Nonconvex and Nonsmooth Optimization for Smart Grids

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2024.11

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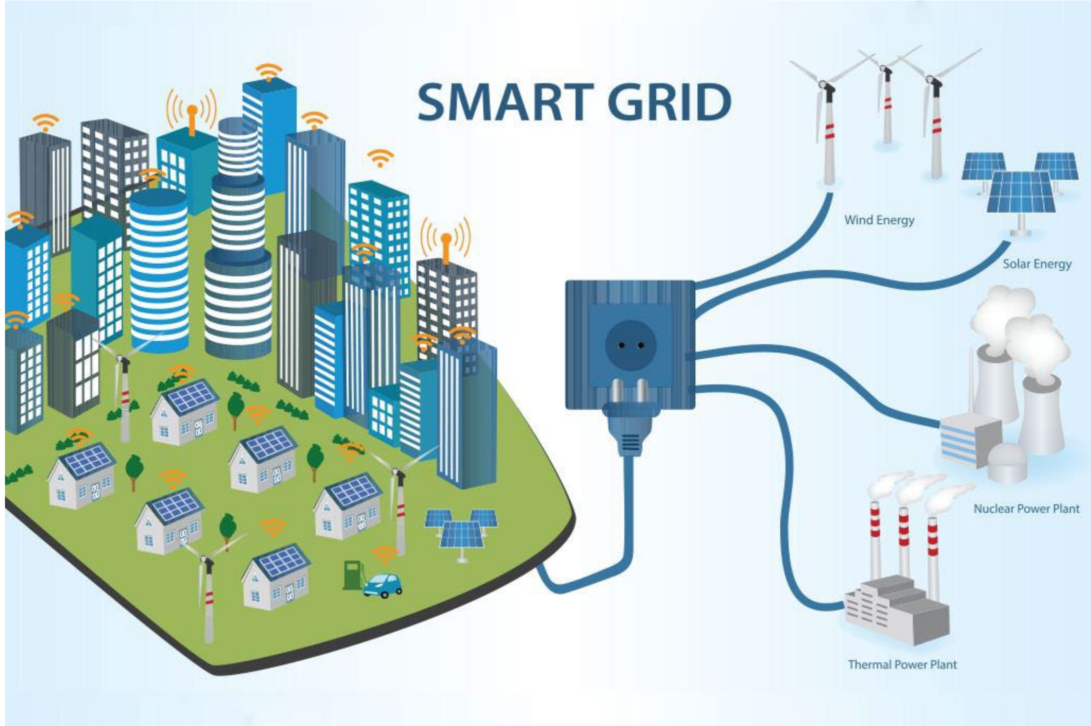
What is the Smart Grid?

- The smart grid is a planned nationwide network that uses information technology to deliver electricity efficiently, reliably, and securely.

- Unlike today's grid, the smart grid will permit the two-way flow of both electricity and information.



Why Smart Grid?

- **Economical efficiency:**

	Gross production (GWh)	Electricity price ($/GWh)	Cost ($billion)	Savings ($billion)
China (2020)	7,623,600	300,000	2287	114
World	27,004,700	300,000	8101	405

The potential cost savings are based on 5% increase of power dispatch efficiency.

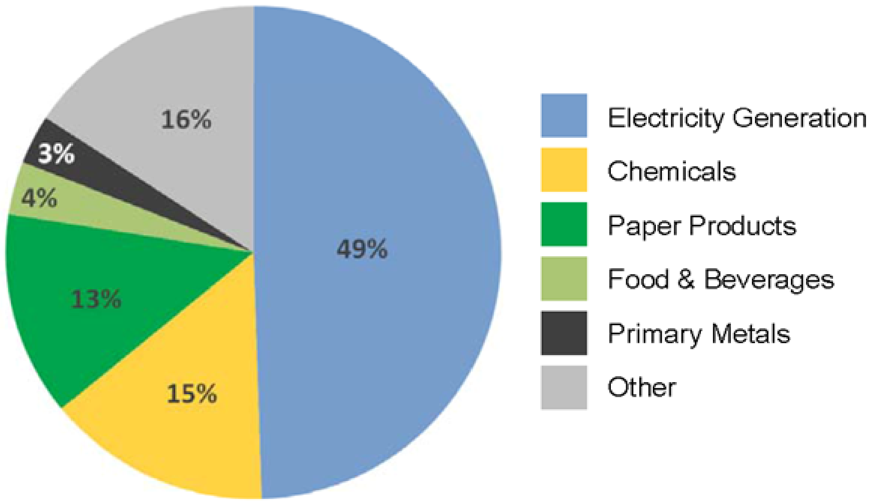
Why Smart Grid?

- **Environmental Urgency:**

  Half of the air pollutions come from electricity generation.

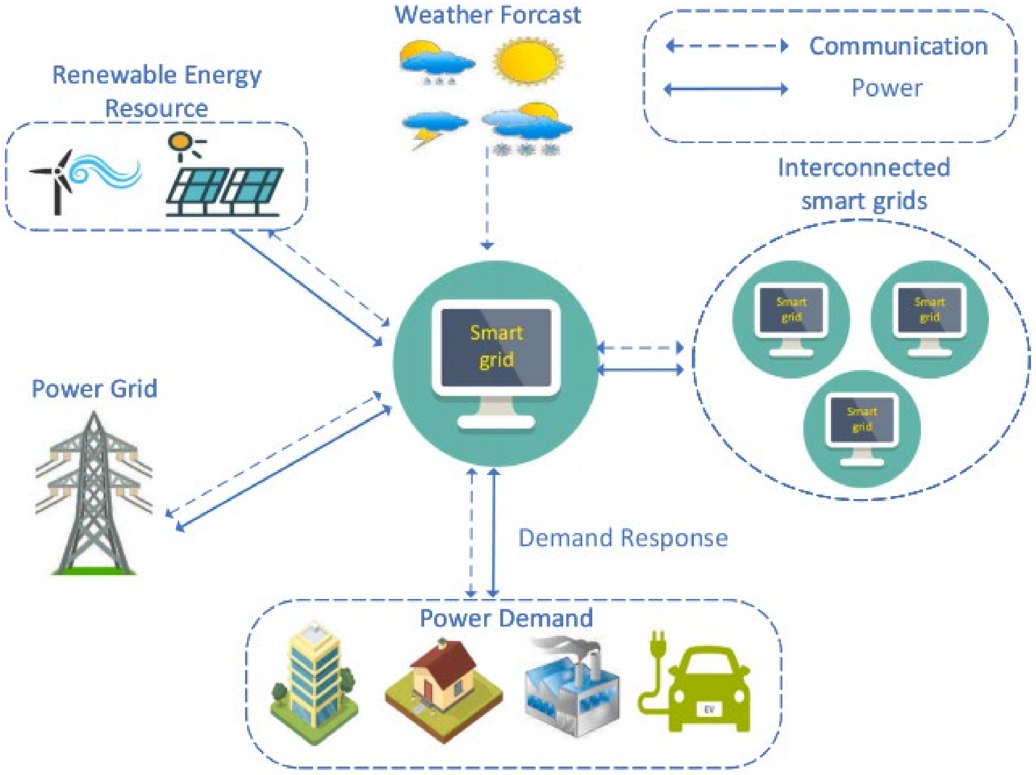
Toxic Industrial Air Pollution in the U.S.

Toxic Air Pollution by Sector



Sector	Toxic Air Pollution (lbs)	% of National Air Pollution
Electricity Generation	381,740,601	49%
Chemicals	112,870,057	15%
Paper Products	103,249,010	13%
Food & Beverages	26,908,977	3%
Primary Metals	24,923,246	3%
Other	121,888,815	16%
Total	**771,580,707**	**100%**

How does Smart Grid work?



Smart Grid Conceptual Model

What are the benefits of the smart grid?

- **For consumers, the smart grid will:**
  - Offer up-to-the-moment information on their energy usage;
  - Enable smart devices to be charged and run during off-peak hours to lower energy bills;
  - Open up a wider range of electricity pricing options.
- **For utilities and other stakeholders, the smart grid will:**
  - Increase grid reliability and reduce the frequency of power blackouts;
  - Reduce inefficiencies in energy delivery;
  - Integrate the sustainable resources more fully into the grid;
  - Improve management of distributed energy resources.

This Talk

- **Part I: Optimal Power flow (OPF)**
  - Semidefinite Programming (SDP) reformulation for OPF
  - Nonconvex and Nonsmooth optimization for SDP relaxation
  - OPF for large scale smart grid
- **Part II: EV charging and Demand response**
  - Joint OPF-EV charging
  - Distributed MPC for joint OPF and DR
- **Part III: PMU placement**
  - Mixed-integer convex programming (MICP)

Part I: Optimal Power Flow

- Semidefinite Programming (SDP) reformulation for OPF
- Nonconvex and Nonsmooth optimization for SDP relaxation
- OPF for large scale smart grid

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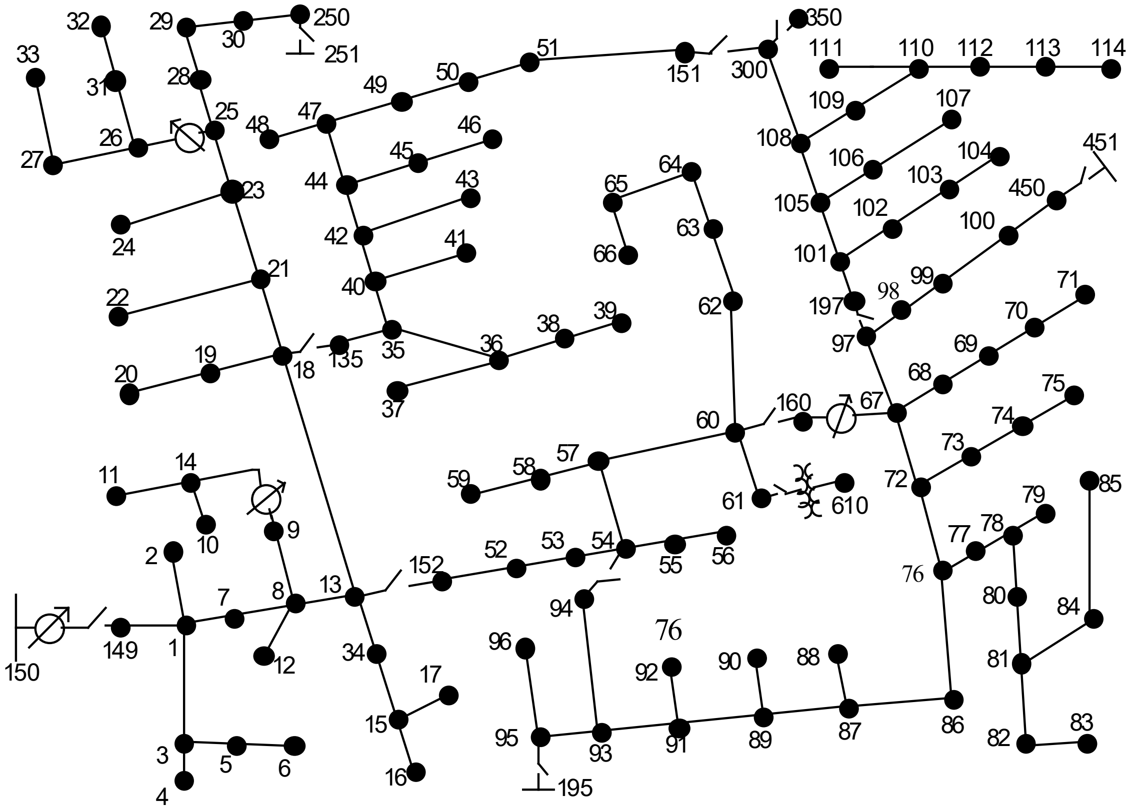
1. Optimal power flow problem in smart grid

- To locate nodal buses' voltages for minimizing the cost of generation subject to operating constraints.



Mathematical formulation of the OPF

- **Objective function:**
  — Minimize total generating cost

$$f(V) = \sum_{k\in\mathcal{G}}[c_{k2}(P_{L_k} + \Re(\sum_{m\in\mathcal{N}(k)} V_k V_m^* y_{km}^*))^2$$
$$+c_{k1}(P_{L_k} + \Re(\sum_{m\in\mathcal{N}(k)} V_k V_m^* y_{km}^*)) + c_{k0}].$$

- **Equality constraints:**
  — Power balance at each node

$$-P_{L_k} - jQ_{L_k} = \sum_{m\in\mathcal{N}(k)} V_k V_m^* y_{km}^*, \forall k \in \mathcal{N} \setminus \mathcal{G}$$

Mathematical formulation of the OPF

- **Inequality constraints:**
  - Limits on active and reactive power at each generator

$$P_{G_k}^{min} \leq P_{L_k} + \Re\Big(\sum_{m \in \mathcal{N}(k)} V_k V_m^* y_{km}^*\Big) \leq P_{G_k}^{max}, \forall k \in \mathcal{G}$$

$$Q_{G_k}^{min} \leq Q_{L_k} + \Im\Big(\sum_{m \in \mathcal{N}(k)} V_k V_m^* y_{km}^*\Big) \leq Q_{G_k}^{max}, \forall k \in \mathcal{G}$$

  - Limits on voltage at each node

$$V_k^{min} \leq |V_k| \leq V_k^{max}, \forall k \in \mathcal{N}$$

$$|\arg(V_k) - \arg(V_m)| \leq \theta_{km}^{max}, \forall (k, m) \in \mathcal{L}$$

Mathematical formulation of the OPF

Indefinite Quadratic Formulation

$$\min_{V \in \mathbb{C}^n} f(V) \quad \text{s.t.} \tag{5a}$$

$$-P_{L_k} - jQ_{L_k} = \sum_{m \in \mathcal{N}(k)} V_k {V_m}^* y_{km}^*, \forall k \in \mathcal{N} \setminus \mathcal{G} \tag{5b}$$

$$P_{G_k}^{min} \leq P_{L_k} + \Re\Big(\sum_{m \in \mathcal{N}(k)} V_k {V_m}^* y_{km}^* \Big) \leq P_{G_k}^{max}, \forall k \in \mathcal{G} \tag{5c}$$

$$Q_{G_k}^{min} \leq Q_{L_k} + \Im\Big(\sum_{m \in \mathcal{N}(k)} V_k {V_m}^* y_{km}^* \Big) \leq Q_{G_k}^{max}, \forall k \in \mathcal{G} \tag{5d}$$

$$V_k^{min} \leq |V_k| \leq V_k^{max}, \forall k \in \mathcal{N} \tag{5e}$$

$$|S_{km}| = |V_k {V_m}^* y_{km}^*| \leq S_{km}^{max}, \forall (k,m) \in \mathcal{L} \tag{5f}$$

$$|V_k - V_m| \leq V_{km}^{max}, \forall (k,m) \in \mathcal{L} \tag{5g}$$

$$|\arg(V_k) - \arg(V_m)| \leq \theta_{km}^{max}, \forall (k,m) \in \mathcal{L} \tag{5h}$$

It is obvious that (5a) is minimization of nonconvex objective function over quadratic equality constraints (5b) and (nonconvex) indefinite quadratic constraints (5c)-(5h).

