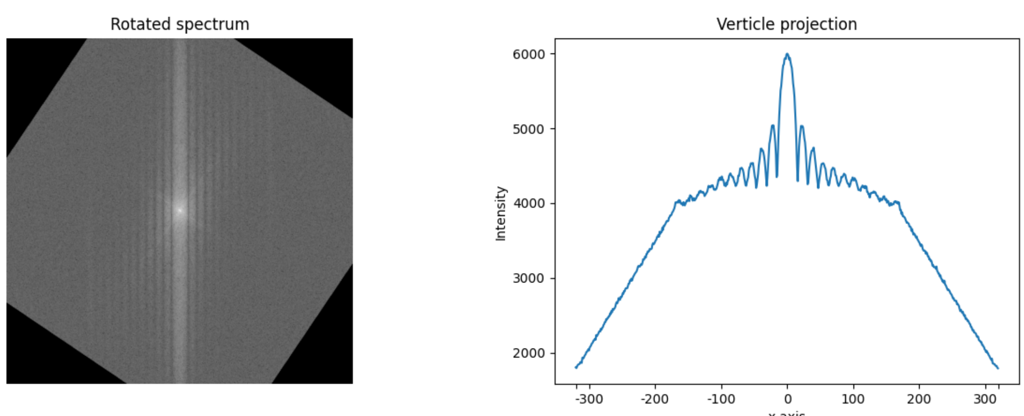

Figure 11. Radon Transformed image

From Figure 11, we can see that the two similar dark strips are $d = 16$ pixels apart. So we have $N = 640, L = \dfrac{N}{d} = 40$.

So above all, the parameters of the motion blur are $\theta = 124°, L = 40$.

(c) We construct the frequency domain Wiener filter using the formula:

$$W = \frac{H^*}{|H|^2 + K}$$

where H is the Fourier transform of the point spread function, which was generated by the motion blur model with parameters $\theta = 124°, L = 40$. And the help of 'psf2otf'.

We here take $K = 0.004$, and we do not apply 'fftshift' during the filtering. Then the restored image by applying Wiener filter is shown in Figure 12.

Figure 12. Restored image