HW 9

1. Determine the Z-transform for each of following sequences. Sketch the pole zero plot and indicate the ROC.

a. $6^n u[-n] + (\frac{1}{6})^n u[n-2]$

Ans: $x_1[n] = 6^n u[-n]$. $X_1(z) = \sum_{-\infty}^{0} (6)^n z^{-n} = \sum_{0}^{+\infty} (6)^{-n} z^{n} = \frac{-6z^{-1}}{1-6z^{-1}}$, $|z| > 6$.

$x_2[n] = (\frac{1}{6})^n u[n-2]$. $X_2(z) = \sum_{n=2}^{+\infty} (\frac{1}{6})^n z^{-n} = \sum_{n=0}^{+\infty} (\frac{1}{6})^{n+2} z^{-n-2} = \frac{1}{36z^2} \cdot \sum_{n=0}^{+\infty} (\frac{1}{6} \cdot z^{-1})^n = \frac{1}{36z^2} \cdot \frac{z}{(z-\frac{1}{6})}$

$= \frac{1}{36z^2 - 6z}$

$|z| < \frac{1}{6}$

$x[n] = x_1[n] + x_2[n]$

$X(z) = \frac{-6}{z-6} + \frac{1}{36z^2 - 6z} = \frac{-216z^2 + 36z + z - 6}{6z(6z-1)(z-6)} = \frac{-216z^2 + 37z - 6}{36z^3 - 222z^2 + 36z}$

zeros: $0.0856 + 0.1430j$, $0.0856 - 0.1430j$,

poles: $6, \frac{1}{6}, 0$

ROC: $\frac{1}{6} < |z| < 6$

b. $3^n \cos[\frac{\pi}{3} n + \frac{1}{3}\pi]\, u[n-1]$

Ans: $x[n] = 3^n \cdot \frac{e^{j\frac{\pi}{3}n + \frac{\pi}{3}} + e^{-j\frac{\pi}{3}n + \frac{\pi}{3}}}{2} u[n-1]$

$X(z) = \frac{1}{2} e^{j\pi/3} \cdot \frac{3e^{j\frac{1}{3}\pi} z^{-1}}{1-3e^{j\frac{1}{3}\pi} z^{-1}} + \frac{1}{2} e^{-j\pi/3} \cdot \frac{3e^{-j\frac{\pi}{3}} z^{-1}}{1-3e^{-j\cdot\frac{\pi}{3}} z^{-1}}$, $|z| > 3$

ROC: $|z| > 3$

Poles: $3e^{\frac{1}{3}j\pi}$, $3e^{\frac{-1}{3}j\pi}$; zeros: -3

c. $n(\frac{1}{3})^{|n|}$

Ans: $x[n] = n(\frac{1}{3})^n u[n] + n \cdot 3^n \cdot u[-n-1]$

$X(z) = \frac{z^{-1}/3}{(1-\frac{1}{3}z^{-1})^2} - \frac{3z^{-1}}{(1-3z^{-1})^2}$

ROC: $\frac{1}{3} < |z| < 3$

zeros: $0, 1, -1$; Poles: $\frac{1}{3}, 3$

2. The following facts are given about a real signal $x[n]$ with z-transform $X(z)$

a. $x[n]$ is left-sided

b. $X(z)$ has two poles

c. $X(z)$ has no zeros in finite z-plane

d. $X(z)$ has a poles at $\frac{1}{6}e^{-j\pi/3}$

e. $X(0) = 7$

Ans: real $\Rightarrow$ poles are conjugate pairs

$$X(z) = \frac{A}{(z-\frac{1}{6}e^{j\pi/3})(z-\frac{1}{6}e^{-j\pi/3})}$$

$$X(0) = 36A = 7 . \quad A = \frac{7}{36}$$

$$\Rightarrow X(z) = \frac{7}{36(z-\frac{1}{6}e^{j\pi/3})(z-\frac{1}{6}e^{-j\pi/3})} , \quad \text{ROC}: |z| < \frac{1}{6}$$

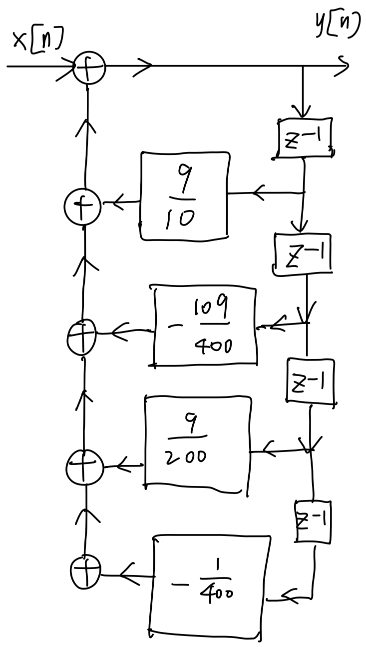
Consider the following [redacted] system function[redacted] corresponding to causal LTI systems:

$$H(z) = \frac{1}{(1-\frac{1}{2}z^{-1}+\frac{1}{16}z^{-2})} \cdot \frac{1}{(1-\frac{2}{5}z^{-1}+\frac{1}{25}z^{-2})}$$

(a) For each system function, draw a direct-form block diagram.
(b) For each system function, draw a block diagram that corresponds to the cascade connection of two second-order block diagrams. Each second-order block diagram should be in direct form.
(c) For each system function, determine whether there exists a block diagram representation which is the cascade of four first-order block diagrams with the constraint that all the coefficient multipliers must be real.