Mathematical formulation of the OPF

- **Introduce new variable**

Define the Hermitian symmetric matrix of outer product:

$$W = VV^H \in \mathbb{C}^{n\times n}.$$

which must satisfy

$$W \succeq 0 \text{ and } \mathsf{rank}(W) = 1.$$

to be qualified as the self-outer-product of vectors *V.*

Mathematical formulation of the OPF

◆ Recast to linear constraints + rank-one constraint

$$\min_{W \in \mathbb{C}^{n \times n}} \quad F(W) \quad \text{s.t.} \quad (6a)$$

$$-P_{L_k} - jQ_{L_k} = \sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^* \; \forall k \in \mathcal{N} \setminus \mathcal{G}, \quad (6b)$$

$$P_{G_k}^{min} \leq P_{L_k} + \Re\Big(\sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^*\Big) \leq P_{G_k}^{max}, \forall k \in \mathcal{G} \quad (6c)$$

$$Q_{G_k}^{min} \leq Q_{L_k} + \Im\Big(\sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^*\Big) \leq Q_{G_k}^{max}, \forall k \in \mathcal{G} \quad (6d)$$

$$(V_k^{min})^2 \leq W_{kk} \leq (V_k^{max})^2, \forall k \in \mathcal{N} \quad (6e)$$

$$|W_{km} y_{km}^*| \leq S_{km}^{max}, \forall (k,m) \in \mathcal{L} \quad (6f)$$

$$W_{kk} + W_{mm} - W_{km} - W_{mk} \leq (V_{km}^{max})^2, \forall (k,m) \in \mathcal{L} \quad (6g)$$

$$\Im(W_{km}) \leq \Re(W_{km}) \tan \theta_{km}^{max}, \forall (k,m) \in \mathcal{L} \quad (6h)$$

$$W \succeq 0, \quad (6i)$$

$$\text{rank}(W) = 1, \quad (6j)$$

Dropping the rank-one constraint to treat OPF as a semi-definite program.

- When the optimal solution is not of rank-one, it has no physical meaning.
- Even when there is an optimal rank-one solution, the existing software often output rank-more-than-one solution.

Our Method

- We express the discrete rank-one constraint by d.c. constraint

$$\mathrm{rank}(W) = 1 \quad + \quad W \geq 0 \quad = \quad \mathrm{Trace}(W) = \lambda_{max}(W)$$

- As Trace(W) ≥ $\lambda_{max}(W)$ is always, the rank-one constraint is just the reverse convex constraint

$$\lambda_{max}(W) - \mathrm{Trace}(W) \geq 0$$

Then we incorporate the reverse convex constraint into the objective to consider the penalized optimization.

Our Method

- **Exact penalized optimization**

$$\min \ F(W) + \mu\big(Trace(W) - \lambda_{\max}(W)\big)$$
$$\text{s.t.} \ \ \text{linear constraints}$$

- F(W) is convex in W, so the above objective is a d.c. function (but nonsmooth).
- As the problem is high dimensional, it is challenging to use global search methods for computation.

Our Method

- **Exact penalized optimization**

Go to local search iterations: initialized by point $W^{(0)}$ of the SDR problem, at the κ–th iteration to solve the SDP.

$$\min \ F(W) + \mu\left(Trace(W) - w_k^H W w_k\right)$$
$$\text{s.t.} \ \ \text{linear constraints}$$

By doing this, the rank of W^{κ} will tend to 1, yielding an optimal solution of OPF.

How to know the found solution is global

Network	Lower Bound	Found Value
WB5	946.53	946.58
Case9	5305.5	5305.68
Case14	8088.71	8088.71
Case30	574.87	574.87
Case57	41,213.99	41,313.72
Case118mod	129,682.50	129,686.03

Practically global optimal solutions found.

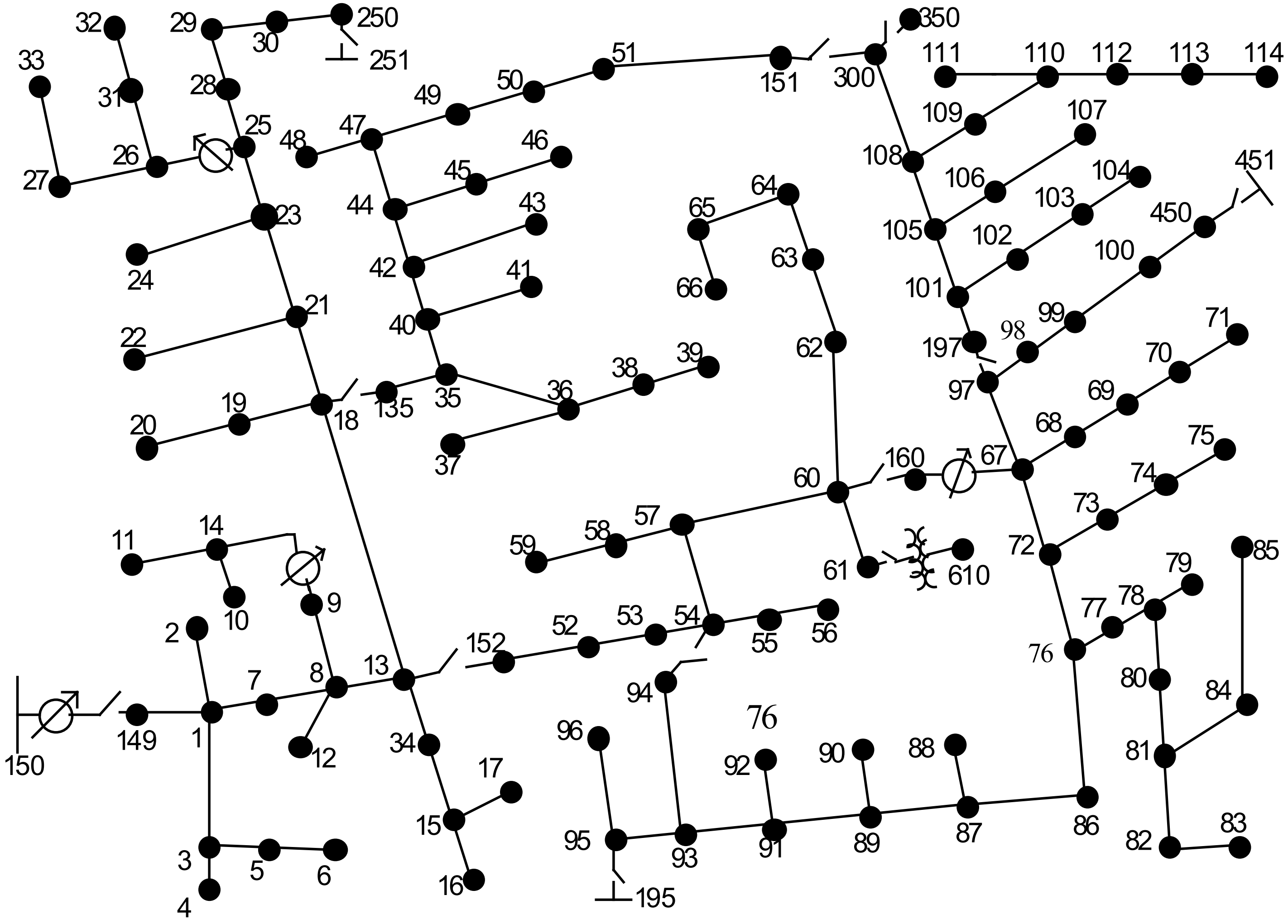
2. OPF for large scale smart grid

- For large scale algorithm, there are more than 2000 nodes, i.e. the dimension of V is more than 2000 and the dimension of $W = VV^H$ is more that 2,000,000.
  SDP solvers are unable to address.

- Fortunately, large scale grids' connection are sparse.

- It is impossible to design large scale grids with many connections.

IEEE 123 nodes test feeder



Sparseness of W

- It means that there are not so many W_{km}.

$$\min_{W \in \mathbb{C}^{n \times n}} \quad F(W) \quad \text{s.t.} \quad W \succeq 0 \text{(4a)}$$

$$-P_{L_k} - jQ_{L_k} = \sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^*, \; k \in \mathcal{N} \setminus \mathcal{G} \text{(4b)}$$

$$P_{G_k}^{min} \leq P_{L_k} + \Re\left(\sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^*\right) \leq P_{G_k}^{max}, k \in \mathcal{G} \text{(4c)}$$

$$Q_{G_k}^{min} \leq Q_{L_k} + \Im\left(\sum_{m \in \mathcal{N}(k)} W_{km} y_{km}^*\right) \leq Q_{G_k}^{max}, k \in \mathcal{G} \text{(4d)}$$

$$(V_k^{min})^2 \leq W_{kk} \leq (V_k^{max})^2, k \in \mathcal{N} \text{(4e)}$$

$$|W_{km} y_{km}^*| \leq S_{km}^{max}, (k, m) \in \mathcal{L} \text{(4f)}$$

$$W_{kk} + W_{mm} - W_{km} - W_{mk} \leq (V_{km}^{max})^2, (k, m) \in \mathcal{L} \text{(4g)}$$

$$\Im(W_{km}) \leq \Re(W_{km}) \tan \theta_{km}^{max}, (k, m) \in \mathcal{L} \text{(4h)}$$

$$\text{rank}(W) = 1, \text{(4i)}$$

Bags Decomposition

- Decompose the set of buses' voltages $\mathcal{N} := \{V_1, \dots, V_n\}$ into I possibly overlapped subsets $\mathcal{N}_i = \{V_{i_1}, \dots, V_{i_{\mathrm{N}_i}}\}$ of connected buses called by bags of voltages.
- We resort N = {1, 2..., n} as N = $\{N_1, \dots, N_n\}$ such that the cardinality $|\mathcal{N}(N_k)|$ is in decreased order:

$$|\mathcal{N}(N_1)| \geqslant |\mathcal{N}(N_2)| \geqslant \cdot \cdot \cdot \geqslant |\mathcal{N}(N_n)|.$$

- Accordingly, the first bag of buses is defined as $\mathcal{N}_1 = \mathcal{N}(N_1)$.

- The second bag is defined as $\mathcal{N}_2 = \{i \in \mathcal{N}(N_2) : \{i, \mathcal{N}_2\} \notin \mathcal{N}_1\}$.

New rank-one formulations

- Define

$$V_{\mathrm{N}_i} = [V_{i_1}, \dots, V_{i_{\mathrm{N}_i}}]^H$$
$$W^i = V_{\mathrm{N}_i}(V_{\mathrm{N}_i})^H$$

Entries of W^i may be overlapped

Replacing $W^i_{km} = V_{i_k} V^H_{i_m}$, we have the following equivalent optimization reformulation

$$\min \ F(W) \ \text{s.t.} \ (4\mathrm{b}) - (4\mathrm{h}), \mathrm{rank}(\mathrm{W}^{\mathrm{i}}) = 1$$

Advantages and challenges

- **Advantages**: Each W^i is of moderate dimension and the total dimension of all W^i is dramatically reduced compared with W.
- **Challenges**: Many rank-one constraints (many d.c. constraints), not easy to handle.
- For example, consider

$$\min F(W) + \mu \sum_{i=1}^{N} (Trace(W^i) - \lambda_{max}(W^i))$$

and use iterations:

$$\min F(W) + \mu \sum_{i=1}^{N} (Trace(W^i) - (w_{max}^i)^H W^i w_{max}^i)$$

It is difficult to get all W^i of rank-one.

Large Scale OPF Iteration

- After reaching rank-one how to keep W^i of rank-one?

$$\text{Trace}(W^i) - (w_{\max}^{i,(\kappa)})^H W^i w_{\max}^{i,(\kappa)} \leq \epsilon_{tol}, i \in \mathcal{L}^{(\kappa)}$$

- Iteration

$$\min_{W=\text{diag}\{W^i\}} F(W) + \mu \sum_{i \notin \mathcal{L}^{(\kappa)}} [\text{Trace}(W^i)$$
$$-(w_{\max}^{i,(\kappa)})^H W^i w_{\max}^{i,(\kappa)}] \quad \text{s.t.} \quad (4b)-(4h), (6b),$$

Simulation results

- Global optimality tolerance (GOT) of its found solution defined as:

$$GOT = \frac{\text{found value} - \text{lower bound}}{\text{lower bound}}$$

System	Lower bound	Found value	GOT
Polish-2383wp	1.8490 E6	1.8408 E6	4.3267 E-4
Polish-2736sp	1.3041 E6	1.3042 E6	7.6681 E-5
Polish-2737sop	7.7571 E5	7.7572 E5	1.2891 E-5
Polish-2746wop	1.2039 E6	1.2040 E6	8.3063 E-5
Polish-2746wp	1.6266 E6	1.6266 E6	6.1478 E-7
Polish-3012wp	2.5717 E6	2.5727 E6	3.8885 E-4
Polish-3120sp	2.1314 E6	2.1391 E6	3.6321 E-3

Global optimal solution found.

Reference

[1] Ye Shi, Hoang Duong Tuan*, Hoang Tuy and Steven W. Su, 'Global Optimization for Optimal Power Flow over Transmission Networks', Journal of Global Optimization, vol. 69, pp. 745-760, 2017.

[2] Ye Shi, Hoang Duong Tuan*, Andrey V. Savkin, Steven W. Su, 'Optimal Power Flow over Large-Scale Transmission Networks', Systems & Control Letters, vol. 118, pp. 16-21, 2018.

Part II: EV charging and Demand response

- Joint OPF-EV charging
- Distributed MPC for joint OPF and DR

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3. Joint OPF and EV charging

- **Motivations:**
  - Electrical vehicles (EVs) as a promising solution to resolve both the economic and environmental concerns in the transportation industry.
  - Using a smart power grid to serve residences and charge EVs constitutes one of the most important applications of the smart grid technology.
  - The massive integration of plug-in EVs (PEVs) into the grid causes many potential impacts such as voltage deviation, increased load variations and power loss of the grid.

