

Lecture 3

Intensity transformation &

Spatial Filtering

Dr. Xiran Cai

Email: caixr@shanghaitech.edu.cn

Office: 3-438 SIST

Tel: 20684431

ShanghaiTech University



上海科技大学
ShanghaiTech University

Intensity Transformation-Outline

□ Histogram (直方图)

- Definition
- Property

□ Intensity Transformation (灰度变换)

- Linear transform
- Non-linear transform

□ Histogram Processing (直方图处理)

- Histogram Equalization
- Histogram Matching



Definition

$$h(r_k) = n_k$$

Where r_k : the k th intensity value in the level range of $[0, L-1]$

n_k : the number of pixels in the image with intensity r_k

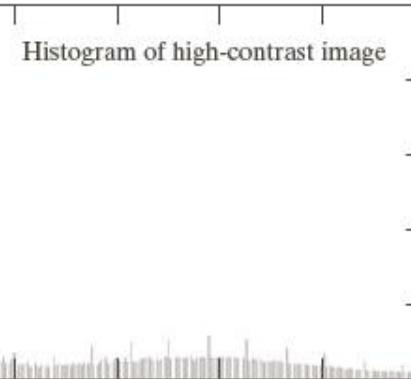
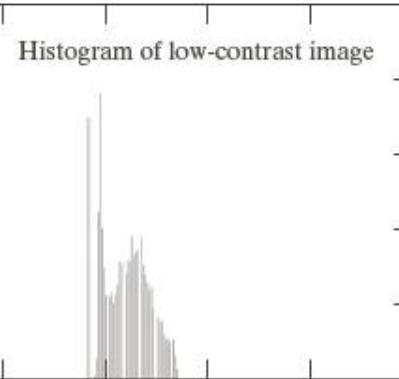
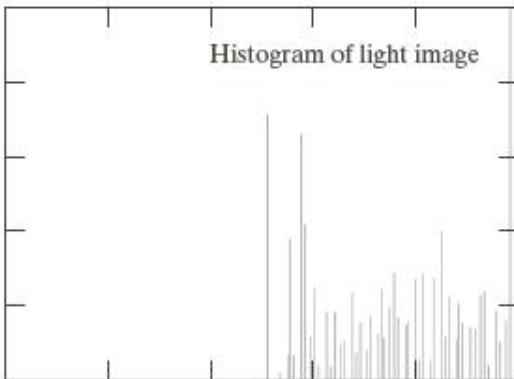
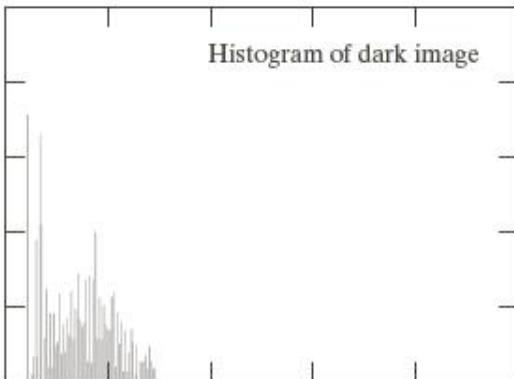
Normalized Histogram (归一化直方图)

$$p(r_k) = \frac{n_k}{MN}$$

Where $p(r_k)$: the probability of occurrence of intensity r_k in an image

M, N : the row and column dimensions of the image

Basic Image Type



Properties

□ The histogram of an image

- describe the number or probability of intensity, **NO** location (spatial) information
- can be same as other images
- $\sum_0^{L-1} n_k = M \cdot N$ or $\sum_0^1 p(r_k) = 1$
- If Region C=A \cup B, A and B are disjoint, $H_C = H_A + H_B$



Intensity Transformation

□ Simplest image processing techniques

$$s = T(r)$$

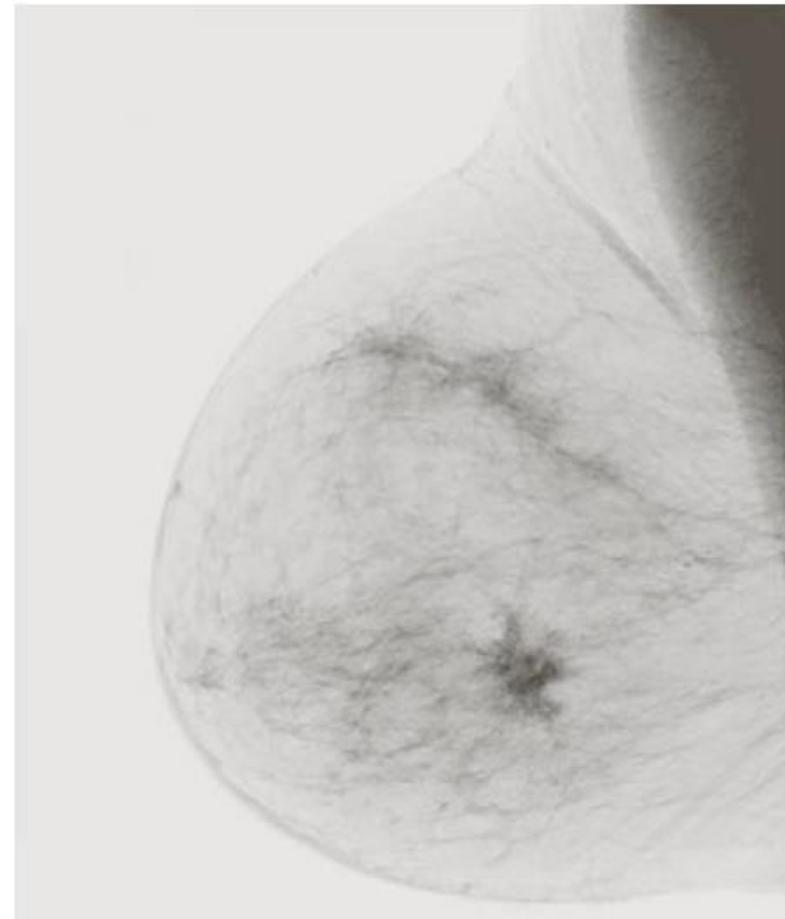
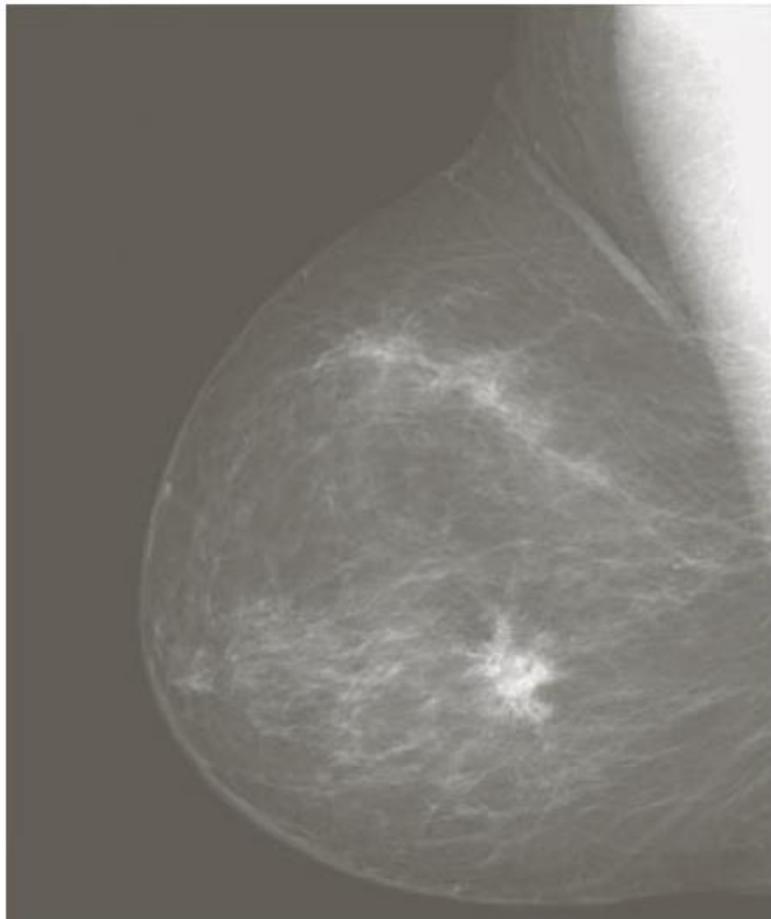
□ Types of Intensity Transformation

- Image Negatives (图像反转)
- Log Transformation (对数变换)
- Power-law (gamma) Transformation (幂律/伽马变换)
- Piecewise-Linear Transformation (分段线性变换)



