Numerical Optimization

Lecture 2: Linear Programming

王浩

信息科学与技术学院

Email: wanghao1@shanghaitech.edu.cn

1、Examples

Linear Programming

Objective: linear
Constraint: linear equalities or/and inequalities
Variables: real value

$$
\begin{array}{ll}
\text{minimize} & 3x_1 + x_2 \\
\text{subject to} & x_1 + 2x_2 \geq 2 \\
& 2x_1 + x_2 \geq 3 \\
& x_1 \geq 0, x_2 \geq 0
\end{array}
\qquad
\begin{array}{ll}
\text{minimize} & c^T x \\
\text{subject to} & Ax \geq b \\
& x \geq 0
\end{array}
$$

$$
\mathbf{c} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}
$$

Notice: vectors are all column vectors and inequalities are component wise

Example 1. The Diet Problem

- The first studies applying LP to diets were published between 1950 and 1960
- The search for diet solutions started with Jerry Cornfield, who formulated "The Diet Problem" for the Army during World War II (1941–1945), in search of a low-cost diet that would meet the nutritional needs of a soldier.
- The economist George Stigler (1982 Nobel Prize), endeavored optimization techniques to establish the cheapest diet delivering enough energy, proteins, vitamins, and minerals. According to Buttriss et al., this diet should be composed by the available list of 77 US foods of which the costs and nutrient composition were measured

George Joseph Stigler (1911 – 1991)

Example 1. The Diet Problem

问题：确定食品数量，满足士兵营养需求，花费最小?

n种食品，m种营养成份；c_j为第 j 种食品的单价, $j = 1,\dots,n$;

b_i 为了健康，士兵每天必须食用第 i 种营养的数量, $i = 1,\dots,n$;

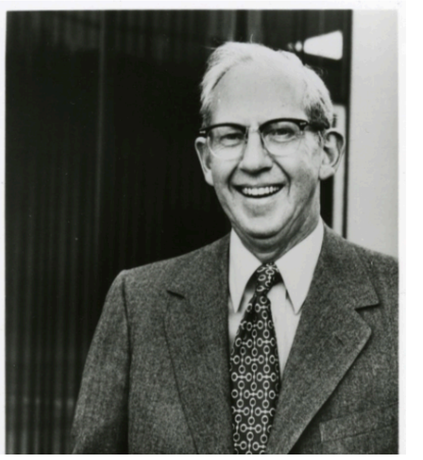
a_{ij}为每单位第 j 种食品所含第 i 种营养的数量.

变量：x_j – 第 j 种食品的数量

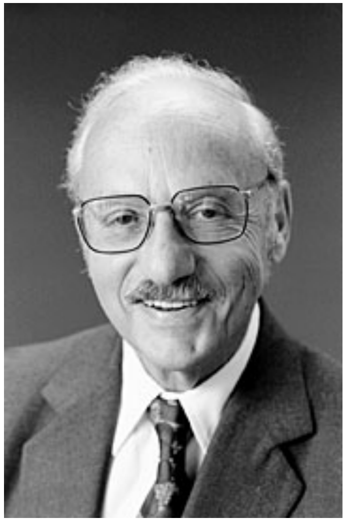
建模：

$$
\begin{array}{ll}
\text{minimize} & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
\text{subject to} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq b_2 \\
& \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq b_m \\
& x_1 \geq 0, x_2 \geq 0, \cdots, x_n \geq 0
\end{array}
$$

- For the duration of World War II, the Air Force and other parts of the army were hiring mathematicians to solve the important diet problem and to plan affordable meals. Among the researchers involved in solving this problem was George Dantzig.
- George Dantzig proposed a new algorithm he had developed. It took him until 1947, being the first to deliver the correct mathematical result

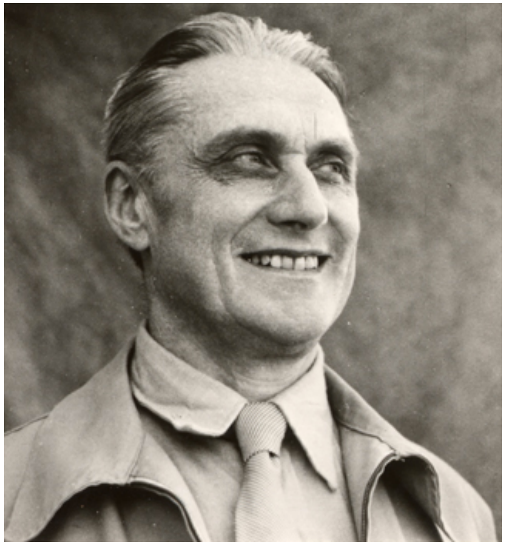


George Joseph Stigler (1911 – 1991)
1982年诺贝尔经济学奖获得者之一
运筹学贡献：线性规划



George Bernard Dantzig (1914 - 2005)

2020年诺贝尔经济学奖获得者之一
Robert B.Wilson（1937- ）
运筹学贡献：非线性规划SQP

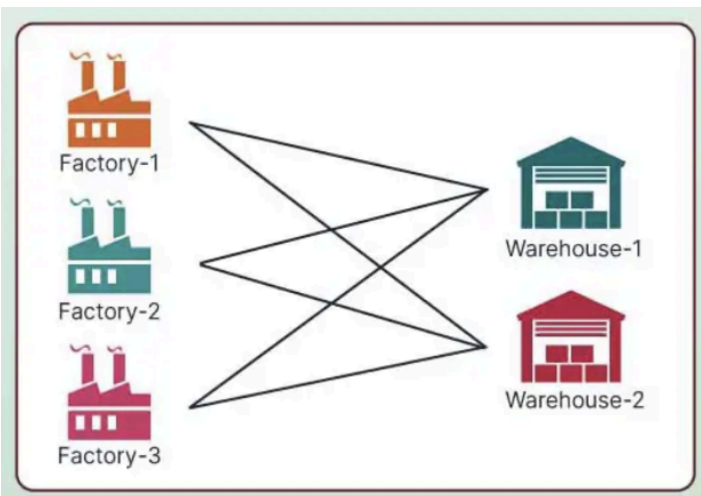


Ragnar Anton Kittil Frisch (1895 – 1973)
1969年诺贝尔经济学奖获得者之一
运筹学贡献：线性非线性规划内点法

Example 2. transportation problem

问题：确定从仓库(warehouse)到厂家(factory)运输某产品的量。每个仓库的货物量为$a_1, \ldots, a_m$; 每个厂家货物需求量为$b_1, \ldots, b_n$. 假设x_{ij}为仓库i到厂家j的运输量，运输单价为c_{ij}。现在选择运输方案，使得总运费达到最小，并且满足运输需求。

