1. Determine the energy E_∞ and power P_∞ of those signals. Which are energy signals? Which are power signals? (10')

(a) $x(t) = e^{j(2t+\frac{\pi}{4})}$

(b) $x[n] = (\frac{1}{2})^n u[n]$

解答：

(a)

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left| e^{j(2t+\frac{\pi}{4})} \right|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 1^2 dt = 1 \quad 2'$$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{j(2t+\frac{\pi}{4})} \right|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} 1^2 dt = \infty \quad 2'$$

$\therefore$ $x(t)$ is a finite-power signal 1'

(b)

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[(\tfrac{1}{2})^n u[n] \right]^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (\tfrac{1}{4})^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1-(\frac{1}{4})^{N+1}}{1-\frac{1}{4}} = 0 \quad 2'$$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} \left[(\tfrac{1}{2})^n u[n] \right]^2 = \lim_{N\to\infty} \sum_{n=0}^{N} (\tfrac{1}{4})^n = \lim_{N\to\infty} \frac{1-(\frac{1}{4})^{N+1}}{1-\frac{1}{4}} = \frac{4}{3} \quad 2'$$

$\therefore$ $x[n]$ is a finite-energy signal. 1'

2. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period. (10')

(a) $x_1(t) = 2e^{j(t+\frac{\pi}{4})}u(t)$

(b) $x_2[n] = e^{j7\pi n}$

(c) $x_3[n] = 3e^{j\frac{3}{5}(n+\frac{1}{2})}$

解答:

(a).

$$x_1(t)=\begin{cases}2\left[\cos(\frac{\pi}{4}+t)+j\sin(\frac{\pi}{4}+t)\right] & t>0 \quad 2' \\ 0 & t<0 \quad 2'\end{cases}$$

∴ $x_1(t)$ is aperiodic. 1'

(b) $x_2[n] = \cos(7\pi n)+j\sin(7\pi n)$ 2'

$\frac{m}{N} = \frac{w_0}{2\pi}$

∴ $N = \frac{2\pi}{w_0}\cdot m$ 2'

$= \frac{2\pi}{7\pi}\cdot m$

∴ $N = 2$ 1'

(c) $x_3[n] = 3e^{j\frac{3}{5}(n+\frac{1}{2})} = 3e^{j(\frac{3}{5}n+\frac{3}{10})} = 3\cos(\frac{3}{5}n+\frac{3}{10})+3j\sin(\frac{3}{5}n+\frac{3}{10})$ 2'

∵ $\frac{w_0}{2\pi} = \frac{3}{10\pi}$ (无理数) 2'

∴ $x_3[n]$ is aperiodic. 1'

3. (15')

i. A continuous-time signal $x(t)$ is shown in Figure P1.21. Sketch and label carefully each of the following signals:

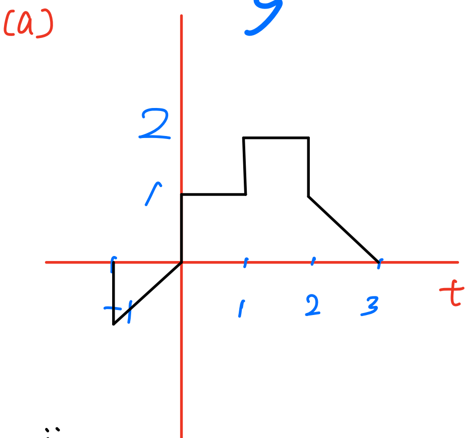
左加右减. 从外往内看.

(a) $x(t-1)$

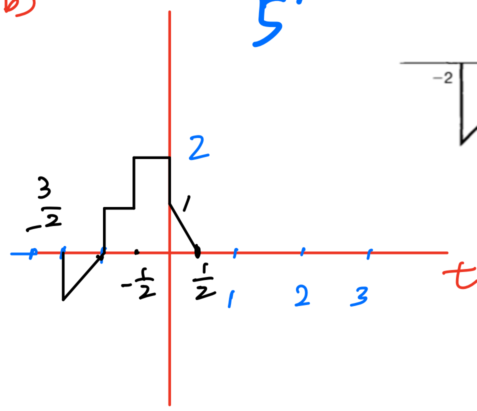
(b) $x(2t+1)$

$x(2(t+\frac{1}{2}))$ 先缩小 再平移.

(a) 5'



(b) 5'



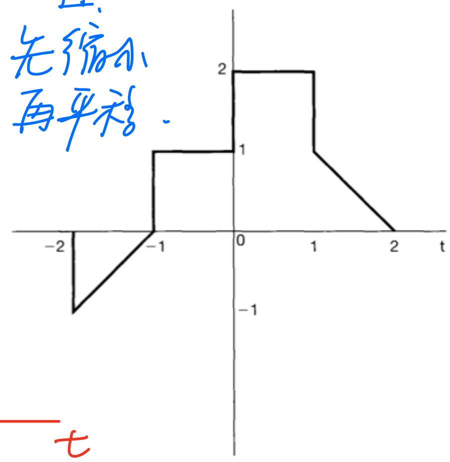
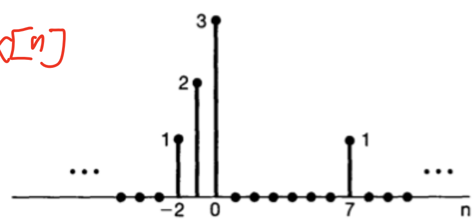


Figure P1.21

ii.