Image Negatives

$$s = T(r) = L - 1 - r$$

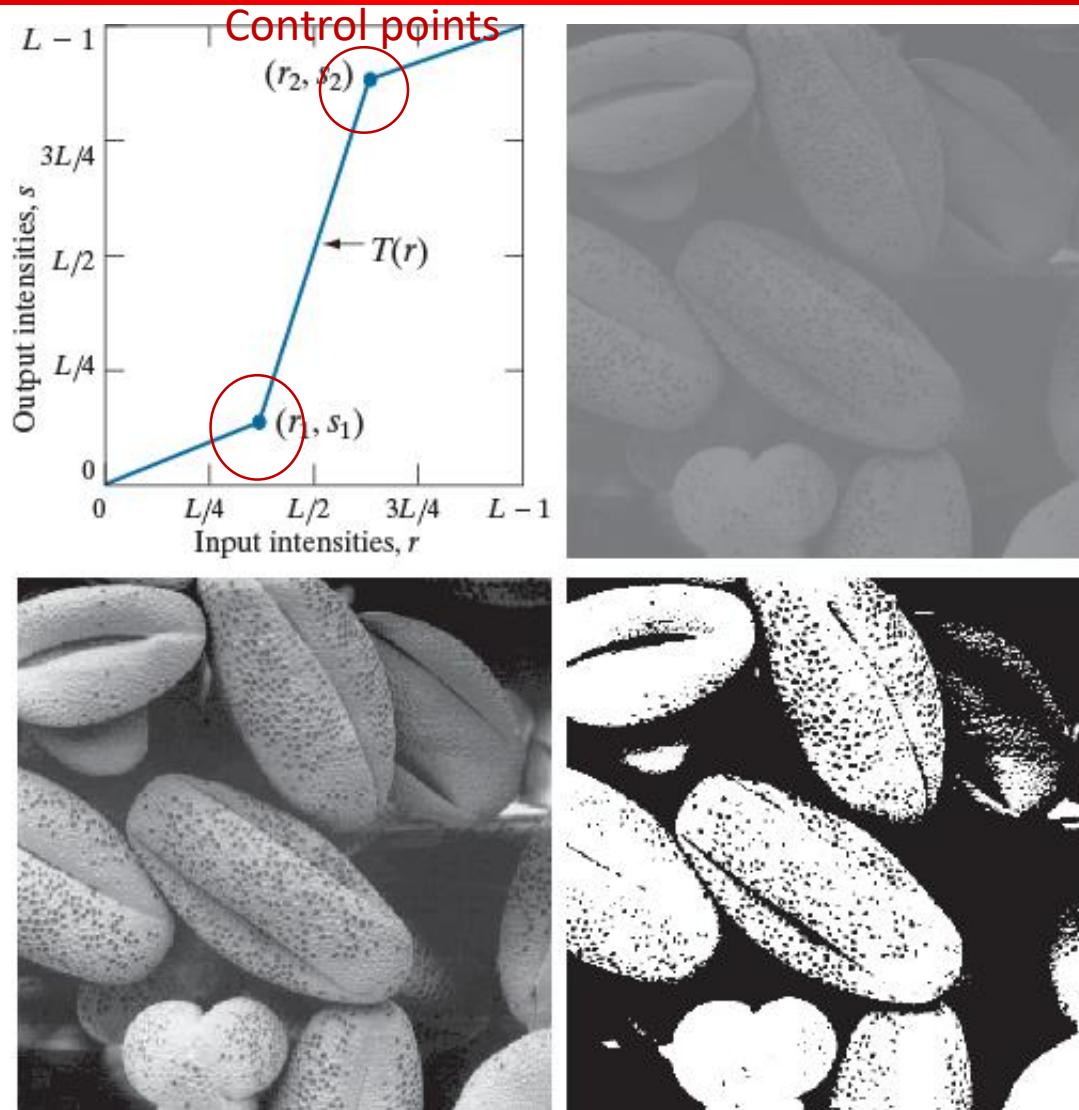


Contrast Stretching

a
b
c
d

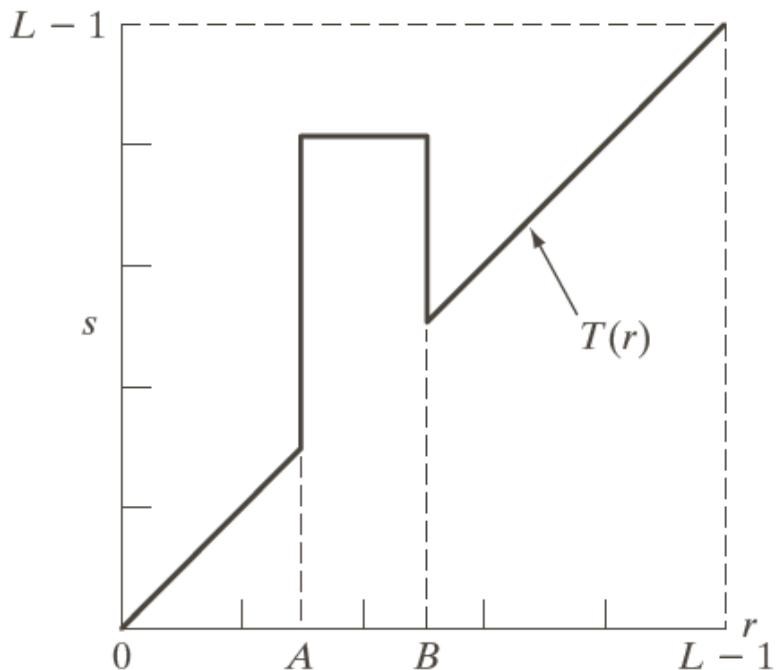
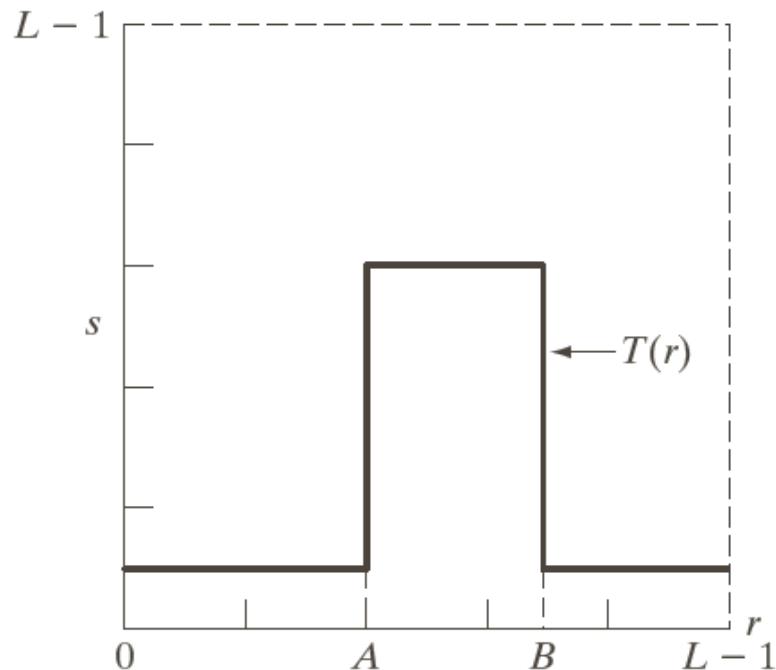
FIGURE 3.10

Contrast stretching.
(a) Piecewise linear transformation function.
(b) A low-contrast electron microscope image of pollen, magnified 700 times.
(c) Result of contrast stretching.
(d) Result of thresholding.
(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

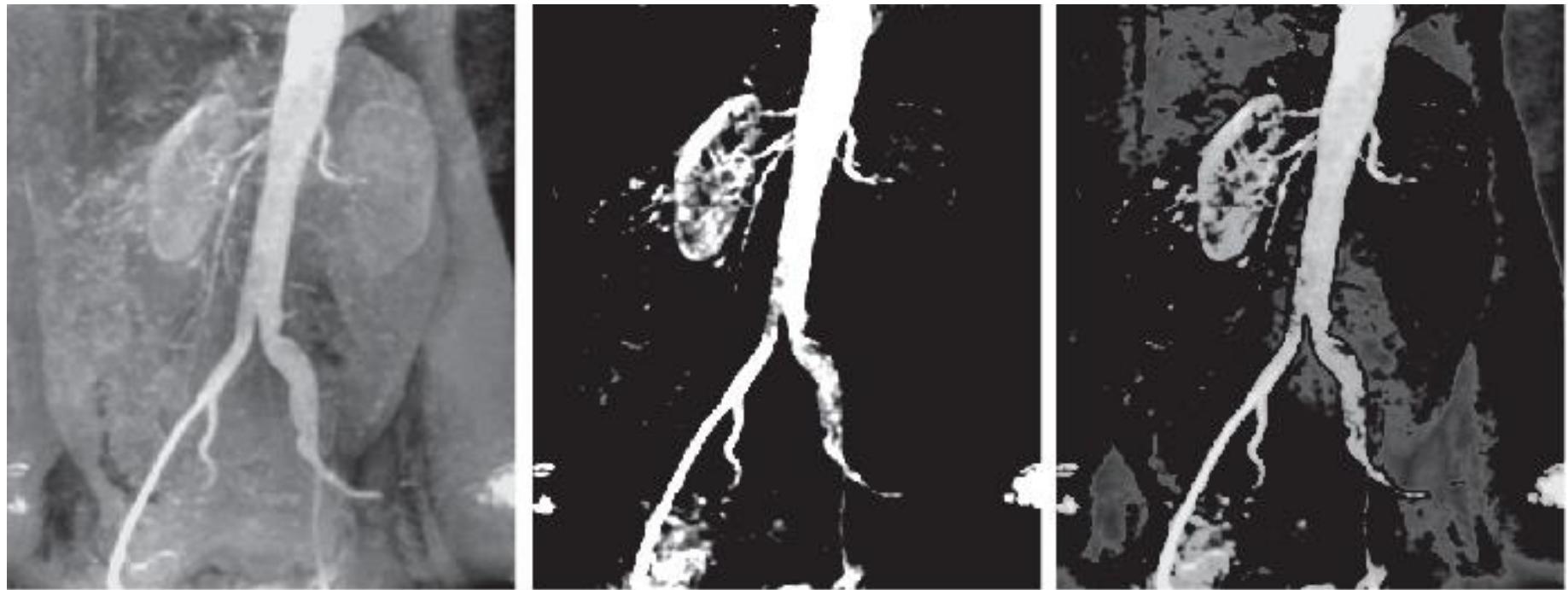


Intensity-level slicing

- What's the function of the transform $s = T(r)$ in the figures below?



Intensity-level slicing



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Log Transformation

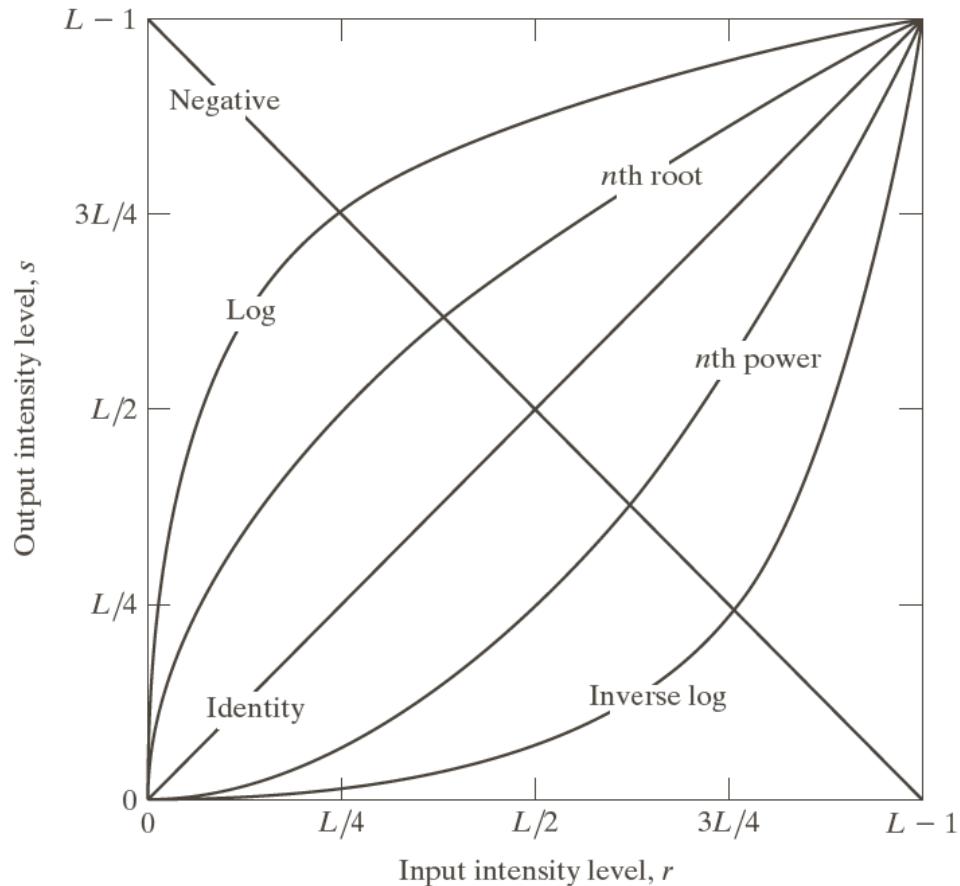
➤ Log Transformation

(对数变换)

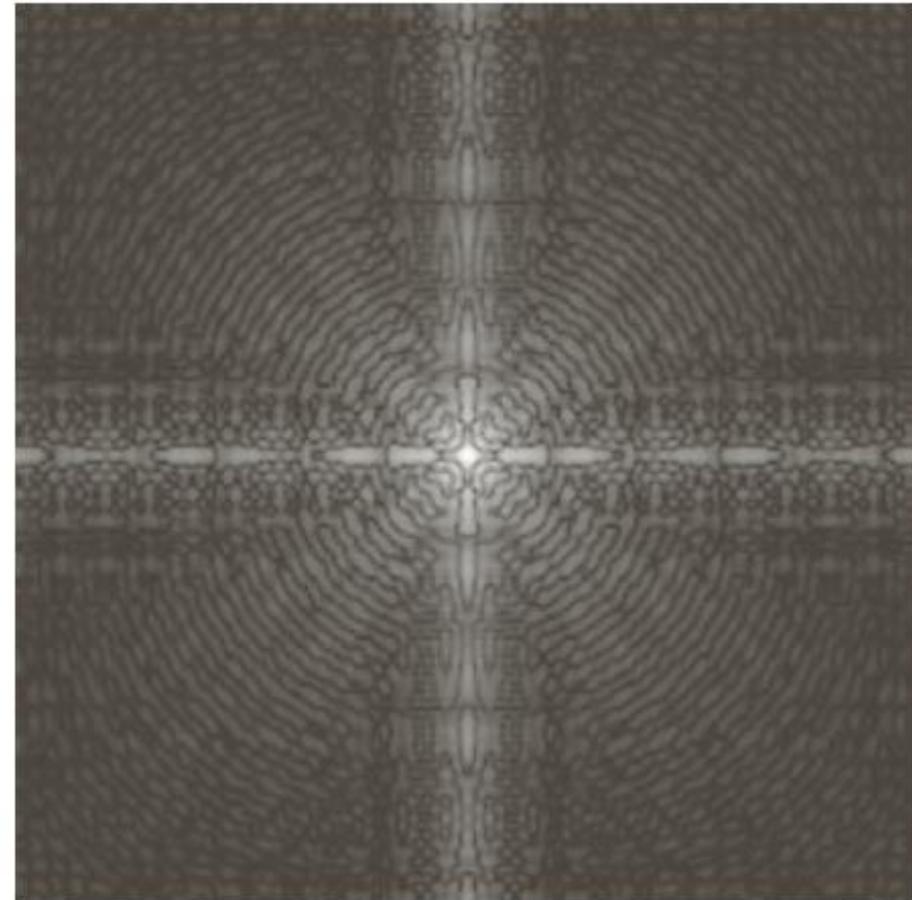
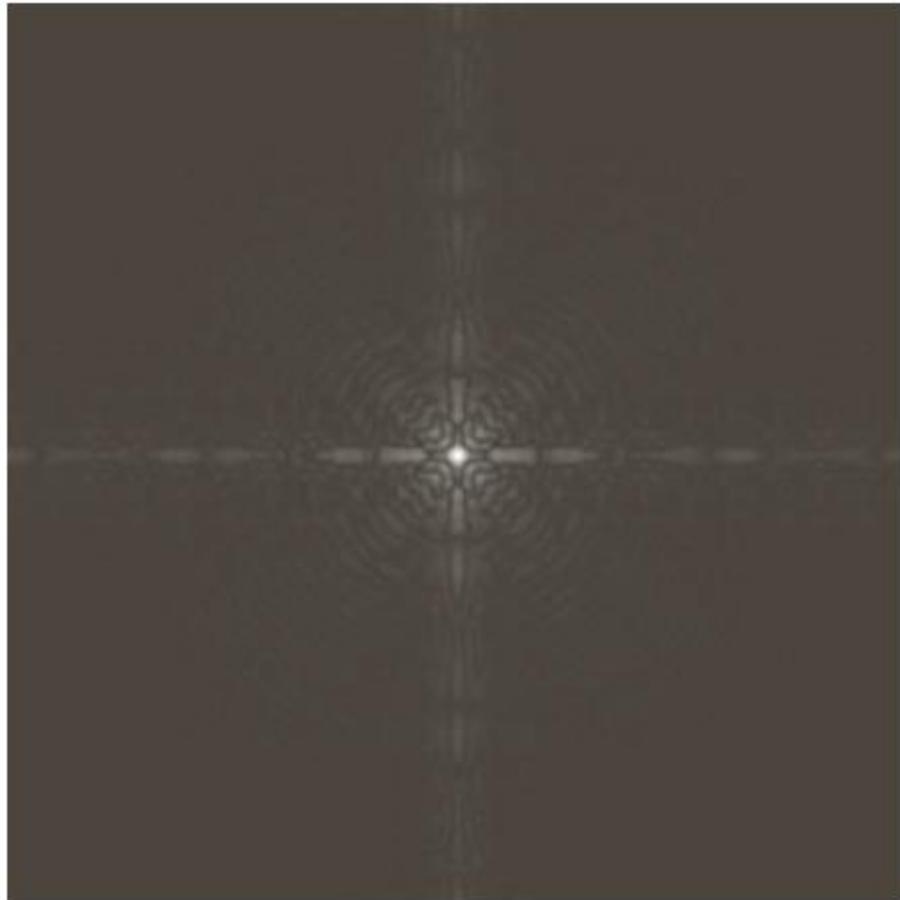
$$s = c \log(1 + r)$$