建模：



j i

$$\min \sum_{ij} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = a_i, i = 1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,\ldots,n$$

$$x_{ij} \geq 0, i = 1,\ldots,m; j = 1,\ldots,n$$

Example 3. basis pursuit (基追踪)

问题：基追踪问题是寻找满足线性系统的稀疏解，一般可以写为：

$$\begin{aligned} \text{min} \quad & \|x\|_1 \\ \text{s.t.} \quad & Ax = b \end{aligned}$$

建模：

$$\begin{aligned} \text{min} \quad & \sum_{i=1}^{n} z_i \\ \text{s.t.} \quad & Ax \geq b \\ & x_i \leq z_i, i = 1,\ldots,n \\ & -x_i \leq z_i, i = 1,\ldots,n \end{aligned}$$

$$\begin{aligned} \text{min} \quad & \sum_{i=1}^{n} (x_i^+ + x_i^-) \\ \text{s.t.} \quad & Ax^+ - Ax^- = b \\ & x^+, x^- \geq 0 \end{aligned}$$

- 其他案例: AVE (absolute value error) linear regression:

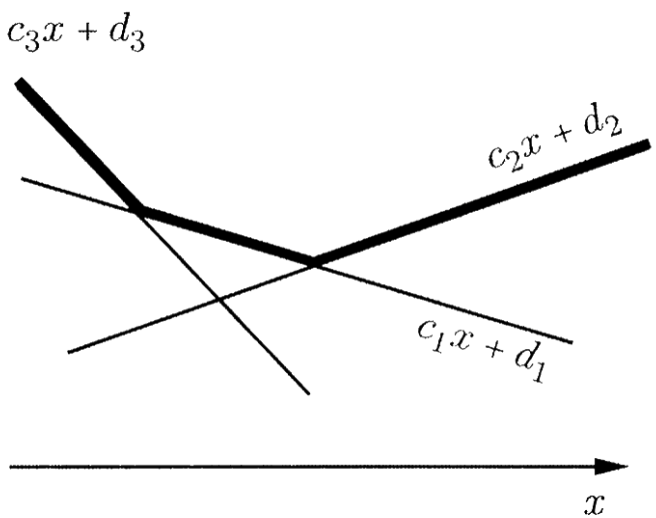
$$\min_{x \in \mathbb{R}^n} \|Ax - b\|_1, \quad \text{given } A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, m > n$$

如果绝对值在约束上呢?

- 案例: robust linear regression given $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m, m > n$:

$$\min_{x\in\mathbb{R}^n} \ \|Ax - b\|_\infty$$

- 最优化问题的等价表述指由其中一个问题的最优解可以得到另一个的最优解。



$$\begin{array}{ll} \text{minimize} & \displaystyle\max_{i=1,\ldots,m} (\mathbf{c}_i'\mathbf{x} + d_i) \\ \text{subject to} & \mathbf{A}\mathbf{x} \geq \mathbf{b} \end{array}$$

$$\Downarrow$$

$$\begin{array}{llll} \text{minimize} & z & & \\ \text{subject to} & z \geq \mathbf{c}_i'\mathbf{x} + d_i, & & i = 1, \ldots, m, \\ & \mathbf{A}\mathbf{x} \geq \mathbf{b} & & \end{array}$$

2、Geometric Properties

General formulation of LP

$$
\begin{aligned}
\min \quad & c^T x \\
\text{s.t.} \quad & a_i^T x \geq b_i, i \in M_1 \\
& a_i^T x \leq b_i, i \in M_2 \\
& a_i^T x = b_i, i \in M_2 \\
& x_j \geq 0, j \in N_1 \\
& x_j \leq 0, j \in N_2
\end{aligned}
$$

事实上这里的形式可以更简单

其中c, a_i是n维向量，b_i是实数，这些均是给定的数据; x是变量

Active, inactive

consider those constraints at x^*

$$
\begin{aligned}
\mathbf{a}_i'\mathbf{x} &\geq b_i, & i \in M_1\\
\mathbf{a}_i'\mathbf{x} &\leq b_i, & i \in M_2\\
\mathbf{a}_i'\mathbf{x} &= b_i, & i \in M_3
\end{aligned}
$$

非积极约束
inactive constraint

$$\boxed{\begin{aligned} &\mathbf{a}_i'\mathbf{x}^* > b_i, i \in M_1\\ &\mathbf{a}_i'\mathbf{x}^* < b_i, i \in M_2 \end{aligned}}$$

$$\{i \mid \mathbf{a}_i'\mathbf{x}^* > b_i, i \in M_1\\ \mathbf{a}_i'\mathbf{x}^* < b_i, i \in M_2\}$$

非积极集
inactive set

积极约束
active constraint

$$\boxed{\begin{aligned} &\mathbf{a}_i'\mathbf{x}^* = b_i, i \in M_1\\ &\mathbf{a}_i'\mathbf{x}^* = b_i, i \in M_2\\ &\mathbf{a}_i'\mathbf{x}^* = b_i, i \in M_3 \end{aligned}}$$

$$\{i \mid \mathbf{a}_i'x^* = b_i, i \in M_1\\ \mathbf{a}_i'x^* = b_i, i \in M_2\\ \mathbf{a}_i'x^* = b_i, i \in M_3\}$$

