1. let $E = p \to \neg q \lor r \to \neg(\neg r \to s \land p)$

$\equiv [p \to ((\neg q) \lor r)) \to [\neg((\neg r) \to (s \land p)))$

let $A = (\neg q) \lor r$, $B = p \to A$

$C = s \land p$, $D = (\neg r) \to C$

so $E = B \to (\neg D)$

so the truth table is:

p	q	r	s	$\neg q$	$\neg r$	A	B	C	D	$\neg D$	E
T	T	T	T	F	F	T	T	T	T	F	F
T	T	T	F	F	F	T	T	F	T	F	F
T	T	F	T	F	T	F	F	T	T	F	T
T	T	F	F	F	T	F	F	F	F	T	T
T	F	T	T	T	F	T	T	T	T	F	F
T	F	T	F	T	F	T	T	F	T	F	F
T	F	F	T	T	T	T	T	T	T	F	F
T	F	F	F	T	T	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T	F	F
F	T	T	F	F	F	T	T	F	T	F	F
F	T	F	T	F	T	F	T	F	F	T	T
F	T	F	F	F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T	F	T	F	F
F	F	T	F	T	F	T	T	F	T	F	F
F	F	F	T	T	T	T	T	F	F	T	T
F	F	F	F	T	T	T	T	F	F	T	T

2.(1) $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$

$$\equiv ((\neg p) \vee q) \wedge (q \rightarrow ((\neg r) \wedge (\neg p))) \wedge (p \vee r) \triangleq A$$

notice that parts of the truth table

p	q	r	$\neg p$	$\neg r$	$(\neg p) \vee q$	$(\neg r) \wedge (\neg p)$	$q \rightarrow ((\neg r) \wedge (\neg p))$	$p \vee r$	A
T	T	T	F	F	T	F	F	T	F
F	F	T	T	F	T	F	T	T	T

So when $(p, q, r) = (T, T, T)$, $A = F$

when $(p, q, r) = (F, F, T)$, $A = T$

So A (the formula) is contingency

(2) $(p \to q) \land (q \to r) \to (p \to r)$

$\equiv ((p\to q) \land (q \to r)) \to (p \to r)$

let $A = p \to q$

$B = q \to r$

$C = A \land B$

$D = p \to r$, $E = C \to D$

So $E = (p\to q) \land (q \to r) \to (p \to r)$

the truth table is:

p	q	r	A	B	C	D	E
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

above all

for all $(p, q, r) \in \{T, F\}^3$

the truth assignment is T

So the formula is tautology

(3) $(p \to r) \land (q \to s) \land (p \lor q) \to (r \lor s)$

$= ((p \to r) \land (q \to s) \land (p \lor q)) \to (r \lor s)$

so let $A = p \to r$, $B = q \to s$, $C = p \lor q$

$D = A \land B \land C$

$E = r \lor s$

$F = D \to E$

So $F = (p \to r) \land (q \to s) \land (p \lor q) \to (r \lor s)$

So the truth table is

p	q	r	s	A	B	C	D	E	F
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	F	T	T
T	T	F	T	F	T	T	F	T	T
T	T	F	F	F	F	T	F	F	T
T	F	T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	F	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	F	T	T
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	F	T	F	F	T
F	F	T	T	T	T	F	F	T	T
F	F	T	F	T	T	F	F	T	T
F	F	F	T	T	T	F	F	T	T
F	F	F	F	T	T	F	F	F	T

above all

for all $(p, q, r, s) \in \{T, F\}^4$

the truth assignment is T

So the formula is tautology

3. a: "Alice attends the meeting" b: "Bob ...." c: "Charlie ..." , d: "Davaid ..."

(1) $d \leftrightarrow (c \wedge (\neg a))$

(2) $((\neg d) \rightarrow c) \wedge (d \rightarrow (\neg b))$

(3) $a \rightarrow (((\neg b) \wedge (\neg c)) \rightarrow d)$

~~(4)~~ $\cancel{(a \wedge b \wedge c) \Leftrightarrow}$

(4) $((a \wedge b \wedge c) \leftrightarrow (\neg d)) \wedge (((\neg a) \wedge (\neg b)) \rightarrow (d \rightarrow c))$

4.

