EE 150L
Signals and Systems Lab

Lab3 Analysis of Periodic Signals in the Frequency Domain

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1. Get to know the frequency domain:

Find out the amplitude-frequency and phase-frequency of the signal:

$$f(t) = 1 + 2\sin(\pi t) - \sin(3\pi t) + \sin(4\pi t) + \cos(3\pi t) - \frac{1}{2}\cos\left(5\pi t - \frac{\pi}{4}\right)$$

The necessary steps need to be given.

提示:

利用三角、和差化积等公式将 f(t)转换为$f(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_1 t + \varphi_n)$，或利用欧拉公式转换成$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t + \varphi_n}$的形式后，找出角频率与幅度，角频率与相位的对应关系。如:

$\omega = 0$时，$c_0 = 1$，$\varphi_0 = 0$

$$f(t) = 1 - 2\cos(1\cdot\pi t + \pi) + \sqrt{2}\cos\left(3\cdot\pi t + \frac{\pi}{4}\right) - \cos(4\cdot\pi t + \pi) - \frac{1}{2}\cos\left(5\cdot\pi t - \frac{\pi}{4}\right)$$

when $\omega = 0$, $C_0 = 1$, $\varphi_0 = 0$

when $\omega = \pi$, $C_1 = -2$, $\varphi_1 = \pi$

when $\omega = 3\pi$, $C_3 = \sqrt{2}$, $\varphi_3 = \frac{\pi}{4}$

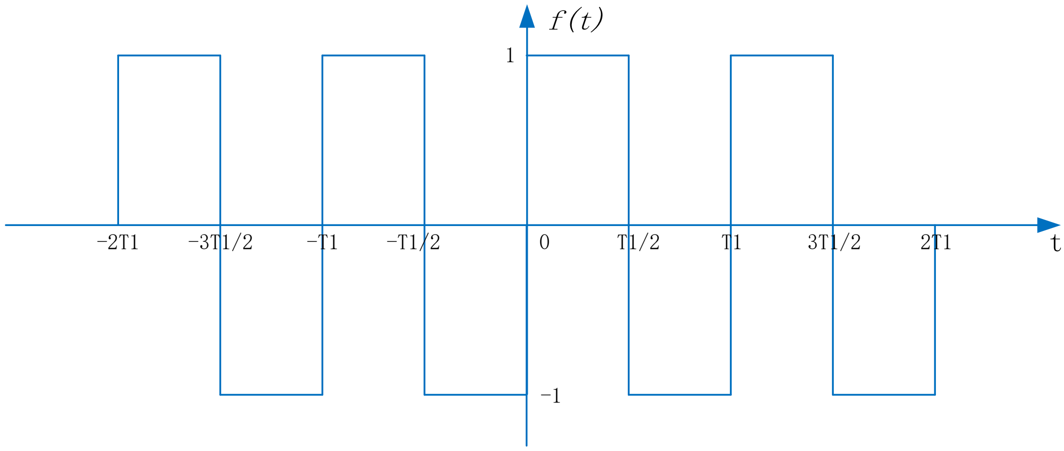
when $\omega = 4\pi$, $C_4 = -1$, $\varphi_4 = \pi$

when $\omega = 5\pi$, $C_5 = -\frac{1}{2}$, $\varphi_5 = -\frac{\pi}{4}$

when $\omega = k\pi$, $(k \neq 0, 1, 3, 4, 5)$, $C_k = 0$, $\varphi_k = 0$

2. Get to know the Fourier Series:

Find the Fourier series of the following period signal. $T_1 = 2$.



提示:

a) 使用三角或指数形式将上述周期函数展开为傅里叶级数，详细方法请参考Lab 3 Analysis of Periodic Signals in the Frequency Domain 2022-2.pdf。

三角形式：$f(t) = a_{0+}\sum_{n=1}^{\infty}(a_n \cos n\omega_1 t + b_n \sin n\omega_1 t)$

指数形式：$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t}$

b) 请手算（不需要 MATLAB 代码）。

$$\omega_1 = \frac{2\pi}{T_1} = \pi$$

$$a_0 = \frac{1}{T_1}\int_{t_0}^{t_0+T_1} f(t)\,dt$$

$$= \frac{1}{2}\int_0^2 f(t)\,dt$$

$$= 0$$

$$a_n = \frac{1}{T_1}\int_{t_0}^{t_0+T_1} f(t)\cos n\omega_1 t\,dt$$

$$= \frac{1}{2}\int_0^2 f(t)\cos n\pi t\,dt$$

$$= \frac{1}{2}\int_0^1 \cos n\pi t\,dt - \frac{1}{2}\int_1^2 \cos n\pi t\,dt$$

$$= \frac{1}{2}\cdot\frac{1}{n\pi}\cdot \sin n\pi t\Big|_0^1 - \frac{1}{2}\cdot\frac{1}{n\pi}\cdot \sin n\pi t\Big|_1^2$$

$$= \frac{1}{2n\pi}\cdot \sin n\pi - \frac{1}{2n\pi}(\sin 2n\pi - \sin n\pi)$$

$= 0 \qquad (n \neq 0)$

$$b_n = \frac{1}{T_1}\int_{t_0}^{t_0+T_1} f(t)\sin n\omega_1 t\,dt$$

$$= \frac{1}{2}\int_0^2 f(t)\sin n\pi t\,dt$$

$$= \frac{1}{2}\int_0^1 \sin n\pi t\,dt - \frac{1}{2}\int_1^2 \sin n\pi t\,dt$$

$$= \frac{1}{2}\cdot\frac{-1}{n\pi}\cdot\cos n\pi t\Big|_0^1 - \frac{1}{2}\cdot\frac{-1}{n\pi}\cdot\cos n\pi t\Big|_1^2$$

$$= -\frac{1}{2n\pi}(\cos n\pi - 1) + \frac{1}{2n\pi}(1-\cos n\pi)$$

$$= \frac{1}{2n\pi}\cdot[2-2(-1)^n]$$

$$= \frac{1-(-1)^n}{n\pi} \qquad (n\neq 0)$$

so above all

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{1-(-1)^n}{n\pi}\right]\sin(n\pi t)$$