
Machine Learning, 2024 Spring

Assignment 6

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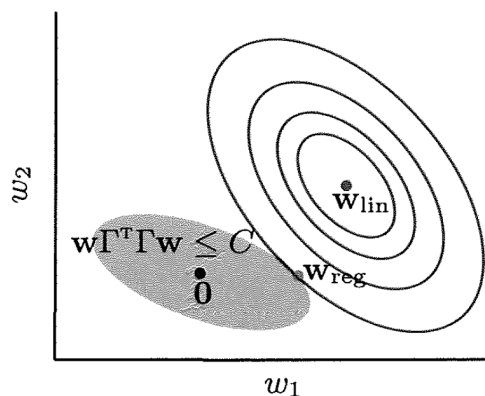
Notice

Plagiarizer will get 0 points.
 \LaTeX is highly recommended. Otherwise you should write as legibly as possible.

Problem 1 In this problem, you will investigate the relationship between the soft order constraint and the augmented error. The regularized weight \mathbf{w}_{reg} is a solution to

$$\begin{aligned} & \min E_{\text{in}}(\mathbf{w}) \\ & \text{subject to } \mathbf{w}^T \Gamma^T \Gamma \mathbf{w} \leq C \end{aligned}$$

- (a) If $\mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}} \leq C$, then what is \mathbf{w}_{reg} ?
(b) If $\mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}} > C$, the situation is illustrated below,



The constraint is satisfied in the shaded region and the contours of constant E_{in} are the ellipsoids (why ellipsoids?). What is $\mathbf{w}_{\text{reg}}^T \Gamma^T \Gamma \mathbf{w}_{\text{reg}}$?

(c) Show that with

$$\lambda_C = -\frac{1}{2C} \mathbf{w}_{\text{reg}}^T \nabla E_{\text{in}}(\mathbf{w}_{\text{reg}})$$

\mathbf{w}_{reg} minimizes $E_{\text{in}}(\mathbf{w}) + \lambda_C \mathbf{w}^T \Gamma^T \Gamma \mathbf{w}$. [Hint: use the previous part to solve for \mathbf{w}_{reg} as an equality constrained optimization problem using the method of Lagrange multipliers.]

(d) Show that the following hold for λ_C :

- (i) If $\mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}} \leq C$ then $\lambda_C = 0$ (\mathbf{w}_{lin} itself satisfies the constraint).
(ii) If $\mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}} > C$, then $\lambda_C > 0$ (the penalty term is positive).
(iii) If $\mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}} > C$, then λ_C is a strictly decreasing function of C . [Hint: show that $\frac{d\lambda_C}{dC} < 0$ for $C \in [0, \mathbf{w}_{\text{lin}}^T \Gamma^T \Gamma \mathbf{w}_{\text{lin}}]$.]

Solution

Problem 2 [The Lasso algorithm] Rather than a soft order constraint on the squares of the weights, one could use the absolute values of the weights:

$$\begin{aligned} & \min E_{\text{in}}(\mathbf{w}) \\ & \text{subject to } \sum_{i=0}^d |w_i| \leq C \end{aligned}$$

The model is called the lasso algorithm.

- (a) Formulate and implement this as a quadratic program.
- (b) What is the augmented error and discuss the algorithm for solving it. You can solve this problem using iterative soft-thresholding algorithm or a gradient projection method and present your pseudocode.

Solution

Problem 3 Similar to problem 3 in assignment 3 and assignment 4, you need to use the SUV dataset to implement (using Python or MATLAB) the L_1/L_2 regularization (penalty/augmented).

- Present your code.
- Present the path plots of L_1 and L_2 regularization. (Notice: you need to mark the selected value of the regular parameter)
- Analyze the weight difference between L_1 and L_2 regularization. (Notice: you need to describe the similarities and differences between the solutions of path plots)
- If you only want to build a model that contains 2 variables, which two features would you choose?

Solution