SI 120 Discrete Mathematics (Spring 2022), Final Exam

Instructions

- Time: 10:30am–12:30pm (110 minutes 答卷+ 10 minutes 交卷)
- This exam is open-book with the following restriction: you can only bring course ppt slides (printed or on screen) and one A4 cheat sheet.
- You can write your answers in either English or Chinese.
- See the online exam guideline ppt for detailed instructions.

Note: Question Sheet has 4 pages and 10 questions in total.

1 From natural language to formulas (10 pt)

a) Propositional Logic (7 pt):

Denote by A the proposition: "I am hungry", in B, "it is three o'clock now", and in C, "it is time to have dinner". Translate the following statements into formulas in propositional logic (命题逻辑), including A, B, and C:

(1) If it is three o'clock now or I am hungry, then it is time to have dinner.

(2) If I am hungry, then it is time to have dinner. But I am not hungry. So,

either it is not three o'clock now, or it is not time to dinner.

b) Predicate Logic (3 pt):

The number A is a limit (极限) of the function $f(x)$ at the point x_0 if and only if

$$\forall \varepsilon > 0\ \exists \delta > 0\ \forall x\ \ (|x - x_0| < \delta \rightarrow |f(x) - A| < \varepsilon)$$

With the help of quantifiers (量词) and De Morgan's Laws, write the statement: "the limit of the function $f(x)$ at the point x_0 is not equal to A".

2 Propositional formulas (10 pt)

Let p, q and r be propositional (命题) variables. Determine the types of the followingformulas (tautology (重言式), contradiction (矛盾式) or contingency (可能式)). Explain your answers.

(1) $(p \leftrightarrow \neg r) \rightarrow (q \leftrightarrow r)$

(2) $\left(\neg(p \leftrightarrow q) \rightarrow \left((p \wedge \neg q) \vee (\neg p \wedge q)\right)\right) \vee r$

3 Logic Equivalence (10 pt)

a) Propositional Logic (5 pt):

Let p, q and r be propositional variables, and

$$A = (p \wedge q) \vee (\neg p \wedge q \wedge r)$$

$$B = \big(p \vee (q \wedge r)\big) \wedge \big(q \vee (\neg p \wedge r)\big)$$

Show that $A \equiv B$ using the rule of replacement. (You can use any laws in lec2.)

b) Predicate Logic (5 pt):

Identify two mistakes in the following logic equivalence:

$$\neg\exists x \forall y \Big(F(x) \wedge \big(G(y) \to H(x,y)\big)\Big) \equiv \forall x\, \exists y\, (F(x)\ \wedge (G(y) \to H(x,y))$$
$$\equiv \forall x \exists y \Big(\big(F(x) \wedge G(y)\big) \to H(x,y)\Big)$$

4 Rule of Inference in Propositional Logic (10 pt)

Show the following inference is valid using the tautological implications (and the resulting valid argument forms)

$$(\neg p \vee r) \wedge (\neg q \vee s) \wedge (p \wedge q) \Rightarrow (t \to (r \wedge s))$$

5 Predicate Logic (10 pt)

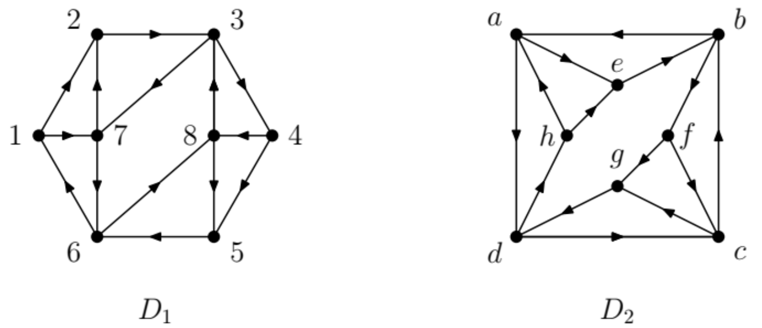
a) Show that $\Big(\big(\forall x F(x) \to \exists y G(y)\big) \wedge \forall x F(x)\Big) \to \exists y G(y)$ is logically valid (普遍有效).

b) Show that $\neg\big(\forall x\, F(x) \to \exists y G(y)\big) \wedge \exists y G(y)$ is unsatisfiable (不可满足).