➤ Inverse Log Transformation (反 对数变换)

$$s = c \cdot 2^r - 1$$



Fourier Spectrum(log trans)



上海科技大学
ShanghaiTech University

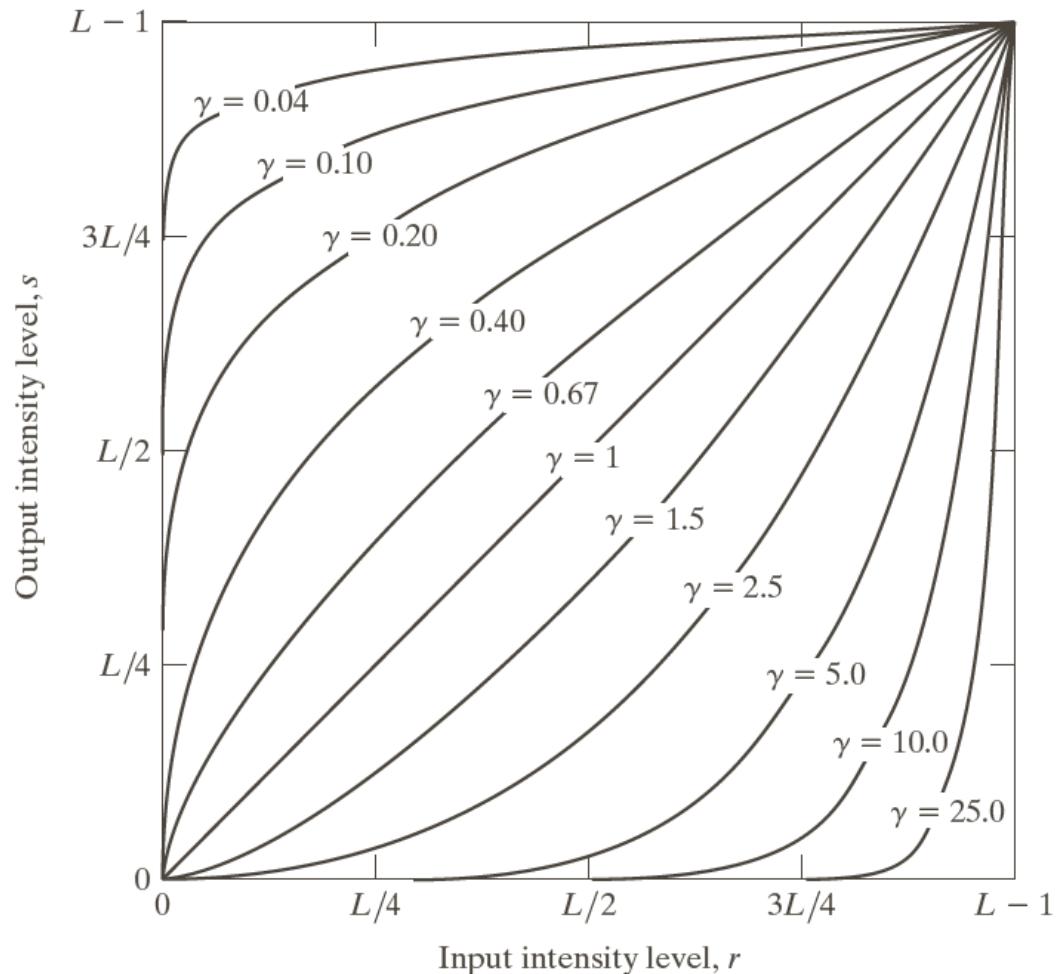
Gamma Transformation

► Gamma Transformation
(伽马变换)

$$s = c \cdot r^\gamma$$

or

$$s = c \cdot (r + \varepsilon)^\gamma$$



Gamma Transformation (gamma<1)



Fractured spine



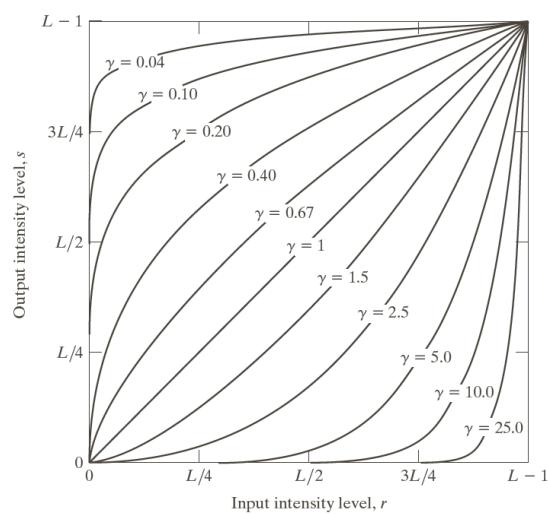
$\gamma = 0.6$

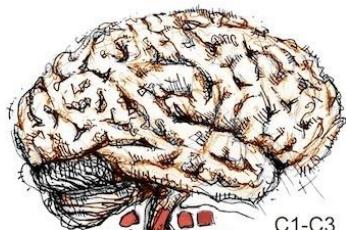


$\gamma = 0.4$



$\gamma = 0.3$





Cervical

C1-C3 Neck Muscles
C4 Diaphragm
C5 Deltoid (shoulder)
C6 Wrist
C7 Triceps
C7-C8 Fingers

Thoracic

T1 Hand
T2-T12 Intercostals (Trunk)
T7-L1 Abdominals
T11-L2 Ejaculation

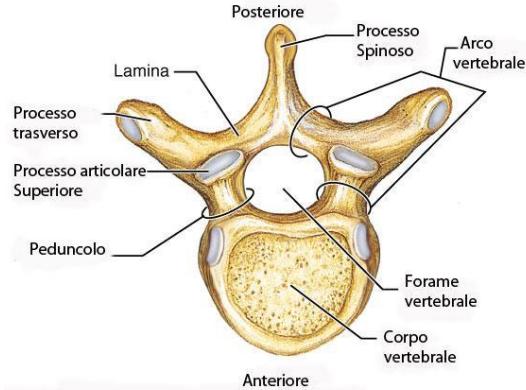
Lumbar

L2 Hips
L3 Quadriceps
L4-L5 Hamstrings - Knee
L4-S1 Foot

Sacral

S2 Penile erection
S2-S3 Bowel and bladder

Coccygeal



正常腰椎间盘

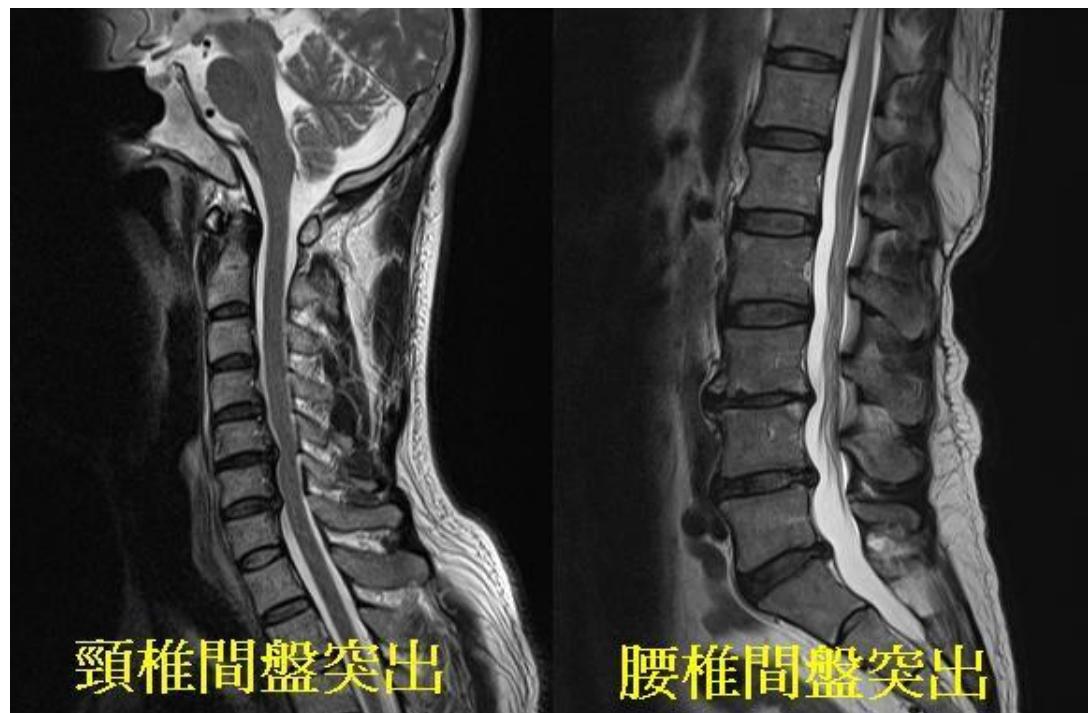


腰椎间盘突出

腰椎间盘脱出

腰椎间盘退化伴钙化

椎体病变示意图



颈椎间盘突出

腰椎间盘突出

Gamma Transformation($\gamma > 1$)

