1. (a) $\frac{du(t)}{dt} = \delta(t)$ 1'

so $\frac{d}{dt}\{u(-2-t) + u(t-2)\} = -\delta(t+2) + \delta(t-2)$ 2'

$X(jw) = \int_{-\infty}^{\infty} (\delta(t-2) - \delta(t+2)) e^{-jwt} dt$

$= e^{-2jw} - e^{2jw} = -2j\sin 2w$ 4'

(b) $w_0 = 6\pi$ 1'

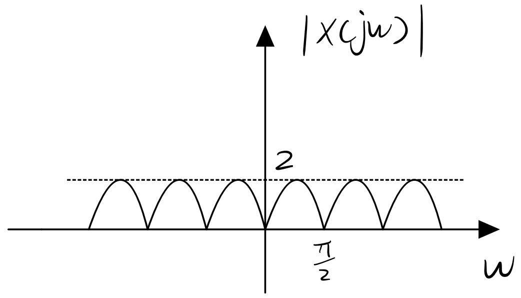
Consider the Fourier series:

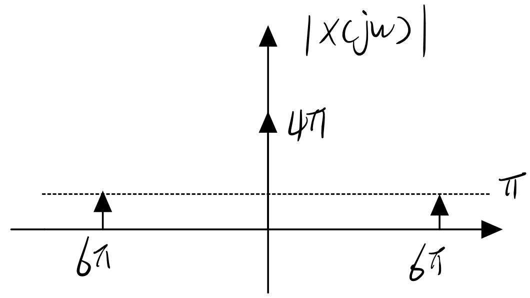
$x(t) = 2 + \frac{1}{2}e^{j(6\pi t + \frac{\pi}{8})} + \frac{1}{2}e^{-j(6\pi t + \frac{\pi}{8})}$

$= 2 + \frac{1}{2}e^{j\frac{\pi}{8}} e^{j6\pi t} + \frac{1}{2}e^{-j\frac{\pi}{8}} e^{-j6\pi t}$ 2'

where $a_0 = 2$, $a_1 = \frac{1}{2}e^{j\frac{\pi}{8}}$, $a_{-1} = \frac{1}{2}e^{-j\frac{\pi}{8}}$. $a_k = 0$ for other

so $X(jw) = 4\pi\delta(w) + \pi e^{j\frac{\pi}{8}}\delta(w-6\pi) + \pi e^{-j\frac{\pi}{8}}\delta(w+6\pi)$ 4'





2、(a) $x(t) \xleftrightarrow{\mathcal{F}} X(j\omega) \Rightarrow x(3t) \xleftrightarrow{\mathcal{F}} \frac{1}{3} X(j\frac{\omega}{3})$

$\Rightarrow x(3(t-2)) \xleftrightarrow{\mathcal{F}} e^{-2j\omega}\frac{1}{3} X(j\frac{\omega}{3}) \Rightarrow x^*(3t-6) \xleftrightarrow{\mathcal{F}} e^{-2j\omega}\frac{1}{3} X^*(-j\frac{\omega}{3})$

6'

For invertible system: $H_1(j\omega)H_2(j\omega)=1$

so the Fourier transform of inverse is:

$3e^{2j\omega}/X^*(-j\frac{\omega}{3})$ 4'

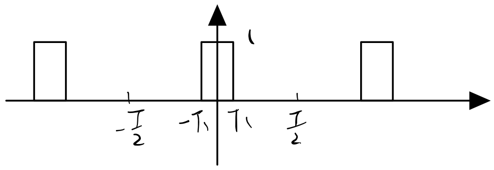
(b) $x(t) \xleftrightarrow{\mathcal{F}} X(j\omega) \Rightarrow \frac{dx(t)}{dt} \xleftrightarrow{\mathcal{F}} j\omega X(j\omega)$

$\Rightarrow \frac{d^2x(t)}{dt^2} \xleftrightarrow{\mathcal{F}} -\omega^2 X(j\omega)$ 5'