Determine and sketch the even and odd parts of the signals depicted in Figure P1.24. Label your sketches carefully.

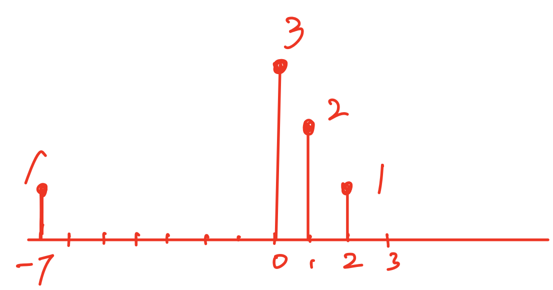
$x[n]$



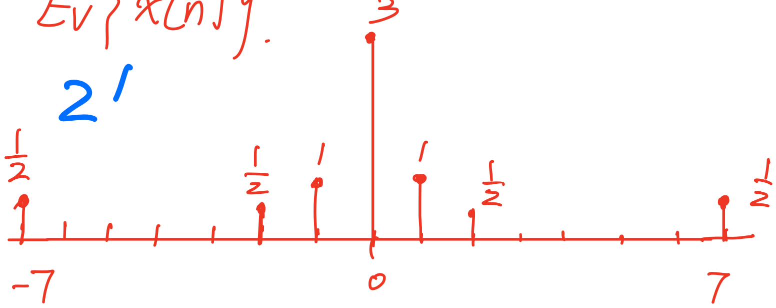
$Ev\{x[n]\} = \frac{1}{2}\{x[n]+x[-n]\}$

$Od\{x[n]\} = \frac{1}{2}\{x[n]-x[-n]\}$

$x[-n]$

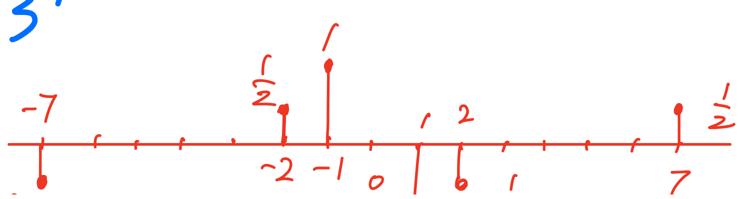


$Ev\{x(n)\}$. 2'



3'

$Od\{x[n]\}$



4. $-\frac{1}{2}$ -1 -2

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be (20')

(1) Memoryless
(2) Time invariant
(3) Linear
(4) Causal
(5) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y[n] = n x[n]$

(b) $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$.

(a) Output is dependent only on the current input
(1)(4)
So the system is memoryless and Causal. 4'(2'+2')

(2) if: $x_1[n] = x[n-n_0]$

$\therefore y_1[n] = n x_1[n] = n x[n-n_0] \neq [n-n_0] x[n-n_0] = y[n-n_0]$

$\therefore$ the system is time-variant. 2'

(3) if $y_1[n] = n x_1[n]$

$y_2[n] = n x_2[n]$ 2'

$x_3[n] = a x_1[n] + b x_2[n]$

$\therefore y_3[n] = n(a x_1[n] + b x_2[n])$

$= a n x_1[n] + b n x_2[n]$

$= a y_1[n] + b y_2[n]$

$\therefore$ the system is linear

(5) if $|x[n]| = |u[n]| < \infty$ 2'

but $\lim\limits_{n\to\infty} n u[n] = \infty$

$\therefore$ the system is not Stable.

(b)

(1) $y(t)=\int_{-\infty}^{2t} x(\tau)d\tau$, the output depends on the past input, so the system is memorable. 2'

(2) if $x_1(t)=x(t-t_0)$ $\therefore y_1(t)=\int_{-\infty}^{2t} x_1(\tau)d\tau$

$=\int_{-\infty}^{2t} x(\tau-t_0)d\tau = \int_{-\infty}^{2t-t_0} x(\tau')d\tau'$ 2'

$\because y(t-t_0)=\int_{-\infty}^{2t-2t_0} x(\tau)d\tau$

$\therefore y_1(t)\neq y(t-t_0)$

$\therefore$ the system is time-variant.

(3). if $y_1(t)=\int_{-\infty}^{2t} x_1(\tau)d\tau$, $y_2(t)=\int_{-\infty}^{2t} x_2(\tau)d\tau$

Set $x_3(t)=ax_1(t)+bx_2(t)$ 2'

So $y_3(t)=\int_{-\infty}^{2t} x_3(\tau)d\tau=\int_{-\infty}^{2t}[ax_1(\tau)+bx_2(\tau)]d\tau$

$=a\int_{-\infty}^{2t} x_1(\tau)d\tau+b\int_{-\infty}^{2t} x_2(\tau)d\tau$

$= ay_1(t)+by_2(t)$

$\therefore$ the system is linear.

