# EE 150L
# Signals and Systems Lab

## Lab6 Laplace Transform

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1. Please briefly describe the difference and relationship between Laplace transform and Fourier transform.

the exponential term of Laplace transform is a complex number $s = \sigma + j\omega$

the Fourier tramsform is the purely imaginary number $j\omega$;

and the exponential factor $\sigma$ has the effect of forcing the signal to converge.

In Fourier transform, the signal in time domain and frequency domain are two -dimensional, while Laplace transform add a s-plane to become three - dimensional.

when $\sigma = 0$, Laplace transform is same as Fourier transform.

$$x(t) \overset{L}{\longleftrightarrow} X(s)$$

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$$

$s = j\omega \Downarrow$ $\qquad\qquad$ $\Rightarrow s = \sigma + j\omega$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-(\sigma + j\omega)t} dt$$

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} [x(t) e^{-\sigma t}] e^{-j\omega t} dt$$

$$X(s)\big|_{s = j\omega} = F\{x(t)\}$$

$$X(s)\big|_{s = \sigma + j\omega} = F\{x(t) e^{-\sigma t}\}$$

So $L\{x(t)\} = F\{x(t) e^{-\sigma t}\}$

2. $y''(t)+3y'(t)+2y(t)=2f'(t)+6f(t), f(t)=u(t), y(0_-)=2, y'(0_-)=1$

a) Find out the transfer function H(s).

b) What is the relationship between H(s) and h(t).

c) Find out the zero state response with H(s).

提示:

➢ 系统的传递函数 H(s) 是指在零初始条件下系统响应（即输出）与激励（即输入）之比。

即当$y(0_-)=0, y'(0_-)=0$时:

$$H(s)=\frac{Y(s)}{F(s)}$$

➢ 要从微分方程获得系统传递函数，需对微分方程两边进行拉普拉斯变换，同时利用拉普拉斯变换的时域微分性质。

a) do unilateral Laplace transform to both side of the differential equation

$$[s^2Y(s)-sy(0_-)-y'(0_-)]+3[sY(s)-y(0_-)]+2Y(s)$$

$$=2[sF(s)-f(0_-)]+6F(s)$$

$$Y(s)=\frac{2s+7}{s^2+3s+2}+\frac{2s+6}{s^2+3s+2}F(s)$$

while initial state is zero

$$H(s)=\frac{Y(s)}{F(s)}=\frac{2s+6}{s^2+3s+2}$$

b) H(s) is a Laplace pair with h(t)

$$H(s)=\mathcal{L}\{h(t)\}$$

c) the zero state response

$$Y_{zi}=\frac{2s+6}{s^2+3s+2}F(s)$$

since $H(s)=\dfrac{2s+6}{s^2+3s+2}$

and $f(t)=u(t)$ so $F(s)=\dfrac{1}{s}$

$$Y_{zi}=H(s)\cdot\frac{1}{s}=\frac{2s+6}{(s^2+3s+2)s}$$