Fundamentals of Information Theory Homework 1

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- 2.1 Coin flips. A fair coin is flipped until the first head occurs. Let X denote the number of flips required.
 - (a) Find the entropy H(X) in bits. The following expressions may be useful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad \sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2}$$

(b) A random variable X is drawn according to this distribution. Find an "efficient" sequence of yes-no questions of the form, "Is X contained in the set S?" Compare H(X) to the expected number of questions required to determine X.

Solution

- (a)
- (b)

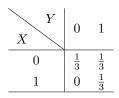
2.3 Minimum entropy. What is the minimum value of $H(p_1, ..., p_n) = H(\mathbf{p})$ as \mathbf{p} ranges over the set of n-dimensional probability vectors? Find all \mathbf{p} 's that achieve this minimum.

Solution

2.5 Zero conditional entropy. Show that if $H(Y \mid X) = 0$, then Y is a function of X [i.e., for all x with p(x) > 0, there is only one possible value of y with p(x, y) > 0].

Solution

2.12 Example of joint entropy. Let p(x,y) be given by



Find:

- (a) H(X), H(Y).
- (b) H(X | Y), H(Y | X).
- (c) H(X, Y).
- (d) $H(Y) H(Y \mid X)$.
- (e) I(X;Y).
- (f) Draw a Venn diagram for the quantities in parts (a) through (e).

Solution

- (a)
- (b)
- (c)
- (d)
- (e)

- 2.14 Entropy of a sum. Let X and Y be random variables that take on values x_1, x_2, \ldots, x_r and y_1, y_2, \ldots, y_s , respectively. Let Z = X + Y.
- (a) Show that $H(Z \mid X) = H(Y \mid X)$. Argue that if X, Y are independent, then $H(Y) \leq H(Z)$ and $H(X) \leq H(Z)$. Thus, the addition of independent random variables adds uncertainty.
- (b) Give an example of (necessarily dependent) random variables in which H(X) > H(Z) and H(Y) > H(Z).
 - (c) Under what conditions does H(Z) = H(X) + H(Y)? Solution
 - (a)
 - (b)
 - (c)