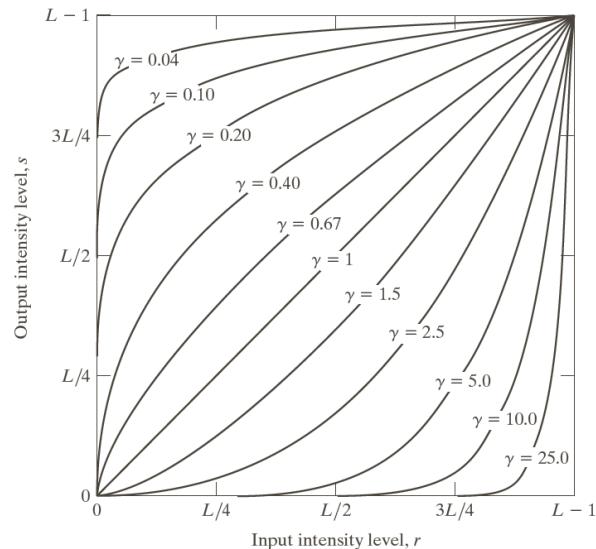


Aerial image

$\gamma = 3.0$

$\gamma = 4.0$

$\gamma = 5.0$



Bit-Plane Slicing

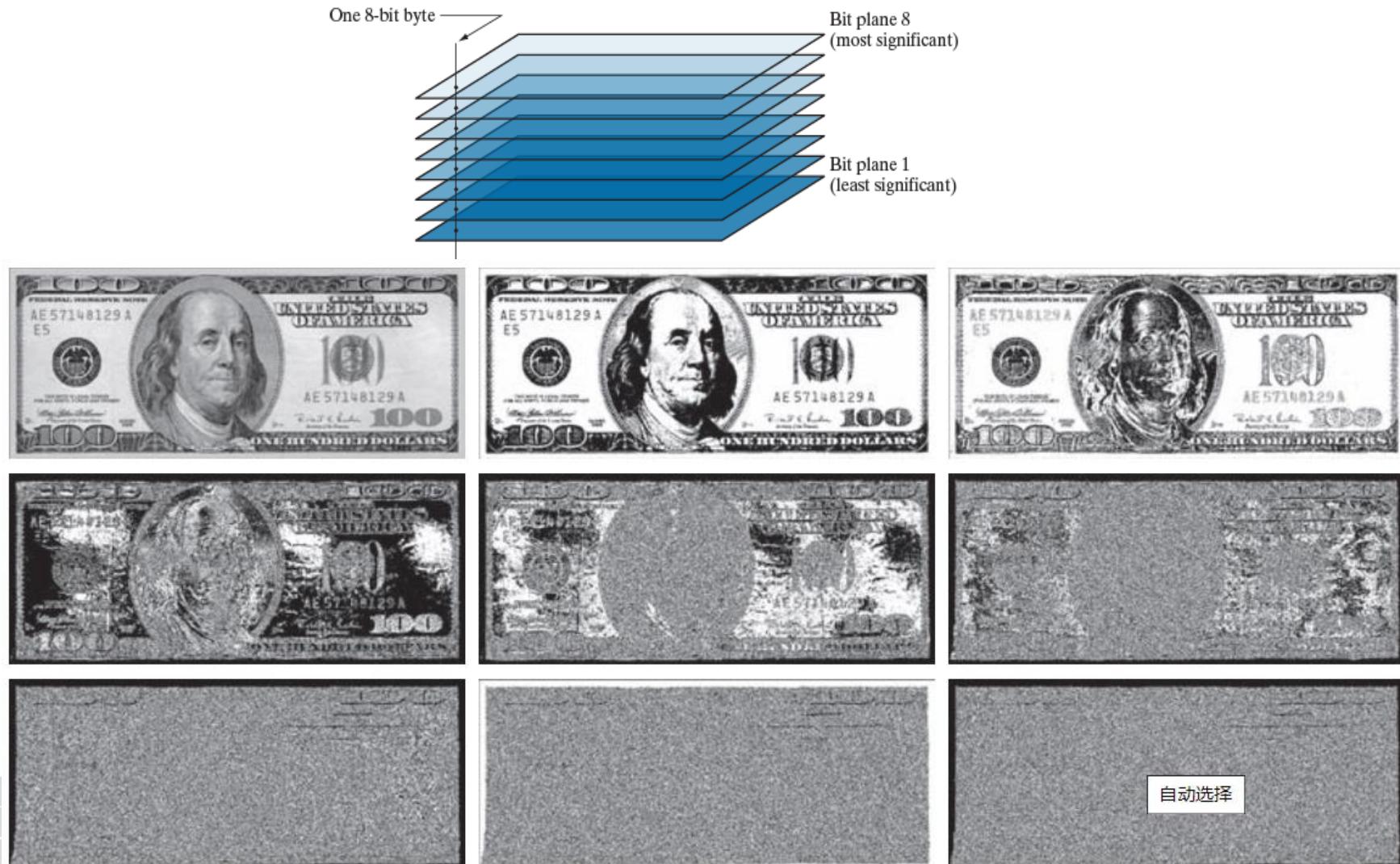


FIGURE 3.14 (a) An 8-bit gray-scale image of size 550×1192 pixels. (b) through (i) Bit planes 8 through 1, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image..

Bit-Plane Slicing

- ❑ Useful in image compression



Original image



bit planes 8, 7 and 6



bit planes 8 and 7

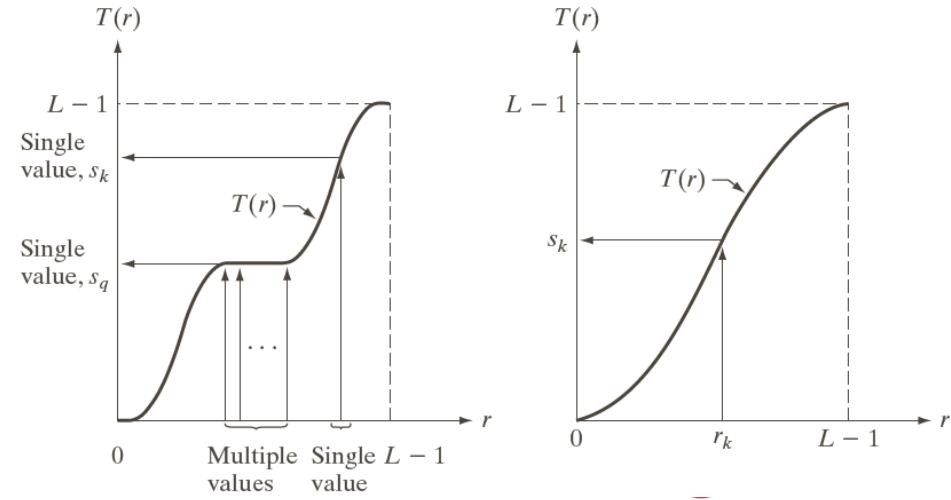


bit planes 8, 7, 6 and 5



Basis of Histogram Processing

- Given intensity transformation $s = T(r)$, where $T(r)$
 - $T(r)$ is strictly monotonically increasing function (严格单调递增函数, $T(r_2) > T(r_1)$ if $r_2 > r_1$) in the interval $0 \leq r \leq L - 1$
 - $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$
- The inverse transform $r = T^{-1}(s)$



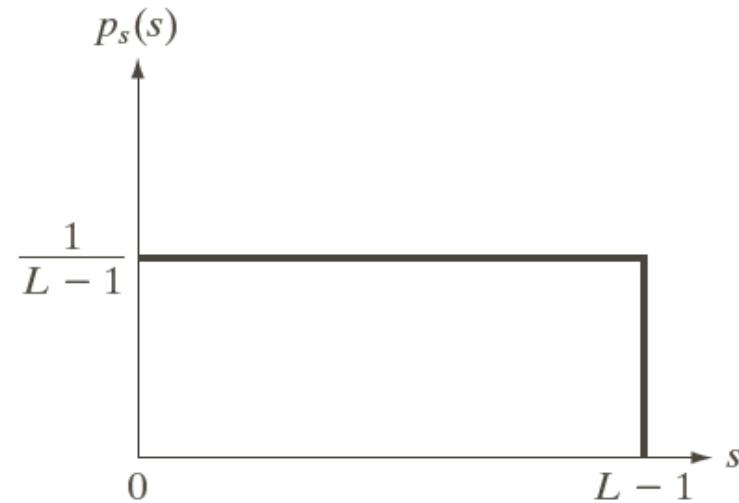
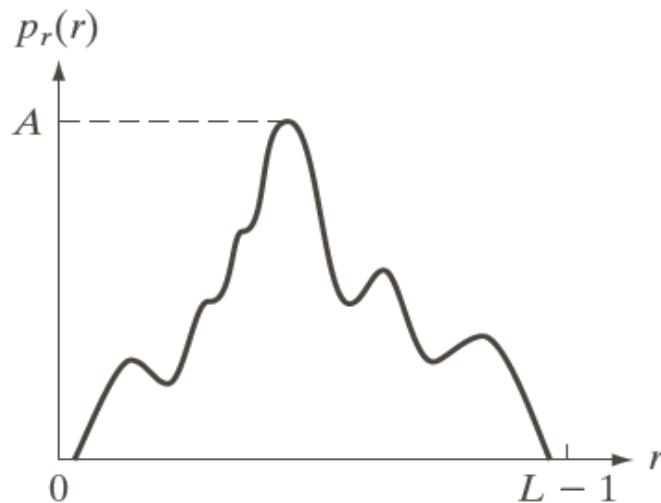
Histogram Equalization

➤ Uniform Probability density function : $p_s(s) = \frac{1}{L-1}$

➤ The probability density function (PDF) of s is

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds} \Rightarrow p_r(r) \cdot \frac{dr}{ds} = \frac{1}{L-1} \Rightarrow (L-1)p_r(r) \cdot dr = ds$$

➤ Transformation function : $s = T(r) = (L-1) \int_0^r p_r(w) dw$



Complementary prove

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds}$$

➤ Since $S = T(r)$ is strictly monotonically increasing function

⇒ We have $s = T(r)$, $v = T(w)$, if $v < s$ then we have $v < s \Leftrightarrow w < r$

⇒ $P(v < s) = P(w < r)$

⇒ $(\int_{-\infty}^s p_s(v) dv)' = (\int_{-\infty}^r p_r(w) dw)'$

⇒ $p_s(s) ds = p_r(r) dr$

⇒ $p_s(s) = p_r(r) \cdot \frac{dr}{ds}$

1) If $f(x)$ is continuous on $[a,b]$, then
 $F(x) = \int_a^x f(t) dt$ is differentiable,
and $F'(x) = f(x)$.

2) If $f(x)$ is continuous on $[a,b]$, and
 $\varphi(x)$ is differentiable, then

$$(\int_a^{\varphi(x)} f(t) dt)' = f[\varphi(x)]\varphi'(x).$$



Histogram Equalization

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw = (L - 1) \sum_{j=0}^k p_r(r_j) = (L - 1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{L - 1}{MN} \sum_{j=0}^k n_j k$$
$$= 1, 2, \dots, L - 1$$

r _k	n _k	p _r (r _k)	s _k			s' _k	p _s (s _k)
0	790	0.19	1.33	1		0	0
1	1023	0.25	3.08	3		1	0.19
2	850	0.21	4.55	5		2	0
3	656	0.16	5.67	6		3	0.25
4	329	0.08	6.23	6		4	0
5	245	0.06	6.65	7		5	0.21
6	122	0.03	6.86	7		6	0.24
7	81	0.02	7.00	7		7	0.11



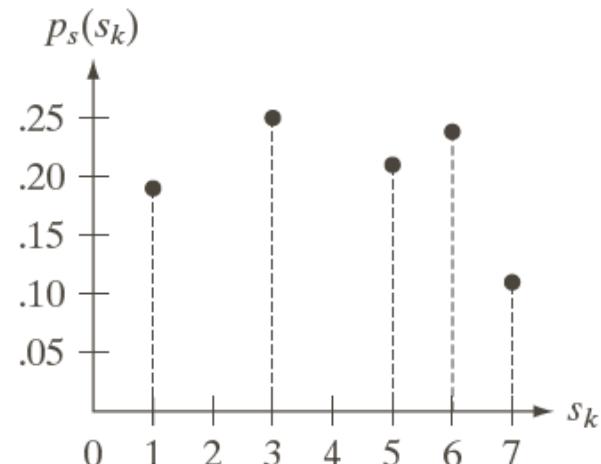
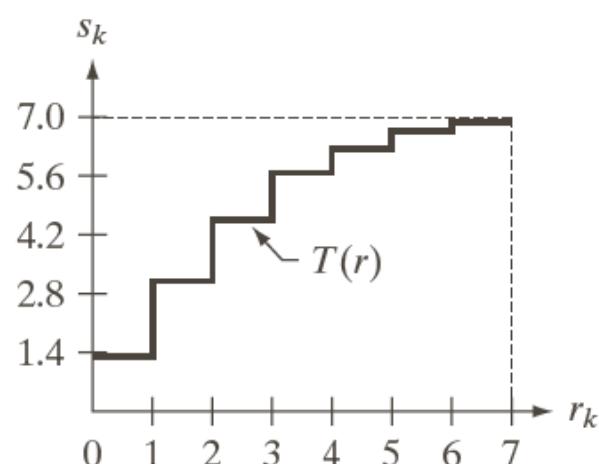
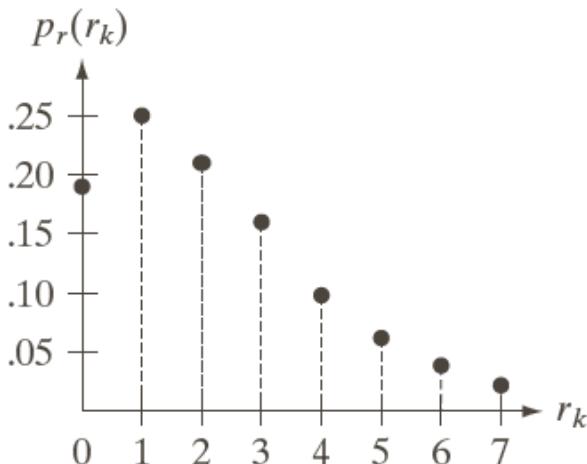
Histogram Equalization

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw = (L - 1) \sum_{j=0}^k p_r(r_j) = (L - 1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{L - 1}{MN} \sum_{j=0}^k n_j k$$
$$= 1, 2, \dots, L - 1$$

