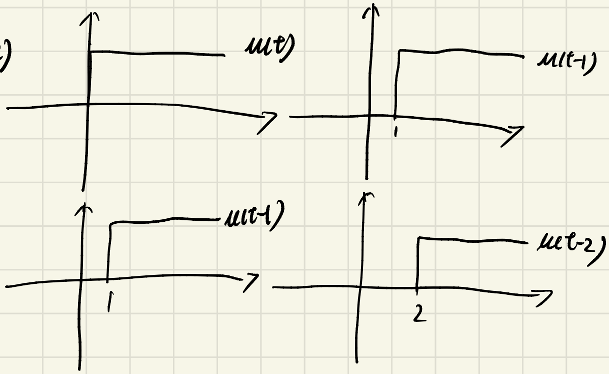
DT F F F F

casual	linear	time-invariant	stable
No	Yes.	Yes.	No.

2)



$$f_1(t) = x_0(t) * x_0(t-1)$$
$$= \int_{-\infty}^{+\infty} x_0(k)\cdot x_0(t-k-1)\,dk.$$

let $x_0(t) = u(t) - u(t-1) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{elsewhere.} \end{cases}$

$x_1(t) = u(t-1) - u(t-2) = \begin{cases} 1 & 1 \le t \le 2 \\ 0 & \text{elsewher} \end{cases} = x_0(t-1)$

1° $t-1 \le 0$ $f_1(t) = 0$

2° $0 < t-1 \le 1$ $f_1(t) = \int_0^{t-1} 1\,dk.$

$= t-1$ $1-t+2$

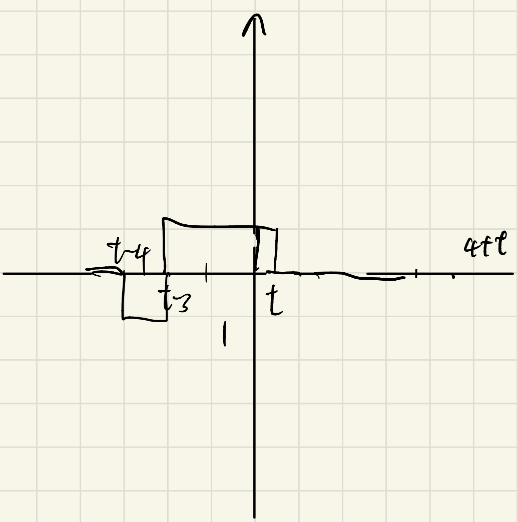
3° $0 < t-2 < 1$ $f_1(t) = \int_{t-2}^{1} 1\,dk$

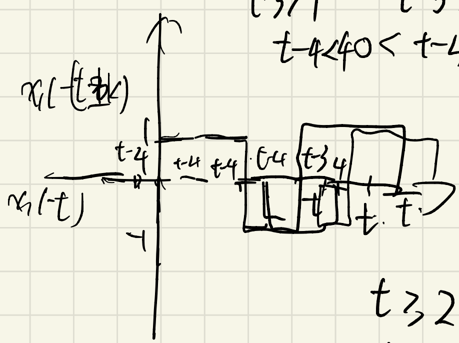
$= 1+t$

4° $t-2 \ge 1$ $f_1(t) = 0$

$t-3 > 4$ $t-3 \le 2.$

$t-4 < 4\, 0 < t-4$





$2 \le t \le 3$

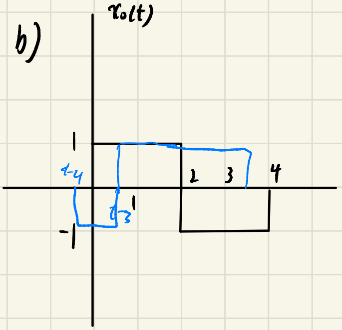
$t \ge 2.$

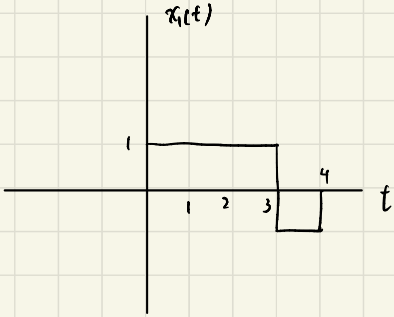
$t-3 \le 0$

$0 < t-3$

$3 \le 4$

b)





$t>0$

$-(k-t)$

$t-4\le 2$

$t-3>2$

$f_2(t) = x_0(t) * x_1(t) = \int_{-\infty}^{+\infty} x_0(k)\cdot x_1(t-k)\,dk.$

1° $t\le 0$ $f_2(t)=0$

2° $0<t\le 2$ $f_2(t)=\int_0^t 1\,dk = t.$

3° $2\le t\le 3$ $f_2(t)=\int_0^2 1\,dk + \int_2^t -1\,dk = 2-t+2 = 4-t.$

3° $3<t\le 4$ $f_2(t) = \int_0^{t-3} -1\,dk + \int_{t-3}^2 1\,dk + \int_2^t -1\,dk$

$= 3-t + 5-t + 2-t$

$= -3t+10$ $-3t+10$

4° $4<t\le 5$ $f_2(t) = \int_{t-4}^{t-3} -1\,dk + \int_{t-3}^2 1\,dk + \int_2^4 -1\,dk.$

