Student Name: _____

Student Number: _____

School: _____

Year of Entrance: _____

ShanghaiTech University Final Examination Cover Sheet

Academic Year : __2021__to__2022__ Term: ___Spring___

Course-offering School: _____SIST_____

Instructor: ___Yong Zhou____

Course Name: ___Signals and Systems___

Course Number: _____EE150_____

Exam Instructions for Students:

1. All examination rules must be strictly observed throughout the entire test, and any form of cheating is prohibited.

2. Other than allowable materials, students taking closed-book tests must place their books, notes, tablets and any other electronic devices in places designated by the examiners.

3. Students taking open-book tests may use allowable materials authorized by the examiners. They must complete the exam independently without discussion with each other or exchange of materials.

For Marker' s Use:

Section	1	2	3	4	5	6	7	8	9	10	Total
Marks											
Recheck											

Marker' s Signature: **Rechecker' s Signature:**

Date: **Date:**

Instructions for Examiners:

1. The format of the exam papers and answer sheets shall be determined by the school and examiners according to actual needs. All pages should be marked by the page numbers in order (except the cover page). All text should be legible, visually comfortable and easy to bind on the left side. A4 double-sided printing is recommended for the convenience of archiving (There are all-in-one printers in the university).

2.The examiners should make sure that exam questions are correct and appropriate, If errors are found in exam questions during the exam, the examiners should be responsible to respond on site, which will be taking into account in the teaching evaluation.

EE150: Signals and Systems, Spring 2022
Final Exam
(**Exam Duration: June 16 13:30-15:30**)
(**Submit to Gradescope Due: June 16 15:40**)

1. [16 points] Consider a discrete-time linear time-invariant (LTI) system defined by frequency response
$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}.$$
   (a) [5 points] What is the unit impulse response of this system?
   (b) [5 points] What is the system's response to the input $x[n] = (\frac{1}{7})^n u[n]$? Note that $u[n]$ is the discrete-time unit step sequence;
   (c) [6 points] What is the system's response to the input $x[n] = \cos(\frac{\pi}{2}n)$?

2. [20 points] Suppose $f(t)$ is a band-limited signal and its spectrum $F(j\omega)$ is shown in Figure 1.
   (a) [8 points] Calculate the Nyquist frequency ω_s of signals $f(2t)$, $f(\frac{1}{2}t)$, $f(2t)f(\frac{1}{2}t)$, and $f(2t) * f(\frac{1}{2}t)$, where $*$ denotes convolution;
   (b) [12 points] We use sampling function $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{n\pi}{4})$ to sample signals $f(t)$, $f(2t)$, and $f(\frac{1}{2}t)$, respectively. For each sampled signal, please sketch its spectrum and determine whether aliasing occurs. Note that $\delta(t)$ is the continuous-time unit impulse function.

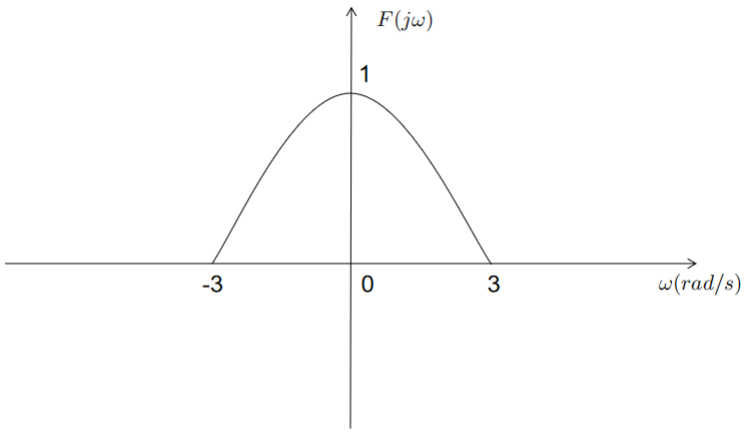

Figure 1: Spectrum of signal $f(t)$.

3. [18 points] Find the Laplace Transform of the following signals. Note that $u(t)$ is the continuous-time unit step function.
   (a) [6 points] $x_1(t) = (t-1)[u(t-1) - u(t-2)]$;
   (b) [6 points] $x_2(t) = \sin(3t)u(t-1)$;
   (c) [6 points] $x_3(t) = \begin{cases} \sin(\omega t), & \text{if } 0 < t < \dfrac{T}{2} \\ 0, & \text{otherwise} \end{cases}$, where $T = \frac{2\pi}{\omega}$.

4. [12 points] The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block diagram representation shown in Figure 2.

   (a) [6 points] Determine the transfer function $H(s)$ of this system; Determine a differential equation relating $x(t)$ and $y(t)$; Is this system stable?

   (b) [6 points] Determine the inverse Laplace Transform of $e^{-2s}H(s)$.

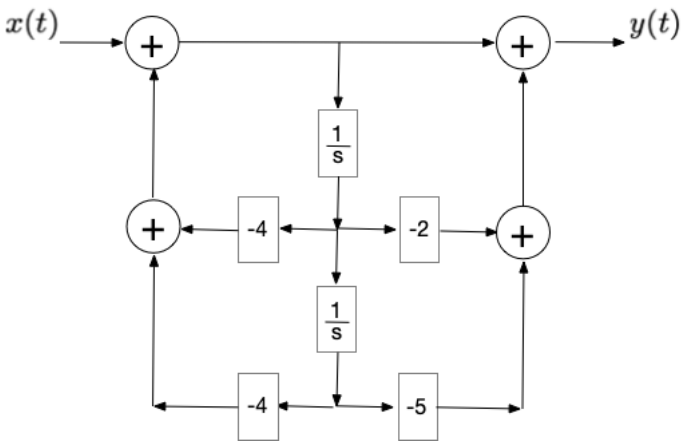

Figure 2: Block diagram of Problem 4.

5. [18 points] Consider a sequence $x[n]$ with Z-Transform

$$X(Z) = \frac{-8z^{-1}}{3 - 10z^{-1} + 3z^{-2}}.$$

   (a) [6 points] Plot the pole-zero pattern for $X(Z)$;

   (b) [6 points] Determine three possible choices of region of convergence (ROC);

   (c) [6 points] For each choice of ROC, determine the expression of $x[n]$.

6. [16 points] An LTI system has an impulse response $h[n]$ for which the Z-Transform is

$$H(Z) = \frac{1}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{4}z^{-1})}, \quad |z| > \frac{1}{3}.$$

   (a) [6 points] Draw a direct-form block diagram for $H(Z)$;

   (b) [10 points] Determine the system output $y[n]$ for all n if the input $x[n]$ is

$$x[n] = \left(\frac{2}{3}\right)^n + 5 \times 2^n.$$