3. Joint OPF and EV charging

Development plan

- **China:**

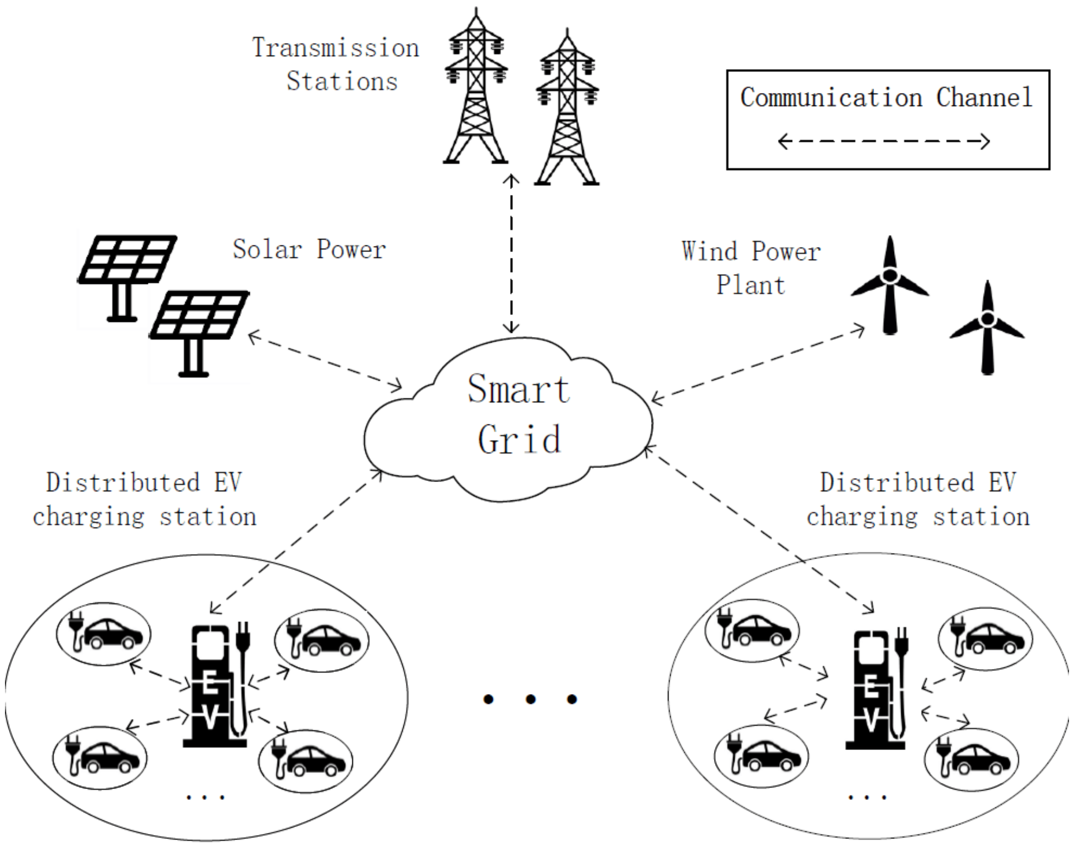
  By 2025, New buses and Heavy-duty trucks will be fully electrified.

  By 2030, EVs will be fully popularized.

- **Europe:**

  By 2040, No new diesel and petrol vehicles permitted.

Structure



New challenges

- EVs arrive and depart randomly at different CSs.
- The arrival and departure information of EVs are not known in advance. Thus, the conventional model predictive control (MPC) is not applicable.
- Our contribution is to develop a novel MPC-based approach to address this problem.

Joint OPF-PEV Charging

Objective

$$F(\mathcal{R}, \mathcal{P}^{PEV}) = \sum_{t' \in \mathcal{T}} \sum_{k \in \mathcal{G}} f(P_{g_k}(t')) + \sum_{t' \in \mathcal{T}} \sum_{k \in \mathcal{N}} \sum_{n \in \mathcal{H}_k} \beta_t P_{k_n}(t'),$$

where $f(P_{g_k}(t'))$ is the cost function of real power generation by DGs, which is linear or quadratic in $P_{g_k}(t')$, and β_t is the known PEV charging price during the time interval $(t', t'+1]$.

Dynamic PEV charging

$$\sum_{t'=t_{k_n,a}}^{t_{k_n,d}} u_h P_{k_n}(t') = C_{k_n}(1 - s_{k_n}^0), \qquad (2)$$

where u_h is the charging efficiency of the battery and $P_{k_n}(t')$ is a decision variable representing the power charging rate of PEV $k_n \in \mathcal{H}_k$ at time t'.

Power balance

$$V_k(t')\Big(\sum_{m \in \mathcal{N}(k)} y_{km}(V_k(t') - V_m(t'))\Big)^* = (P_{g_k}(t') - P_{l_k}(t') - \sum_{n \in \mathcal{H}_k} P_{k_n}(t')) + j(Q_{g_k(t')} - Q_{l_k}(t')), k \in \mathcal{G}.$$

Similarly,

$$V_k(t')\Big(\sum_{m \in \mathcal{N}(k)} y_{km}(V_k(t') - V_m(t'))\Big)^* = -P_{l_k}(t') - jQ_{l_k}(t'), k \notin \mathcal{G}.$$

Joint OPF-PEV Charging

- **Prediction horizon**
  - For each $k_n \in C(t)$, let $P_{kn}(t)$ be its remaining demand for charging by the departure time $t_{kn,d}$.

    Define

$$\Psi(t) = \max_{k_n \in C(t)} t_{kn,d}$$

  - At time t we solve the optimal power flow (OPF) problem over the prediction horizon [t, $\Psi(t)$] but then take only V(t), P(t), R(t) for online updating solution.

Joint OPF-PEV Charging

◆ Joint OPF-PEV charging formulation

$$\min_{V(t'),R(t'),P_{k_n}(t'),t'\in[t,\Psi(t)],k_n\in C(t)} F_{[t,\Psi(t)]} \quad (13a)$$

$$\text{s.t.} \quad \text{network balance and bound constraints in} \quad [t,\Psi(t)], \quad (13b)$$

$$V_k(t')\Big(\sum_{m\in\mathcal{N}(k)} y_{km}(V_k(t')-V_m(t'))\Big)^* = (P_{g_k}(t') - P_{l_k}(t') - \sum_{k_n\in C(t)} P_{k_n}(t')) + j(Q_{g_k}(t') - Q_{l_k}(t')), \quad (13c)$$

$$\sum_{t'=t}^{t_{k_n,d}} u_h P_{k_n}(t') = \mathcal{P}_{k_n}(t), \quad (13d)$$

◆ Joint OPF-PEV charging Re-cast

$$\min_{W(t'),R(t'),P_{k_n}(t'),t'\in[t,\Psi(t)],k_n\in C(t)} F_{[t,\Psi(t)]} \quad (14a)$$

$$\text{s.t.} \quad \text{network balance and bound constraints in} \quad [t,\Psi(t)], \quad (14b)$$

$$\sum_{m\in\mathcal{N}(k)} (W_{kk}(t') - W_{km}(t'))y^*_{km} = (P_{g_k}(t') - P_{l_k}(t') - \sum_{k_n\in C(t)} P_{k_n}(t')) + j(Q_{g_k}(t') - Q_{l_k}(t')), \quad k\in\mathcal{G}, \quad (14c)$$

$$\sum_{m\in\mathcal{N}(k)} (W_{kk}(t') - W_{km}(t'))y^*_{km} = -P_{l_k}(t') - jQ_{l_k}(t'), k\notin\mathcal{G}, \quad (14d)$$

$$\underline{V}_k^2 \le W_{kk}(t') \le \overline{V}_k^2, \quad k\in\mathcal{N}, \quad (14e)$$

$$\Im(W_{km}(t')) \le \Re(W_{km}(t'))\tan(\theta_{km}^{max}), (k,m)\in\mathcal{L}, \quad (14f)$$

$$|(W_{kk}(t') - W_{km}(t'))y^*_{km}| \le S_{km}, \quad (14g)$$

$$W(t') \succeq 0, \quad (14h)$$

$$\text{rank}(W(t')) = 1. \quad (14i)$$

Our method

- Relax the rank constraint (14i), we solve the SDR:

$$\min_{W(t'),R(t'),P_{k_n}(t')} F_{[t,\Psi(t)]} \quad \text{s.t. } (14b)-(14h).$$

- If the solution of W is not rank-one, solve the following problem by previous algorithm for OPF at time t.

$$\min_{W(t),R(t)} \ F(P_g(t))) := \sum_{k\in\mathcal{G}} f(P_{g_k}(t)) \qquad (16a)$$

$$\text{s.t.} \quad (3)-(4),(7),(14d)-(14h) \quad \text{for } t'=t, \qquad (16b)$$

$$\sum_{m\in\mathcal{N}(k)} (W_{kk}(t)-W_{km}(t))y^*_{km} = (P_{g_k}(t)-P_{l_k}(t)$$

$$-\sum_{k_n\in C(t)} \hat{P}_{k_n}(t)) + j(Q_{g_k}(t)-Q_{l_k}(t)), \quad k\in\mathcal{G}, \qquad (16c)$$

$$\text{rank}(W(t))=1. \qquad (16d)$$

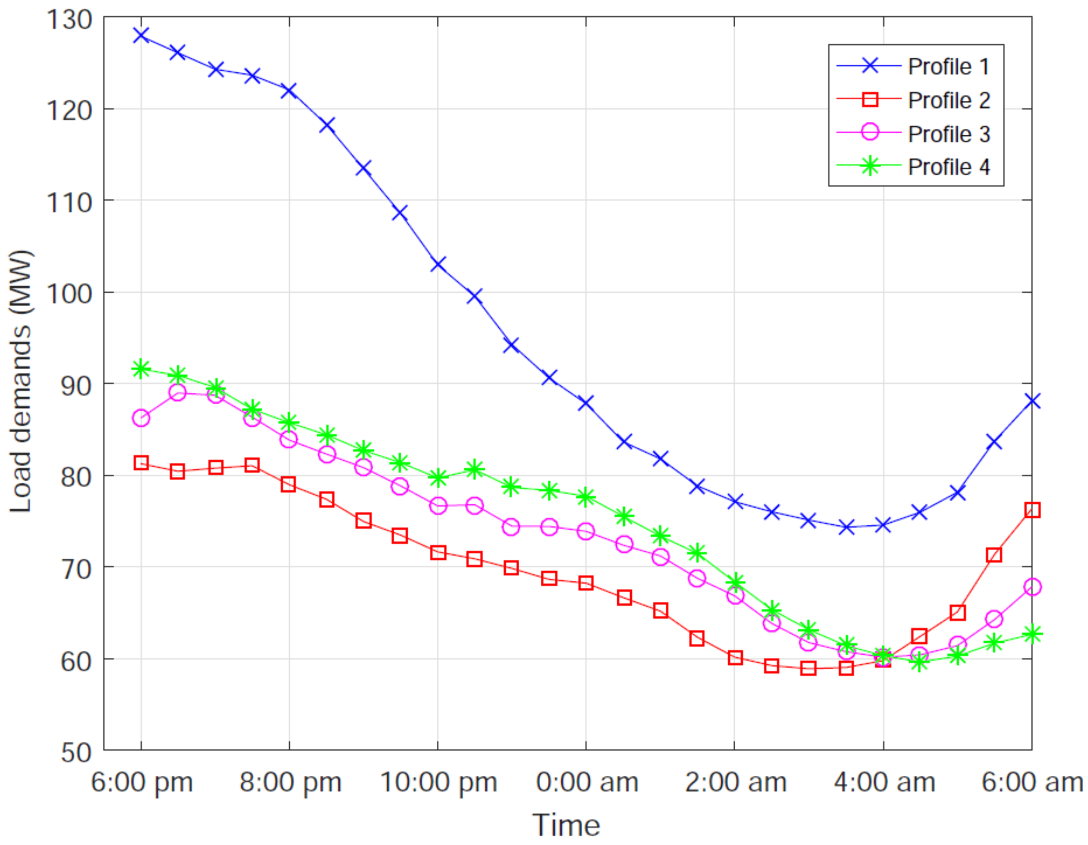
Global solutions found

- Some cases are tested with the residential data of profile 2 to analyze the efficiency of off-line computation. The computational results are summarized in this Table.

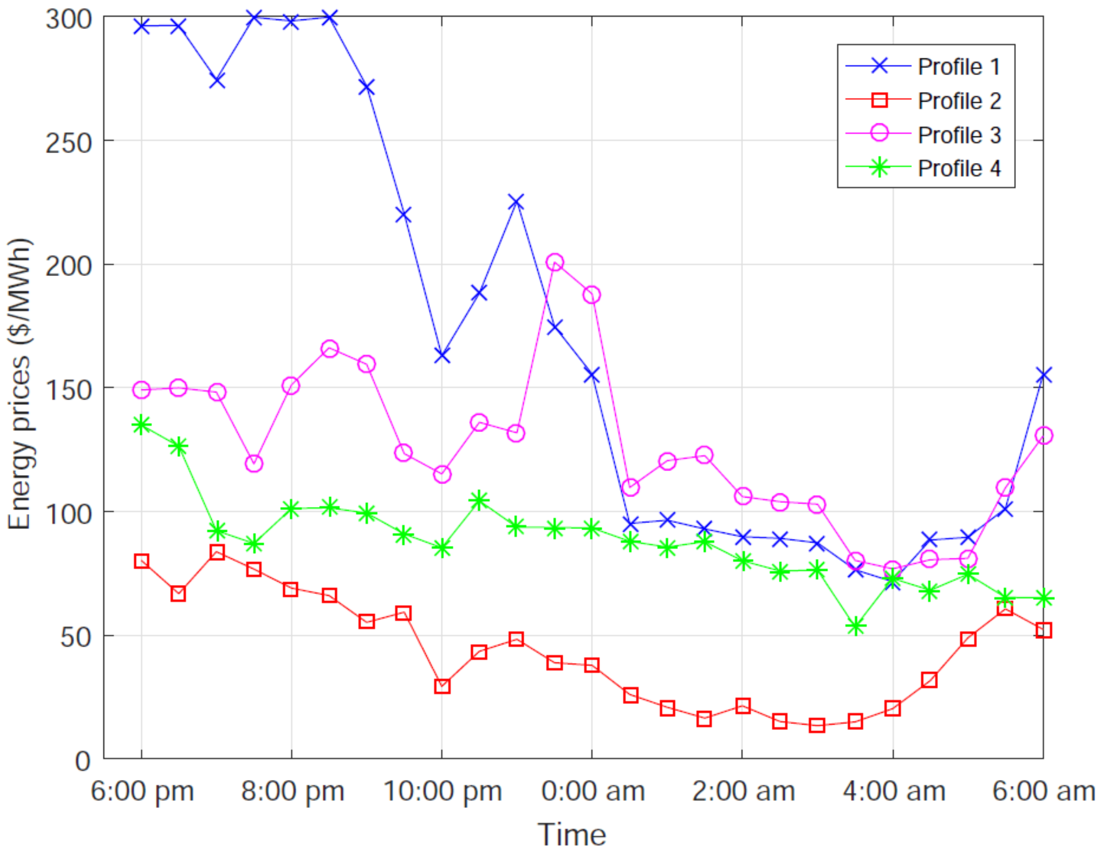
	Rank	μ	Lower bound	Computed value	Opt. degree
Case9	9	1	27978.1	27978.1	100%
Case14	1	-	40800.7	40800.7	100%
Case30	1	-	4935.6	4935.6	100%
Case118mod	2	50	644225.3	644233.9	99.999%

