$E_{(x,y)}$

$\min \underline{E_{(x,y)}[y \neq f(x)]}$.

$= \mathbb{P}[f(x) \neq y]$

$\pi(x) = \mathbb{P}[Y = \cancel{0}1 \mid x]$

$f(x) = \begin{cases} 1 & \text{if } \pi(x) \geq \frac{1}{2} \\ -1 & \text{else. } \pi(x) < \frac{1}{2} \end{cases}$

If $\pi(x) \geq \frac{1}{2} \Rightarrow f(x) = 1$.

~~$f(x) \neq y \Rightarrow f(x) = +1,\ y = -1$.~~

$\Rightarrow \mathbb{P}(f(x) \neq y) = \mathbb{P}(y = -1) = 1 - \pi(x)$

If $\pi(x) < \frac{1}{2}$, $\Rightarrow f(x) = -1$

$\mathbb{P}(f(x) \neq y) = \mathbb{P}(y = 1) = \pi(x)$

$\Rightarrow \mathbb{P}[f(x) \neq y] = \min(\pi(x), 1 - \pi(x))$

For other $h(x)$.

令 $p = \mathbb{P}[h(x) = +1 \mid x]$ determinitic or not

$\Rightarrow \mathbb{P}[h(x) \neq y]$

$= \mathbb{P}[h(x) \neq y \mid y = 1]\,\mathbb{P}(y = 1]$

$+ \mathbb{P}[h(x) \neq y \mid y = -1]\,\mathbb{P}(y = -1)$

$= (1 - p(x))\pi(x) + p(x)(1 - \pi(x))$

$\geq [(1 - p(x)) + p(x)] \min(\pi(x), 1 - \pi(x))$

$= E_{(x,y)}[f(x) \neq y]$