$= -1 + 5-t - 2.$

$4-(t-3)$

$= 2-t.$ $2-t+4$

$7-t$

$t-7$

5° $5<t\le 6$ $f_2(t)=\int_{t-4}^2 -1\,dk + \int_2^{t-3} 1\,dk + \int_{t-3}^4 -1\,dk$

$2<t-4$ $= t-6 + t-5 + t-7$

$t-3<4$ $= 3t-18$

6° $6 < t \leq 7$ $f_2(t) = \int_{t-4}^{t-3} 1\,dk + \int_{t-3}^{4} -1\,dk$

$4-t+3$

$7-t$

$t-7$

$= 1+t-7$

$= t-6.$

7° $7 < t \leq 8$ $f_3(t) = \int_{t-4}^{4} 1\,dk = 8-t.$

3. (a) $z(t) = x(t)y(t)$

$$= \sum_{k=-\infty}^{+\infty} a_k \cdot e^{jk\omega_0 t} \cdot \sum_{k=-\infty}^{+\infty} b_k e^{jk\omega_0 t}$$

$$= (\dots + a_{-N} e^{-jN\omega_0 t} + \dots + a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + \dots + a_N e^{jN\omega_0 t} + \dots)$$

$$\cdot (\dots + b_{-N} e^{-jN\omega_0 t} + \dots + b_{-1} e^{-j\omega_0 t} + b_0 + b_1 e^{j\omega_0 t} + \dots + b_N e^{jN\omega_0 t} + \dots)$$

Let $z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

then for a fixed k, we have $c_k e^{jk\omega_0 t} = \dots + a_{-N} e^{-jN\omega_0 t} \cdot b_{k+N} e^{j(k+N)\omega_0 t} + \dots$

$$+ a_{-1} e^{-j\omega_0 t} \cdot b_{k+1} e^{j(k+1)\omega_0 t} + \dots + a_1 e^{j\omega_0 t} \cdot b_{k-1} e^{j(k-1)\omega_0 t}$$

$$+ \dots + a_N e^{jN\omega_0 t} \cdot b_{k-N} e^{j(k-N)\omega_0 t} + \dots$$

$$= \left(\sum_{n=-\infty}^{+\infty} a_n \cdot b_{k-n}\right) \cdot e^{jk\omega_0 t}$$

so, $c_k = \sum_{n=-\infty}^{+\infty} a_n \cdot b_{k-n}$.

b) $b_k = a_{-k}^*$

$$\frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt = \frac{1}{T_0} \int_0^{T_0} x(t) \cdot x^*(t)\, dt$$

$$= \frac{1}{T_0} \int_0^{T_0} x(t) \cdot \sum_{k=-\infty}^{+\infty} a_k^* e^{-jk\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{+\infty} a_k^* \frac{1}{T_0} \int_0^{T_0} x(t) \cdot e^{-jk\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{+\infty} a_k^* \cdot a_k$$

$$= \sum_{k=-\infty}^{+\infty} |a_k|^2$$

4. (1) $x(t) \xleftrightarrow{F} X(jw)$

$x(-t) \xleftrightarrow{F} X(-jw)$

$$e^{-a|t|} = x(t) + x(-t)$$

$$= X(jw) + X(-jw)$$

$$X(jw) = \int_{-\infty}^{+\infty} x(t)\, e^{-jwt} = \frac{1}{a+jw}$$

$$e^{-a|t|} = \frac{1}{a+jw} + \frac{1}{a-jw}$$

$$= \frac{2a}{a^2+w^2}$$

(2) Suppose $X(jw) = \frac{1}{1+w^2}$

then we know $x(t) = \frac{1}{2} \cdot e^{-|t|} \xleftrightarrow{F} \frac{1}{1+w^2}$

so we have $\frac{1}{2} \cdot e^{-|t|} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(jw)\, e^{jwt}\, dw$

$$\pi \cdot e^{-|t|} = \int_{-\infty}^{+\infty} \frac{1}{1+w^2}\, e^{jwt}\, dw$$

$$\text{so } \pi \cdot e^{-|t|} = \int_{-\infty}^{+\infty} \frac{1}{1+w^2}\, e^{-jwt}\, dw$$

$$\text{so } \pi \cdot e^{-|w|} = \int_{-\infty}^{+\infty} \frac{1}{1+t^2}\, e^{-jwt}\, dt.$$

(c) $X(jw) = \frac{1}{1+(3w)^2}$ $\quad \frac{1}{2}\left(\frac{1}{1+3jw} + \frac{1}{1-3jw}\right)$

time and frequency scaling

$x(t) \xleftrightarrow{F} X(jw)$

$x(at) \xleftrightarrow{F} \frac{1}{|a|} X\left(\frac{jw}{a}\right)$

$e^{-b|t|} = \frac{1}{b+jw} + \frac{1}{b-jw}$

$b=1.$ $a=\frac{1}{3}$ $\quad \frac{1}{6} e^{-|\frac{1}{3}t|} \xleftrightarrow{F} \frac{1}{2}\left(\frac{1}{1+3jw} + \frac{1}{1-3jw}\right)$

so. $\frac{1}{6} e^{-|\frac{1}{3}t|} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{1+(3w)^2} e^{jwt} dw$

$\frac{\pi}{3} e^{-|\frac{1}{3}t|} = \int_{-\infty}^{+\infty} \frac{1}{1+(3w)^2} e^{-jwt} dw$

$\frac{\pi}{3} e^{-|\frac{1}{3}w|} = \int_{-\infty}^{+\infty} \frac{1}{1+(3t)^2} e^{-jwt} dt.$