(4). thanks to the value of $y(t)$ depends on the past input to the 2t moment input, so the system is noncausal. 2'

(5). if $|x(t)|<M$ such as $x(t)=u(t)$

$y(t)=\int_{-\infty}^{2t} u(\tau)d\tau = 2tu(t)$ $y(\infty)=\infty$ is not stable

so the system is not stable. 2'

5.

(a) Show that the discrete-time system whose input $x[n]$ and output $y[n]$ are related by $y[n] = \mathcal{R}e\{x[n]\}$ is additive. Does this system remain additive if its input-output relationship is changed to $y[n] = \mathcal{R}e\{e^{j\pi n/4}x[n]\}$? (Do not assume that $x[n]$ is real in this problem.) (20')

(b) In the text, we discussed the fact that the property of linearity for a system is equivalent to the system possessing both the additivity property and homogeneity property. Determine whether [redacted] the systems defined below is additive and/or homogeneous. Justify your answers by providing a proof for each property if it holds or a counterexample if it does not. (20')

(i) $y(t) = \frac{1}{x(t)}\left[\frac{dx(t)}{dt}\right]^2$

(a).

(i.)

if $y_1[n] = Re\{x_1[n]\}$

$y_2[n] = Re\{x_2[n]\}$.

$x_3[n] = x_1[n] + x_2[n]$

$\therefore y_3[n] = Re\{x_3[n]\} = Re\{x_1[n] + x_2[n]\} = Re\{x_1[n]\} + Re\{x_2[n]\}$

$(\because Re\{a+b\} = Re\{a\} + Re\{b\})$

10'

$\therefore y_3[n] = y_1[n] + y_2[n]$

$\therefore$ the system is additive.

(ii). $y[n] = Re\{e^{j\pi\frac{n}{4}} x[n]\}$.

if: $y_1[n] = Re\{e^{j\pi\frac{n}{4}} x_1[n]\}$

10'

$y_2[n] = Re\{e^{j\pi\frac{n}{4}} x_2[n]\}$

$x_3[n] = x_1[n] + x_2[n]$

$Re\{\cos(\frac{\pi}{4}n) + j\sin(\frac{\pi}{4}n) \underline{x[n]}\}$ $\uparrow (a+bj)$

$= \cos(\frac{\pi}{4}n) Re\{x[n]\} - \sin(\frac{\pi}{4}n) \underline{Im\{x[n]\}}$

$x[n]$ is not real

$\therefore y_3[n] = Re\{e^{j\pi\frac{n}{4}} x_3[n]\}$

$\because e^{j\theta} = \cos\theta + j\sin\theta$

$\therefore y_3[n] = \cos(\frac{\pi n}{4}) Re\{x_1[n] + x_2[n]\} - \sin(\frac{\pi}{4}n) Im\{x_1[n] + x_2[n]\}$.

$= \cos(\frac{\pi}{4}n) Re\{x_1[n]\} - \sin(\frac{\pi}{4}n) Im\{x_1[n]\} + \cos(\frac{\pi}{4}n) Re\{x_2[n]\} - \sin(\frac{\pi}{4}n) Im\{x_2[n]\}$

$= Re\{e^{j\frac{\pi}{4}n} x_1[n]\} + Re\{e^{j\frac{\pi}{4}n} x_2[n]\} = y_1[n] + y_2[n]$

$\therefore$ this system remains additive.

(b)

(i) additive

this system is not additive

10'

if $x(t) = x_1(t) + x_2(t)$

$$y(t) = \frac{1}{x_1(t)+x_2(t)}\left\{\frac{d[x_1(t)+x_2(t)]}{dt}\right\}^2$$

$$= \frac{1}{x_1(t)+x_2(t)}\left\{\left[\frac{dx_1(t)}{dt}\right]^2 + 2\cdot\frac{dx_1(t)}{dt}\cdot\frac{dx_2(t)}{dt} + \left[\frac{dx_2(t)}{dt}\right]^2\right\}$$

$$= \frac{x_1(t)}{x_1(t)+x_2(t)}y_1(t) + \frac{2\frac{dx_1(t)}{dt}\cdot\frac{dx_2(t)}{dt}}{x_1(t)+x_2(t)} + \frac{x_2(t)}{x_1(t)+x_2(t)}\cdot \cancel{y_2(t)}\, y_2(t) \neq y_1(t)+y_2(t)$$

∴ the system is not additive.

(ii) homogeneous.

if $x_4(t) = a\,x(t)$

$$y_4(t) = \frac{1}{x_4(t)}\left[\frac{d\,x_4(t)}{dt}\right]^2 = \frac{1}{a x(t)}\left[\frac{d\{a x(t)\}}{dt}\right]^2$$

10'

$$= \frac{a}{x(t)}\cdot\left[\frac{d(x(t))}{dt}\right]^2 = a y(t)$$

∴ the system is homo...