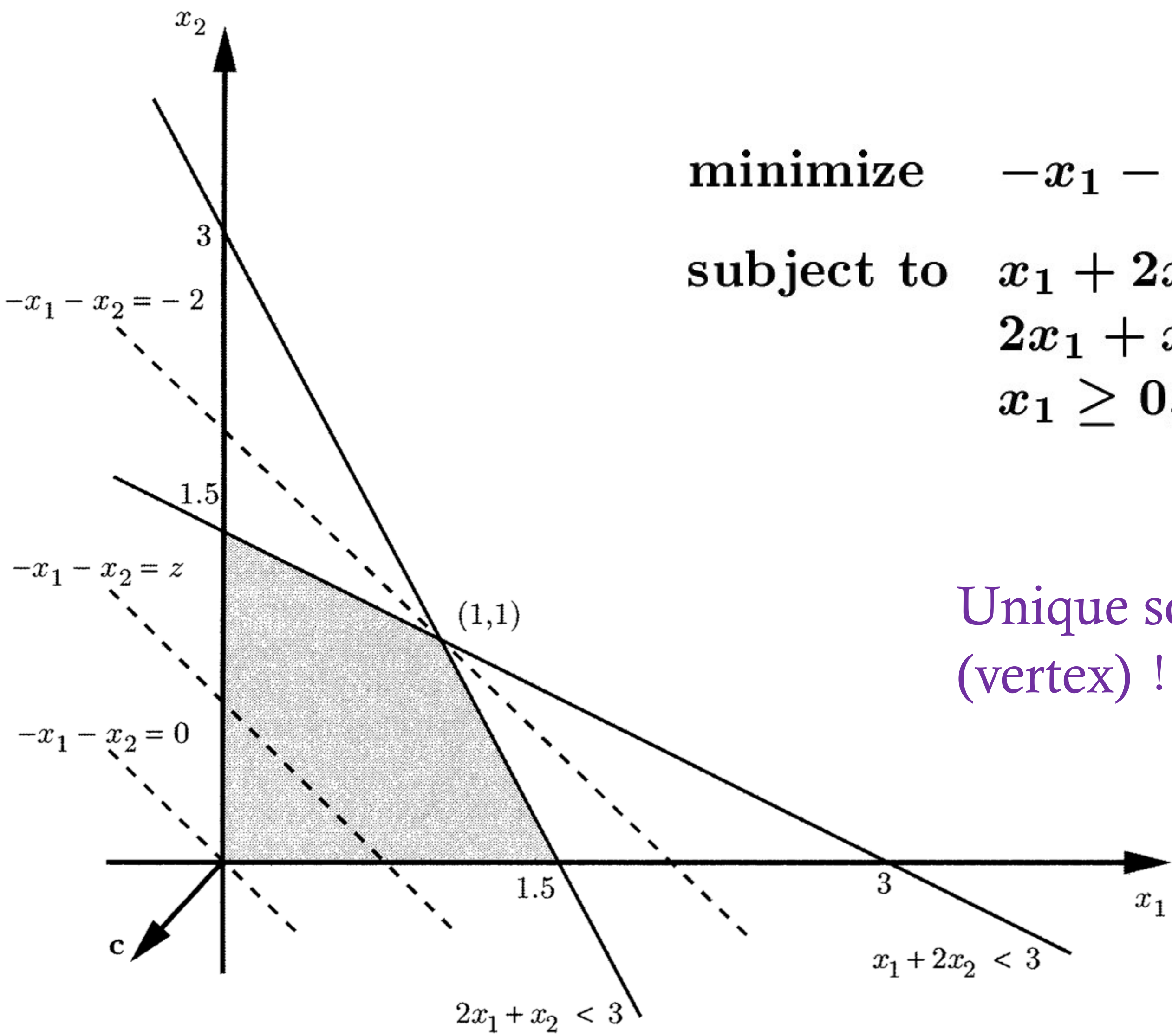
积极集
active set

infeasible/violated constraint

$$\boxed{\begin{aligned} &\mathbf{a}_i'\mathbf{x}^* < b_i, i \in M_1\\ &\mathbf{a}_i'\mathbf{x}^* > b_i, i \in M_2\\ &\mathbf{a}_i'\mathbf{x}^* \neq b_i, i \in M_3 \end{aligned}}$$

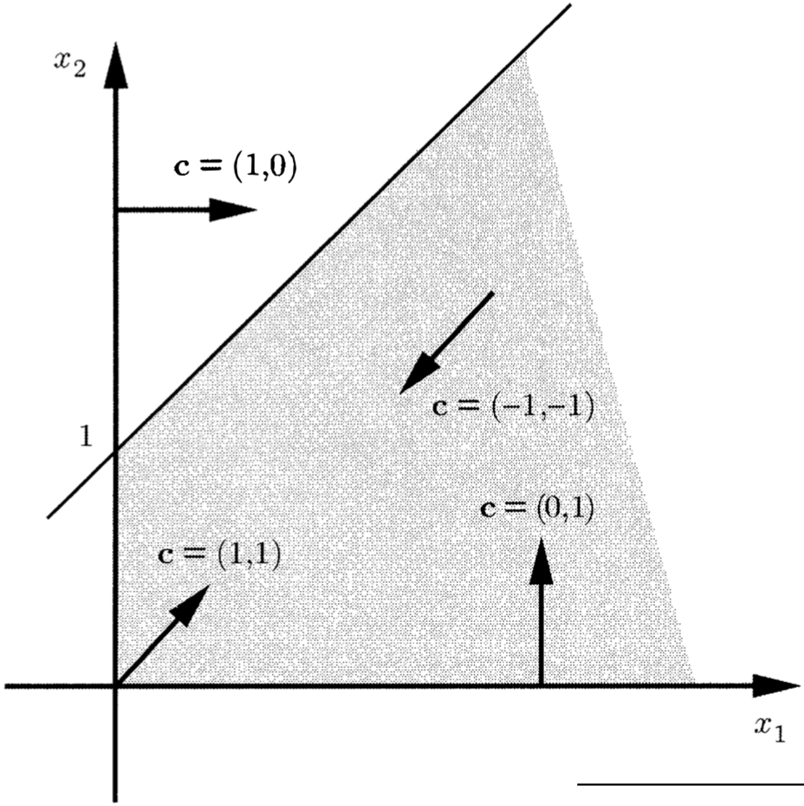
$$\{i \mid \mathbf{a}_i'\mathbf{x}^* < b_i, i \in M_1\\ \mathbf{a}_i'\mathbf{x}^* > b_i, i \in M_2\\ \mathbf{a}_i'\mathbf{x}^* \neq b_i, i \in M_3\}$$

infeasible/violated constraint set



$$
\begin{array}{ll}
\textbf{minimize} & -x_1 - x_2 \\
\textbf{subject to} & x_1 + 2x_2 \le 3 \\
 & 2x_1 + x_2 \le 3 \\
 & x_1 \ge 0, x_2 \ge 0.
\end{array}
$$

Unique solution (vertex) !



$$
\begin{aligned}
\text{minimize} \quad & c_1x_1 + c_2x_2 \\
\text{subject to} \quad & -x_1 + x_2 \le 1 \\
& x_1 \ge 0, x_2 \ge 0
\end{aligned}
$$

