(15 points)

(1) Determine the Laplace transform by definition and the associated ROC and pole-zero plot for each of the following functions of time.

20'

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

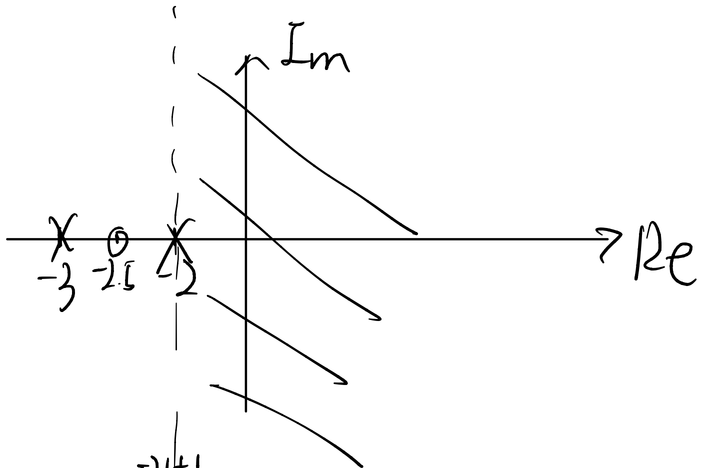
$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$$

$$= \int_{-\infty}^{+\infty} e^{-2t} u(t) e^{-st} dt + \int_{-\infty}^{+\infty} e^{-3t} u(t) e^{-st} dt$$

$$= \frac{1}{s+2} + \frac{1}{s+3} \quad Re\{s\} > -2$$

$$= \frac{2s+5}{(s+2)(s+3)}$$

3'



2'

(b) $x(t) = te^{-2|t|}$

$$x(t) = te^{-2t}u(t) + te^{2t}u(-t)$$

$$e^{-2t}u(t) \xleftrightarrow{L} \frac{1}{s+2} \quad Re\{s\} > -2$$

$$e^{2t}u(-t) \xleftrightarrow{L} -\frac{1}{s-2} \quad Re\{s\} < 2$$

$$te^{-2t}u(t) \xleftrightarrow{L} \frac{1}{(s+2)^2} \quad Re\{s\} > -2$$

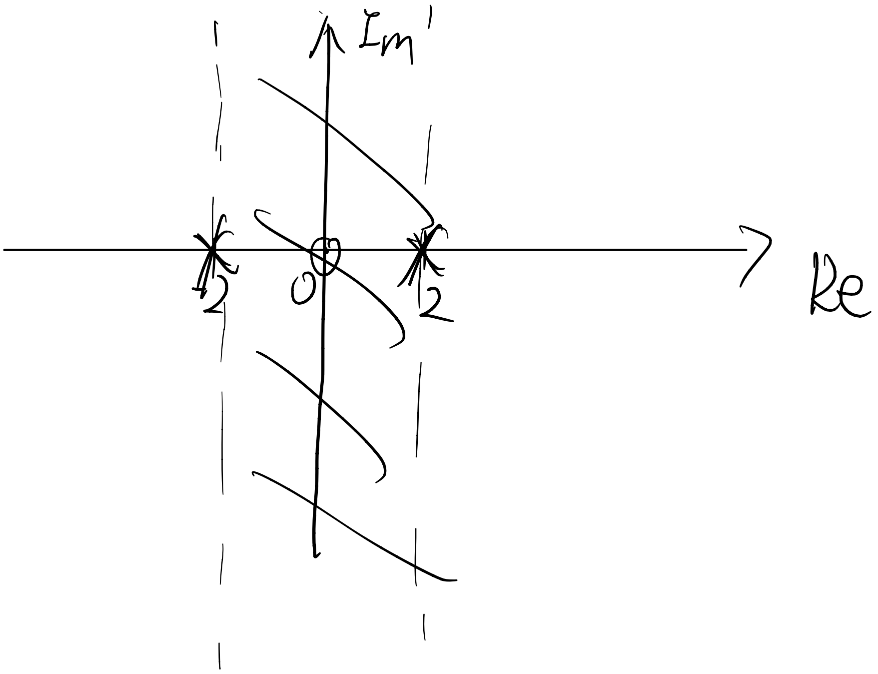
$$te^{2t}u(-t) \xleftrightarrow{L} -\frac{1}{(s-2)^2} \quad Re\{s\} < 2$$