Offline results of optimal pev charging for four networks

Load demands and Energy Price



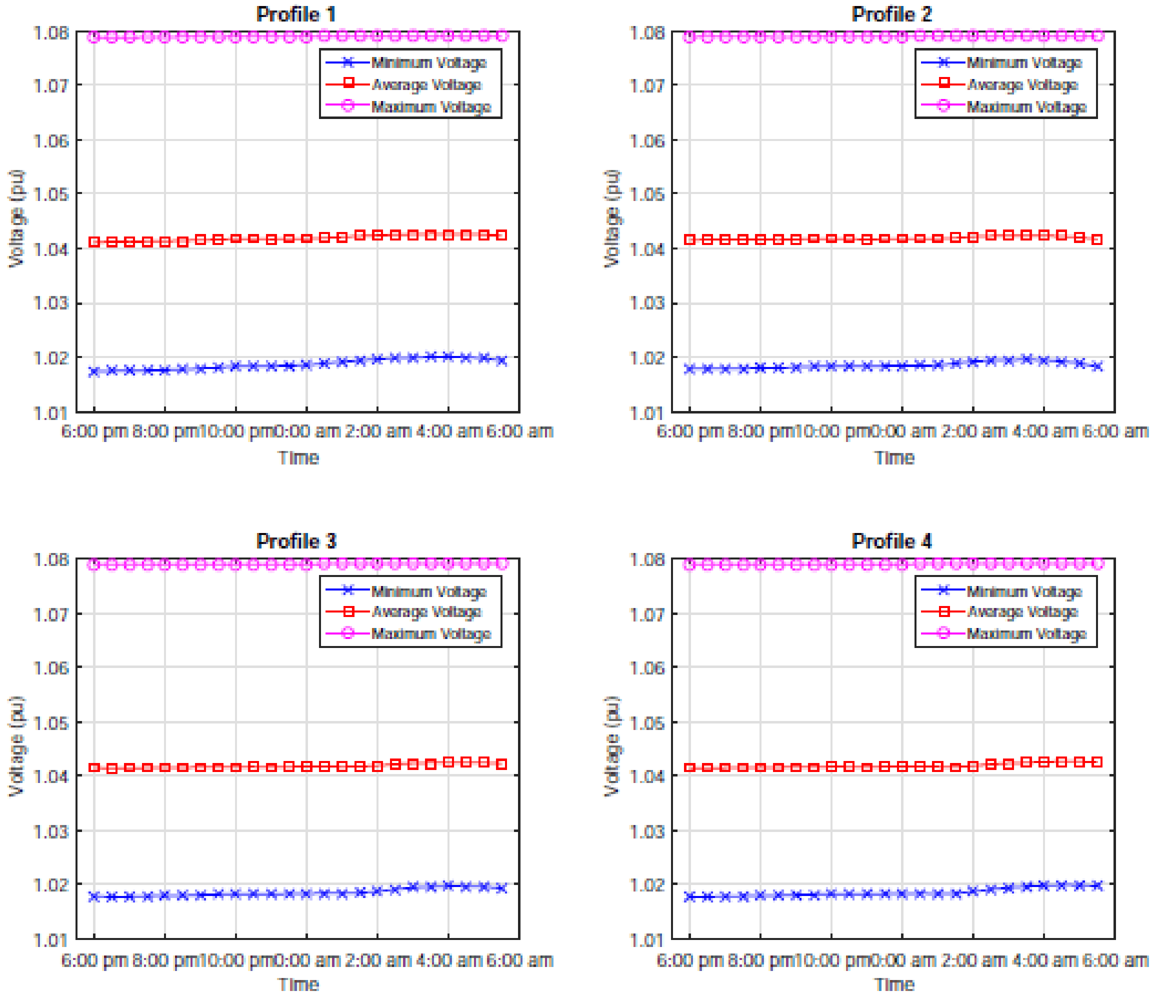
Residential load demands of four profiles.



Energy prices for four profiles.

Voltage

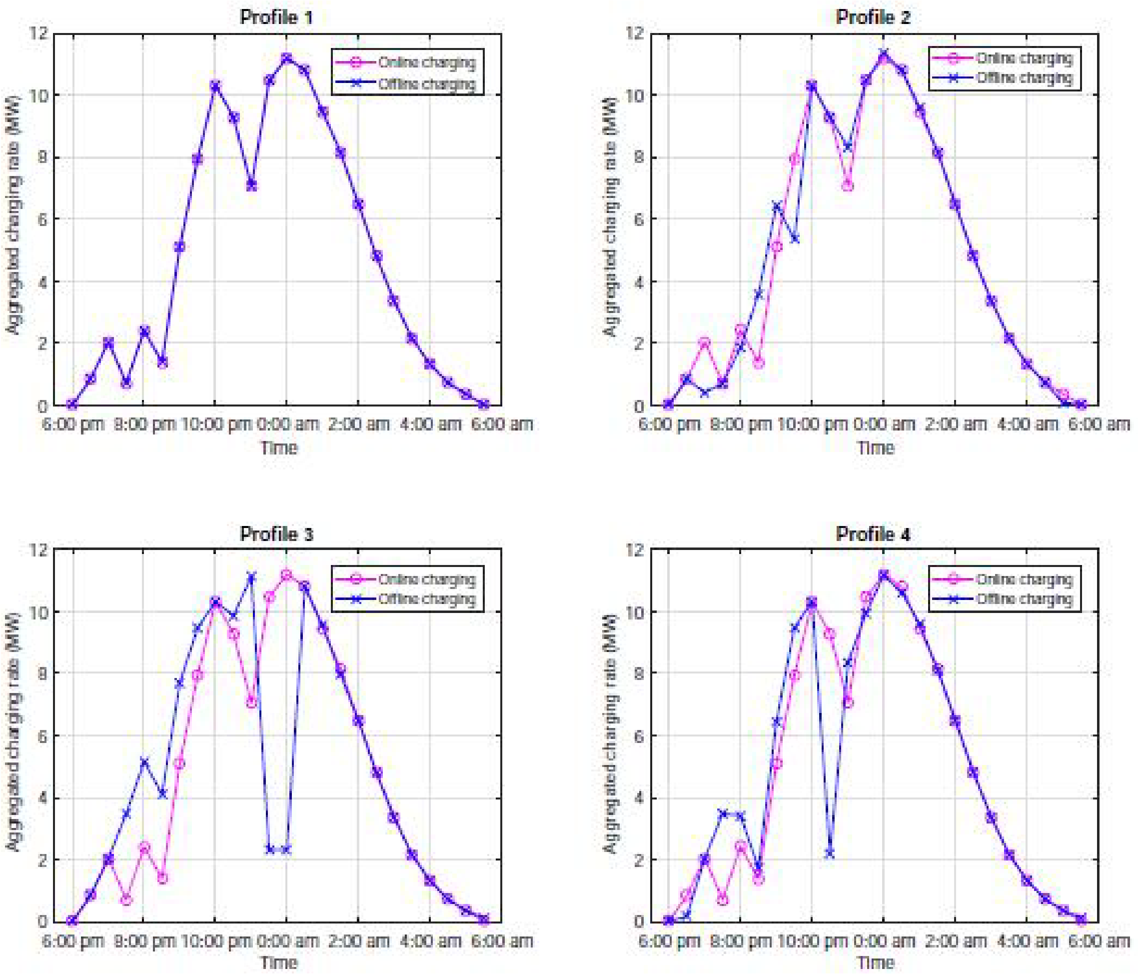
- Voltage profile of online charging for Case30 under four residential profiles.



The stable and smooth voltage profile for these four residential profiles during the charging period are shown in this figure.

Charging rate

◆ PEVs charging load under MPC-based and offline computation for Case30 with four residential profiles.



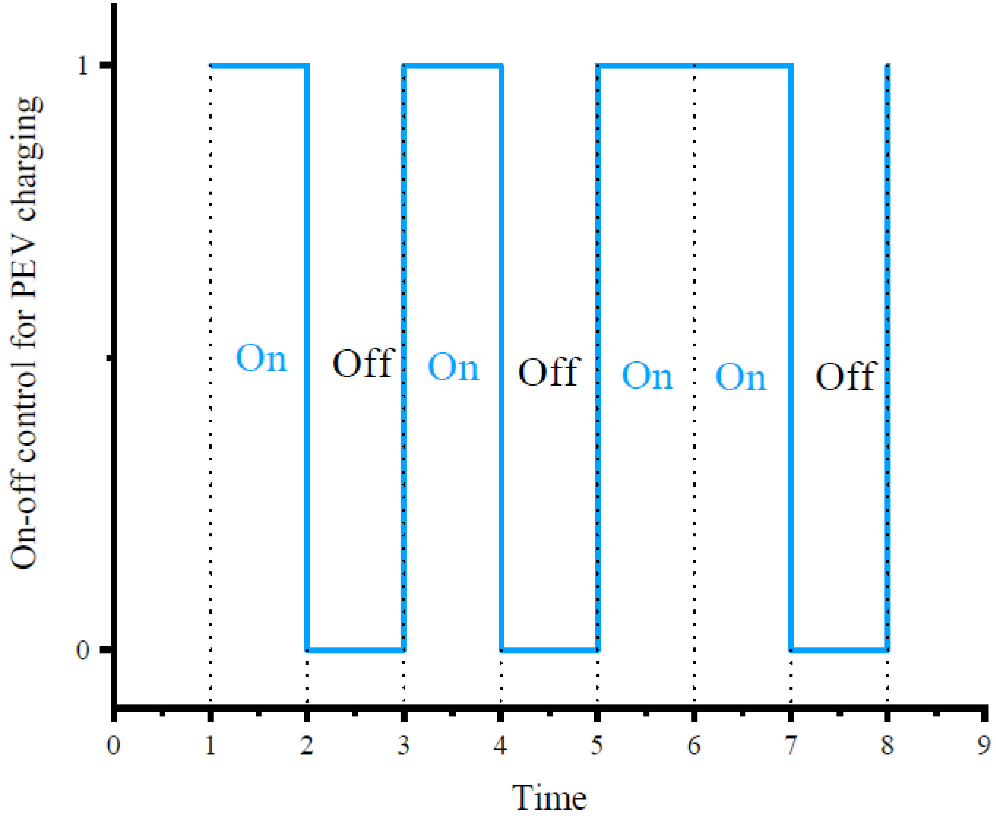
Insights on charging rate:

1. Energy price is not the only impact factor for the aggregating charging rate.
2. The power balance and residential demands also have significant effects on the aggregating charging rate.

On-off charging

- **Easily control**
- **Simple implementation**

On-off EV charging



On–off control for PEV charging in a serving period

Challenges

- **Dynamic AC optimal power flow**
  - Nonconvex nodal voltage constraints
- **PEVs arrive and depart randomly at different Charging Stations**
  - Random nature
- **On-off strategy of PEV charging**
  - Binary charging variables
- **Dynamic AC optimal power flow**
  - Nonconvex constraints in terms of nodal voltage

$$V_k(t')[\sum_{m\in\mathcal{N}(k)} y_{km}(V_k - V_m)]^* = [P_{g_k}(t') - P_{l_k}(t') - \sum_{n\in\mathcal{H}_k} \bar{P}_{k_n}\tau_{k_n}(t')] + j[Q_{g_k}(t') - Q_{l_k}(t')], k \in \mathcal{G},$$

Challenges

- **PEVs arrive and depart randomly at different Charging Stations**
  - Random nature
- **Model predictive control:**
  - During the computational procedure, we solve the dynamic OPF-PEV problem over the prediction horizon [t, T], while only the solution at time slot t is required for online updating.
- **On-off strategy of PEV charging**
  - Binary charging constraints

$$\tau_{k_n}(t') \in \{0, 1\}$$

$$\sum_{t'=t_{k_n,a}}^{t_{k_n,d}} u_h \bar{P}_{k_n} \tau_{k_n}(t') \geq C_{k_n}(1 - s_{k_n}^0).$$

Smart grid operation constraints

- The following constraints are about power balance, power generation, voltage and phase balance, and line capacity:

$$\sum_{m\in\mathcal{N}(k)} (W_{kk}(t') - W_{km}(t'))y^*_{km} = [P_{g_k}(t') - P_{l_k}(t')$$
$$- \sum_{k_n\in C(t)} \bar{P}_{k_n}\tau_{k_n}(t')] + j(Q_{g_k}(t') - Q_{l_k}(t')), \quad k \in \mathcal{G},$$
$$\underline{P}_{g_k} \le P_{g_k}(t') \le \overline{P}_{g_k}, \;\; k \in \mathcal{G},$$
$$\underline{Q}_{g_k} \le Q_{g_k}(t') \le \overline{Q}_{g_k}, \;\; k \in \mathcal{G},$$
$$\underline{V}_k^2 \le W_{kk}(t') \le \overline{V}_k^2, \;\; k \in \mathcal{N},$$
$$\Im(W_{km}(t')) \le \Re(W_{km}(t')) \tan(\theta^{max}_{km}), (k,m) \in \mathcal{L},$$
$$|(W_{kk}(t') - W_{km}(t'))y^*_{km}| \le S_{km}, (k,m) \in \mathcal{L},$$

Mathematical formulation of the MINLP

$$
\begin{aligned}
\min_{\mathcal{W}_P(t),\mathcal{R}_P(t),\tau_P(t)} \quad & F_P(\mathcal{R}_P(t),\tau_P(t)) \\
\text{s.t.} \quad & (1)-(5),(9)-(10) \quad \text{for } t' \in [t,\Psi(t)], && \text{(16a)} \\
& \sum_{t'=t}^{t_{k_n,d}} u_h \bar{P}_{k_n} \tau_{k_n}(t') \geq d_{k_n}(t), k_n \in C(t), && \text{(16b)} \\
& \tau_{k_n}(t') \in \{0,1\}, t' \in [t, t_{k_n,d}], k_n \in C(t), && \text{(16c)} \\
& W(t') \succeq 0, \quad \text{for } t' \in [t,\Psi(t)] && \text{(16d)} \\
& \operatorname{rank}(W(t')) = 1, \quad \text{for } t' \in [t,\Psi(t)]. && \text{(16e)}
\end{aligned}
$$

- The MPC is solved over [t, Ψ(t)] at each time t, only R(t), W(t) and τ(t) are used for updating the solution.
- (16b) and (16c) are the integer constraints.
- (16e) is the nonconvex rank constraint.

