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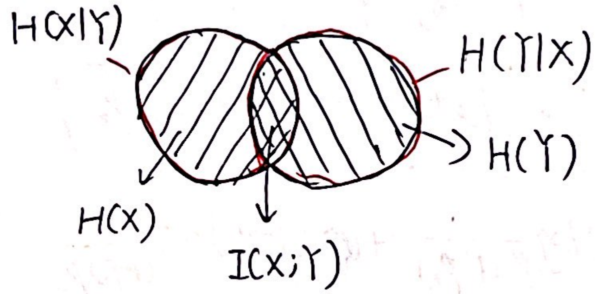
1.



2 <1> $H(X_1, X_2, \cdots, X_n) = \sum_{i=1}^{n} H(X_i \mid X_1, \cdots, X_{i-1})$

$= \sum_{i=1}^{n} H(X_i \mid X_{i+1}, \cdots, X_n)$

<2> $I(X_1, X_2, \cdots, X_n ; Y) = \sum_{i=1}^{n} I(X_i ; Y \mid X_1, \cdots, X_{i-1})$

$= \sum_{i=1}^{n} I(X_i ; Y \mid X_{i+1}, \cdots, X_n)$

3. From the definition of $H(X)$:

<1> $H(X) = \sum_{x \in \mathcal{X}} p(x) \log \frac{1}{p(x)}$, since $p(x) \in (0,1)$, so $\log \frac{1}{p(x)} > 0$, ~~i.e.~~

i.e. $\forall x \in \mathcal{X}$, $p(x) \log \frac{1}{p(x)} > 0$, so $H(X) > 0$

iff $p(x) = 1$, i.e. X is determinstic , $H(X) = 0$

so $H(X) \geq 0$

<2> $I(X;Y) = \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} p(x,y) \frac{\log p(x,y)}{p(x)p(y)} = D(p(x,y) \| p(x)p(y))$

Since $D(p\|q) \geq 0$, iff $p = q$, $D(p\|q) = 0$. $\Rightarrow I(X;Y) \geq 0$

So iff $p(x,y) = p(x)p(y)$, $\forall x \in \mathcal{X}, y \in \mathcal{Y}$, which means that $X \perp Y$, $I(X;Y) = 0$

4. <1> $H(X|Y) \leq H(X)$ ~~False~~ ~~H(X|Y) ≥ I~~ True .

<2> $H(X|Y=y) \leq H(X)$, $\forall x, y$ False . The ~~inequta~~ inequlity could be $<, =, >$

<3> $I(X;Y) \geq I(X;Z)$ if $X - Y - Z$ True .

<4> $I(X;Y)$ is convex of $p(x,y)$ False . Its nonconvex and non-concave

<5> $D(p(x) \| q(x)) = D(q(x) \| p(x))$ False .

$D(p(x) \| q(x)) = \sum_{x \in \mathcal{X}} p(x) \log \frac{p(x)}{q(x)}$, its non swapable .

5.

5°. $X, Y \in \{0,1\}$, $P(X=0)=p_1$, $P(Y=0)=p_2$, $X \perp Y$

<1> $H(X,Y) = \sum_{x,y} p(x,y) \log \frac{1}{p(x,y)}$ | Let $q_1 = 1-p_1$, $q_2 = 1-p_2$ |

$$= p_1p_2 \log\frac{1}{p_1p_2} + p_1(1-p_2)\log\frac{1}{p_1(1-p_2)} + (1-p_1)p_2\log\frac{1}{(1-p_1)p_2}$$

$$+ (1-p_1)(1-p_2)\log\frac{1}{(1-p_1)(1-p_2)}$$

$$= p_1p_2\log\frac{1}{p_1p_2} + p_1q_2\log\frac{1}{p_1q_2} + q_1p_2\log\frac{1}{q_1p_2} + q_1q_2\log\frac{1}{q_1q_2}$$

<2> $H(X \oplus Y)$

$$X \oplus Y = \begin{cases} 0, & \text{w.p. } \cancel{p_1q_2+} p_1p_2 + q_1q_2 \\ 1, & \text{w.p. } p_1q_2 + \cancel{p_1p_2}\, q_1p_2 \end{cases}$$

$$= (p_1p_2+q_1q_2)\log\frac{1}{p_1p_2+q_1q_2} + (p_1q_2+q_1p_2)\log\frac{1}{p_1q_2+q_1p_2}$$

<3> $\cancel{H}$

$$X+Y = \begin{cases} 0, & \text{w.p. } p_1p_2 \\ 1, & \text{w.p. } p_1q_2 + q_1p_2 \\ 2, & \text{w.p. } q_1q_2 \end{cases}$$

$$H(X+Y) = p_1p_2\log\frac{1}{p_1p_2} + (p_1q_2+q_1p_2)\log\frac{1}{p_1q_2+q_1p_2} + q_1q_2\log\frac{1}{q_1q_2}$$