Name:
ID:
E-mail:

Digital Image Processing

Quiz 2

Problem 1: Circular convolution (26 pts)

Suppose now we have sequence $h(n) = \{\underline{1}, 1, 1, 1\}$ and sequence $x(n) = \{\underline{1}, 2, 3, 4\}$

(a) Calculate $h[n] * x[n]$ (full convolution) and $h[n] ⑧ x[n]$ respectively. (16 pts)

hint: If $L > n$, $h[n] \text{Ⓛ} x[n]$ means to zeropad h[n] and x[n] to the length of L, then do the circular convolution.

(b) Consider two sequences, f[n] and g[n] composed of A and B samples,can we use the following approach to calculate the full convolution of f[n] and g[n] ?

$$f[n] * g[n] = IDFT\{DFT\{f[n]\} \cdot DFT\{g[n]\}\}$$

If not, explain why, and make some modification so we can calculate $f[n] * g[n]$ using DFT and IDFT. (No need for calculation, just make explanation.) (10 pts)

Solution:

(a) $h[n] * x[n] = \{1, 3, 6, 10, 9, 7, 4\}$ (8 pts)

$$h[n] ⑧ x[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 10 \\ 9 \\ 7 \\ 4 \\ 0 \end{bmatrix} \text{ (8 pts)}$$

(b) It's not correct, due to the implicit period and period continuation of DFT, the wraparound problem will occur. (5 pts) To solve this problem, we can append zeros to both functions so that they have the same length, denoted by P, then wraparound is avoided by choosing

$$P >= A + B - 1$$

Finally we have

$$f[n] * g[n] = IDFT\{DFT\{f_{zp}[n]\} \cdot DFT\{g_{zp}[n]\}\}$$

(5 pts)

Problem 2: Morphological processing (24 pts)

(a) Image I consists of a set (object) A, and background. The size of A is 7×7. B is the structuring element. The size of B is 5×3. Please calculate the size of Erosion and Dilation result of A by B.(12 pts)

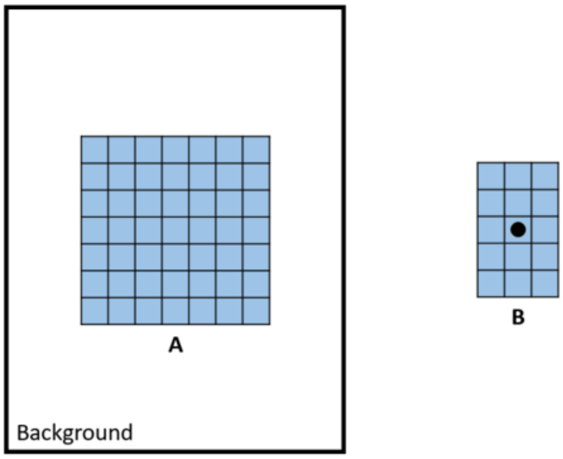
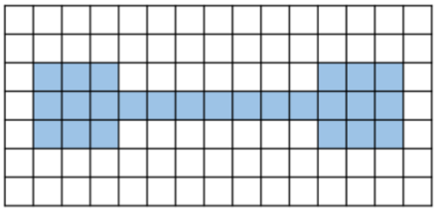
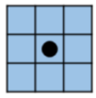


Image I SE

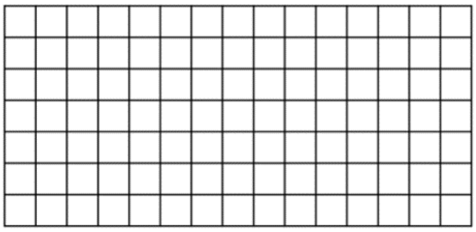
(b) Show all intermediate steps of your computations for the following: Obtain the opening of the figure C using the structuring element D. (12 pts)

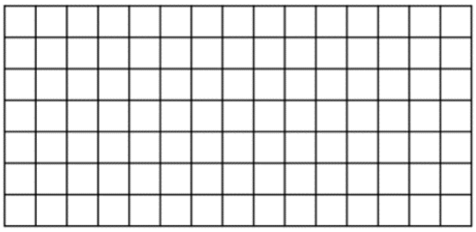


C



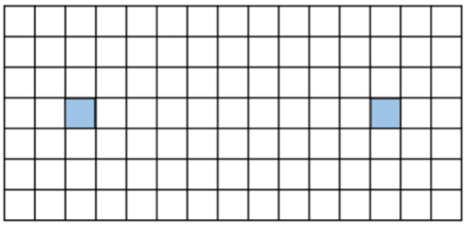
D



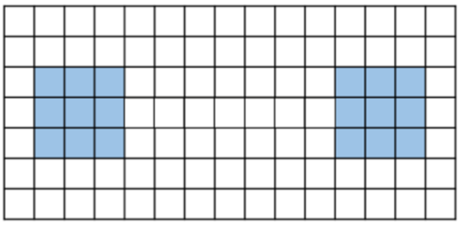


Solution:

(a) size of erosion result: $(7-5+1) \times (7-3+1) = 3 \times 5$ (6 pts)
size of dilation result: $(7+5-1) \times (7+3-1) = 11 \times 9$ (6 pts)



(b) (6 pts)



(6 pts)

Problem 3: Image domain transformation (30 pts)

(a) Prove the following 2-D discrete Fourier transform pairs (15 pts):

$$cos(2\pi\mu_0 x/M + 2\pi v_0 y/N) \Leftrightarrow (MN/2)[\delta(u+\mu_0, v+v_0) + \delta(u-\mu_0, v-v_0)]$$

hint:

$$\delta(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \frac{1}{2\pi} e^{jux} e^{jvy}$$