Two-Stage Optimization

◆**Stage 1: Mixed-integer convex programming (MICP)**

➢Relax the rank-one constraint (16e) and solve the MICP.

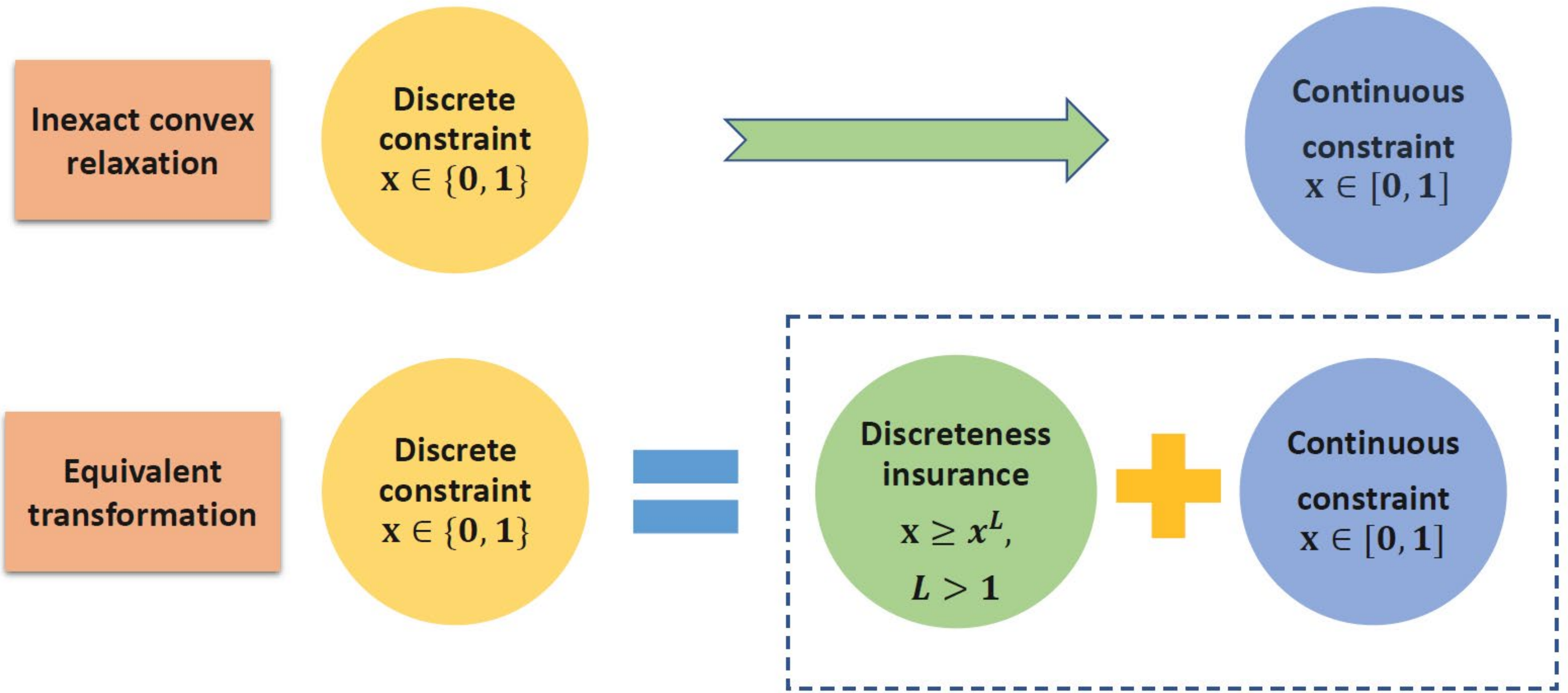
$$\min_{W(t),R(t)} \quad F(P_g(t)) + \nu(\mathrm{Trace}(W(t)) - \lambda_{\max}(W(t))), \quad (28a)$$

$$\text{s.t.} \quad (18b), W(t) \succeq 0. \quad (28b)$$

◆**Stage 2: Nonconvex rank-one optimization**

➢If the solution of Stage 1 is rank-one, then skip Stage 2; otherwise, use the following penalized optimization to handle the rank-one constraints, which was published in our previous work.

Equivalent transformation for MICP



MICP

Lemma and Proposition

Lemma 1: The binary constraint (14) can be fulfilled with the following continuous constraints with linear constraint (15),

$$0 \le \tau_{k_n}(t') \le 1, t' \in [t, t_{k_n,d}], k_n \in C(t), \tag{19}$$

$$\begin{aligned} g(\tau_P(t)) &\ge \bar{\tau}(t) \\ &\triangleq \sum_{k_n \in C(t)} \bar{\tau}_{k_n}(t), \end{aligned} \tag{20}$$

for $L > 1$ and $g(\tau_P(t)) \triangleq \sum_{k_n \in C(t)} \sum_{t'=t}^{t_{k_n,d}} \tau_{k_n}^L(t')$.

Proposition 1: Under the linear constraint (15), the function

$$g_1(\tau_P(t)) \triangleq \frac{1}{g(\tau_P(t))} - \frac{1}{\bar{\tau}(t)} \tag{21}$$

is a measure to evaluate the satisfaction of binary constraint (14) with $g_1(\tau_P(t)) \ge 0 \ \forall \ \tau_{k_n}(t') \in [0,1]$ and $g_1(\tau_P(t)) = 0$ if and only if $\tau_{k_n}(t')$ are binary (i.e. satisfying (14)).

Find a tight lower bound

$$\begin{aligned} g(\tau_P(t)) &\ge g^{(\kappa)}(\tau_P(t)) \\ &\triangleq g(\tau_P^{(\kappa)}(t)) + \langle \nabla g(\tau_P^{(\kappa)}(t)), \tau_P(t) - \tau_P^{(\kappa)}(t) \rangle \\ &= -(L-1) \sum_{k_n \in C(t)} \sum_{t'=t}^{t_{k_n,d}} (\tau_{k_n}^{(\kappa)}(t'))^L \\ &\quad + L \sum_{k_n \in C(t)} \sum_{t'=t}^{t_{k_n,d}} (\tau_{k_n}^{(\kappa)}(t'))^{L-1} \tau_{k_n}(t'). \end{aligned} \tag{23}$$

Hence, an approximation of the upper bounding for $g_1(\tau_P(t))$ at the variable $\tau_P^{(\kappa)}(t)$ can be easily obtained as

$$g_1(\tau_P(t)) \le g_1^{(\kappa)}(\tau_P(t)) \triangleq \frac{1}{g^{(\kappa)}(\tau_P(t))} - \frac{1}{\bar{\tau}(t)} \tag{24}$$

over the trust region

$$g^{(\kappa)}(\tau_P(t)) > 0. \tag{25}$$

Path following algorithm

Then, the following convex problem is solved at the κ-th iteration to obtain the next iterative point $(\mathcal{W}_P^{(\kappa+1)}(t), \mathcal{R}_P^{(\kappa+1)}(t), \tau_P^{(\kappa+1)}(t))$:

$$\min_{\mathcal{W}_P(t),\mathcal{R}_P(t),\tau_P(t)} \Phi^{(\kappa)}(\mathcal{R}_P(t), \tau_P(t)) \triangleq$$
$$F_P(\mathcal{R}_P(t), \tau_P(t)) + \mu g_1^{(\kappa)}(\tau_P(t)) \quad \text{s.t.} \quad (1)-(5)$$
$$\text{for } t' \in [t, \Psi(t)], (15), (16b), (16c), (19), (25). \tag{26}$$

Algorithm 1 MICP Solver

Set $\kappa = 0$, choose a feasible $\tau_P^{(0)}(t)$ for the following optimization problem:

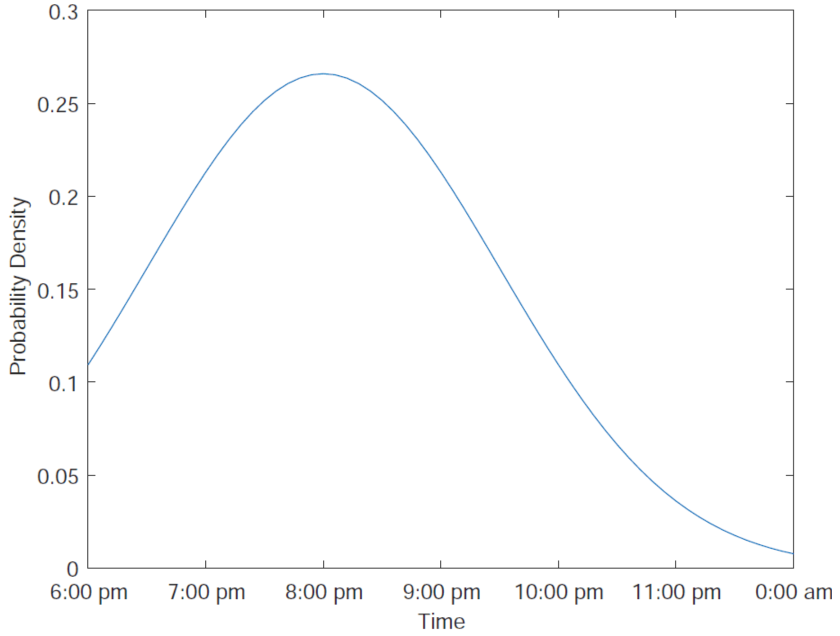
$$\min_{\mathcal{W}_P(t),\mathcal{R}_P(t),\tau_P(t)} F_P(\mathcal{R}_P(t), \tau_P(t)) \quad .$$
$$\text{s.t. } (15), (16b), (16c), (19). \tag{27}$$

κ-th iteration. Solve the optimization problem (26),
if

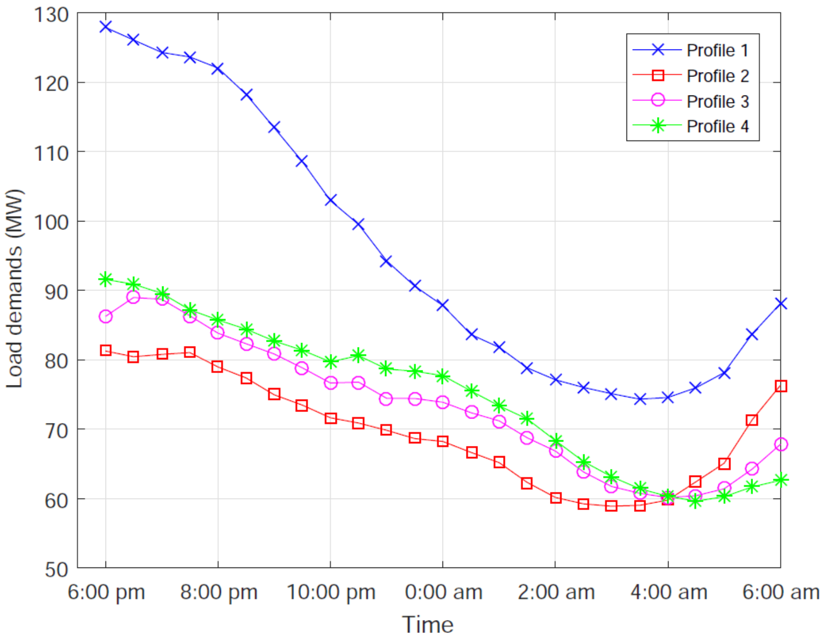
$$\sum_{k_n \in C(t)} \sum_{t'=t}^{t_{k_n,d}} \left(\tau_{k_n}^{(\kappa+1)}(t') - \left(\tau_{k_n}^{(\kappa+1)}(t')\right)^L \right) \approx 0,$$

then accept $\tau_P^{(\kappa+1)}(t)$ as the found solution.
else $\kappa = \kappa + 1$, go to the next iteration.
end if