(1) $(\neg l) \rightarrow q$

(2) $((\neg l) \rightarrow n) \wedge (n \rightarrow (\neg l))$

(3) $(\neg q) \rightarrow b$

(4) $(\neg l) \rightarrow b$

(5) $\neg b$

suppose that there is a system satisfied the five specifications

then (5) $\neg b = T$

so $b = F$

from (3) we know that $(\neg q) \rightarrow b = T$

and since $b = F$

so $(\neg q) = F$

so $q = T$

similarly from (4) we know that $(\neg l) \rightarrow b = T$ and $b = F$

so $(\neg l) = F$

$l = T$

above all from (3), (4), (5) we have $l = T$, $q = T$, $b = F$

so $(\neg l) = F$ so $(\neg l) \rightarrow q = T$

so (1) satisfied these results

from (2) we have $((\neg l) \rightarrow n) \wedge (n \rightarrow (\neg l)) = T$

so $(\neg l) \rightarrow n = T$, $n \rightarrow (\neg l) = T$

since $l = T$ so $(\neg l) = F$

from $n \rightarrow (\neg l) = T$, we have $n = F$

and when $n = F$, $(\neg l) \rightarrow n = T$ satisfied (2)

so above all ~~n=F~~ $l = T$, $q = T$, $n = F$, $b = F$ put in into (1) $l = T$ so $(\neg l) = F$

so $(\neg l) \rightarrow q = T$ satisfied (1)

so $l = T$, $q = T$, $n = F$, $b = F$ is a system satisfying all the five specifications.

the hypothesis established.

5. let a, b, c be three proposition

and $B = a \wedge \cancel{q}\, b \wedge c$

so $B = T$ if and only if $(a, b, c) = (T, T, T)$

from the form and the result above, we have

$A_1 = p \wedge q \wedge r$

$A_2 = p \wedge q \wedge (\neg r)$

$A_3 = p \wedge (\neg q) \wedge r$

$A_4 = p \wedge (\neg q) \wedge (\neg r)$

$A_5 = (\neg p) \wedge q \wedge r$

$A_6 = (\neg p) \wedge q \wedge (\neg r)$

$A_7 = (\neg p) \wedge (\neg q) \wedge r$

$A_8 = (\neg p) \wedge (\neg q) \wedge (\neg r)$

and $A = A_2 \vee A_4 \vee A_6 \vee A_7 \vee A_8$

$$\equiv (p \wedge q \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge (\neg q) \wedge (\neg r))$$

$$\equiv ((p \wedge (\neg r) \wedge q) \vee (p \wedge (\neg r) \wedge (\neg q))) \vee (((\neg p) \wedge (\neg r) \wedge q) \vee ((\neg p) \wedge (\neg r) \wedge (\neg q))) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$\cancel{\equiv (p \wedge q \wedge}$

$$\equiv ((p \wedge (\neg r)) \wedge (q \vee (\neg q))) \vee (((\neg p) \wedge (\neg r)) \wedge (q \vee (\neg q))) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv ((p \wedge (\neg r)) \wedge T) \vee (((\neg p) \wedge (\neg r)) \wedge T) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv (p \wedge (\neg r)) \vee ((\neg p) \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv (((\neg r) \wedge p) \vee ((\neg r) \wedge (\neg p))) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv ((\neg r) \wedge (p \vee (\neg p))) \vee ((\neg p) \wedge (\neg q) \wedge r) \equiv ((\neg r) \wedge T) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv (\neg r) \vee ((\neg p) \wedge (\neg q) \wedge r)$$

$$\equiv ((\neg r) \vee (\neg p)) \wedge ((\neg r) \vee (\neg q)) \wedge ((\neg r) \vee r)$$

$$\equiv ((\neg r) \vee (\neg p)) \wedge ((\neg r) \vee (\neg q)) \wedge T \equiv ((\neg r) \vee (\neg p)) \wedge ((\neg r) \vee (\neg q))$$

