Test 1 in Class:

1. What's the relationship between $H(X), H(Y)$, $H(X|Y)$, $H(Y|X)$ and $I(X;Y)$? Draw a Venn diagram to explain the relationship.
2. Chain Rules:

$$H(X_1, X_2, \ldots, X_n) = ? \tag{1}$$
$$I(X_1, X_2, \ldots, X_n; Y) = ? \tag{2}$$

3. Prove $H(X) \geq 0, I(X;Y) \geq 0$ for any RVs X, Y.
4. Check whether the following statements are correct or not.
   - $H(X|Y) \leq H(X)$;
   - $H(X|Y=y) \leq H(X)$ for any X, y;
   - $I(X;Y) \geq I(X;Z)$ if $X - Y - Z$ forms Markov chain;
   - $I(X;Y)$ is convex of $p(x,y)$;
   - Relative Entropy $D(p(x)||q(x)) = D(q(x)||p(x))$ for any distributions $p(x), q(x), x \in \mathcal{X}$.
5. Given two independent RVs $X, Y \in \{0,1\}$ where $P(X=0) = p_1$, $P(Y=0) = p_2$. Compute $H(X,Y), H(X \oplus Y), H(X+Y)$.