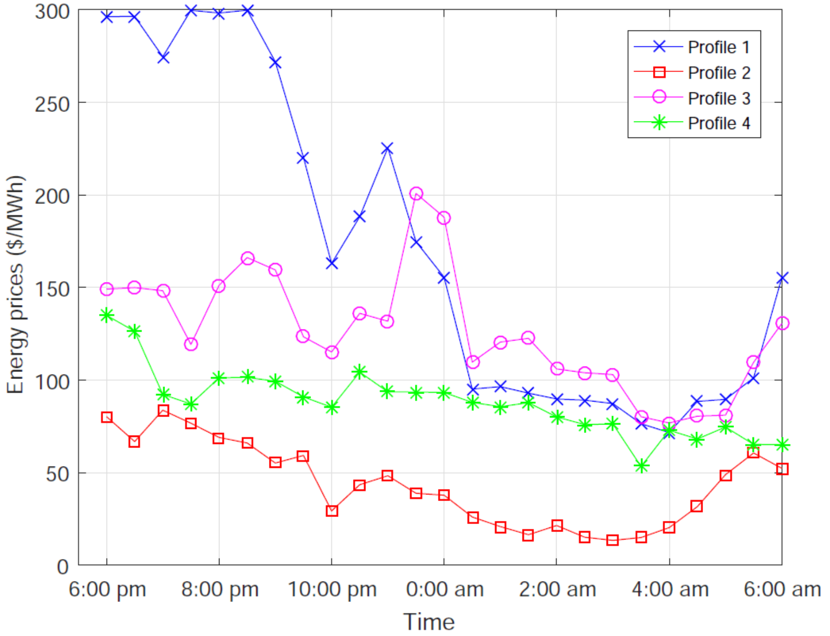
Simulation



PEV distribution

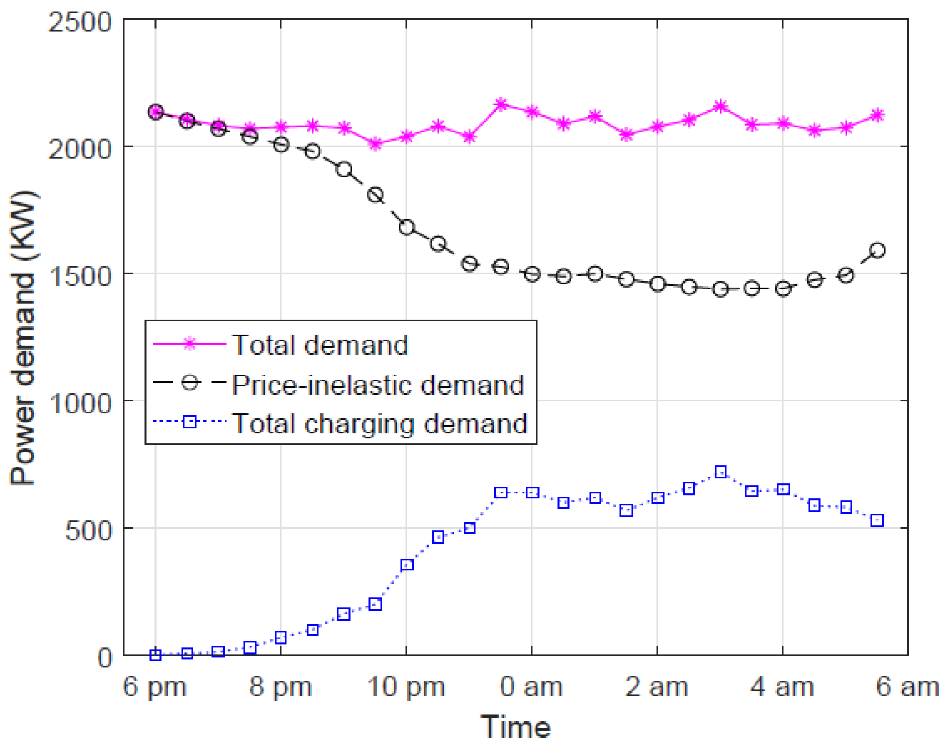


Residential load demands

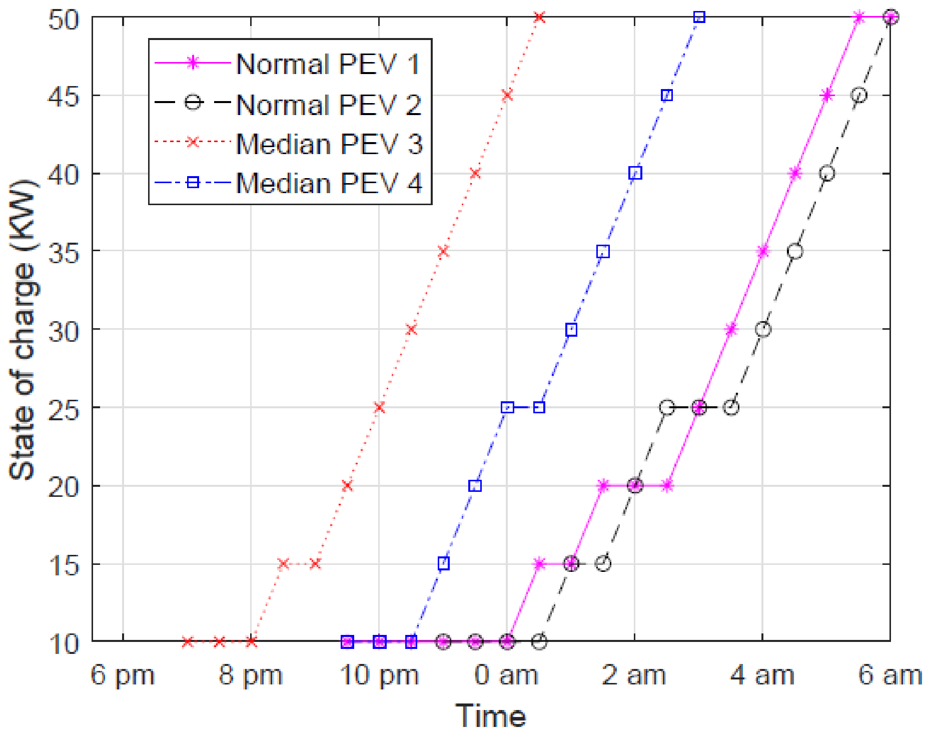


Energy Price

Simulation



Total power demand, charging power demand and real price-inelastic demand under Profile 1 during the serving time period.

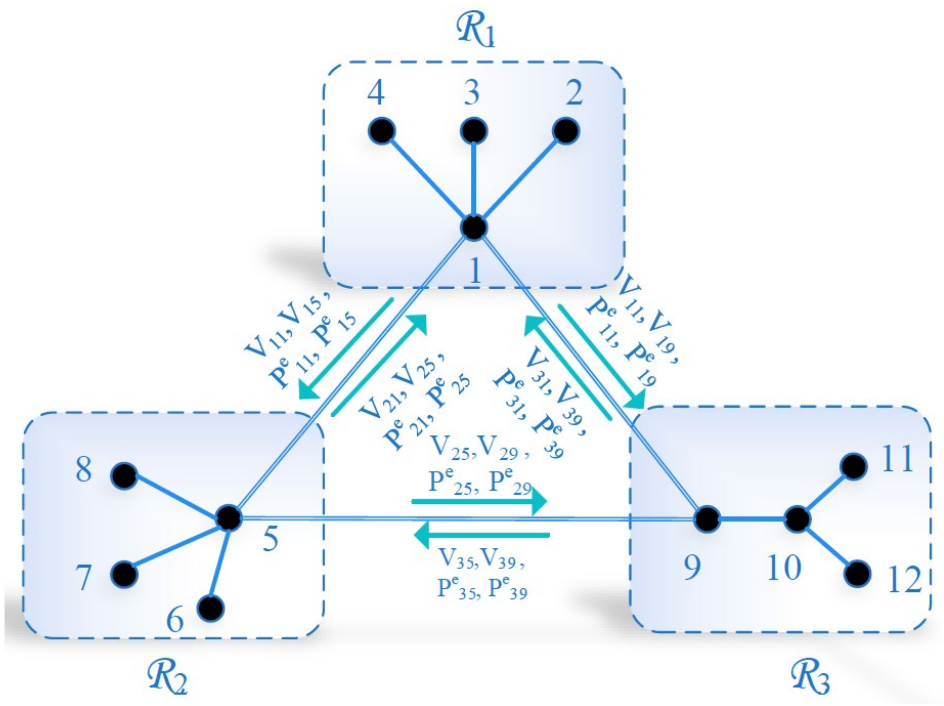


The Sate of charge of PEVs under Profile 1 during the serving time period.

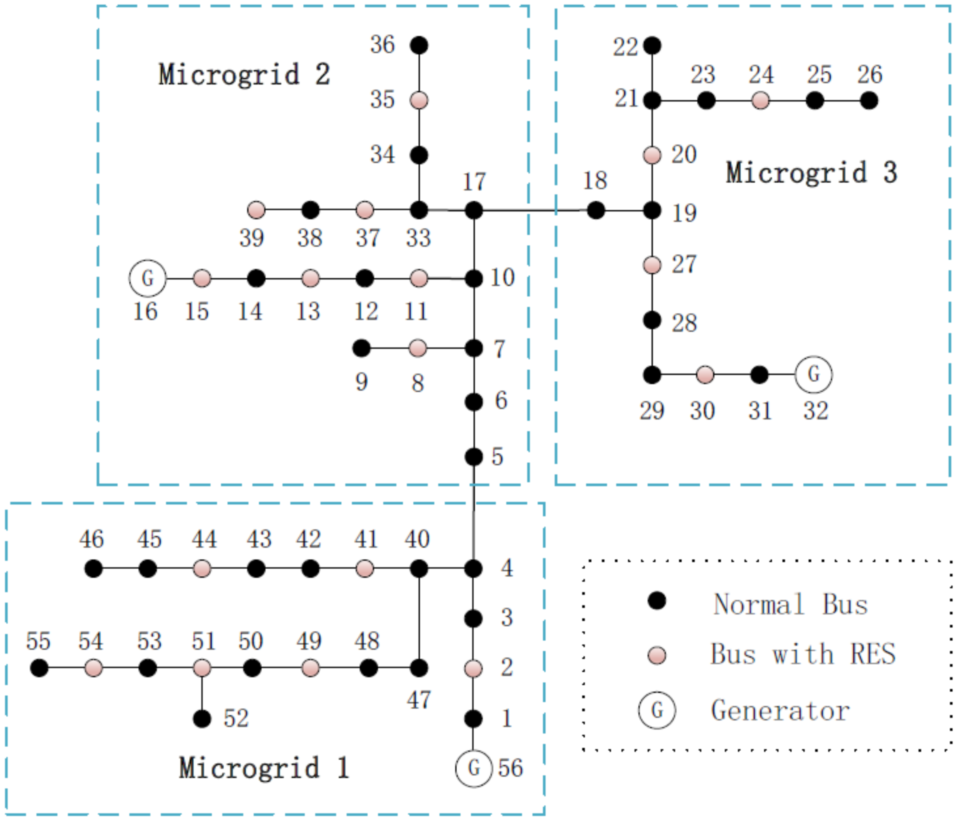
Distributed MPC for joint coordination of OPF and DR

- **Motivations**
  - The system is prone to disruption once the centralized controller is in outage.
  - Distributed controller is capable of improving the system robustness and reliability even when contingency occurs.
  - The computational and communication cost increases dramatically with the increase of system size since the centralized controller has to exchange the information with all the participants.
  - Importantly, unlike the centralized controller, which relies on gathering users' private data such as load profiles, the distributed controller can alleviate the potential concerns of privacy and security problems.

Distributed Networks



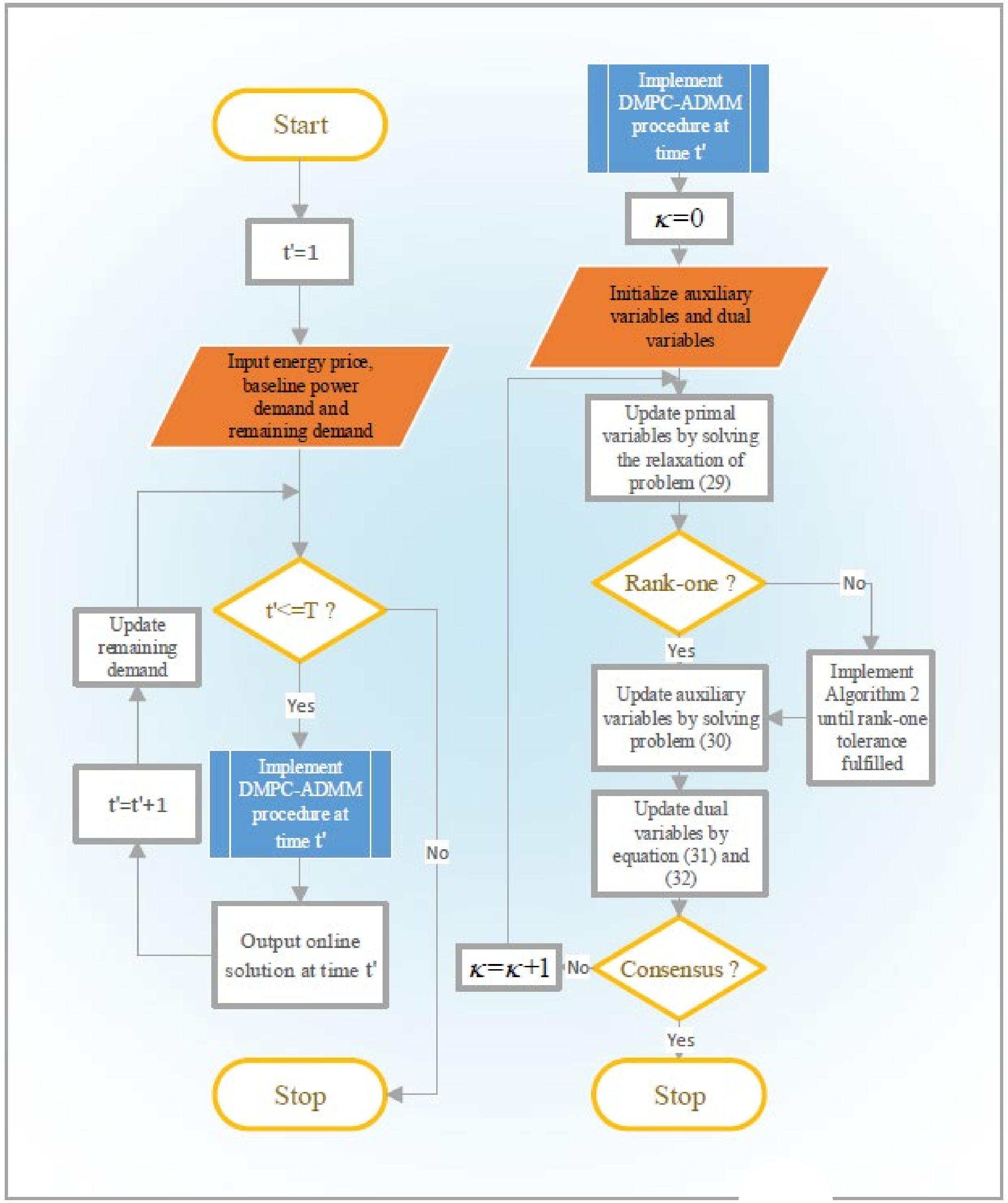
Information exchange between subsystems.



Three subsystems in a simplified IEEE-123 test feeder.

ADMM-based DMPC

- The figure illustrates the proposed DMPC. It is worth noting that we do not just apply a basic ADMM to solve the DMPC. Instead, we develop a new ADMM-based DMPC algorithm for the distributed solution of this challenge DR-OPF coordination problem.



ADMM-based DMPC

Algorithm 1 ADMM-based DMPC (19)

1) Initialization: Set $\kappa = 0$ and initialize $\mathbf{W}_{ji}^{(\kappa)}(t)$, $(\mathbf{P}_{ji}^{e}(t))^{(\kappa)}$, $\Gamma_{ij}^{(\kappa)}(t)$ and $\gamma_{ij}^{(\kappa)}(t)$ for all $i \in \mathcal{K}$.

