1. (20 points) Consider a stable LTI system descripted by a differential equation

$$\frac{dy(t)}{dt} + ay(t) = 1000\pi \cdot x(t), \qquad a > 0$$

We know that for an input $x_1(t) = 1$, the output of the system $y_1(t) = 10$.

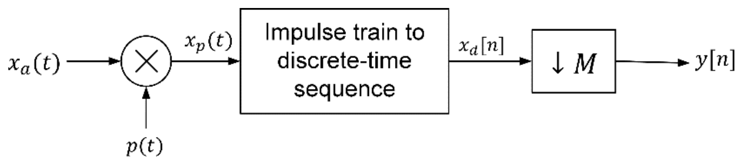
(a) Determine the value of a, and the frequency response of this system, i.e., $H(j\omega)$.

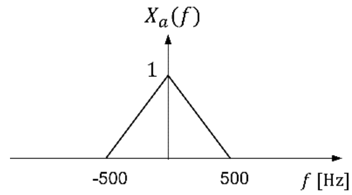
(b) Sketch the Bold-plot of $H(j\omega)$ (straight-line approximation of), mark clearly the x and y axis. Hint: first express $H(j\omega)$ in the form of $A\frac{1}{1+j\omega\tau}$.

(c) Compute the approximate output of the system $y_2(t)$ for an input

$$x_2(t) = 5e^{j10\pi t} + 10e^{j1000\pi t}e^{j\frac{\pi}{4}}$$ based on the straight-line Bold-plot in (b).

2. (30 points) Consider a system shown in the figure below ($\downarrow \mathrm{M}$ means factor-M decimation) . Assume that the input continuous-time signal $x_a(t)$ is bandlimited, i.e., $X_a(f) = 0$ for $|f| \geq 500\,\mathrm{Hz}$, and $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_1)$. For M = 2, answer the following questions:





(a) Write the time-domain expression of $x_p(t)$, $x_d[n]$, and $y[n]$ in terms of $x_a(.)$.

(b) What constraint must be placed on T_1 in order to avoid aliasing in $y[n]$?

(c) Chose a value of T_1 satisfying (b) and sketch the spectrum of $p(t)$ $x_p(t)$, $x_d[n]$, and $y[n]$, i.e., $P(f)$ $X_p(f)$, $X_d(e^{j\Omega})$, and $Y[e^{j\Omega}]$, mark clearly the x and y axis.

3. (30 points) Consider a continuous LTI system characterized by the differential equation,

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} - 15y(t) = 9\frac{dx(t)}{dt} + Ax(t).$$

We know that for an input signal $x(t) = 15$, the output of the system $y(t) = -29$.

Answer the following questions:

(a) Determine the system function $H(s)$ and the value of A.

(b) Sketch the zero-pole plot of $H(s)$, determine its region of convergence.

(c) Is $H(s)$ stable? Is $H(s)$ Causal?

(d) Draw the direct-form block diagram of $H(s)$.

(e) Determine the impulse response of this system.

4. (20 points) Consider a LTI system characterized by a difference equation

$$y[n] + 7y[n-1] = x[n] - 2x[n-1].$$

For an input $x[n] = \alpha u[n]$ with the initial condition $y[-1] = \beta$, the output of the system

$$y[n] = \left[-\frac{7}{4}(-7)^n - \frac{1}{4}\right]u[n].$$

Answer the following questions:

(a) Determine the values of α and β.

(b) Determine the zero-state response $y_{zs}[n]$ and the zero-input response $y_{zi}[n]$.