EE150 Signal and System
Homework 2

Due on 4 Oct 23:59 UTC+8

> Note:
> - Please provide enough calculation process to get full marks.
> - Please submit your homework to Gradescope.
> - It's highly recommended to wirte every exercise on single sheet of page.

Exercies 1. (20 pts)

卷积

Compute the following convolutions:

(a) $\boldsymbol{x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]}$ and $\boldsymbol{h[n] = 2\delta[n+1] + 2\delta[n-1]}$

(b) $\boldsymbol{x[n] = 0.5^n u[n]}$ and $\boldsymbol{h[n] = u[n+3]}$

(c) $\boldsymbol{x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]}$ and $\boldsymbol{h[n] = \left(\frac{1}{4}\right)^n u[n+3]}$

(d) $\boldsymbol{x[n]}$ and h[n] are in Figure P1

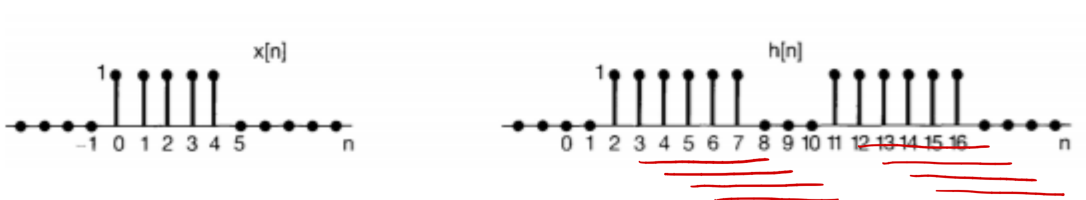

Figure 1: P1

Exercies 2. (20 pts)

$$\boldsymbol{x(t) = u(t-3) - u(t-5)} \text{ and } \boldsymbol{h(t) = e^{-3t}u(t)}.$$

(a) Compute $y(t) = x(t) * h(t)$ and sketch the result.

(b) Compute $g(t) = \frac{dx(t)}{dt} * h(t)$.

$g(t) = \frac{dy(t)}{dt}$

解 (a) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

$= \int_{-\infty}^{\infty} u(\tau-3)e^{-3(t-\tau)}u(t-\tau)d\tau - \int_{-\infty}^{\infty} u(\tau-5)e^{-3(t-\tau)}u(t-\tau)d\tau$

$= \int_3^t e^{-3(t-\tau)}d\tau - \int_5^t e^{-3(t-\tau)}d\tau$

$= e^{-3t}[\frac{1}{3}(e^{3t}-e^9)u(t-3) - \frac{1}{3}(e^{3t}-e^{15})u(t-5)]$

? → $= \frac{1}{3}[1-e^{-3(t-3)}]u(t-3) - \frac{1}{3}[1-e^{-3(t-5)}]u(t-5)$

$= \begin{cases} 0, & t \leqslant 3 \\ \frac{1}{3}[1-e^{-3(t-3)}], & 3 < t \leqslant 5 \\ \frac{1}{3}(1-e^{-6})e^{-3(t-5)}, & t > 5 \end{cases}$

(b) $\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$

$g(t) = \frac{dx(t)}{dt} * h(t) = [\delta(t-3)-\delta(t-5)] * e^{-3t}u(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$

(c) $\boxed{y(t) = \frac{1}{3}[1-e^{-3(t-3)}]u(t-3) - \frac{1}{3}[1-e^{-3(t-5)}]u(t-5)}$

Exercies 3. (20 pts)

Consider the cascade interconnection of three causal LTI systems, illustrated in Figure P2(a). The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n-2]$$

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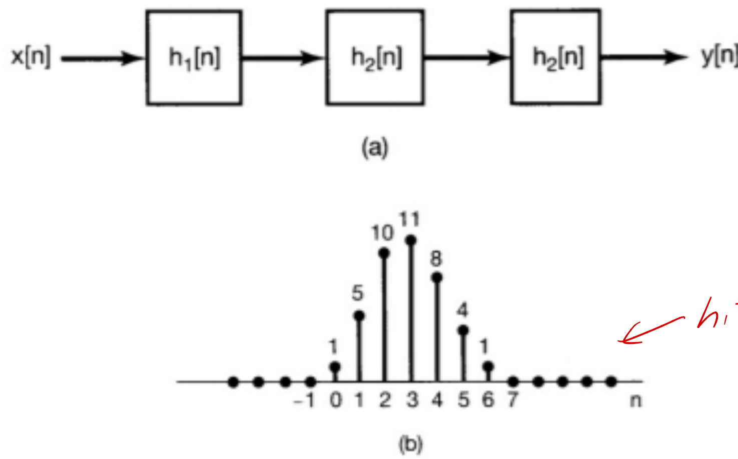
and the overall impulse response is as shown in Figure P2(b).

$h[n]$



$h_1[n] * h_2[n] * h_2[n]$

Figure 2: P2

(a) Find the impulse response $h_1[n]$.

(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1]$$

Exercies 4. (20 pts)

Determine whether each of the following statements concerning LTI systems is true or false. Justify your answers(if True, please prove briefly; if False, please give a counter-example). 级联

$\delta[n-1] * \delta[n+1] = \delta[n]$

(a) The inverse of a causal LTI system is always causal. 因果

(b) If $|h[n]| \leq K$ for each n, where K is a given number, then the LTI system with $h[n]$ as its impulse response is stable. × $h[n]=k$ $x[n]=1$

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(c) If a discrete-time LTI system has an impulse response $h[n]$ of finite duration, the system is stable.

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(d) The cascade of a noncausal LTI system with a causal one is necessarily noncausal. × $\delta[n-1] * \delta[n+2]$

(e) A continuous-time LTI system is stable iff its step response $s(t)$ is absolutely integrable —— that is, if and only if

$x(t) = u(t)$

×

$$\int_{-\infty}^{\infty} |s(t)|dt < \infty$$

Exercies 5. (20 pts)

Draw block diagram representations for causal LTI systems described by the following differential equations, and determine the system output $y[n]$ **or** $y(t)$

(a) $y(t) = -(\frac{1}{2})\frac{dy(t)}{dt} + 4x(t)$ when $x(t) = e^{3t}u(t)$

(b) $y[n] = \frac{1}{3}y[n-1] + \frac{1}{2}x[n-1]$ when $x[n] = K\delta[n]$