$$\equiv (\neg r) \vee ((\neg p) \wedge (\neg q))$$

So $A = ((\neg p) \wedge (\neg q)) \vee (\neg r)$

6. P, Q, R, S are propositional formulas

Show that $(P\wedge Q\wedge S)\vee(P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge\neg S)\vee\neg(P\wedge R\to Q)\equiv P$

use the rule of replacement

we have <1> $(P\wedge Q\wedge S)\vee(P\wedge Q\wedge\neg S)$

$$\equiv((P\wedge Q)\wedge S)\vee((P\wedge Q)\wedge(\neg S))$$

$$\equiv (P\wedge Q)\wedge(S\vee(\neg S))$$

$$\equiv (P\wedge Q)\wedge T$$

$$\equiv (P\wedge Q)$$

<2> $\cancel{\neg((P\wedge R)\to Q)}$ $\neg(P\wedge R\to Q)\equiv\neg((P\wedge R)\to Q)$

$$\equiv\neg(\neg(P\wedge R)\vee Q)$$

$$\equiv(\neg(\neg(P\wedge R)))\wedge(\neg Q)$$

$$\equiv(P\wedge R)\wedge(\neg Q)$$

<3> So $(P\wedge\neg Q\wedge\neg R)\vee(\neg(P\wedge R\to Q))$

$$\equiv (P\wedge(\neg Q)\wedge(\neg R))\vee(P\wedge R\wedge(\neg Q))$$

$$\equiv((P\wedge(\neg Q))\wedge(\neg R))\vee((P\wedge(\neg Q))\wedge R)$$

$$\equiv (P\wedge\neg Q)\wedge((\neg R)\vee R)$$

$$\equiv (P\wedge(\neg Q))\wedge T$$

$$\equiv (P\wedge(\neg Q))$$

combine <1>, <3>

So $(P\wedge Q\wedge S)\vee(P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge\neg S)\vee\neg(P\wedge R\to Q)$

$$\equiv((P\wedge Q\wedge S)\vee(P\wedge Q\wedge\neg S))\vee((P\wedge\neg Q\wedge\neg R)\vee(\neg(P\wedge R\to Q)))$$

$$\equiv (P\wedge Q)\vee(P\wedge(\neg Q))$$

$$\equiv P\wedge(Q\vee(\neg Q))$$

$$\equiv P\wedge T$$

$$\equiv P$$

So above all $(P\wedge Q\wedge S)\vee(P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge\neg S)\vee\neg(P\wedge R\to Q)\equiv P$

□

7.

p	$\triangle p$
T	F
F	F

(a) $\neg p \equiv p \rightarrow (\triangle p)$

prove: the truth table:

p	$\neg p$	$\triangle p$	$p \rightarrow (\triangle p)$
T	F	F	F
F	T	F	T

the ~~result~~ truth assignment of $\neg p$ and $p \rightarrow (\triangle p)$ on the truth table is same

so $\neg p \equiv p \rightarrow (\triangle p)$

(b) use the truth table

p	q	$\neg q$	$p \to q$	$p \wedge (\neg q)$	$\neg(p \to q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

from the truth table we can see that the truth assignment of $p \wedge (\neg q)$ and $\neg(p \to q)$ is the same

so $p \wedge (\neg q) \equiv \neg(p \to q)$

So $p \wedge q \equiv \neg(p \to (\neg q))$

~~Since~~ from (a) we have $\neg p \equiv p \to (\circ p)$

So $p \to (\neg q) \equiv p \to (q \to (\circ q))$

So $\neg(p \to (\neg q)) \equiv (p \to (\neg q)) \to (\circ(p \to (\neg q)))$

$\equiv \big(p \to (q \to (\circ q))\big) \to \big(\circ(p \to (q \to (\circ q)))\big)$

So above all $p \wedge q \equiv (p \to (q \to (\circ q))) \to (\circ(p \to (q \to (\circ q))))$

(c) from the logical equivalences we know that $p \to q \equiv (\neg p) \vee q$

So $(\neg p) \to q \equiv (\neg(\neg p)) \vee q \equiv p \vee q$

So $p \vee q \equiv (\neg p) \to q$

from (a) we know that $\neg p \equiv p \to (\triangle p)$

So $p \vee q \equiv (p \to (\triangle p)) \to q$