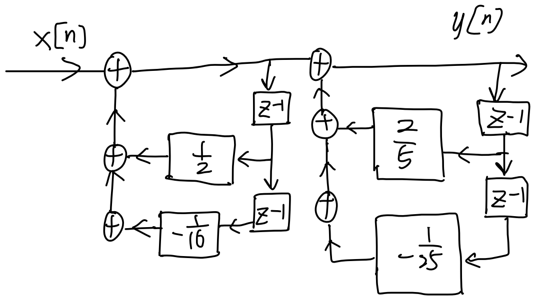
(a)

$$H(z) = \frac{1}{1-\frac{9}{10}z^{-1}+\frac{109}{400}z^{-2}-\frac{9}{200}z^{-3}+\frac{1}{400}z^{-4}}$$



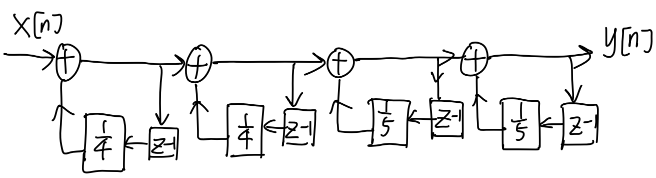
(b)

$$H(z)=\frac{1}{(1-\frac{1}{2}z^{-1}+\frac{1}{16}z^{-2})}\cdot\frac{1}{(1-\frac{2}{5}z^{-1}+\frac{1}{25}z^{-2})}$$



(c)

$$H(z)=\frac{1}{(1-\frac{1}{4}z^{-1})}\cdot\frac{1}{(1-\frac{1}{4}z^{-1})}\cdot\frac{1}{(1-\frac{1}{5}z^{-1})}\cdot\frac{1}{(1-\frac{1}{5}z^{-1})}$$



4.

A LTI system associate input $x[n]$ and output $y[n]$ with the differential equation:

$$y[n-1]-\frac{3}{2}y[n]+\frac{1}{2}y[n+1]=x[n]$$

The stability of system is uncertain.

By considering the pole-zero pattern associated with the preceding difference equation, determine three possible choices for the unit ~~sample~~ impulse response of the system. Show that each choice satisfies the difference equation.

Answer:

$$H(z)=\frac{1}{z^{-1}-\frac{3}{2}+\frac{1}{2}z}=\frac{z^{-1}}{z^{-2}-\frac{3}{2}z^{-1}+\frac{1}{2}}$$

The partial fraction expansion of $H(z)$ is

$$H(z)=\frac{-2}{1-z^{-1}}+\frac{2}{1-2z^{-1}}$$

If ROC is $|z|>2$, $h_1[n]=-2u[n]+2^{n+1}u[n]$

If ROC is $1<|z|<2$, $h_2[n]=-2u[n]-2^{n+1}u[-n-1]$

If ROC is $|z|<1$, $h_3[n]=2u[-n-1]-2^{n+1}u[-n-1]$

The impulse response of this system is $h[n]$

$$x_1[n]=h_1[n-1]-\frac{3}{2}h_1[n]+\frac{1}{2}h_1[n+1]$$
$$=-2u[n-1]+2^{n}u[n-1]+3u[n]-3\times2^{n}u[n]-u[n+1]+2^{n+1}u[n+1]$$
$$=(2^{n}-2)u[n-1]+3(1-2^{n})u[n]+(2^{n+1}-1)u[n+1]$$

$$x_1[n]=\begin{cases}0, & n\le -1\\ 1, & n=0\\ 0, & n\ge 1\end{cases}\Rightarrow x_1=\delta$$

$$x_2[n]=h_2[n-1]-\frac{3}{2}h_2[n]+\frac{1}{2}h_2[n+1]=\delta$$

$x_3[n]=h_3[n-1]-\frac{3}{2}h_3[n]+\frac{1}{2}h_3[n+1]=\delta$, so each choice satisfies the difference equation.

5.

Consider the system characterized by the difference equation:

$$y[n-2] + 3y[n-1] + 2y[n] = x[n]$$

(a) Determine the zero input response of this system where $y[-2] = 4$, $y[-1] = 0$

(b) Determine the zero state response of this system to the input $x[n] = 4\delta[n]$

(c) Determine the output of this system for $n \geq 0$ when $x[n] = 4\delta[n]$, $y[-2] = 4$ $y[-1] = 0$

(a) $z^{-2}(Y(z) + zy[-1] + z^2 y[-2]) + 3z^{-1}(Y(z) + zy[-1]) + 2Y(z) = 0$

$$z^{-2}Y(z) + z^{-1}y[-1] + y[-2] + 3z^{-1}Y(z) + 3y[-1] + 2Y(z) = 0$$

$$(z^{-2} + 3z^{-1} + 2)Y(z) + y[-2] = 0$$

$$Y(z) = \frac{4}{z^{-2}+3z^{-1}+2} = \frac{4}{(z^{-1}+2)(z^{-1}+1)}$$

5. $\alpha^n u[n]$ $\frac{1}{1-\alpha z^{-1}}$ $|z| > |\alpha|$

$$= \frac{-4}{z^{-1}+2} + \frac{4}{z^{-1}+1}$$

so, $y_{zi}[n] = -2\cdot\left(-\frac{1}{2}\right)^n u[n] + 4(-1)^n u[n]$

(b) zero state, $y[-2] = y[-1] = 0$.

$$z^{-2}Y(z) + 3z^{-1}Y(z) + 2Y(z) = 4$$

$$Y(z) = \frac{-4}{z^{-1}+2} + \frac{4}{z^{-1}+1}$$

$$y_{zs}[n] = -2\left(-\frac{1}{2}\right)^n u[n] + 4(-1)^n u[n].$$

(c) sum above up. $y[n] = y_{zi}[n] + y_{zs}[n] = -4\left(-\frac{1}{2}\right)^n u[n] + 8(-1)^n u[n]$