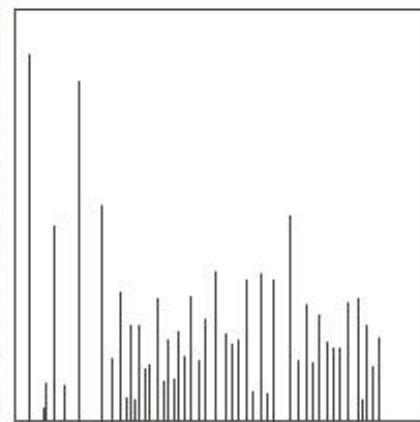
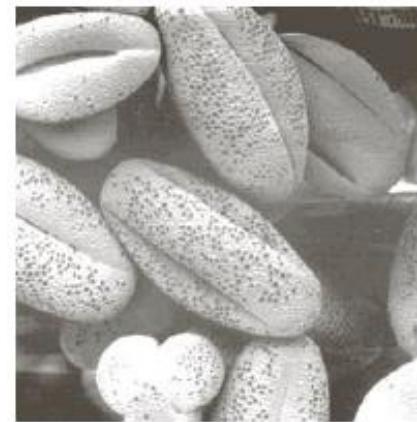
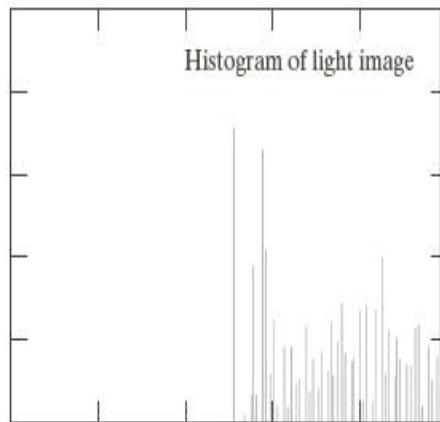
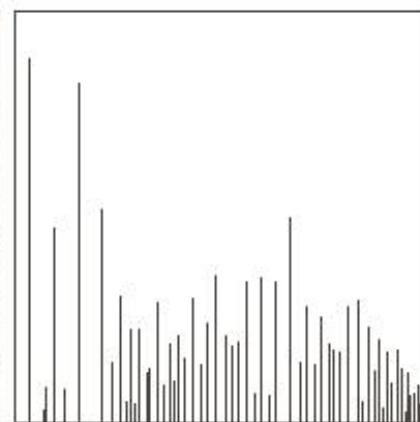
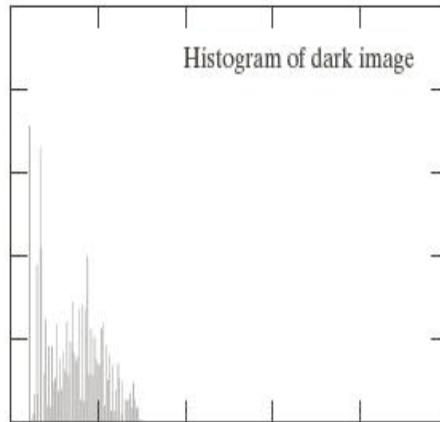
r _k	n _k	p _r (r _k)	s _k		s' _k	p _s (s _k)
0	790	0.19	1.33	1	0	0
1	1023	0.25	3.08	3	1	0.19
2	850	0.21	4.55	5	2	0
3	656	0.16	5.67	6	3	0.25
4	329	0.08	6.23	6	4	0
5	245	0.06	6.65	7	5	0.21
6	122	0.03	6.86	7	6	0.24
7	81	0.02	7.00	7	7	0.11



Example

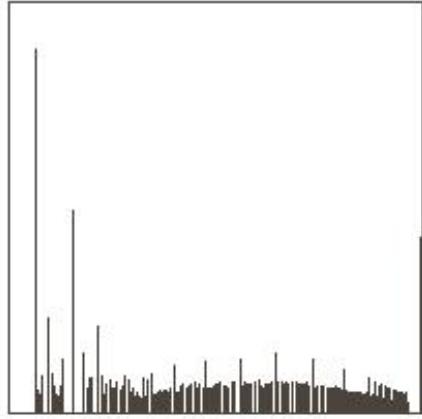
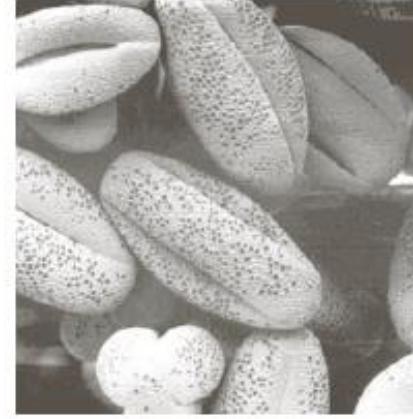
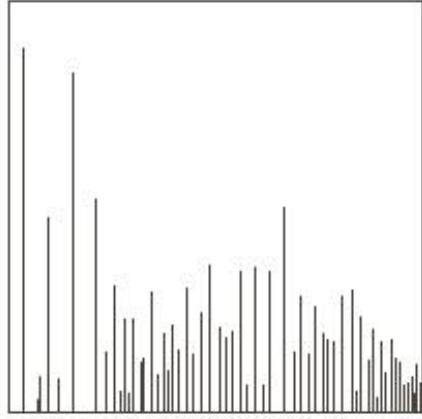
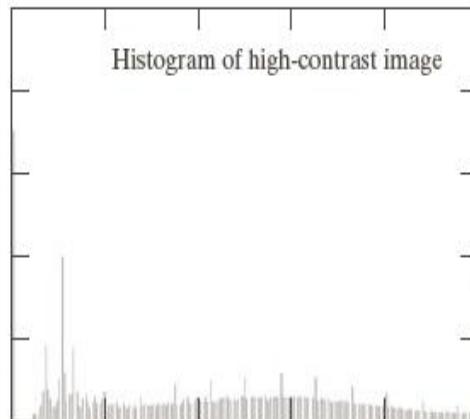
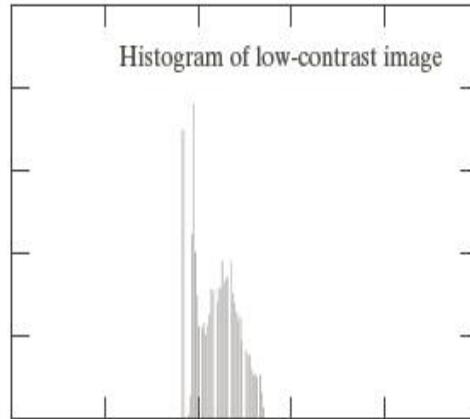


Example



上海科技大学
ShanghaiTech University

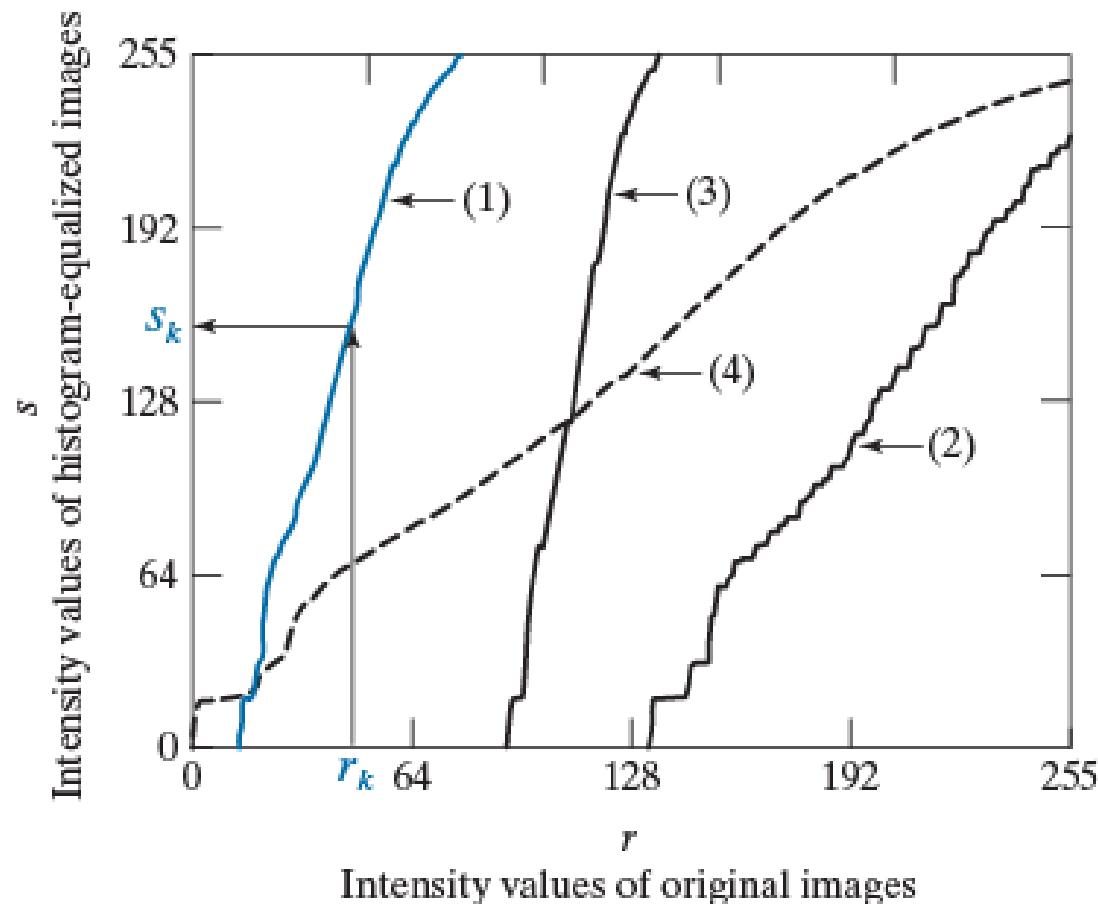
Example



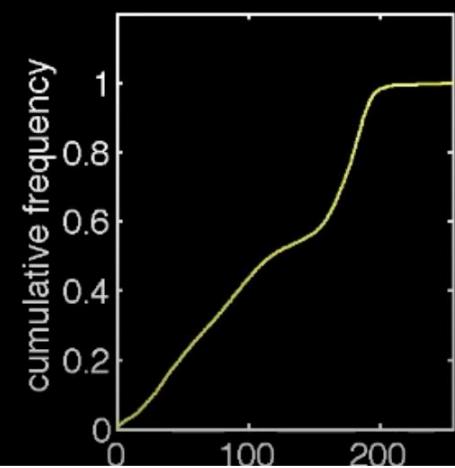
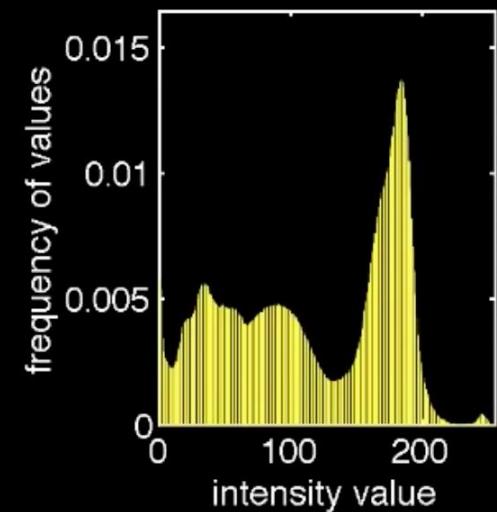
Transformation Function

FIGURE 3.21

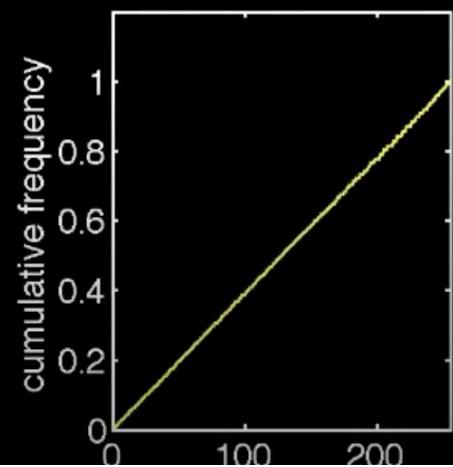
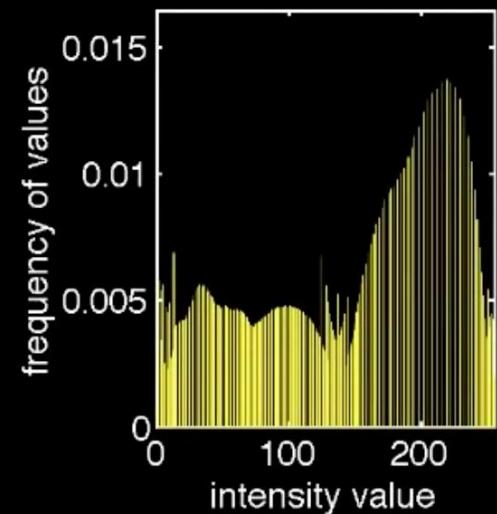
Transformation functions for histogram equalization. Transformations (1) through (4) were obtained using Eq. (3-15) and the histograms of the images on the left column of Fig. 3.20. Mapping of one intensity value r_k in image 1 to its corresponding value s_k is shown.



