Homework 1

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Problem i. Write the gradient and Heissan matrix of the following formula. [10pts]

$$\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} + \mathbf{b}^{\mathrm{T}}\mathbf{x} + \mathrm{c} \quad (\mathbf{A} \in \mathbf{R}^{\mathbf{n*n}}, \mathbf{b} \in \mathbf{R}^{\mathbf{n}}, \mathrm{c} \in \mathbf{R})$$

Let $f(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} + \mathbf{b}^{\mathrm{T}}\mathbf{x} + \mathrm{c}$

1. the gradient of the formula is

$\nabla f = \dfrac{\partial f}{\partial \mathbf{x}} = (\mathbf{A} + \mathbf{A}^{\mathbf{T}})\mathbf{x} + \mathbf{b}$

2. the Heissan matrix of the formula is

$\nabla^2 f = \nabla(\nabla f) = \dfrac{\partial(\nabla f)}{\partial \mathbf{x}} = \mathbf{A} + \mathbf{A}^{\mathbf{T}}$

Problem ii. Write the gradient and Heissan matrix of the following formula. [10pts]

$$\|\mathbf{Ax}-\mathbf{b}\|_2^2 \quad (\mathbf{A}\in\mathbf{R^{m*n}}, \mathbf{b}\in\mathbf{R^m})$$

Let $f(\mathbf{x}) = \|\mathbf{Ax}-\mathbf{b}\|_2^2 = (\mathbf{Ax}-\mathbf{b})^T(\mathbf{Ax}-\mathbf{b}) = \mathbf{x^TA^TAx}-\mathbf{2x^TA^Tb}+\mathbf{b^Tb}$

1. the gradient of the formula is

$\nabla f = \dfrac{\partial f}{\partial \mathbf{x}} = \mathbf{2A^TAx}-\mathbf{2A^Tb}$

2. the Heissan matrix of the formula is

$\nabla^2 f = \nabla(\nabla f) = \dfrac{\partial(\nabla f)}{\partial \mathbf{x}} = \mathbf{2A^TA}$

Problem iii. Convert the following problem to linear programming. [10pts]

$$\min_{\mathbf{x}\in\mathbf{R^n}} \|\mathbf{Ax}-\mathbf{b}\|_1 + \|\mathbf{x}\|_\infty \quad (\mathbf{A}\in\mathbf{R^{m*n}}, \mathbf{b}\in\mathbf{R^m})$$

$$\begin{array}{ll} \underset{x\in\mathbb{R}^n}{\text{minimize}} & f(x) \\ \text{subject to} & c_i(x) = 0, \quad i\in\mathcal{E} \\ & c_i(x) \leq 0, \quad i\in\mathcal{I} \end{array}$$

Problem vi. Proof the convergence rates of the following point sequences. [30pts]

$$\mathbf{x}^{\mathrm{k}} = \frac{1}{\mathrm{k}}$$

$$\mathbf{x}^{\mathrm{k}} = \frac{1}{\mathrm{k}!}$$

$$\mathbf{x}^{\mathrm{k}} = \frac{1}{2^{2^{\mathrm{k}}}}$$

(Hint: Given two iterates $\mathbf{x}^{\mathrm{k}+1}$ and $\mathbf{x}^{\mathrm{k}}$, and its limit point $\mathbf{x}^*$, there exists real number $\mathrm{q} > 0$, satisfies

$$\lim_{\mathrm{k}\to\infty} \frac{\left\|\mathbf{x}^{\mathrm{k}+1} - \mathbf{x}^*\right\|}{\|\mathbf{x}^{\mathrm{k}} - \mathbf{x}^*\|} = \mathrm{q}$$

if $0 < \mathrm{q} < 1$, then the point sequence Q-linear convergence; if $\mathrm{q} = 1$, then the point sequence Q-sublinear convergence; if $\mathrm{q} = 0$, then the point sequence Q-superlinear convergence)

Problem v. Select the Haverly Pool Problem or the Horse Racing Problem in the courseware, compile the program using AMPL model language and submit it to https://neos-server.org/neos/solvers/index.html.(Hint: both AMPL solver and NEOS solver can be used, please indicate the type of solver used in the submitted job, show the solution results (eg: screenshots attached to the PDF file), and submit the source code together with the submitted job, please package as .zip file, including your PDF and source code.) [40pts]