

SI 120 (Spring 2022), Final Exam Practice

Name (in Chinese): _____

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Instructions

- Time: 10:30–10:40am (10 minutes)
- This exam is closed-book, you may bring nothing but a pen. Put all the study materials and electronic devices into your bag and put your bag in the front, back, or sides of the classroom.
- You can write your answers in either English or Chinese.
- Two blank pieces of paper are attached, which you can use as scratch paper. Raise your hand if you need more paper.

1 (50 points) Calculation:

During a sport competition, we want to organize tournaments of Tennis (T), Badminton (B), Ping pong (P), Swimming (S), Golf (G), Running (R). Several people are registered to play B, T, P, others for P, S, R, and finally others for T, G, R. The simultaneous participation is impossible and the organizers want to satisfy everybody.

- a. What is the maximum number of tournaments that can happen simultaneously?
- b. Knowing that each tournament must last at most 3 hours, suggest a schedule for the tournaments with a minimum time respecting all the choice of the participants.

2 (50 points) Proof:

Let $G = (V, E)$ be a simple graph, then define $\beta(G)$ to be the size of the largest subset $A \subset V$ such that any two vertices in A are joined by an edge. Suppose we are given a subset $A \subset V$, then define a graph G_1 with vertex set A , and two vertices are joined by an edge \Leftrightarrow they are joined by an edge in G , and a graph G_2 whose vertex set is $V \setminus A$ and two vertices are joined by an edge \Leftrightarrow they are joined by an edge in G .

Question: Let G be a simple graph with $\beta(G)$ even. Prove that there exists A such that $\beta(G_1) = \beta(G_2)$.