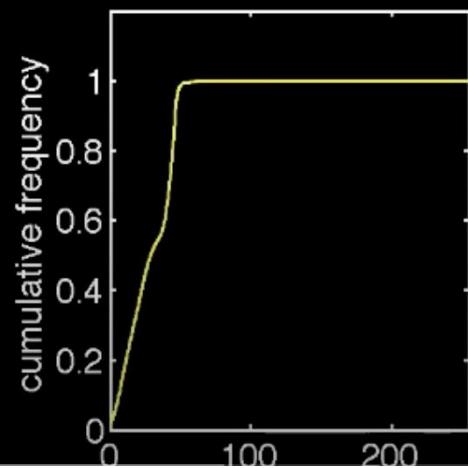
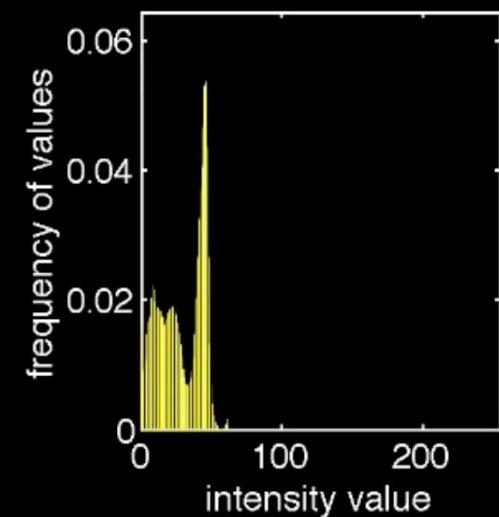
Example



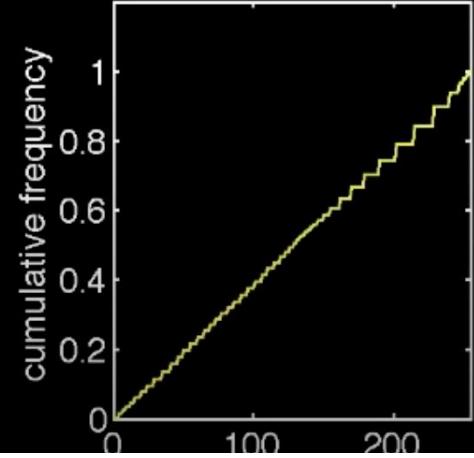
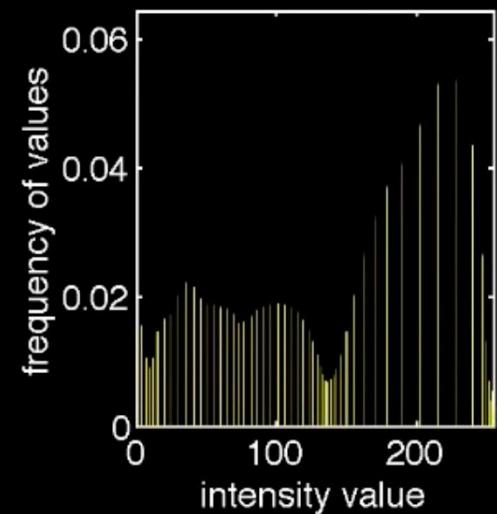
Example



Example



Example



Histogram Matching

- Generate a processed image with a specified histogram

For input : $s = T(r) = (L - 1) \int_0^r p_r(w) dw$

For output : $G(z) = (L - 1) \int_0^z p_z(t) dt = s$

Therefore $z = G^{-1}(s) = G^{-1}[T(r)]$



Histogram Matching

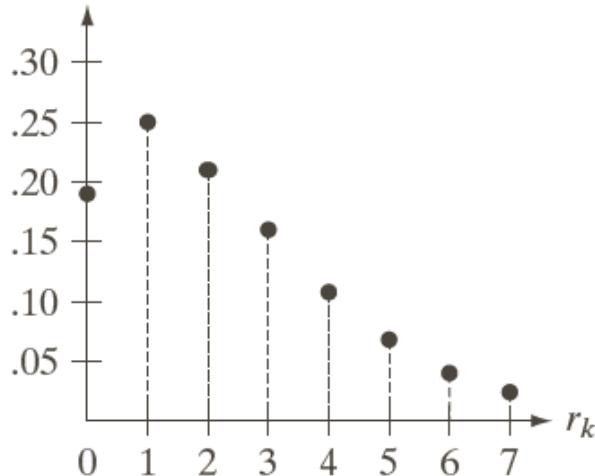
r_k	$p(r_k)$	$s'_k = T(r_k)$	z_k	$p(z_k)$	$s_k = G(z_k)$	$s_k \rightarrow z_k$	$r_k \rightarrow z_k$	z_k	$p(z_k)$
0	0.19	1	0	0	0	0 → 0, 1, 2	0 → 3	0	0
1	0.25	3	1	0	0	1 → 3	1 → 4	1	0
2	0.21	5	2	0	0	2 → 4	2 → 5	2	0
3	0.16	6	3	0.15	1		3 → 6	3	0.19
4	0.08	6	4	0.20	2		4 → 6	4	0.25
5	0.06	7	5	0.30	5	5 → 5	5 → 7	5	0.21
6	0.03	7	6	0.20	6	6 → 6	6 → 7	6	0.24
7	0.02	7	7	0.15	7	7 → 7	7 → 7	7	0.11

input
specified
actual (output)

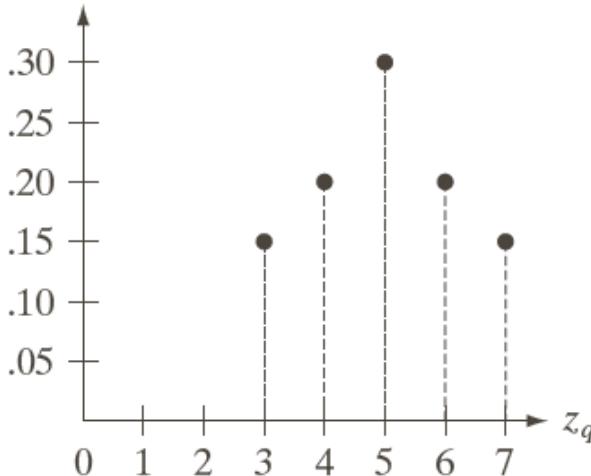


Histogram Matching

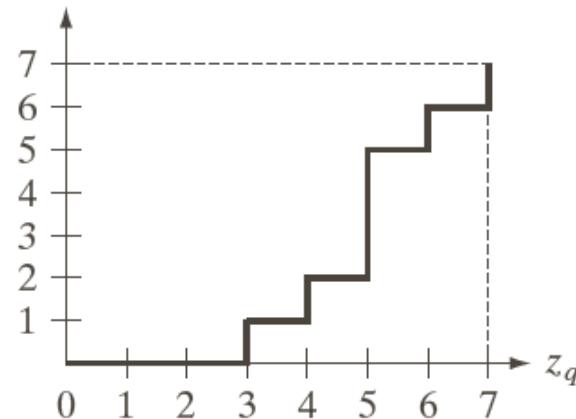
$$p_r(r_k)$$



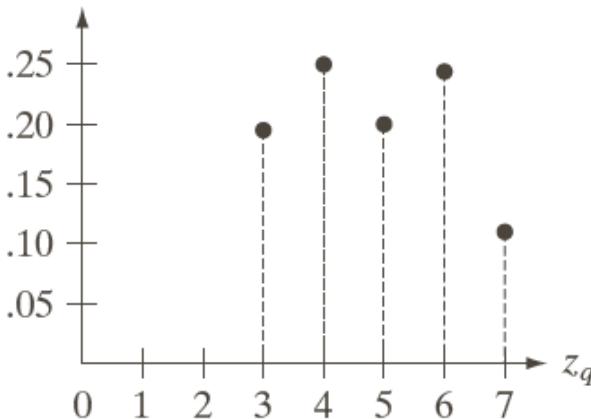
$$p_z(z_q)$$



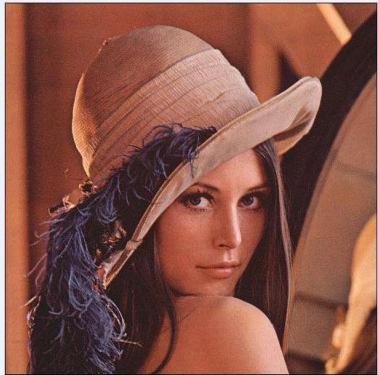
$$G(z_q)$$



$$p_z(z_q)$$



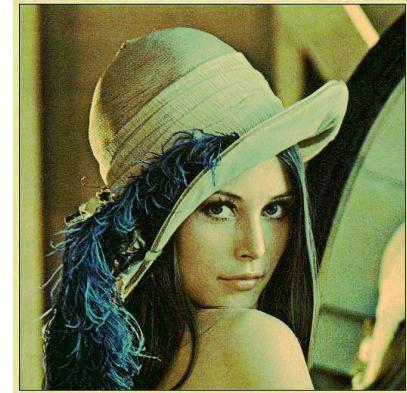
Histogram Matching Application



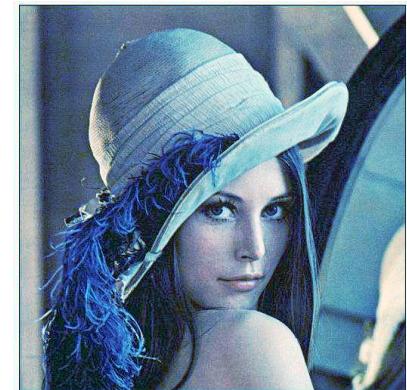
input



specified



output



Take home message

- ❑ Histogram describes intensity property of image, **NO** location (spatial) information.
- ❑ Simplest image processing technique- intensity transform.

$$s=T(r)$$

- ❑ The main purpose of intensity transform is to modify image histogram, to make the image contrast look more comfortable.
- ❑ Common intensity transform: contrast stretching, log, gamma, histogram equalization, histogram matching.

Spatial Filtering-Outline

- Spatial filtering definition

- Smoothing (平滑)

- Linear filter
- Non-linear filter

- Sharpening (锐化)

- Spatial differentiation
- Laplace filter



Spatial Filtering

- A Spatial filter is directly applied on the image
- A Spatial filter is also called spatial masks (掩模), kernels (核), templates(模板), windows (窗口)
- A Spatial filter consists of
 - 1) neighborhood
 - 2) a predefined operation
- A Spatial filter can be linear and nonlinear
 - Linear spatial filter corresponds to spectral filter in frequency domain
 - Nonlinear spatial filter cannot be accomplished in frequency domain



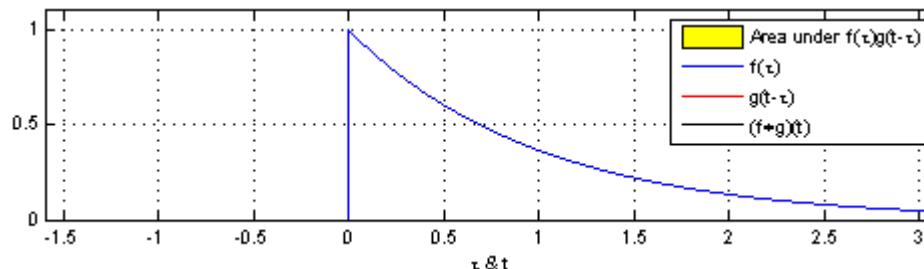
Time-domain Convolution

- Convolution of two signals $x(t)$ and $h(t)$, denoted by $x(t) * h(t)$, is defined by

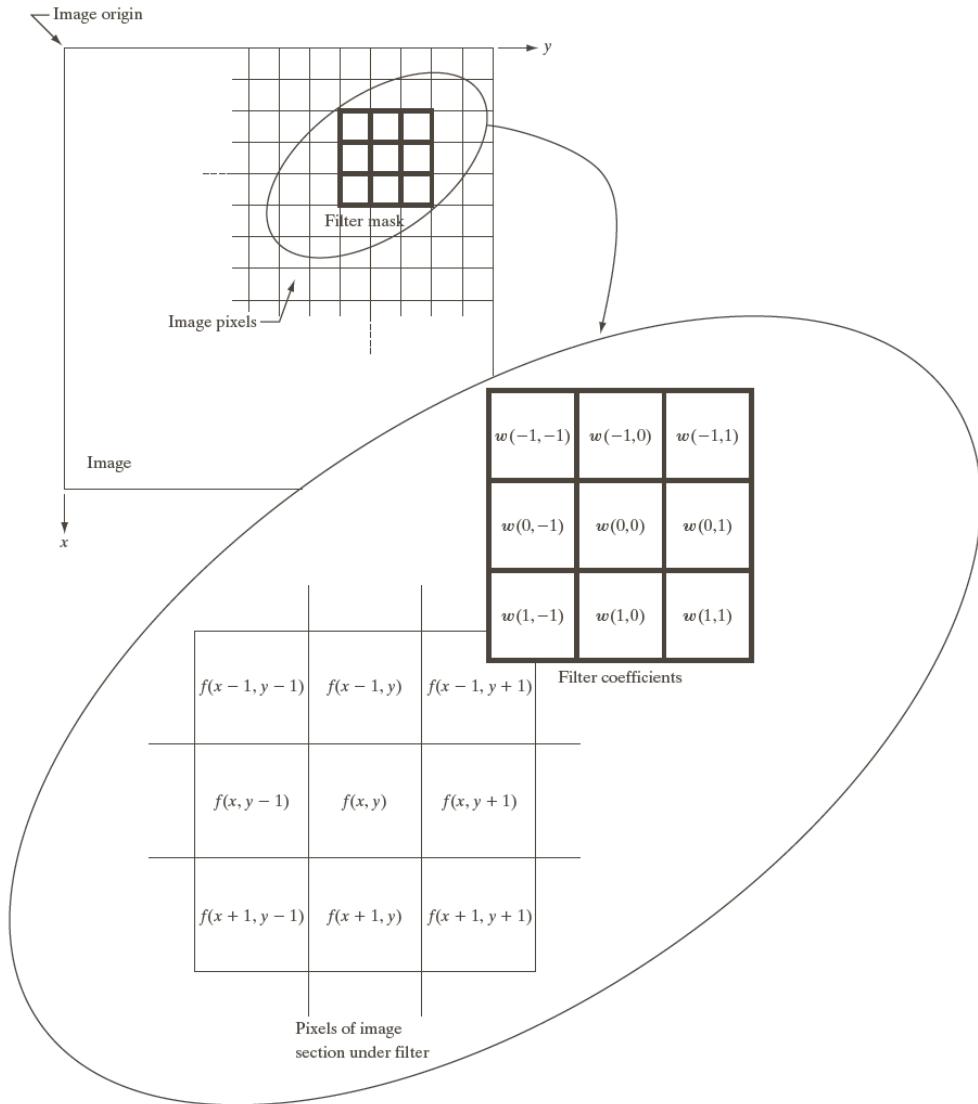
$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t - \tau)d\tau$$

