EE150 Signal and System
Homework 3

Due on 20 Oct 23:59 UTC+8

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Gradescope.
- It's highly recommended to wirte every exercise on

Exercies 1. (20pt)

(a) For the continuous-time periodic signal

$$x(t) = 2 + \cos(\frac{2\pi}{3}t) + 4\sin(\frac{5\pi}{3}t)$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

(b) Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 0 & , k = 0 \\ j\left(\frac{1}{2}\right)^{|k|} & , otherwise \end{cases}$$

Use Fourier series properties to answer the following questions:

(1) Is $x(t)$ real?

(2) Is $x(t)$ even?

(3) Is $\int_{-\infty}^{t} x(\tau)d\tau$ even? And determine its Fourier series.

(4) Is $x(\alpha t) \quad \alpha > 0$(periodic with period $\frac{T}{\alpha}$) even? And determine its Fourier series.

Exercies 2. (30pt)

(a) Suppose we are given the following information about a signal $x[n]$:

1. $x[n]$ is a real and even signal.
2. $x[n]$ has period $N = 10$ and Fourier coefficients a_k
3. $a_{11} = 5$
4. $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$

Show that $x[n] = A\cos(Bn + C)$, and specify numerical values for the constants A, B and C.

(b) Determine whether the following periodic signals can be represented in Fourier series form

(1) $x(t) = \tan(2\pi t)$

(2) $x(t) = 2\cos(\frac{2\pi}{t}) + \sin(\frac{2\pi}{t}) \quad 0 < t \leq 1$

(3)

$$x(t) = \begin{cases} 0 & t \notin Q \\ 1 & t \in Q \end{cases} \qquad for \quad 0 < t \leq 1$$

Exercies 3. (10pt)

Let

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2 - t, & 1 \leq t \leq 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation fo $\frac{dx(t)}{dt}$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$

Exercies 4. (20pt) 3.34

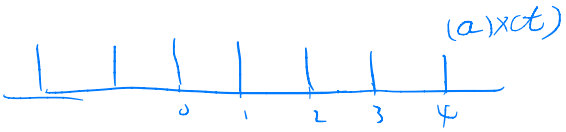
Consider a continuous-time LTI system with impulse response

$$h(t) = e^{-4|t|}$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(a) $x(t) = \sum_{n=-\infty}^{\infty}\delta(t-n)$

(b) $x(t) = \sum_{n=-\infty}^{\infty}(-1)^n\delta(t-n)$



Exercies 5. (20pt) 3.36

Consider a causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] = x[n]$$

Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sin(\frac{3\pi}{4}n)$

(b) $x[n] = \cos(\frac{\pi}{4}n) + 2\cos(\frac{\pi}{2}n)$

and plot phase angle and magnitude of the Fourier Series.