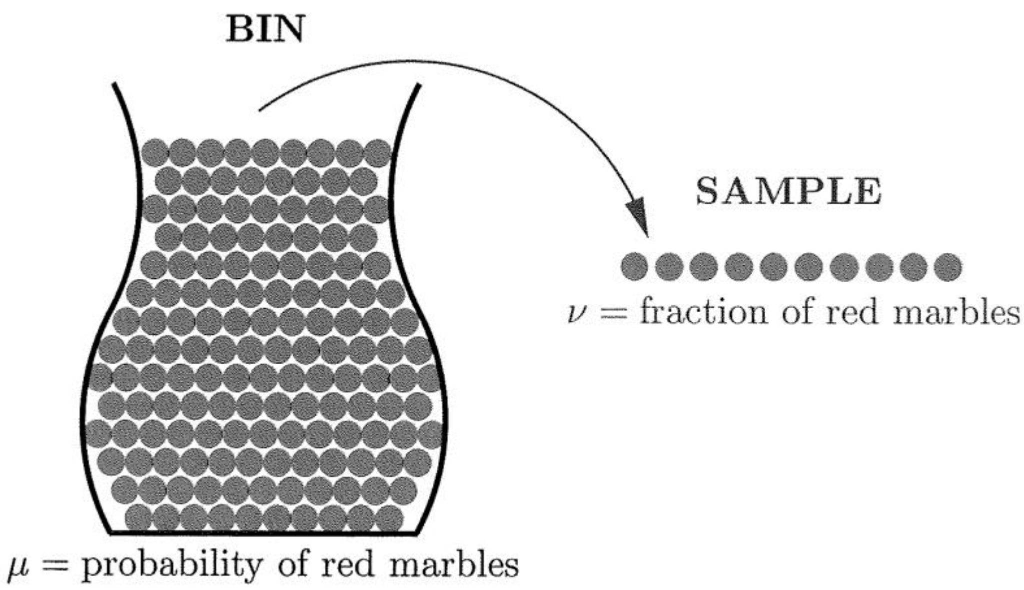
The contents of this homework are Exercise 1.8, 1.9 and Problem 1.10, 1.12 in the "Learning from data (Yaser)" textbook. For your convenience, the questions are listed below.

The following picture is for Exercise 1.8 and 1.9.



Exercise 1.8

If $\mu = 0.9$, what is the probability that a sample of 10 marbles will have $\nu \leq 0.1$? *[Hints: 1. Use binomial distribution. 2. The answer is a very small number.]*

Exercise 1.9

If $\mu = 0.9$, use the Hoeffding Inequality to bound the probability that a sample of 10 marbles will have $\nu \leq 0.1$ and compare the answer to the previous exercise.

Problem 1.10 Assume that $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, \mathbf{x}_{N+1}, \ldots, \mathbf{x}_{N+M}\}$ and $\mathcal{Y} = \{-1, +1\}$ with an unknown target function $f\colon \mathcal{X} \to \mathcal{Y}$. The training data set $\mathcal{D}$ is $(\mathbf{x}_1, y_1), \cdots, (\mathbf{x}_N, y_N)$. Define the *off-training-set error* of a hypothesis h with respect to f by

$$E_{\text{off}}(h, f) = \frac{1}{M} \sum_{m=1}^{M} [\![h(\mathbf{x}_{N+m}) \neq f(\mathbf{x}_{N+m})]\!] \,.$$

(a) Say $f(\mathbf{x}) = +1$ for all $\mathbf{x}$ and

$$h(\mathbf{x}) = \begin{cases} +1, & \text{for } \mathbf{x} = \mathbf{x}_k \text{ and } k \text{ is odd and } 1 \leq k \leq M + N \\ -1, & \text{otherwise} \end{cases} \,.$$

What is $E_{\text{off}}(h, f)$?

(b) We say that a target function f can 'generate' $\mathcal{D}$ in a noiseless setting if $y_n = f(\mathbf{x}_n)$ for all $(\mathbf{x}_n, y_n) \in \mathcal{D}$. For a fixed $\mathcal{D}$ of size N, how many possible $f\colon \mathcal{X} \to \mathcal{Y}$ can generate $\mathcal{D}$ in a noiseless setting?

(c) For a given hypothesis h and an integer k between 0 and M, how many of those f in (b) satisfy $E_{\text{off}}(h, f) = \frac{k}{M}$?

(d) For a given hypothesis h, if all those f that generate $\mathcal{D}$ in a noiseless setting are equally likely in probability, what is the expected off-training-set error $\mathbb{E}_f[E_{\text{off}}(h, f)]$?

(continued on next page)

(e) A deterministic algorithm A is defined as a procedure that takes $\mathcal{D}$ as an input, and outputs a hypothesis $h = A(\mathcal{D})$. Argue that for any two deterministic algorithms A_1 and A_2,

$$\mathbb{E}_f\big[E_{\text{off}}(A_1(\mathcal{D}), f)\big] = \mathbb{E}_f\big[E_{\text{off}}(A_2(\mathcal{D}), f)\big].$$

You have now proved that in a noiseless setting, for a fixed $\mathcal{D}$, if all possible f are equally likely, any two deterministic algorithms are equivalent in terms of the expected off-training-set error. Similar results can be proved for more general settings.

Problem 1.12 This problem investigates how changing the error measure can change the result of the learning process. You have N data points $y_1 \leq \cdots \leq y_N$ and wish to estimate a 'representative' value.

(a) If your algorithm is to find the hypothesis h that minimizes the in-sample sum of squared deviations,

$$E_{\text{in}}(h) = \sum_{n=1}^{N} (h - y_n)^2,$$

then show that your estimate will be the in-sample mean,

$$h_{\text{mean}} = \frac{1}{N} \sum_{n=1}^{N} y_n.$$

(b) If your algorithm is to find the hypothesis h that minimizes the in-sample sum of absolute deviations,

$$E_{\text{in}}(h) = \sum_{n=1}^{N} |h - y_n|,$$

then show that your estimate will be the in-sample median h_{med}, which is any value for which half the data points are at most h_{med} and half the data points are at least h_{med}.

(c) Suppose y_N is perturbed to $y_N + \epsilon$, where $\epsilon \to \infty$. So, the single data point y_N becomes an outlier. What happens to your two estimators h_{mean} and h_{med}?