so, $X(s) = \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2}$

$$= \frac{-8s}{(s^2-4)^2}$$

3'

$$-2 < Re\{s\} < 2$$



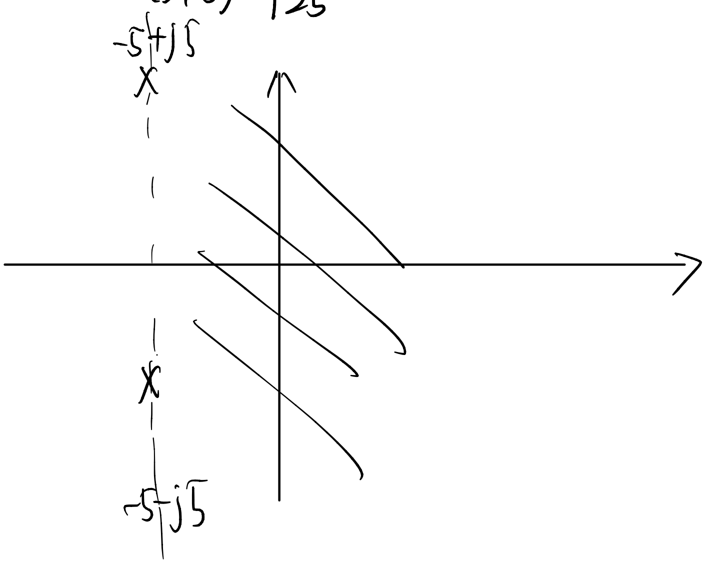
2'

(c) $x(t) = e^{-5t}(\sin 5t)u(t)$

$(\sin w_0 t)u(t) \overset{L}{\longleftrightarrow} \dfrac{w_0}{s^2+w_0^2}$, $Re\{s\} > 0$

$e^{s_0 t} g(t) \overset{L}{\longleftrightarrow} G(s-s_0)$

so, $x(t) \overset{L}{\longleftrightarrow} \dfrac{5}{(s+5)^2+25}$ $\quad Re\{s\} > -5$ 3'



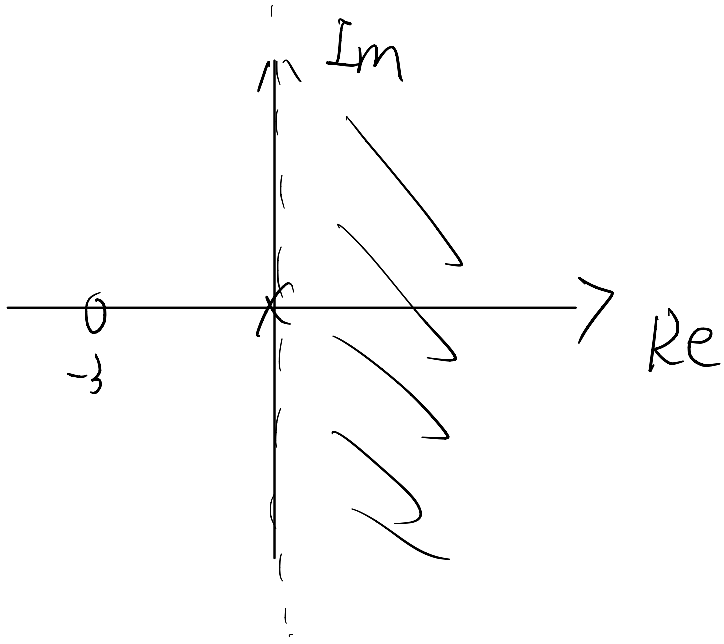
2'

(d) $x(t) = \delta(3t) + u(3t)$

$\delta(t) \xleftrightarrow{L} 1$. $\delta(3t) \xleftrightarrow{L} \frac{1}{3}$ $\forall s$.

