

Numerical Optimization, 2023 Fall

Homework 2

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1 Standard Form

Convert the following problem to a linear program in standard form. [20pts]

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}^4} \quad & 2x_1 - x_3 + x_4 \\ \text{s.t.} \quad & x_1 + x_2 \geq 5 \\ & x_1 - x_3 \leq 2 \\ & 4x_2 + 3x_3 - x_4 \leq 10 \\ & x_1 \geq 0 \end{aligned} \tag{1}$$

2 Two-Phase Simplex

Use the two-phase simplex procedure to solve the following problem. [40pts]

$$\begin{array}{ll} \min_{\mathbf{x} \in \mathbb{R}^4} & -3x_1 + x_2 + 3x_3 - x_4 \\ \text{s.t.} & x_1 + 2x_2 - x_3 + x_4 = 0 \\ & 2x_1 - 2x_2 + 3x_3 + 3x_4 = 9 \\ & x_1 - x_2 + 2x_3 - x_4 = 6 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array} \tag{2}$$

3 Extreme Point

3.1 Q1

Prove that the extreme points of the following two sets are in one-to-one correspondence. [20pts]

$$\begin{aligned} S_1 &= \{\mathbf{x} \in \mathbb{R}^n : \mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq 0\} \\ S_2 &= \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^n \times \mathbb{R}^m : \mathbf{Ax} + \mathbf{y} = \mathbf{b}, \mathbf{x} \geq 0, \mathbf{y} \geq 0\} \end{aligned} \tag{3}$$

, where $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$.

3.2 Q2

Does the set $P = \{\mathbf{x} \in \mathbb{R}^2 : 0 \leq x_1 \leq 1\}$ have extreme points? What is its standard form? Does it have extreme points in its standard form? If so, give a extreme point and explain why it is a extreme point.
[20pts]