2) Update the primal variables: For each region i, input $\mathbf{W}_{ji}^{(\kappa)}(t)$, $(\mathbf{P}_{ji}^{e}(t))^{(\kappa)}$, $\Gamma_{ij}^{(\kappa)}(t)$ and $\gamma_{ij}^{(\kappa)}(t)$ and update $\mathbf{W}_{i}^{(\kappa+1)}(t)$, $\mathcal{R}_{i}^{(\kappa+1)}(t)$ and $(\mathbf{P}_{i}^{e}(t))^{(\kappa+1)}$ concurrently by solving the sub-problem:

$$\min F^{i}_{[t',T]}(\{\mathbf{W}_i(t)\}, \{(\mathbf{P}_i^e(t))\}, \{\mathbf{W}_{ji}^{(\kappa)}(t)\},$$
$$\{\Gamma_{ij}^{(\kappa)}(t)\}, \{(\mathbf{P}_{ji}^{e}(t))^{(\kappa)}(t)\}, \{\gamma_{ij}^{(\kappa)}(t)\}) \text{ s.t. } (21b), (21c). \quad (22)$$

3) Update the auxiliary variables: Update $\mathbf{W}_{ji}^{(\kappa+1)}(t)$ and $(\mathbf{P}_{ji}^{e}(t))^{(\kappa+1)}$ by solving the unconstrained problem:

$$\min \sum_{i=1}^{K} (\mathrm{Trace}(\Gamma_{ij}^{H}(t') \otimes (\mathbf{W}_{ij}^{(\kappa+1)}(t') - \mathbf{W}_{ji}(t'))) +$$
$$\gamma_{ij}^{H}(t')((\mathbf{P}_{ij}^{e}(t'))^{(\kappa+1)} - \mathbf{P}_{ji}^{e}(t')) + \frac{\delta}{2}||\mathbf{W}_{ij}^{(\kappa+1)}(t') -$$
$$\mathbf{W}_{ji}(t')||^2 + \frac{\delta}{2}||(\mathbf{P}_{ij}^{e}(t'))^{(\kappa+1)} - \mathbf{P}_{ji}^{e}(t')||^2). \quad (23)$$

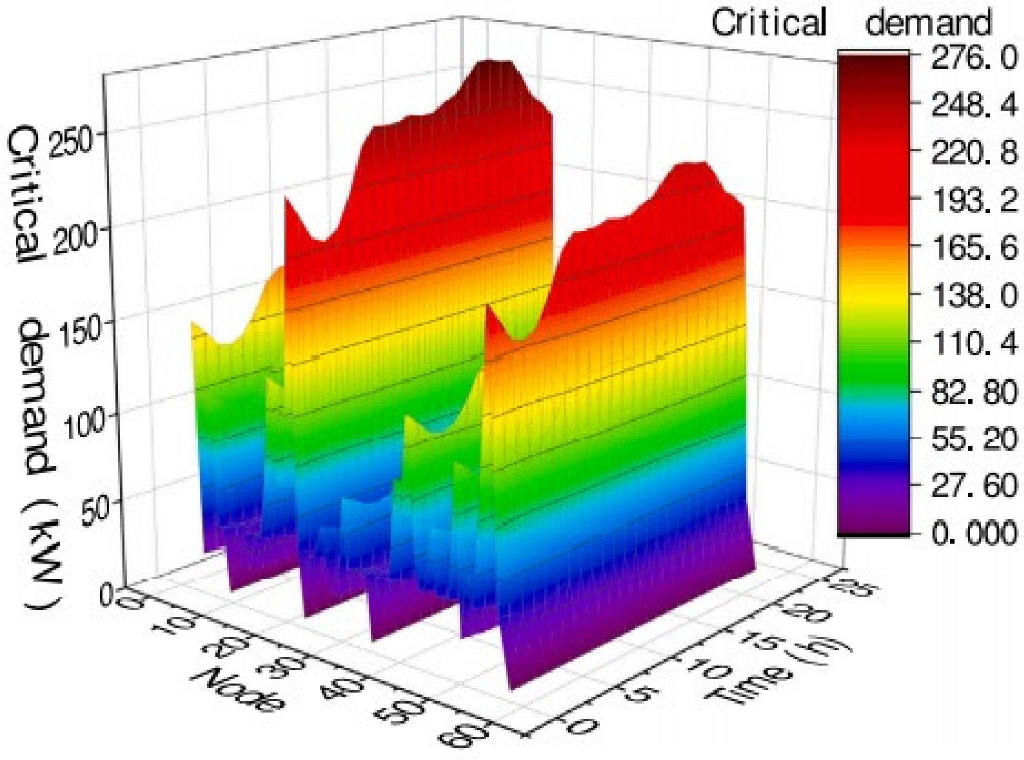
4) Update the dual variables: For each region i, update $\gamma_{il}^{(\kappa+1)}$ by the following procedure,

$$\Gamma_{ij}^{(\kappa+1)}(t') = \Gamma_{ij}^{(\kappa)}(t') + \delta(\mathbf{W}_{ij}^{(\kappa+1)}(t') - \mathbf{W}_{ji}^{(\kappa+1)}(t')),$$
$$\gamma_{ij}^{(\kappa+1)}(t') = \gamma_{ij}^{(\kappa)}(t') + \delta((\mathbf{P}_{ij}^{e}(t'))^{(\kappa+1)} - (\mathbf{P}_{ji}^{e}(t'))^{(\kappa+1)}).$$

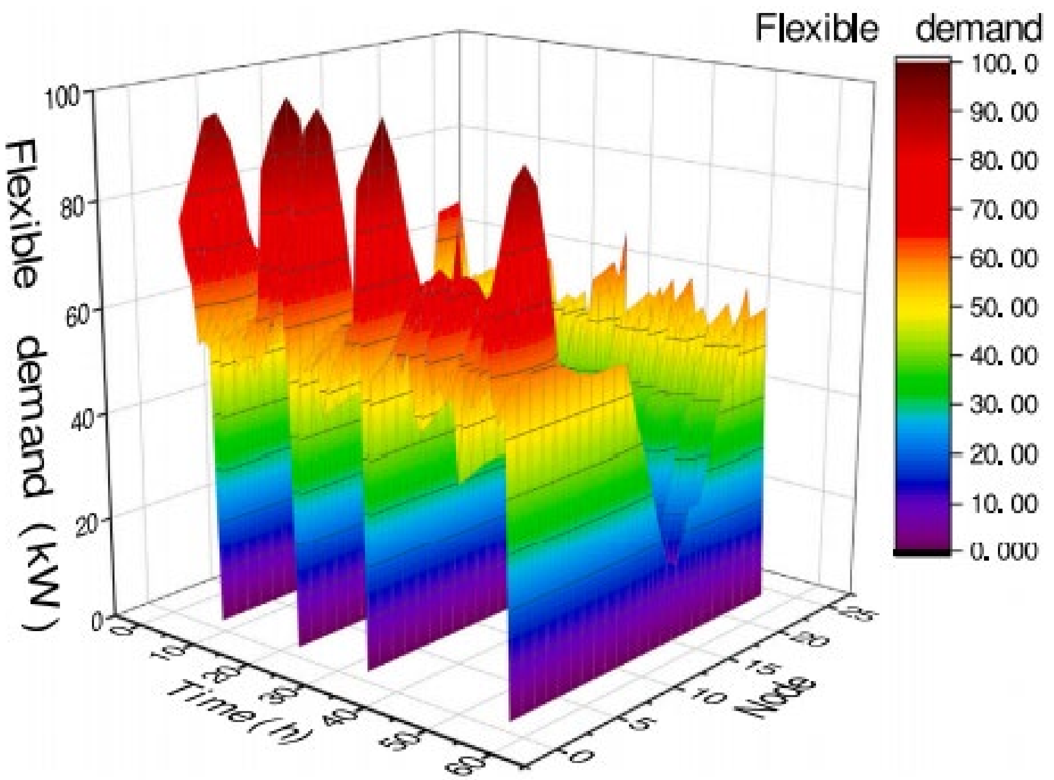
5) Stopping criterion: If $||\mathbf{W}_{ij}^{(\kappa+1)}(t') - \mathbf{W}_{ji}^{(\kappa+1)}(t')|| \leq \epsilon$ and $||(\mathbf{P}_{ij}^{e}(t'))^{(\kappa+1)} - (\mathbf{P}_{ji}^{e}(t'))^{(\kappa+1)}|| \leq \epsilon$, stop the algorithm and output $\mathbf{W}_{i}^{(\kappa+1)}(t')$, $\mathcal{R}_{i}^{(\kappa+1)}(t')$ and $(\mathbf{P}_{i}^{e}(t'))^{(\kappa+1)}$ as the solution of (19); otherwise $\kappa = \kappa + 1$, go to step 2.

The algorithm involves an iterative subroutine computation during the update procedure of primal variables in order to cope with the difficult nonconvex matrix rank-one constraint.

Simulation results



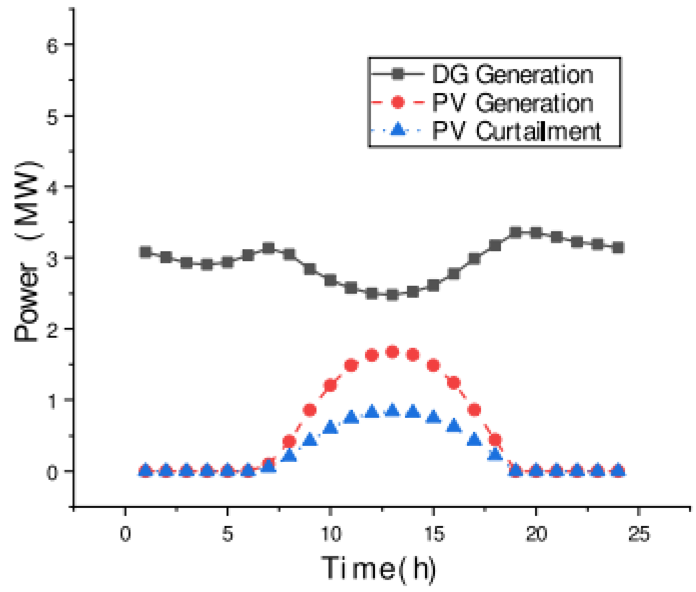
Critical demand



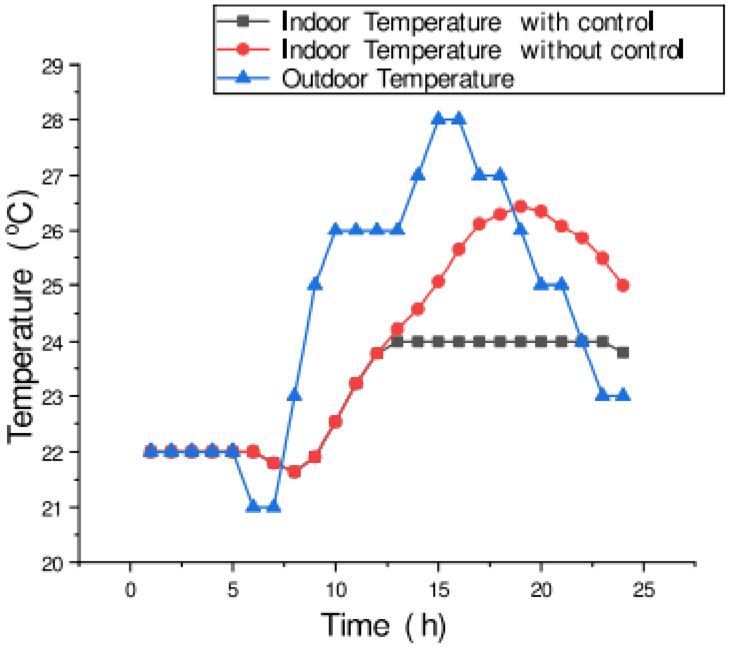
Flexible demand

The obtained flexible demand intends to fill in the valley of critical demand.

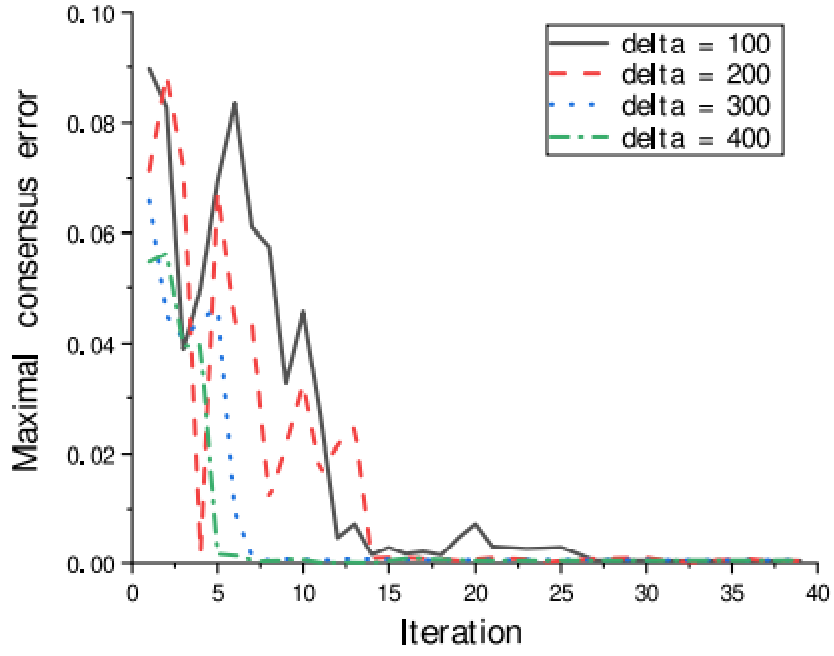
Simulation results



Power generation by DG and PV and power curtailment of PV



Indoor temperature with and without control of the thermal demand



The convergence performance

Reference

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[4] Ye Shi*, Hoang Duong Tuan, Andrey V. Savkin, Trung Q. Duong and H. Vincent Poor, "Model predictive control for on-off Charging of Electrical Vehicles in Smart Grids", accepted by IET ElectricalSystems in Transportation, 2020.

[5] Ye Shi*, Hoang Duong Tuan, Andrey V. Savkin, Chin-Teng Lin, Jian Guo Zhu, H. Vincent Poor, ”Distributed model predictive control for joint coordination of demand response and optimal power flow with renewables in smart grid”, Applied Energy, 290, 2021.