$u(t) \xleftrightarrow{LT} \frac{1}{s}$, $u(3t) = \frac{1}{\cancel{3}} \cdot \frac{1}{\frac{s}{\cancel{3}}}$ $Re\{s\} > 0$

So, $X(s) = \frac{1}{3} + \frac{1}{s} = \frac{s+3}{3s}$ 3'



2'

2.

The following facts are given about a real signal $x(t)$ with Laplace transform $X(s)$. 20'

a. $X(s)$ has no zeros in the finite s-plane

b. $X(0)=4$ and $X(1)=1.6$

c. $X(s)$ has two poles

d. The real part of one pole of $X(s)$ is -1

e. $e^{2t}u(t)$ is not absolutely integrable

Determine $X(s)$ and its ROC

ANS:

From (a) and (c)

$$X(s) = \frac{k}{(s+p_1)(s+p_2)}$$

Since $x(t)$ is real, its poles are conjugate pair.

Besides, by using (d) we have

$$X(s) = \frac{k}{[s-(-1+xj)][s-(-1-xj)]} = \frac{k}{[s+1+xj][s+1-xj]}$$

From (b)

$$\begin{cases} X(0) = \dfrac{k}{1+x^2} = 4 \\ X(1) = \dfrac{k}{4+x^2} = 1.6 \end{cases} \Rightarrow \begin{cases} k=8 \\ x=-1 \text{ or } +1 \end{cases}$$

Therefore, $X(s) = \dfrac{8}{(s+1+j)(s+1-j)} = \boxed{\dfrac{8}{s^2+2s+2}}$

From (e), $y(t) = e^{2t}u(t) \xleftrightarrow{L} Y(s) = X(s-2)$,

so $X(s-2)$ is not absolutly integrable. So the ROC of $X(s-2)$ should not contain $j\omega$-axis. The ROC of $X(s-2)$ is the ROC of $X(s)$ shifted by 2 to the right. Since $X(s)$ has two poles whose real part are both -1, ROC of $X(s)$ should be $\boxed{Re\{s\} > -1}$

(3) 20'

In this problem, we consider the construction of various types of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2 + 2s - 40}{s^2 + 4s + 3}.$$

To derive the direct-form block diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2 + 4s + 3}.$$

With the output of S_1 denoted by $y_1(t)$, the direct-form block diagram representation of S_1 is shown in Figure P9.36. The signals $e(t)$ and $f(t)$ indicated in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

How is $dy_1(t)/dt$ related to $f(t)$?

How is $d^2y_1(t)/dt^2$ related to $e(t)$?

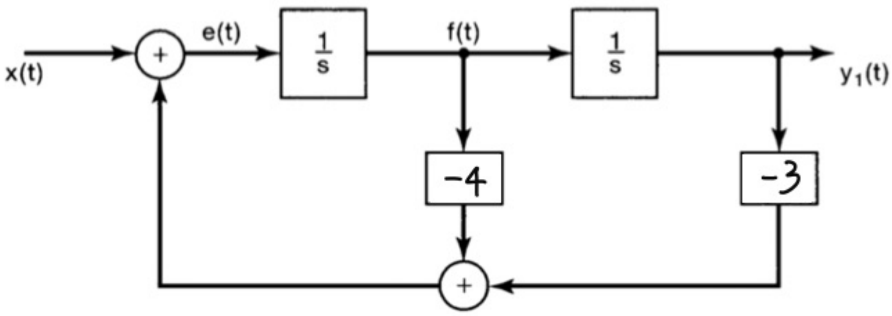
b) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, and $y_1(t)$.

c) Use the result from the previous part to extend the direct-form block diagram representation of S_1 and create a block diagram representation of S.

d) Observing that

$$H(s) = \left(\frac{2(s+5)}{s+3}\right)\left(\frac{s-4}{s+1}\right),$$

draw a block diagram representation for S as a cascade combination of two subsystems.