- For discrete-time

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n - k]$$



Spatial Filter



$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

- $f(x, y)$: input image
- $g(x, y)$: output filtered image
- $w(s, t)$: $m \times n$ spatial filter, where $m = 2a + 1, n = 2b + 1$

Spatial Filter

- For discrete-time convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

- Spatial filter

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t)f(x + s, y + t)$$

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t)f(x - s, y - t)$$

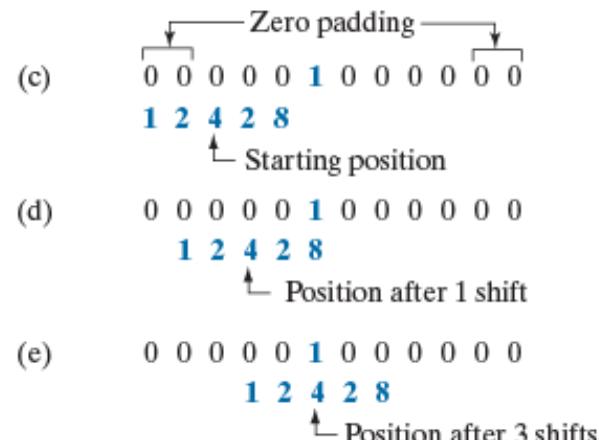
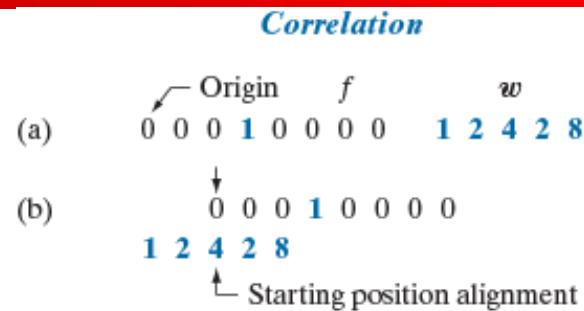


Next: correlation or convolution?

- What is convolution?
- What is correlation?
- What's the difference?
- How?
- Is convolution commutative?
- Is correlation commutative?
- Why convolution?
- Why correlation?

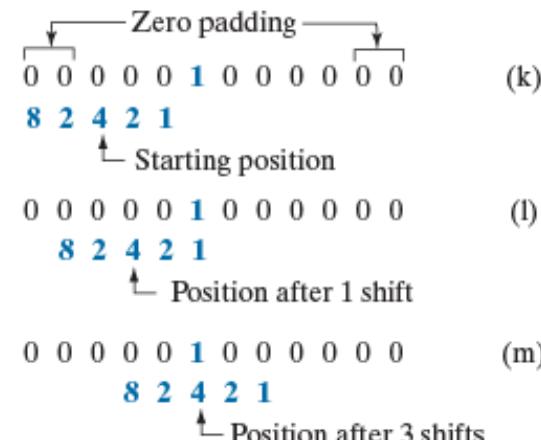
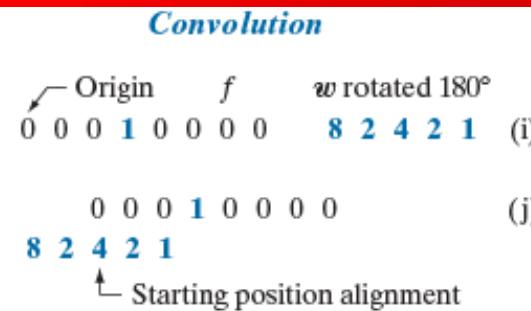


Correlation and Convolution (1D)



(g) Correlation result
 $0 \ 8 \ 2 \ 4 \ 2 \ 1 \ 0 \ 0$

(h) Extended (full) correlation result
 $0 \ 0 \ 0 \ 8 \ 2 \ 4 \ 2 \ 1 \ 0 \ 0 \ 0 \ 0$



(o) Convolution result
 $0 \ 1 \ 2 \ 4 \ 2 \ 8 \ 0 \ 0$

(p) Extended (full) convolution result
 $0 \ 0 \ 0 \ 1 \ 2 \ 4 \ 2 \ 8 \ 0 \ 0 \ 0 \ 0$

Correlation and Convolution (2D)

		Padded f							
Origin f		0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(a)

(b)

Initial position for w		Correlation result				Full correlation result			
1	2	3	0	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0	0
7	8	9	0	0	0	0	9	8	7
0	0	0	1	0	0	0	6	5	4
0	0	0	0	0	0	0	3	2	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(c)

(d)

(e)

Rotated w		Convolution result				Full convolution result			
9	8	7	0	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0	0
3	2	1	0	0	0	0	1	2	3
0	0	0	1	0	0	0	4	5	6
0	0	0	0	0	0	0	7	8	9
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(f)

(g)

(h)



Correlation and Convolution Equations

Correlation

$$w(s, t) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

Convolution

$$w(s, t) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

TABLE 3.5
Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$



Spatial Filter Masks

➤ Linear Spatial Filter (线性滤波器)

$$\text{➤ } R = \frac{1}{9} \sum_{k=1}^9 z_k \text{ (Box filter)}$$

$$\text{➤ } G(s, t) = K e^{-\frac{x^2+y^2}{2\sigma^2}} \text{ (Gaussian filter)}$$

➤ Nonlinear Spatial Filter (非线性滤波器)

□ Max filter (最大值滤波)

□ Median filter (中值滤波)



Smooth Filters (平滑濾波器)

- Blurring – for preprocessing tasks
- Noise deduction
 - Linear filter : average filtering – lowpass filter in frequency domain
 - Nonlinear filter



Smooth Filters (平滑濾波器)

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1



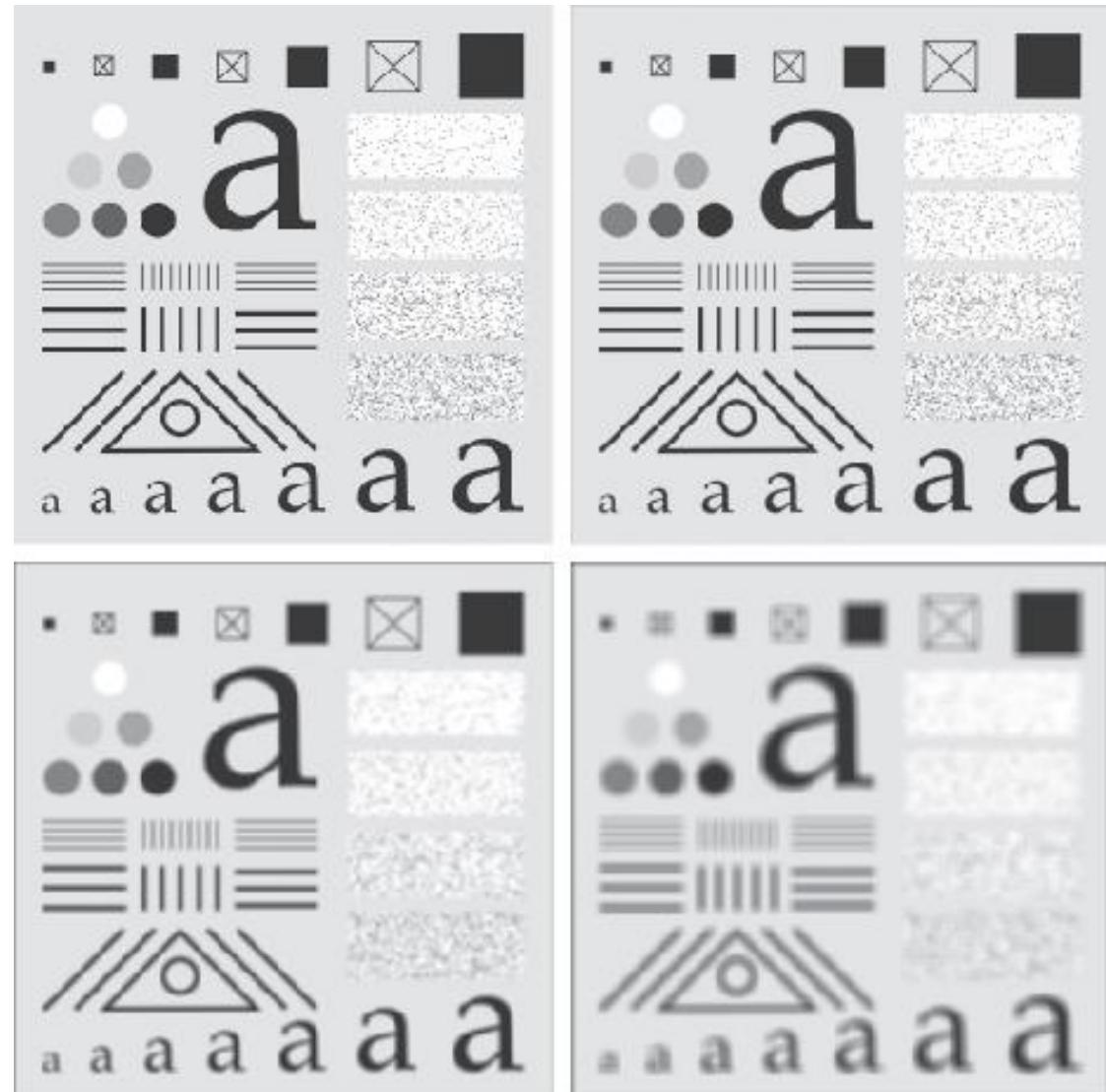
Filter size

a
b
c
d

FIGURE 3.39

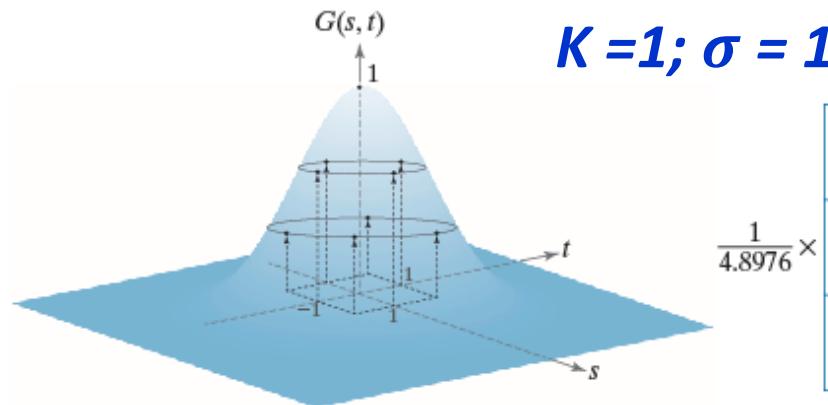
(a) Test pattern of size 1024×1024 pixels.

(b)-(d) Results of lowpass filtering with box kernels of sizes 3×3 , 11×11 , and 21×21 , respectively.



Gaussian Filter

$$G(s, t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$
$$G(r) = Ke^{-\frac{r^2}{2\sigma^2}}$$



0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

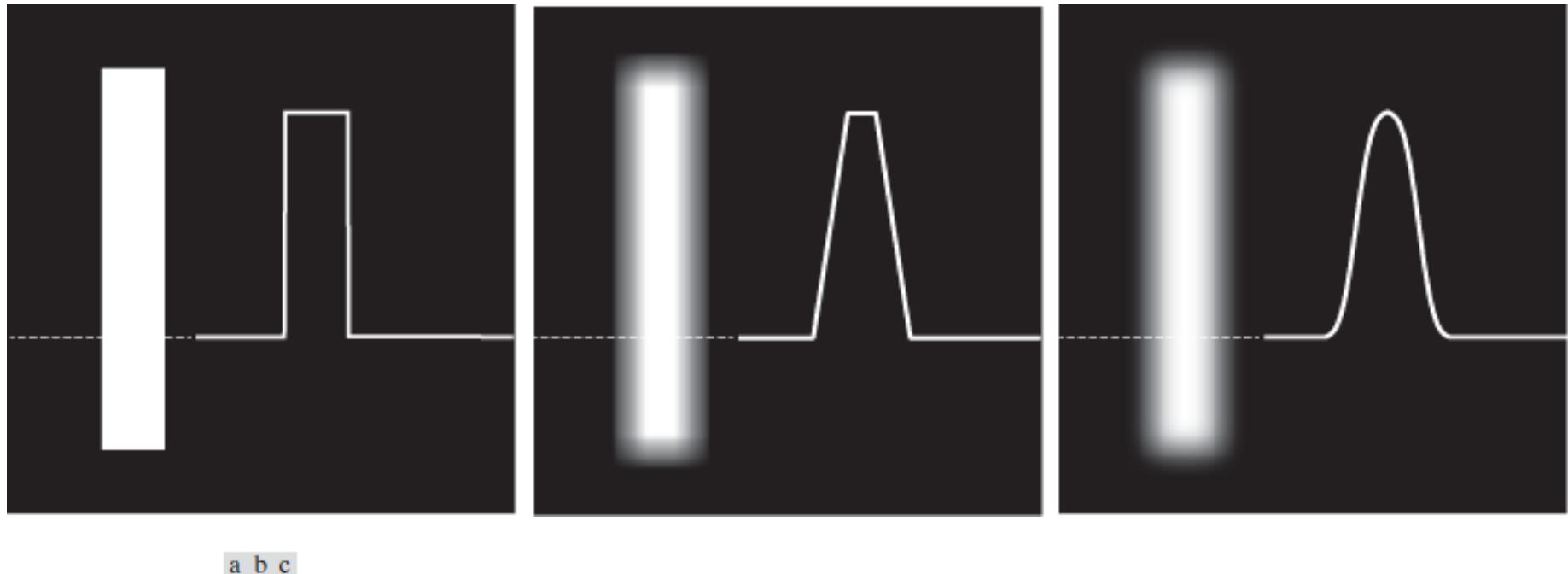


a b c

FIGURE 3.42 (a) A test pattern of size 1024×1024 . (b) Result of lowpass filtering the pattern with a Gaussian kernel of size 21×21 , with standard deviations $\sigma = 3.5$. (c) Result of using a kernel of size 43×43 , with $\sigma = 7$. This result is comparable to Fig. 3.39(d). We used $K = 1$ in all cases.