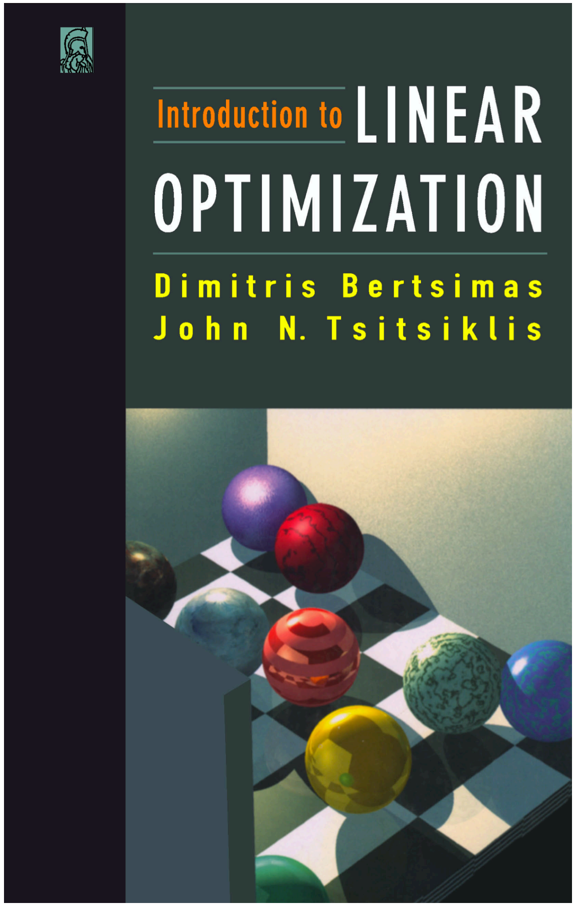
c	x^*	Solution
(1, 1)	(0, 0)	Unique Vertex
(0, 1)	$(x_1, 0), x_1 \ge 0$	An edge
(1, 0)	$(0, x_2), x_2 \in [0,1]$	An edge
$(-1,-1)$	unbounded solution	Unbounded (below)

Observations:

Feasible region：Polyhedron (多面集, 高维空间)

- <u>solution exists（有解）</u>：**unique (vertex) or infinitely many** (a edge, facet, even the entire feasible region)$\Longrightarrow$必然有<u>顶点解</u>
- unbounded（无界）：unbounded optimal solution（极小化时表示无下界，极大化时无上界）$\Longrightarrow$ doesn't have an optimal solution <u>无(最优)解</u>
- infeasbile (不可行)：doesn't have feasible solution $\Longrightarrow$<u>无(最优)解</u>

geometric observations $\Longrightarrow$ algebraic characterization & proof $\Longrightarrow$ algorithm design



Introduction to Linear Optimization (Athena Scientific Series in Optimization and Neural Computation, 6)

多面体(Polyhedron)

Definition 2.1 *A* **polyhedron** *is a set that can be described in the form* $\{\mathbf{x} \in \Re^n \mid \mathbf{Ax} \geq \mathbf{b}\}$*, where* $\mathbf{A}$ *is an* $m \times n$ *matrix and* $\mathbf{b}$ *is a vector in* $\Re^m$.

Definition 2.2 *A set* $S \subset \Re^n$ *is* **bounded** *if there exists a constant* K *such that the absolute value of every component of every element of* S *is less than or equal to* K.

Definition 2.3 *Let* $\mathbf{a}$ *be a nonzero vector in* $\Re^n$ *and let* b *be a scalar.*

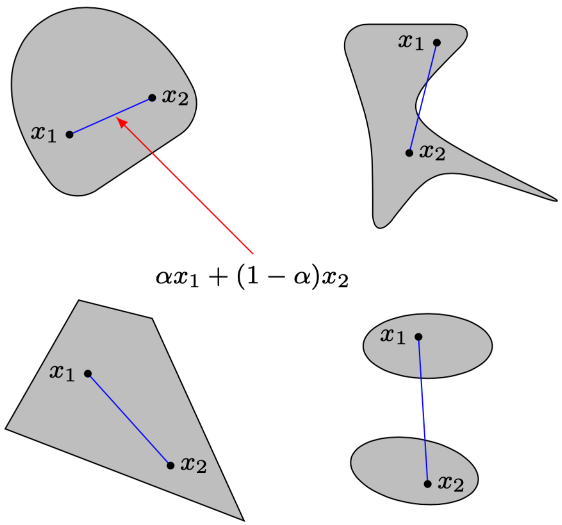
(a) *The set* $\{\mathbf{x} \in \Re^n \mid \mathbf{a}'\mathbf{x} = b\}$ *is called a* **hyperplane**.

(b) *The set* $\{\mathbf{x} \in \Re^n \mid \mathbf{a}'\mathbf{x} \geq b\}$ *is called a* **halfspace**.

Convex Set

Definition 2.4 *A set $S \subset \Re^n$ is* **convex** *if for any $\mathbf{x}, \mathbf{y} \in S$, and any $\lambda \in [0,1]$, we have $\lambda\mathbf{x} + (1-\lambda)\mathbf{y} \in S$.*

几何解释：连接集合中任两点的线段仍含在该集合中

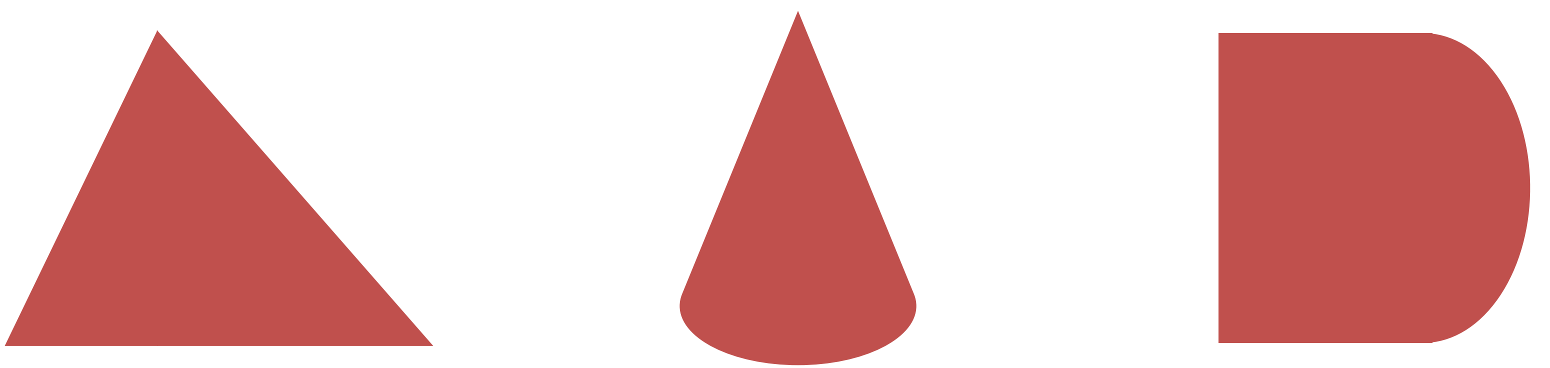


Sets that are convex (left) and not convex (right)

性质：任意多个凸集的交是凸集.

