EE 150L
Signals and Systems Lab

Lab5 Sampling and Reconstruction

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Class Id: Thurs_105

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1. (a) If $\mathcal{F}[f(t)] = F(\omega)$, verify that $\mathcal{F}[f(t)\cos(\omega_0 t)] = \frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$.

   (b) Read the **Nyquist Sampling Theorem** section in the PDF material and briefly explain how the conclusions in (a) relates to Nyquist's Sampling Theorem.

(a) $\mathcal{F}[f(t)] = F(w)$

$= \int_{-\infty}^{+\infty} f(t) e^{-jwt} dt$

linearity of FT

$\mathcal{F}[f(t)\cos(w_0 t)] = \mathcal{F}[f(t)\frac{e^{jw_0 t}+e^{-jw_0 t}}{2}] = \frac{1}{2}\mathcal{F}[f(t)e^{jw_0 t}] + \frac{1}{2}\mathcal{F}[f(t)e^{-jw_0 t}]$

$= \frac{1}{2}\int_{-\infty}^{+\infty} [f(t)e^{jw_0 t}]e^{-jwt} dt + \frac{1}{2}\int_{-\infty}^{+\infty} [f(t)e^{-jw_0 t}]e^{-jwt} dt$

$= \frac{1}{2}\int_{-\infty}^{+\infty} f(t)e^{-j(w-w_0)t} dt + \frac{1}{2}\int_{-\infty}^{+\infty} f(t)e^{-j(w_0+w)t} dt$

$= \frac{1}{2}F(w-w_0) + \frac{1}{2}F(w+w_0)$

$= \frac{1}{2}[F(w+w_0) + F(w-w_0)]$

(b) the Nyquist Sampling Theorem says that

when a real signal $x(t)$ is sampled in the time domain, the sampled signal can be represented as

$x_s(t) = x(t)\,\delta_{T_s}(t)$

since impulse signal $\delta_{T_s}(t)$ is periodic signal with period T, it can be expressed as trigonometric Fourier series

as $\delta_T(t) = \frac{1}{T_s}[1 + 2\cos w_s t + 2\cos 2w_s t + \cdots]$, $w_s = \frac{2\pi}{T_s} = 2\pi f_s$

Therefor, $x_s(t) = x(t)\delta_T(t) = \frac{1}{T_s}[x(t) + 2x(t)\cos w_s t + 2x(t)\cos 2w_s t + \cdots]$

from (a), we know that $\mathcal{F}[f(t)\cos(w_0 t)] = \frac{1}{2}[F(w+w_0) + F(w-w_0)]$

with the linearity of FT,

we have $\mathcal{F}[x_s(t)] = \mathcal{F}\{\frac{1}{T_s}[x(t) + 2x(t)\cos w_s t + 2x(t)\cos 2w_s t + \cdots]\}$

$= \frac{1}{T_s}\{\mathcal{F}[x(t)] + 2\mathcal{F}[x(t)\cos w_s t] + 2\mathcal{F}[x(t)\cos(2w_s t)] + \cdots\}$

$= \frac{1}{T_s}\{F(w) + [F(w+w_s) + F(w-w_s)] + [F(w+2w_s) + F(w-2w_s)] + \cdots\}$

So $\mathcal{F}[x_s(t)] = \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} F(w - nw_s)$

2. According to the Nyquist sampling theorem, the sampling frequency must not be less than Nyquist rate in order to accurately represent the signal before sampling. Read the PDF material to understand the Nyquist rate. Find the Nyquist rate of the following signals and give reasons for your judgment.

(a) $x(t) = \cos(2000\pi t) + \sin(5000\pi t)$

(b) $x(t) = \frac{\sin(500\pi t)}{\pi t}$

(a) $x(t) = \cos(2000\pi t) + \sin(5000\pi t)$

$= \frac{1}{2}e^{j(2000\pi)t} + \frac{1}{2}e^{-j(2000\pi)t} + \frac{1}{2}e^{j(5000\pi t)} - \frac{1}{2}e^{-j(5000\pi)t}$

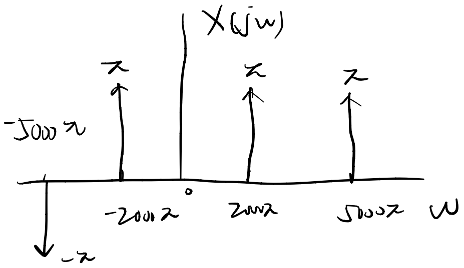
let $w_0 = 1000\pi$

suppose that $x(t) \xleftrightarrow{FS} a_k$

so $a_2 = a_{-2} = \frac{1}{2}$, $a_5 = \frac{1}{2}$, $a_{-5} = -\frac{1}{2}$

so $X(jw) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0)$

$= \pi[\delta(w - 5000\pi) + \delta(w - 2000\pi) + \delta(w + 2000\pi) - \delta(w + 5000\pi)]$



So $w_b = 5000\pi$

and Nyquist rate $= 2w_b = 10000\pi$

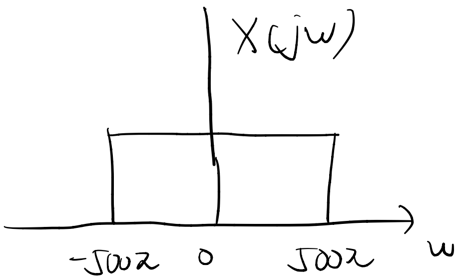
(b) $x(t) = \frac{\sin(500\pi t)}{\pi t}$, $x(t) \xleftrightarrow{F} X(jw)$

let $y(t) = \begin{cases} 1, & |t| < 500\pi \\ 0, & |t| > 500\pi \end{cases}$ $y(t) \xleftrightarrow{F} Y(jw)$, $Y(jw) = \frac{2\sin(500\pi w)}{w}$

since $x(t) = \frac{1}{2\pi} Y(t)$

from duality, we have $X(jw) = \frac{1}{2\pi} \cdot 2\pi \cdot y(w)$

so $X(jw) = \begin{cases} 1. & |w| < 500\pi \\ 0. & |w| > 500\pi \end{cases}$



So $w_b = 500\pi$

and Nyquist rate $= 2w_b = 1000\pi$