$\Rightarrow \frac{d^2x(t-1)}{dt^2} \xleftrightarrow{\mathcal{F}} -e^{-j\omega}\omega^2 X(j\omega)$ 5'

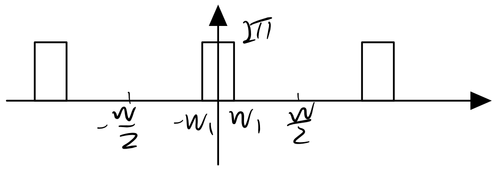
3. (a) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \Rightarrow x(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)e^{j\omega t}dt$ 3'

$\Rightarrow 2\pi x(j\omega) = \int_{-\infty}^{\infty} X(t)e^{-j\omega t}dt$, so $X(t) \xleftrightarrow{\mathcal{F}} 2\pi x(-j\omega)$ 4'

(b) 

$\xleftrightarrow{\mathcal{F}} \sum_{k=-\infty}^{\infty} \frac{2\sin(k\omega_0 T_1)}{k}\delta(\omega-k\omega_0)$

4'

so $X(t) \xleftrightarrow{\mathcal{F}}$ 

4'

4.(a) $\frac{Y(j\omega)}{X(j\omega)} = H(j\omega)$ 1'

so $(j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = j\omega X(j\omega) + 4X(j\omega)$ 2'

$\Rightarrow \frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$ 3'

(b) $H(j\omega) = \frac{j\omega + 4}{6-\omega^2+5j\omega}$ 2'

$= \frac{2}{2+j\omega} - \frac{1}{3+j\omega}$ 2'

Hence, $h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$ 3'

(c) $X(j\omega) = \frac{1}{4+j\omega} - \frac{1}{(4+j\omega)^2}$ 3'

so $Y(j\omega) = \frac{1}{2}\frac{1}{2+j\omega} - \frac{1}{2}\frac{1}{4+j\omega}$ 2'

so $y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t)$ 2'

5. (a) $k_p e(t) + K_I \int_{-\infty}^{t} e(\tau) d\tau + k_d \frac{d e(t)}{dt} = y(t)$ 3'

so $k_p \frac{d(x(t)-y(t))}{dt} + K_I(x(t)-y(t)) + k_d \frac{d^2(x(t)-y(t))}{dt^2} = y'(t)$

$\Rightarrow k_d \frac{d^2 x(t)}{dt^2} + k_p \frac{dx(t)}{dt} + K_I x(t) = k_d \frac{d^2 y(t)}{dt^2} + (k_p+1)\frac{dy(t)}{dt} + K_I y(t)$ 2'

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{k_d (j\omega)^2 + k_p j\omega + k_I}{k_d (j\omega)^2 + (k_p+1) j\omega + k_I} = \frac{-k_d\omega^2 + k_p j\omega + k_I}{-k_d\omega^2 + (k_p+1) j\omega + k_I}$$ 3'

(b) For PID controller:

Assume that the output, input is $y_o(t)$, $x_o(t)$

$y_o(t) = k_p x_o(t) + K_I \int_{-\infty}^{t} x_o(\tau) d\tau + k_d x_o'(t)$

$$\Rightarrow H_{pid}(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{k_p + k_I \frac{1}{j\omega} + k_d j\omega}{1} = \frac{k_p j\omega + k_I - k_d\omega^2}{j\omega}$$ 3'

let $k_I = 0$ $H_{pd}(j\omega) = k_p + k_d j\omega$ 3'

let $k_d = 0$ $H_{pi}(j\omega) = \frac{k_p j\omega + k_I}{j\omega}$ 3'

(c) $H_{pi}(j\omega) H_{pd}(j\omega) = (k_{p1} + \frac{k_I}{j\omega})(k_{p2} + k_d j\omega)$

$= 12 + \frac{5}{j\omega} + 4j\omega$ 6'

so it's equivalent to the pid controller with 2'

$k_p = 12$, $k_i = 5$, $k_d = 4$. proved.