极点(extreme points)，顶点(vertex)

Definition 2.6： Let P be a convex set. A vector $x \in P$ is an extreme point of P if we cannot find two vectors $y, z \in P$, both different from x, and a scalar $\lambda \in [0,1]$, such that $x = \lambda y + (1-\lambda)z$.



几何上：即不能位于连接该集合中其它两点构成的开线段上

Definition 2.7 *Let P be a polyhedron. A vector $\mathbf{x} \in P$ is a* **vertex** *of P if there exists some $\mathbf{c}$ such that $\mathbf{c}'\mathbf{x} < \mathbf{c}'\mathbf{y}$ for all $\mathbf{y}$ satisfying $\mathbf{y} \in P$ and $\mathbf{y} \neq \mathbf{x}$.*

Basic solution, basic feasible solution

基本解、基本可行解

Definition 2.9 *Consider a polyhedron P defined by linear equality and inequality constraints, and let $\mathbf{x}^*$ be an element of $\Re^n$.*

恰好有n个无关方程联立确定

(a) *The vector $\mathbf{x}^*$ is a* **basic solution** *if:*

基本解必可行吗?

- *(i) All equality constraints are active;*
- *(ii) Out of the constraints that are active at $\mathbf{x}^*$, there are n of them that are linearly independent.*

(b) *If $\mathbf{x}^*$ is a basic solution that satisfies all of the constraints, we say that it is a* **basic feasible solution**.

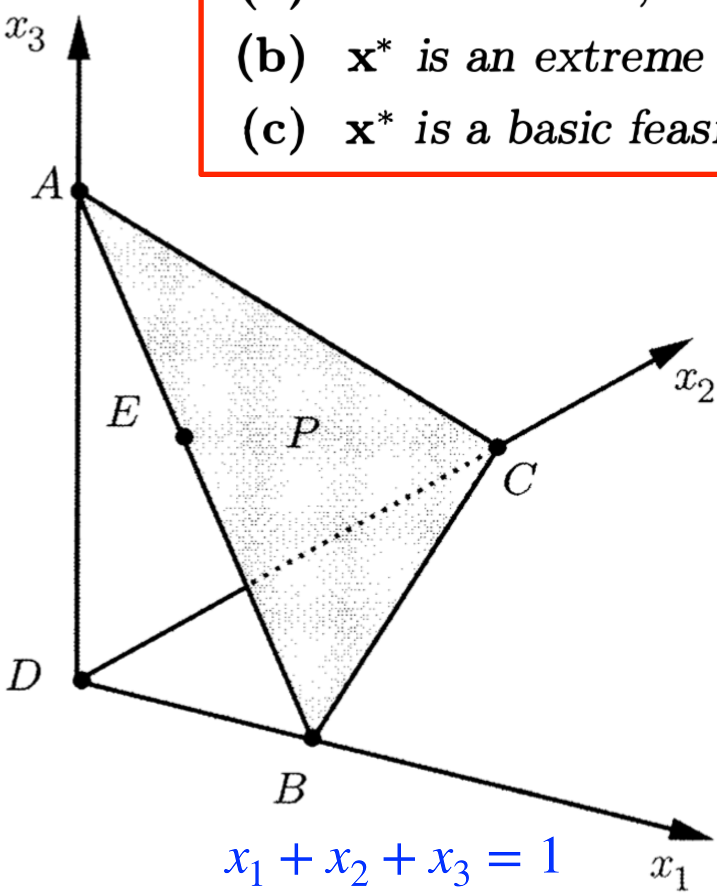
顶点⟺极点⟺基本可行解

Theorem 2.3 *Let P be a nonempty polyhedron and let $\mathbf{x}^* \in P$. Then, the following are equivalent:*

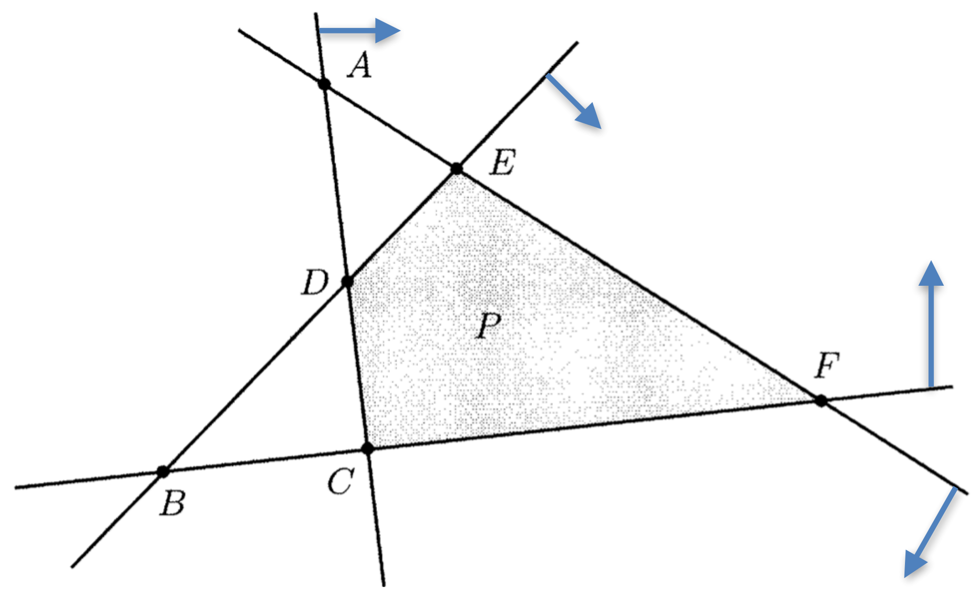
(a) $\mathbf{x}^*$ *is a vertex;*

(b) $\mathbf{x}^*$ *is an extreme point;*

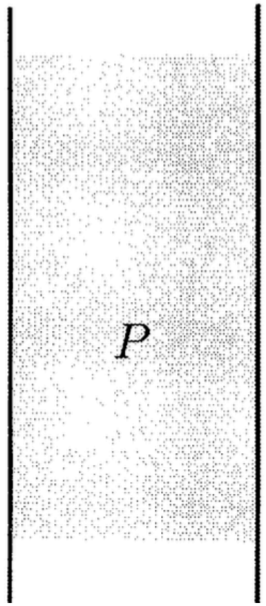
(c) $\mathbf{x}^*$ *is a basic feasible solution.*