(a) $Y(s) = (2s^2+2s-40)Y_1(s)$

$$\therefore y(t) = 2\frac{d^2y_1(t)}{dt^2} + 2\frac{dy_1(t)}{dt} - 40y_1(t).$$

$$\because Y_1(s) = \frac{F(s)}{s}$$

$$\therefore f(t) = \frac{dy_1(t)}{dt}$$

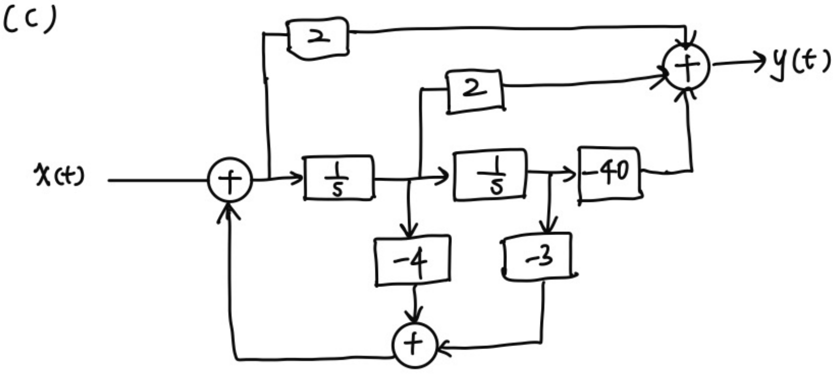
$$\because F(s) = \frac{E(s)}{s}$$

$$\therefore e(t) = \frac{df(t)}{dt} = \frac{d^2y_1(t)}{dt^2}$$

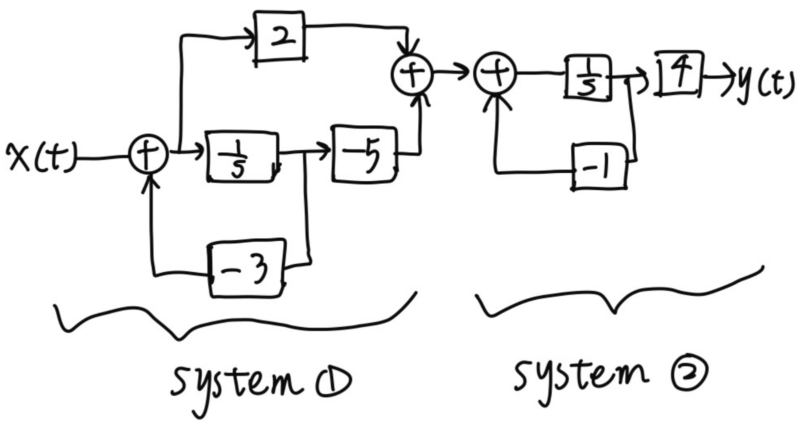
(b)

$$y(t) = 2e(t) + 2f(t) - 40y_1(t)$$

(c)



(d)



20'

4.(b)

$$y(t) = e^{-3t} u(t)$$

is the output of a causal all-pass system for which the system function is

$$H(s) = \frac{s-2}{s+2}.$$

(a) Find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$.

(a) Let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response of a continuous-time LTI system:

$$H(s) = \frac{1}{s^2 - s - 2}$$

Determine $h(t)$ for each of the following cases:

1. The system is stable.
2. The system is causal.
3. The system is *neither* stable *nor* causal.

Answer:

(a) $H(s) = \frac{1/3}{s-2} - \frac{1/3}{s+1}$

(i) If the system is stable, the RoC for $H(s)$ is $-1 < Re\{s\} < 2$.