Part III: PMU placement

- Mixed-integer convex programming (MICP)

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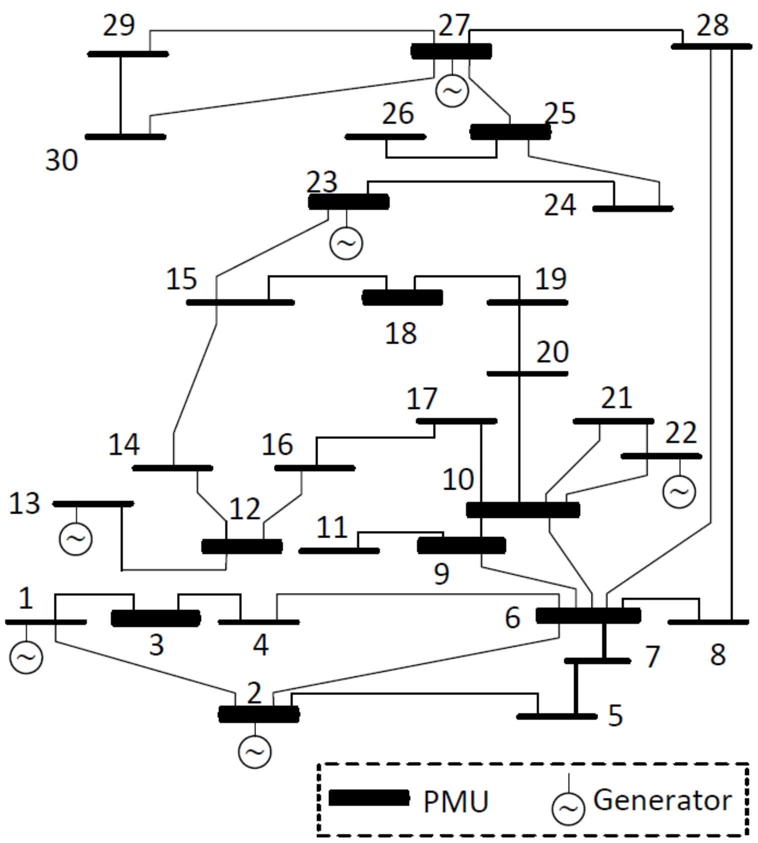
PMU placement

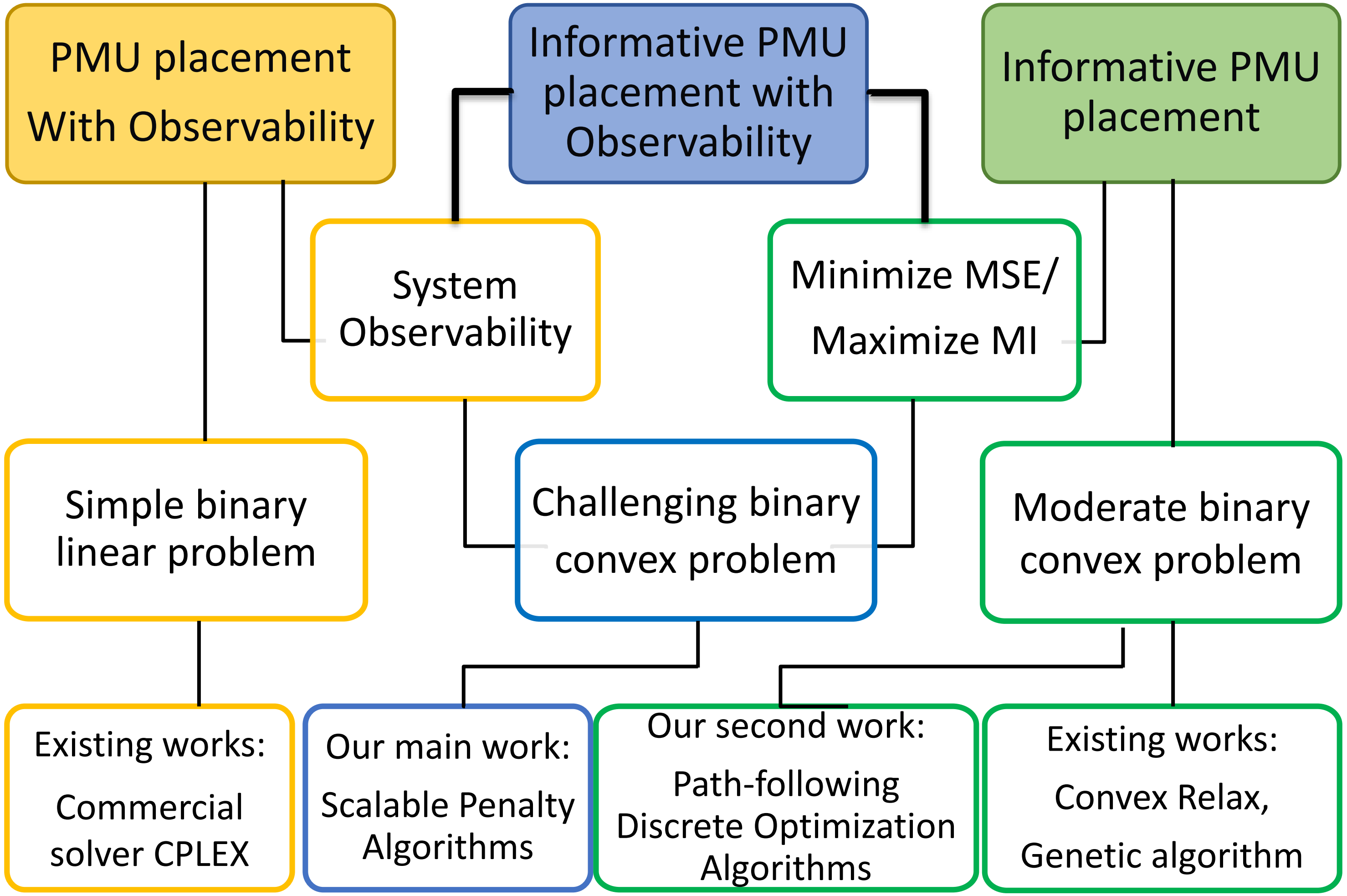
- ◆Phasor measurement unit (PMU) is an advanced digital meter, which is used in smart power grids for real-time monitoring of grid operations.

- ◆PMU placement
  - ➢**System Observability**
  - ➢**Informative state estimation**



IEEE 30-bus power network with PMUs

PMU placement



PMU placement model

◆ Select S PMUs

$$\mathcal{D}_S := \{\boldsymbol{x} \in \{0,1\}^N \ : \ \sum_{k=1}^{N} x_k = S\},$$

to guarantee

➢ **System observability** $\mathcal{A}\boldsymbol{x} \geq 1_N,$

➢ **Minimize Mean square error**

$$f_e(\boldsymbol{x}) \quad = \quad \mathrm{Trace}\left(\left(B^T \Sigma_P^{-1} B + \sum_{k=1}^{N} x_k H_k^T \mathcal{R}_{w_k}^{-1} H_k\right)^{-1}\right),$$

➢ **Maximize Mutual information**

$$\begin{aligned} f_{MI}(\boldsymbol{x}) : &= -\ln|\mathcal{R}_e(\boldsymbol{x})| \\ &= \ln|B^T \Sigma_P^{-1} B + \sum_{k=1}^{N} x_k H_k^T R_{w_k}^{-1} H_k|, \end{aligned}$$

Existing models of PMU placement

- PMU placement with Observability

$$\min_{\boldsymbol{x}} \sum_{k=1}^{N} x_k \ : \ \boldsymbol{x} \in \{0,1\}^N \ \mathcal{A}\boldsymbol{x} \geq 1_N,$$

- Informative PMU placement:
  - **Convex relaxation**
  - **Greedy algorithm**

In optimizing the so called gain matrix [14], the work [15] considered the simple binary convex problem

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{s.t.} \quad \boldsymbol{x} \in \mathcal{D}_S, \tag{25}$$

by solving its convex relaxation problem, which is

$$\min_{\boldsymbol{x}\in\mathbb{R}_+^N, \mathbf{T}\in\mathbb{R}^{N\times N}} \ \text{Trace}(\mathbf{T}) \quad \text{s.t.} \quad \boldsymbol{x} \in \mathsf{Poly}(\mathcal{D}_S), \tag{26a}$$

$$\begin{bmatrix} B^T \Sigma_P^{-1} B + \sum_{k=1}^{N} x_k H_k^T R_{w_k}^{-1} H_k & I_N \\ I_N & \mathbf{T} \end{bmatrix} \succeq 0, \tag{26b}$$

when $f = f_e$, or

$$\max_{\boldsymbol{x}\in\mathbb{R}_+^N} \ \ln |B^T \Sigma_P^{-1} B + \sum_{k=1}^{N} x_k H_k^T R_{w_k}^{-1} H_k|$$
$$\text{s.t.} \quad \boldsymbol{x} \in \mathsf{Poly}(\mathcal{D}_S), \tag{27}$$

when $f = f_{MI}$, for

$$\mathsf{Poly}(\mathcal{D}_S) = \{\boldsymbol{x} \in [0,1]^N : \sum_{k=1}^{N} x_k = S\}, \tag{28}$$

Scale Penalty Algorithms for optimization placement

- Based on our previous work, we transform the nonconvex and discrete binary constraint to continuous linear constraint.
- Propose a scalable algorithm to handle the f(x). Although the function f(x) is already convex, it is not easy to optimize it. For instance, when f = fe, usually fe is expressed by Trace(T), where T is a slack symmetric matrix variable of size N X N satisfying the semi-definite constraint (26b), which is not scalable to x.

$$\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{c}} f(\boldsymbol{x}) + \mu P^{(\kappa)}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{c}), \quad \text{s.t. } (34b), (35), (36), (37), \ (38)$$

with

$$P^{(\kappa)}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{c}) := \left(\frac{1}{g_1^{(\kappa)}(\boldsymbol{x})} - \frac{1}{S}\right) + \left(\frac{1}{g_2^{(\kappa)}(\boldsymbol{y})} - \frac{1}{|\mathcal{Z}|}\right) + \left(\frac{1}{g_3^{(\kappa)}(\boldsymbol{c})} - \frac{1}{N-|\mathcal{Z}|}\right).$$

$$f_e(\boldsymbol{x}) \le f_e^{(\kappa)}(\boldsymbol{x}) := a_0^{(\kappa)} + \sum_{k=1}^{N} \frac{a_k^{(\kappa)}}{x_k + \epsilon} \text{ for}$$

$$0 < a_0^{(\kappa)} := \text{Trace}((\mathcal{R}_e(\boldsymbol{x}^{(\kappa)}))^2 \mathcal{A}_\epsilon),$$

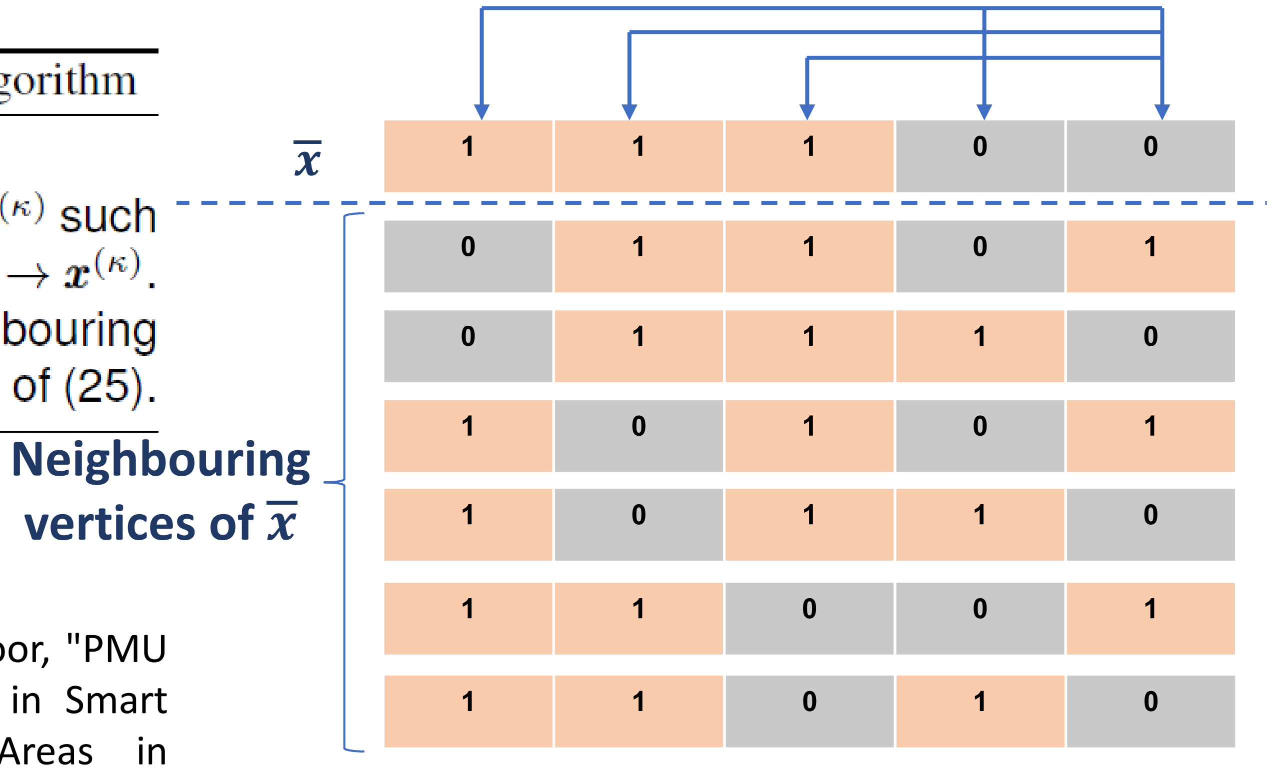
$$0 < a_k^{(\kappa)} := (x_k^{(\kappa)} + \epsilon)^2 \text{Trace}((\mathcal{R}_e(\boldsymbol{x}^{(\kappa)}))^2 \times H_k^T R_{w_k}^{-1} H_k), k = 1, \ldots, N.$$

Tailed path-following discrete optimization algorithms

Recall that point $\boldsymbol{x}$ is a vertex neighbouring the vertex $\bar{\boldsymbol{x}}$ if and only if there exists a pair (i, j) such that $x_i = 0 \neq \bar{x}_i = 1$ and $x_j = 1 \neq \bar{x}_j = 0$ and $x_\ell = \bar{x}_\ell$ whenever $\ell \neq i$ and $\ell \neq j$, i.e. $\bar{\boldsymbol{x}}$ and $\boldsymbol{x}$ are exactly different in two entries and there are $S(N-S)$ neighbouring vertices for each vertex $\bar{\boldsymbol{x}}$.

Algorithm 2 Path-following discrete optimization algorithm

Initialization. Start from a $\boldsymbol{x}^{(0)} \in \mathcal{D}_S$. Set $\kappa = 0$.
κ-th iteration. If there is a $\bar{\boldsymbol{x}} \in \mathcal{D}_S$ neighbouring $\boldsymbol{x}^{(\kappa)}$ such that $f(\bar{\boldsymbol{x}}) < f(\boldsymbol{x}^{(\kappa)})$ then reset $\kappa + 1 \to \kappa$ and $\bar{\boldsymbol{x}} \to \boldsymbol{x}^{(\kappa)}$. Otherwise, if $f(\boldsymbol{x}) \geq f(\boldsymbol{x}^{(\kappa)})$ for all $\boldsymbol{x} \in \mathcal{D}_S$ neighbouring $\boldsymbol{x}^{(\kappa)}$ then stop: $\boldsymbol{x}^{(\kappa)}$ is the global optimal solution of (25).



◆Y. Shi, H. D. Tuan, A. A. Nasir, T. Q. Duong, and H. V. Poor, "PMU Placement Optimization for E_cient State Estimation in Smart Grid" accepted by IEEE Journal on Selected Areas in Communications, 2019. (Impact factor **9.302**)

Reference

[6] Ye Shi, Hoang Duong Tuan*, Trung Q. Duong, H. Vincent Poor and Andrey V. Savkin, ”PMU Placement Optimization for Efficient State Estimation in Smart Grid”, IEEE Journal on Selected Areas in Communications, vol. 38, no. 1, pp. 71-83, 2020.

Thank you!

Questions are welcome!