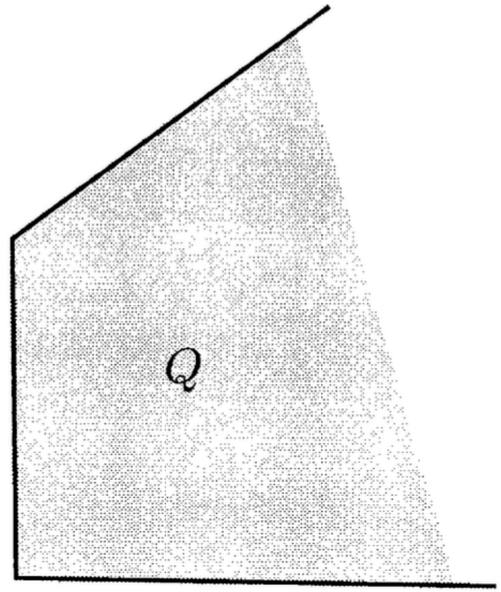


$$x_1 + x_2 + x_3 = 1$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$



Existence of extreme points（section 2.5）





这两种polyhedron都有vertex（extreme points）吗?

他们的最大不同是什么?

这种形式有普适性吗?

Theorem 2.6 *Suppose that the polyhedron* $P = \{\mathbf{x} \in \Re^n \mid \mathbf{a}_i'\mathbf{x} \geq b_i,\ i = 1, \ldots, m\}$ *is nonempty. Then, the following are equivalent:*

(a) *The polyhedron P has at least one extreme point.*

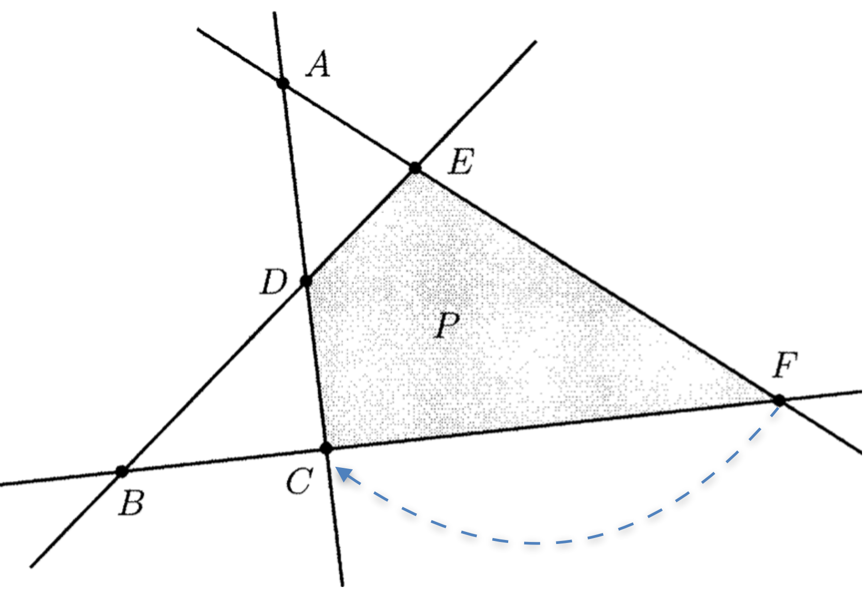
(b) *The polyhedron P does not contain a line.*

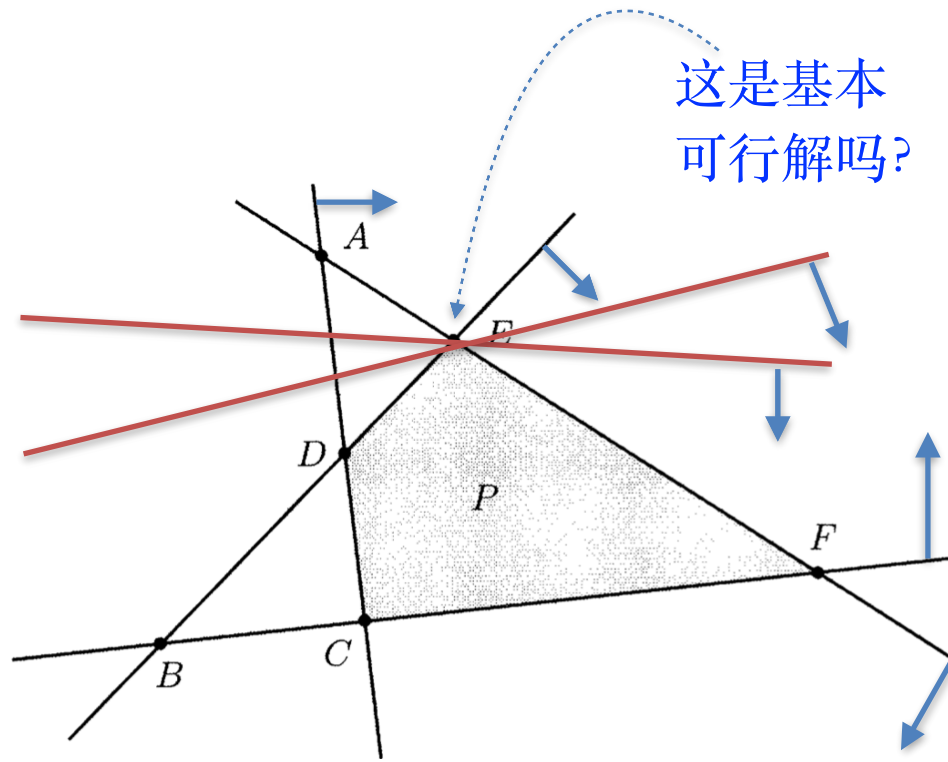
(c) *There exist n vectors out of the family $\mathbf{a}_1, \ldots, \mathbf{a}_m$, which are linearly independent.*