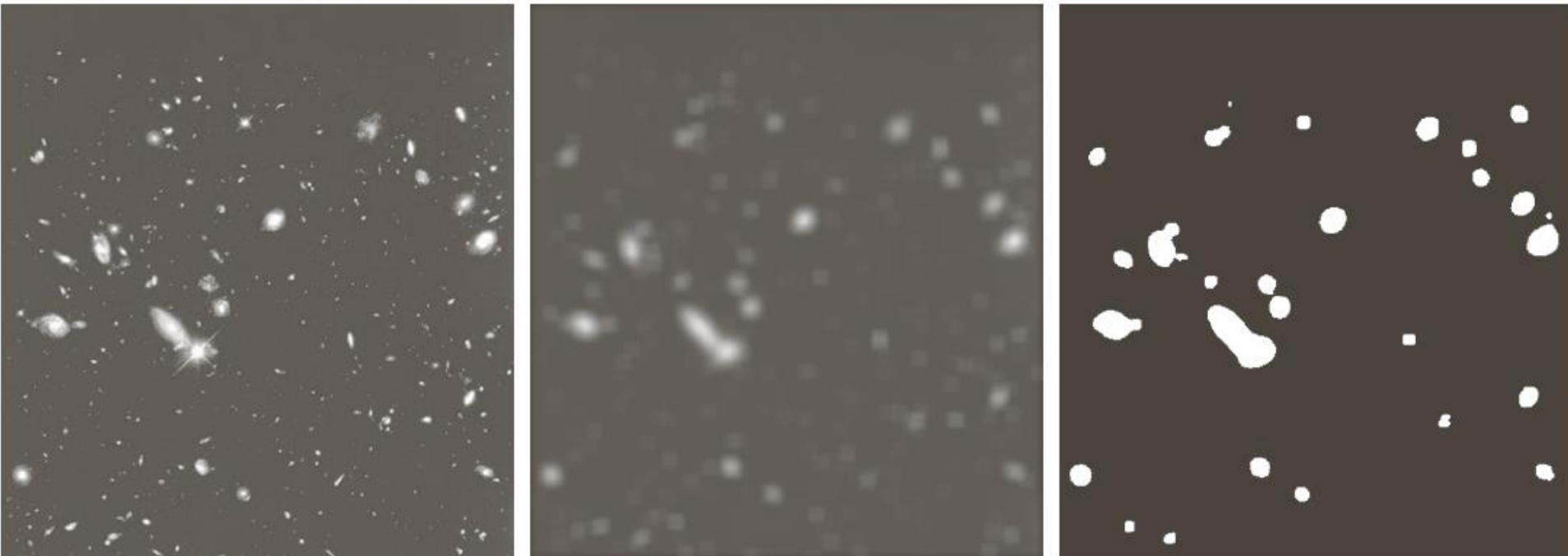
Box v.s. Gaussian



a b c

FIGURE 3.44 (a) Image of a white rectangle on a black background, and a horizontal intensity profile along the scan line shown dotted. (b) Result of smoothing this image with a box kernel of size 71×71 , and corresponding intensity profile. (c) Result of smoothing the image using a Gaussian kernel of size 151×151 , with $K = 1$ and $\sigma = 25$. Note the smoothness of the profile in (c) compared to (b). The image and rectangle are of sizes 1024×1024 and 768×128 pixels, respectively.

Smooth Filter and Thresholding(阈值处理)



Shading correction



a b c

FIGURE 3.48 (a) Image shaded by a shading pattern oriented in the -45° direction. (b) Estimate of the shading patterns obtained using lowpass filtering. (c) Result of dividing (a) by (b). (See Section 9.8 for a morphological approach to shading correction).

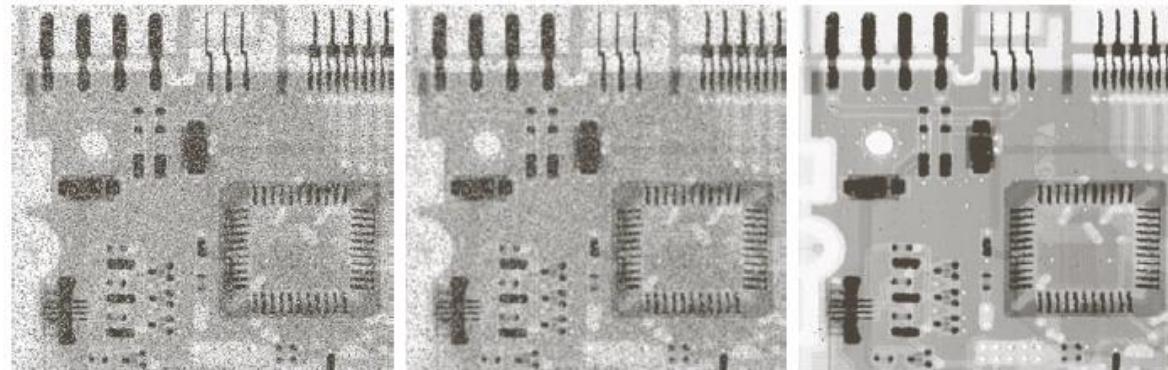
Nonlinear Smooth Filters

➤ Order-statistic filter (统计排序滤波器)

Ex: median filter (中值滤波器)

$g(x, y) = \text{median}\{m \times n \text{ pixel neighbouring around } I(x, y)\}$

50	48	46	42
52	0	50	48
46	47	255	40
51	48	46	42



Sharpening Filter

- Spatial differentiation (空间微分)
- Sharpening filter
 - Laplacian filtering (拉普拉斯算子)



Sharpening Filter

- ❑ To highlight transitions in intensity
- ❑ Accomplished by spatial differentiation
 - First-order derivative: $\frac{\partial f}{\partial x} = f(x + 1) - f(x)$
 - Second-order derivative: $\frac{\partial^2 f}{\partial x^2} = f(x + 1) + f(x - 1) - 2f(x)$



Laplacian(拉普拉斯算子)

For an image function $f(x, y)$,

$$\text{X direction: } \frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\text{Y direction: } \frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x, y+1) + f(x, y-1) + f(x+1, y) + f(x-1, y) - 4f(x, y)$$



Laplacian Filter Masks

0	1	0	1	1	1	0	-1	-1
1	-4	1	1	-8	1	-1	4	-1
0	1	0	1	1	1	0	-1	-1



Image Sharpening with Laplacian

a
b
c
d

FIGURE 3.52

(a) Blurred image of the North Pole of the moon.

(b) Laplacian image obtained using the kernel in Fig. 3.51(a).

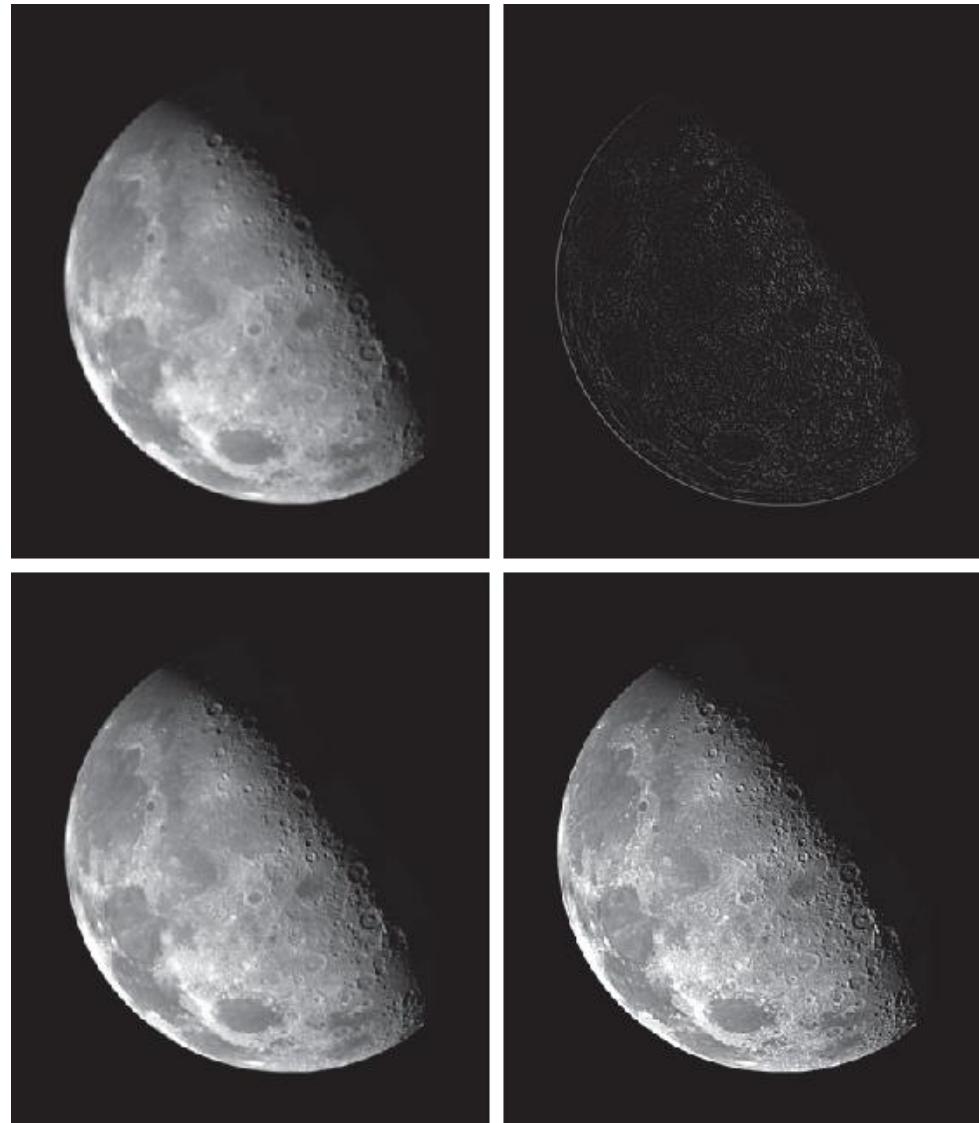
(c) Image sharpened using Eq. (3-63) with $c = -1$.

(d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b).

(Original image courtesy of NASA.)

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1



Laplacian Filter Masks

$$g(x, y) = f(x, y) + c\nabla^2 f(x, y), \quad \text{where } c = \pm 1$$

	1	

—

	1	
1	-4	1
	1	

=

	-1	
-1	5	-1
	-1	

	1	

+

	-1	
-1	4	-1
	-1	

=

	-1	
-1	5	-1
	-1	



Implementations in matlab

Low pass filter example:

```
>> LP = 1/9 *[1,1,1;1,1,1;1,1,1];  
  
>> im3 = imfilter(com,LP);  
  
>>figure; imshow(im3,[]);
```

Sharpening filter example:

```
>> f3 = [-1,-1,-1; -1,8,-1;-1,-1,-1];  
  
>> J1 = imfilter(im,f1);  
  
>>figure; imshow(J1,[]);
```

Median filter example:

```
>> J2 = medfilt2(im,[3 3]);  
  
>> J4 = medfilt2(im,[6 6]);  
  
>> J3 = medfilt2(im,[11 1]);  
  
>> J5 = medfilt2(im,[1 11]);
```



Take home message

- For image processing, the spatial domain processing is familiar with 1-D signal processing in time domain.
- The spatial filter we mentioned in this lecture is actually correlation between image and the filter, however, when using a diagnose symmetric filter, it is equivalent to convolution.
- Common spatial filters involving smoothing filter and sharpening filter.