6 Graph Basics (10 pt)

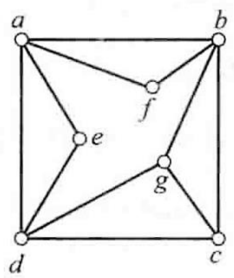
a) Let *G(V,E)* be an undirected graph in which the number of vertices equals the number of edges, i.e., $|V|=|E|$. Assume that it has 2 vertices of degree 2, 2 vertices of degree 3, and the rest of vertices have degree 1. Determine the number of vertices.

b) Find whether the following digraphs D1 and D2 are isomorphic, and why?

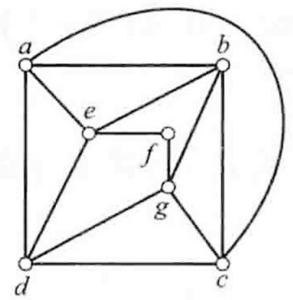


7 Euler Circuit & Connectivity (10 pt)

a) Do the following graphs below admit any Euler path or Euler circuit? If yes, draw one, otherwise, explain why there is no Euler path nor Euler circuit.

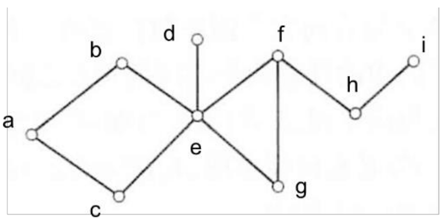


G1

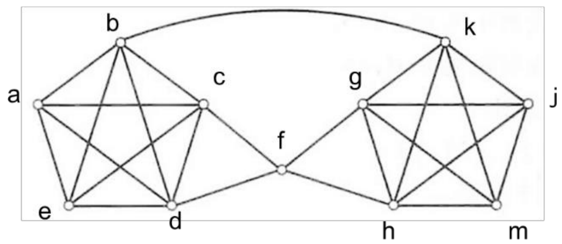


G2

b) Find all the cut vertices and bridges in the following graphs, and compute their κ, λ.



G3



G4

8 Planar Graph & Coloring (10 pt)

a) Let G be a planar simple graph with n=10 vertices, m=8 edges and r=3 regions. Compute the number of connected components p in G.

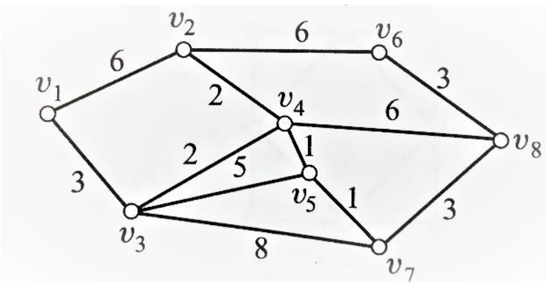
b) In a school, seven lessons are scheduled for Saturday: mathematics, literature, rhetoric, nature, physical education, drawing, and solfeggio. Each lesson takes exactly one hour. Some lessons can be conducted in parallel, and others, for example, those conducted by the same teacher, should be conducted in series. In Table below with “×” are marked the lessons that cannot be conducted simultaneously.

Find the minimal time required to conduct all the seven lessons on Saturday.

Subject	Mathematics	Literature	Rhetoric	Nature	Physical education	Drawing	Solfeggio
Mathematics				×	×		
Literature				×			×
Rhetoric					×	×	×
Nature	×	×					
Physical education	×		×			×	×
Drawing			×		×		×
Solfeggio		×	×		×	×	

9 Shortest Path (10 pt)

For the weighted graph shown in the figure below, use Dijkstra's algorithm to compute the distance $d(v_1, v)$ for every $v \in V$. For each step k of the algorithm write down explicitly the set S_k and the labels $L_k(v)$ for every $v \in V$.



10 Tree (10 pt)

a) It is known that the tree T has one vertex of degree 3, six vertices of degree 2, and seven vertices of degree 1. The rest of the vertices are of degree 4. How many vertices of degree 4 does the tree T have?

b) The tree shown in the figure below represent an arithmetic expression. Use inorder traversal to write out the expression (with parentheses) and also write out its postfix form (reverse Polish notation).