基，basis

Finite number of basic solution (corollary 2.1)

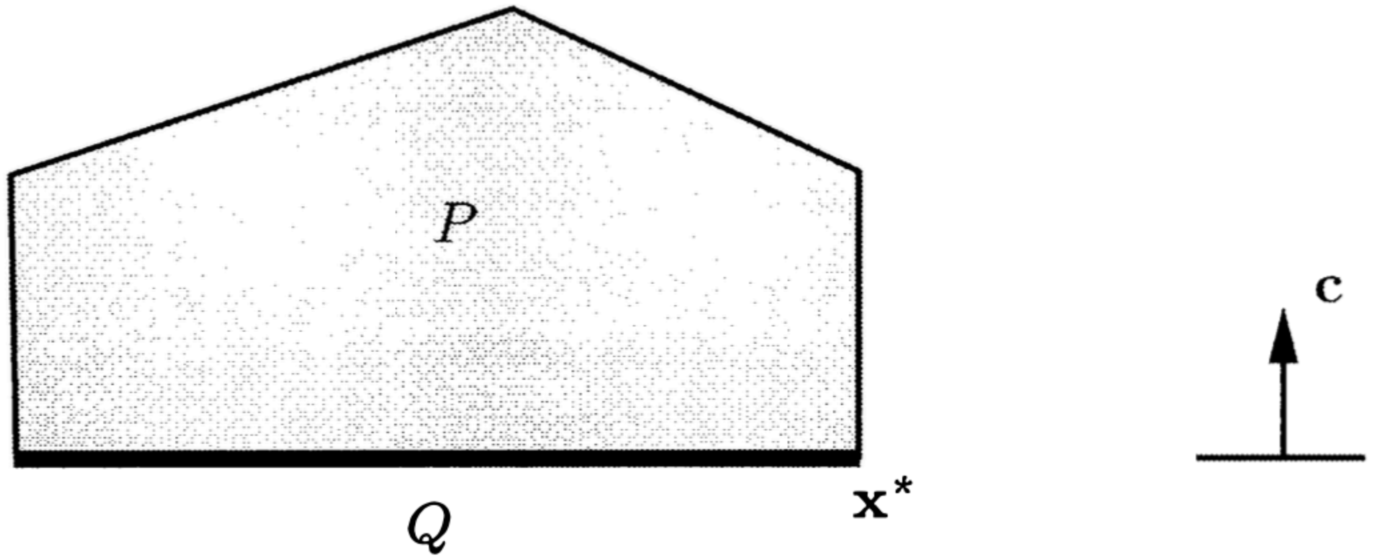
- n维度空间里的多面体最多有多少个基本可行解? 考虑:
  $\{\boldsymbol{x} \in \mathbb{R}^n \mid 0 \leq x_i \leq 1,\ i = 1,\ldots,n\}$
- Adjacent basic solutions: 相邻/临接基础解，2个解共享$n-1$个基。

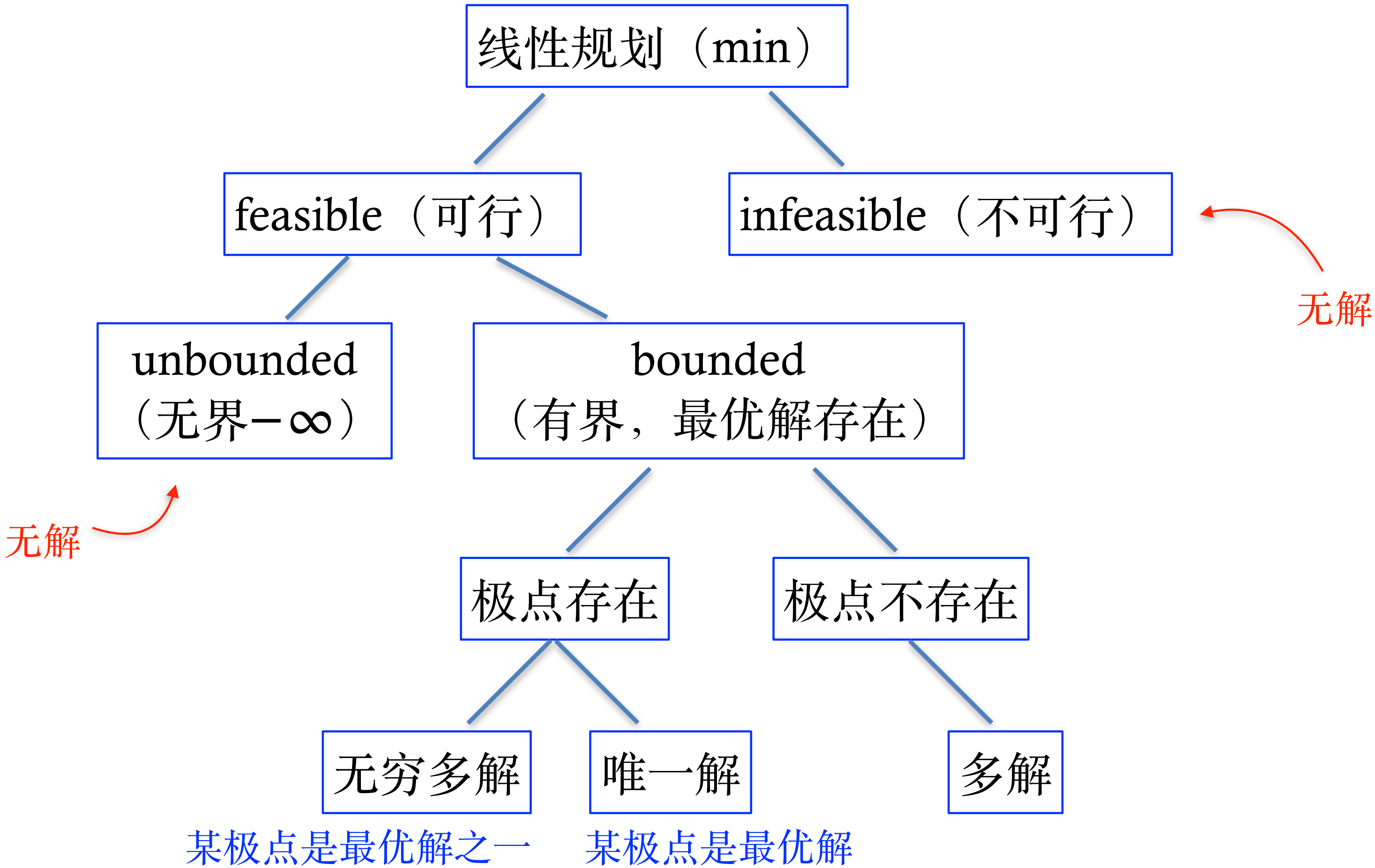




极点(基本可行解)的最优性(optimality)