(b) In CT image reconstruction, directly using the backprojection approach will produce unacceptably fuzzy results. There is a simple way to solve this problem, that is, apply ramp filter to the projection $G(\omega, \theta)$ before calculating the filtered backprojection. Give the derivation of the filter backprojection expression: (15 pts)

$$f(x,y) = \int_0^{\pi} [\int_{-\infty}^{+\infty} |\omega| G(\omega,\theta) e^{j2\pi\omega\rho} d\omega]_{\rho = xcos(\theta)+ysin(\theta))} d\theta$$

where $f(x,y)$ is the reconstructed CT image on a two-dimensional plane (x,y). Here, you can use $F(u,v)$ to represent the Fourier transform of $f(x,y)$.

hint:

$$G(\omega, \theta + 180°) = G(-\omega, \theta)$$

Solution:

(a)

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} cos(\frac{2\pi\mu_0 x}{M} + \frac{2\pi v_0 y}{N}) e^{-j2\pi(\frac{\mu x}{M}+\frac{vy}{N})}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \frac{e^{j2\pi(\frac{\mu_0 x}{M}+\frac{v_0 y}{N})} + e^{-j2\pi(\frac{\mu_0 x}{M}+\frac{v_0 y}{N})}}{2} e^{-j2\pi(\frac{\mu x}{M}+\frac{vy}{N})}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \frac{e^{j2\pi(\frac{(\mu_0-\mu)x}{M}+\frac{(v_0-v)y}{N})} + e^{-j2\pi(\frac{(\mu_0+\mu)x}{M}+\frac{(v_0+v)y}{N})}}{2} (1) (10pts)$$

$$\because \delta(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \frac{1}{2\pi} e^{iux} e^{ivy} dxdy$$

$$\therefore (1) = \frac{MN}{2}[\delta(\mu_0-\mu, v_0-v) + \delta(-(\mu_0+\mu), -(v_0+v))]$$

$$= \frac{MN}{2}[\delta(\mu_0-\mu, v_0-v) - \delta(\mu_0+\mu, v_0+v)]$$

$$= \frac{MN}{2}[\delta(\mu+\mu_0, v+v_0) + \delta(\mu-\mu_0, v-v_0)] (5pts)$$

(b)

$$f(x,y) = \int_{-\infty}^{+\infty} F(u,v) e^{j2\pi(ux+vy)} dudv$$

$$= \int_0^{2\pi}\int_0^{+\infty} F(\omega cos(\theta), \omega sin(\theta)) e^{j2\pi\omega(xcos(\theta)+ysin(\theta))} \omega d\omega d\theta (4pts)$$

$$= \int_0^{2\pi}\int_0^{+\infty} G(\omega,\theta) e^{j2\pi\omega(xcos(\theta)+ysin(\theta))} \omega d\omega d\theta \ (2) (4pts)$$

$$\because G(\omega, \theta+180°) = G(-\omega, \theta)$$

$$\therefore (2) = \int_0^{\pi}\int_{-\infty}^{+\infty} |\omega| G(\omega,\theta) e^{j2\pi\omega(xcos(\theta)+ysin(\theta))} d\omega d\theta (4pts)$$

$$= \int_0^{\pi} [\int_{-\infty}^{+\infty} |\omega| G(\omega,\theta) e^{j2\pi\omega\rho} d\omega]_{\rho = xcos(\theta)+ysin(\theta))} d\theta (3pts)$$

Problem 4: White balance (20 pts)

White balance is a concept in photography and image processing that is used to adjust the colors in a photograph to make the images appear more natural. Now, the following three matrices represent the RGB channels of an image,

(a) Calculate the values of k_R, k_G, and k_B. (Round the resulting values to three decimal places.)(15 pts)

(b) Perform white balance on this image to obtain g_R, g_G, and g_B. (5 pts)

hint:

$$I(x,y) = 0.299 f_R(x,y) + 0.587 f_G(x,y) + 0.114 f_B(x,y)$$

$$k_R = \frac{\bar{I}}{\bar{f}_R}, \quad k_G = \frac{\bar{I}}{\bar{f}_G}, \quad k_B = \frac{\bar{I}}{\bar{f}_B}$$

$$\begin{bmatrix} g_R(x,y) \\ g_G(x,y) \\ g_B(x,y) \end{bmatrix} = \begin{bmatrix} k_R & 0 & 0 \\ 0 & k_G & 0 \\ 0 & 0 & k_B \end{bmatrix} \begin{bmatrix} f_R(x,y) \\ f_G(x,y) \\ f_B(x,y) \end{bmatrix}$$

($\bar{I}$ represents the average value of I.)

$$\mathbf{R} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

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$$\begin{bmatrix} g_R(x,y) \\ g_G(x,y) \\ g_B(x,y) \end{bmatrix} = \begin{bmatrix} k_R & 0 & 0 \\ 0 & k_G & 0 \\ 0 & 0 & k_B \end{bmatrix} \begin{bmatrix} f_R(x,y) \\ f_G(x,y) \\ f_B(x,y) \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

Solution: The average intensity I is calculated as 1.238 (rounded to three decimal places). The white balance coefficients are computed as follows:

$$k_R = 1.114$$

$$k_G = 1.114$$

$$k_B = 0.557$$

$$\mathbf{R} = \begin{bmatrix} 2.228 & 1.114 & 1.114 \\ 0 & 2.228 & 0 \\ 1.114 & 1.114 & 2.228 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 3.342 & 1.114 & 0 \\ 0 & 3.342 & 0 \\ 0 & 0 & 3.342 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1.114 & 1.114 & 1.114 \\ 1.671 & 1.114 & 1.671 \\ 1.114 & 1.114 & 1.114 \end{bmatrix}$$