Therefore, $h(t) = -\frac{1}{3} e^{2t} u(-t) - \frac{1}{3} e^{-t} u(t)$

(ii) If the system is causal, the RoC for $H(s)$ is $Re\{s\} > 2$.

Therefore, $h(t) = \frac{1}{3} e^{2t} u(t) - \frac{1}{3} e^{-t} u(t)$

(iii) If the system is neither stable nor causal, the RoC for $H(s)$ is $Re\{s\} < -1$

Therefore, $h(t) = -\frac{1}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(-t)$

(b) Taking Laplace transform of $y(t)$, we obtain $Y(s) = \frac{\quad}{s+3}$, $Re\{s\} > -3$

Therefore, $X(s) = \frac{Y(s)}{H(s)} = \frac{s+2}{(s+3)(s-2)}$.

The RoC of $H(s)$ is $Re\{s\} > -2$. We know the RoC of $Y(s)$ is at least the intersection of the RoCs of $X(s)$ and $H(s)$. Note that RoC can be larged if poles are canceled out by zeros.

In this case, we choose the RoC of $X(s)$ be either $-3 < Re\{s\} < 2$, or $Re\{s\} > 2$. In both case, the poles at $s=2$, $s=-2$ of $X(s)$, $H(s)$ are canceled each other.

$$X(s) = \frac{1/5}{s+3} + \frac{4/5}{s-2}$$

The RoC of $X(s)$ is $-3 < Re\{s\} < 2$, we get

$$x(t) = \frac{1}{5} e^{-3t} u(t) - \frac{4}{5} e^{2t} u(-t)$$

The RoC of $X(s)$ is $2 < Re(s)$, we get

$$x(t) = \frac{1}{5} e^{-3t} u(t) + \frac{4}{5} e^{2t} u(t)$$

5. Consider the system characterized by the differential equation.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + 6x(t)$$

with initial condition $y(0) = C_0$, $y'(0) = C_1$, 20

(1) when $x(t) = e^{-t}u(t)$, determine the zero-state response

(2) determine zero-input response

(3) determine the output of this system, when input is $x(t) = e^{-t}u(t)$ and initial condition are the same in (2)

(1) $s^2Y(s) + 3sY(s) + 2Y(s) = 2sX(s) + 6X(s)$

and here $x(t) \xleftrightarrow{L} \frac{1}{s+1}$

so, $s^2Y(s) + 3sY(s) + 2Y(s) = \frac{2s+6}{s+1}$ $\quad Re\{s\} > -1$

so $$Y(s) = \frac{2s+6}{(s+1)(s^2+3s+2)}$$

$$= \frac{2s+6}{(s+1)^2(s+2)} \quad Re\{s\} > -1$$

$$= \frac{4}{(s+1)^2} - \frac{2}{s+1} + \frac{2}{s+2}$$

and we know that $\frac{2}{s+1} \xleftrightarrow{L} 2e^{-t}u(t)$

$\frac{2}{s+2} \xleftrightarrow{L} 2e^{-2t}u(t)$

$\frac{4}{(s+1)^2} = 4\frac{d\frac{1}{s+1}}{ds} \xleftrightarrow{L} 4te^{-t}u(t)$

so, $y_{zs}(t) = 2e^{-2t}u(t) + (4t-2)e^{-t}u(t)$

(2) $[s^2Y(s) - sc_0 - c_1] + 3[sY(s) - c_0] + 2Y(s) = 0$

$$Y(s) = \frac{c_0 s + c_1 + 3c_0}{s^2+3s+2} \quad Re\{s\} > -1.$$

$$= \frac{2c_0+c_1}{s+1} - \frac{c_0+c_1}{s+2}$$

therefore $y_{zi}(t) = (2c_0+c_1)e^{-t}u(t) - (c_0+c_1)e^{-2t}u(t)$

(3) just add (1) and (2), we get

$$y(t) = (2-c_0-c_1)e^{-2t}u(t) + (4t-2+2c_0+c_1)e^{-t}u(t)$$