EE150 Signal and System
Homework 6&7

Due on 4 Dec 23:59 UTC+8

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Gradescope.
- It's highly recommended to wirte every exercise on single sheet of page.

Exercies 1. (20pt)

The output $y(t)$ of a causual LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

6.27

(a) Determine the frequency response

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

of the system, and sketch its Bode plot.

(b) Specify, as a function of frequency, the group delay associated with this system.

(c) If the input has its Fourier transform as follows, determine $Y(j\omega)$(the Fourier transform of the output) and the output $y(t)$

(i) $X(j\omega) = \frac{1+j\omega}{2+j\omega}$

(ii) $X(j\omega) = \frac{2+j\omega}{1+j\omega}$

(iii) $X(j\omega) = \frac{1}{(2+j\omega)(1+j\omega)}$

Exercies 2. (15pt)

6.22

(a) Shown in Figure P2(a) is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For the input signals $x(t)$ below, determine the filtered output siganl $y(t)$.

$$x(t) = \cos(2\pi t + \theta) + \cos(4\pi t + \theta)$$

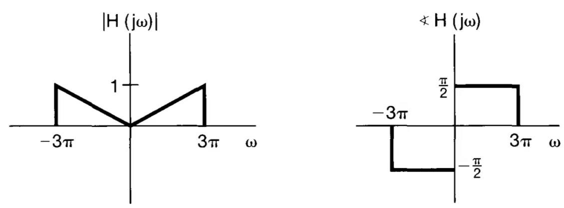


Figure 1: P2(a)

6.35

(b) Shown in Figure P2(b) is the frequency response $H(j\omega)$ of a discrete-time differentiator. Determine the output signal $y[n]$ as a function of ω_0 if the input $x[n]$ is

$$x[n] = \cos[\omega_0 n + \theta] \quad \omega_0 \in [-\pi, \pi]$$

$H(e^{j\omega}) = j\omega_0$

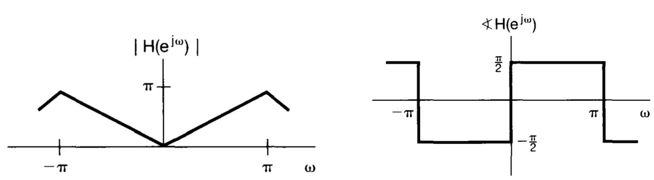


Figure 2: P2(b)

Exercies 3. (25pt)

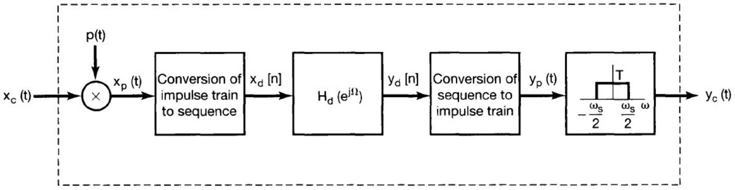
(a) Two signals $x_1(t) = \cos(20\pi t)$ and $x_2(t) = \cos(100\pi t)$ are sampled with the sampling frequency 40 Hz. Obtain the asscoiated time signals $x_1[n]$ and $x_2[n]$ and compare them and briefly explain it(Hint: think about the aliasing effect and the cause of it).

(b) Find two different continuous-time signals that would produce the sequence below when sampled at the frequency $f = 500Hz$. The discrete-time sequency is $x[n] = \cos[n\pi/5]$.

Exercies 4. (20pt)

A major problem in the recording of electrocardiograms (ECGs) is the appearance of unwanted $50Hz$ interference in the output. The causes of this power line interference

干扰



include magnetic mduction, displacement currents in the leads on the body of the patient, and equipment interconnections. Assume that the bandwidth of the signal of interest is $1kHz$, that is,

$$X_a(f) = 0 \quad |f| > 1000Hz$$

CT

The analog signal is converted into a discrete-time signal with an ideal A/D converter operating using a sampling frequency f_s. The resulting signal $x[n] = x_a(nTs)$ is then processed with a discrete-time system that is described by the difference equation $x_a(n \cdot T_s)$

$$y[n] = x[n] + ax[n-1] + bx[n-2]$$

CT

The filtered signal, $y[n]$, is then converted back into an analog signal using an ideal D/A converter. Design a system for removing the $50Hz$ interference by specifying values for f_s, a, and b so that a $50Hz$ signal of the form

指定

$$\omega_a(t) = A\sin(100\pi t)$$

$w = 100\pi = 2\pi f$

$f = 50$

will not appear in the output of the D/A converter.

Exercies 5. (20pt)

Consider the system shown in Figure P5. Assume that the input is bandlimited, $X_a(\omega) = 0 for |\omega| > 2\pi \cdot 1000$.

约束

(a) What constraints must be placed on M, T_1 and T_2 in order for $y_a(t)$ to be equal to $x_a(t)$?

(b) If $f_1 = f_2 = 20kHz$ and $M = 4$, find an expression for $y_a(t)$ in terms of $x_a(t)$.

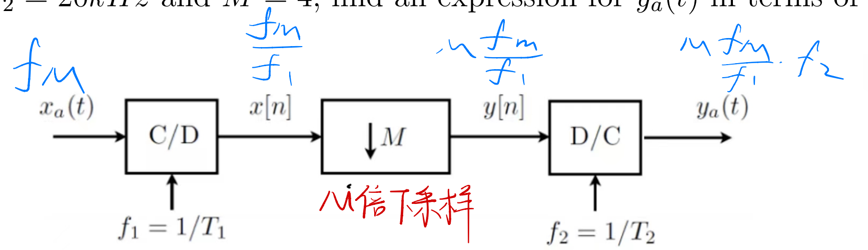


Figure 3: P5

$Y_a(jw) = \frac{1}{M} X_a(\frac{1}{M} jw)$