

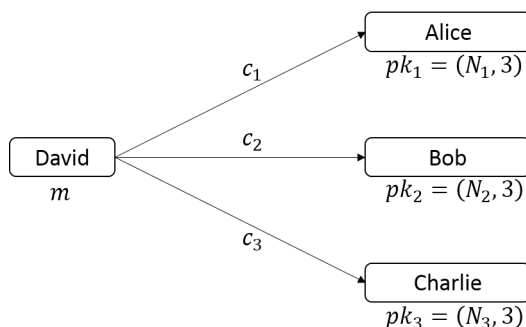
# Discrete Mathematics: Homework 4

(Deadline: 8:00am, March 18, 2022)

- (20 points) Let  $a_1, a_2, a_3, a_4$  be arbitrary integers. Find ALL integer solutions of the following equation system.

$$\begin{cases} x \equiv a_1 \pmod{11}; \\ x \equiv a_2 \pmod{13}; \\ x \equiv a_3 \pmod{17}; \\ x \equiv a_4 \pmod{19}. \end{cases}$$

- (20 points) See the following figure. The RSA public keys of Alice, Bob and Charlie are  $pk_1 = (N_1, 3)$ ,  $pk_2 = (N_2, 3)$  and  $pk_3 = (N_3, 3)$ , respectively. David wants to send a private message  $m$  to Alice, Bob and Charlie, where  $m$  is an integer and  $0 < m < N_i$  for  $i = 1, 2, 3$ . In order to keep  $m$  secret from an eavesdropper Eve, David encrypts  $m$  as  $c_1 = m^3 \pmod{N_1}$ ,  $c_2 = m^3 \pmod{N_2}$  and  $c_3 = m^3 \pmod{N_3}$ ; and then sends  $c_1$  to Alice,  $c_2$  to Bob and  $c_3$  to Charlie.



Suppose that  $N_1, N_2, N_3$  are pairwise relatively prime. Show that with the knowledge of all public keys and all ciphertexts, Eve can decide the value of  $m$ .

- (20 points) Let  $G = \{x : x \in \mathbb{R}, x > 1\}$ . Define  $x \star y = xy - x - y + 2$  for all  $x, y \in \mathbb{R}$ . Show that  $(G, \star)$  is an Abelian group.
- (20 points) Let  $(G, \cdot)$  be a multiplicative (Abelian) group of order  $m$ . Show that  $o(a) | m$  for any  $a \in G$ , i.e., the order of group element must be a divisor of the group's order.
- (20 points) Let  $G = \langle g \rangle$  be a subgroup of  $\mathbb{Z}_p^*$  of order  $q$ , where

$p=1797693134862315907729305190789024733617976978942306572734300811577326758055009$   
 $631327084773224075360211201138798713933576587897688144166224928474306394741243777$   
 $678934248654852763022196012460941194530829520850057688381506823424628814739131105$   
 $40827237163350510684586298239947245938479716304835356329624227998859,$

$q = (p - 1)/2$  and  $g = 3$ . Suppose that in a Diffie-Hellman key exchange protocol Alice and Bob exchanged the following information  $(q, G, g; A, B)$ , where

A=1129835751630026189475896666667354281816845178451448750969029100664347239526230  
166033932125012141273999088232234924787259712660427548927981777812675128216074705  
452830594726890347313130276198642286884664382583275520454375902037906355067286037  
74799021127049872571983254506993921153718739796769296097404717448108;

B=1117727678052102394963651916915168810433949881962970620138536466745747434010427  
364473288861564296291926916015263983660880127367494546266862814675792056750844619  
894945132946240660741372479130373300404872753469132533457334297677819009771026871  
85378411660147190296412313303321533586102552123457499563789255321369.

In particular,  $\log_g A, \log_g B \leq 10^4$ . Find the output of Alice and Bob.