Theorem 2.7 *Consider the linear programming problem of minimizing $\mathbf{c}'\mathbf{x}$ over a polyhedron P. Suppose that P has at least one extreme point and that there exists an optimal solution. Then, there exists an optimal solution which is an extreme point of P.*





*解指的是最优解(optimal solution)

3、standard form

Standard form标准形(便于理论分析和算法设计)

$$\begin{array}{ll} \text{minimize} & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ & \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \\ & x_1 \geq 0, x_2 \geq 0, \cdots, x_n \geq 0 \end{array}$$

Matrix-vector form：

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

定义标准形有必要吗?

where $c \in \mathbb{R}^n, A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$ are given， $x \in \mathbb{R}^n$

标准形的特点：极小化、等式约束、变量非负

Example

$$
\begin{aligned}
\text{maximize} \quad & x_1 - x_2 \\
\text{subject to} \quad & x_1 + x_2 \leq 1, \\
& x_1 + 2x_2 \geq 1, \\
& x_1\text{无限制}, x_2 \geq 0.
\end{aligned}
$$

$\Big\downarrow$ 等价于

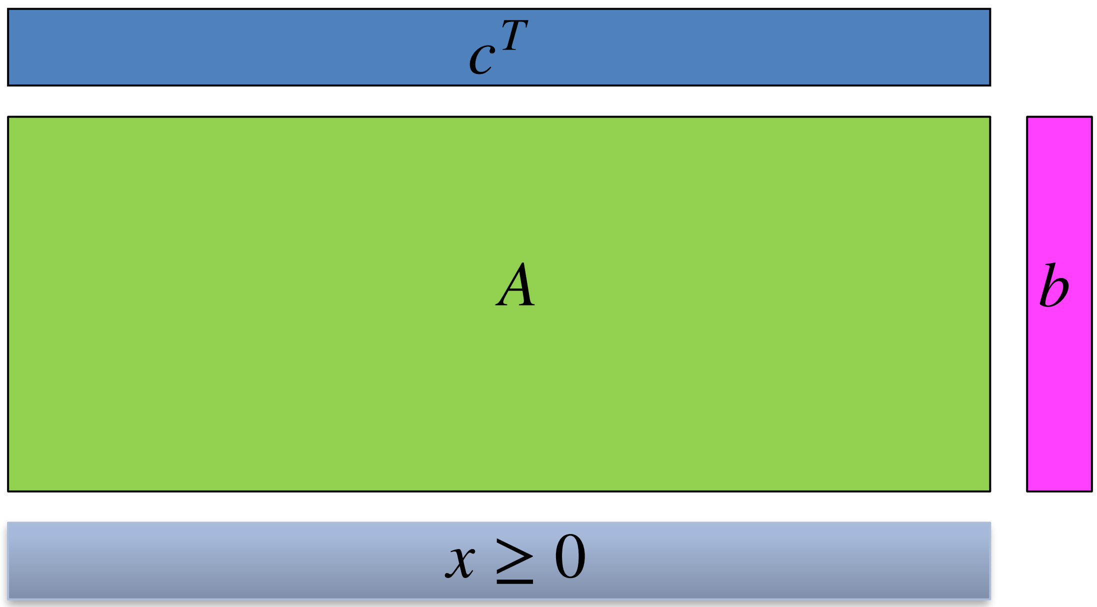
$$
\begin{aligned}
\text{minimize} \quad & -u + v + x_2 \\
\text{subject to} \quad & u - v + x_2 + s_1 = 1, \\
& u - v + 2x_2 - s_2 = 1, \\
& u, v, x_2, s_1, s_2 \geq 0
\end{aligned}
$$

一般形式转化为标准形

$$
\begin{array}{llll}
\text{minimize} & \mathbf{c}'\mathbf{x} & & \\
\text{subject to} & \mathbf{a}_i'\mathbf{x} \geq b_i, & i \in M_1, & \rightarrow s_i = a_i^T x - b_i \\
& \mathbf{a}_i'\mathbf{x} \leq b_i, & i \in M_2, & \rightarrow s_i = b_i - a_i^T x \\
& \mathbf{a}_i'\mathbf{x} = b_i, & i \in M_3, & \\
& x_j \geq 0, & j \in N_1, & \\
& x_j \leq 0, & j \in N_2. & \rightarrow w_i = -x_i \\
& x_j \text{ 无限制} & j \in N_3 & \rightarrow x_i = u_i - v_i
\end{array}
$$

称 s_i 松弛变量(slack variable)/盈余变量(surplus variable)

$$
\begin{aligned}
\text{minimize} \quad & c^T x \\
\text{subject to} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$



BFS of standard form: $Ax = b, \quad x \geq 0$

Full rank assumption (满秩假定)：$m < n$，且A的行向量线性无关

定义　设B是A的m个线性无关的列组成的矩阵，　记其余列所成矩阵为N，与B和N所对应的变量分别为 x_B和 x_N。

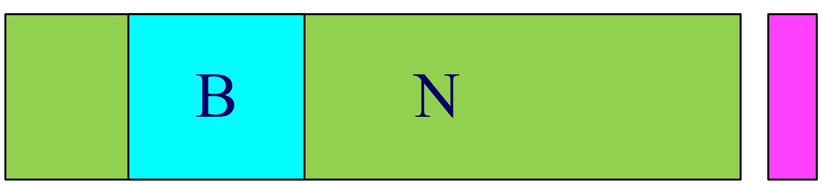
称方程组$Bx_B = b$, x_N=0的解是 $Ax = b$ 的基本解(basic solution) ;

非负基本解是标准形问题的基本可行解(basic feasible solution).

称 B 是基矩阵(basis matrix);

称与 B 的列对应的变量为基变量(basic variables);

而与N的列对应的变量为非基变量(nonbasic variables)



basic variables

basis

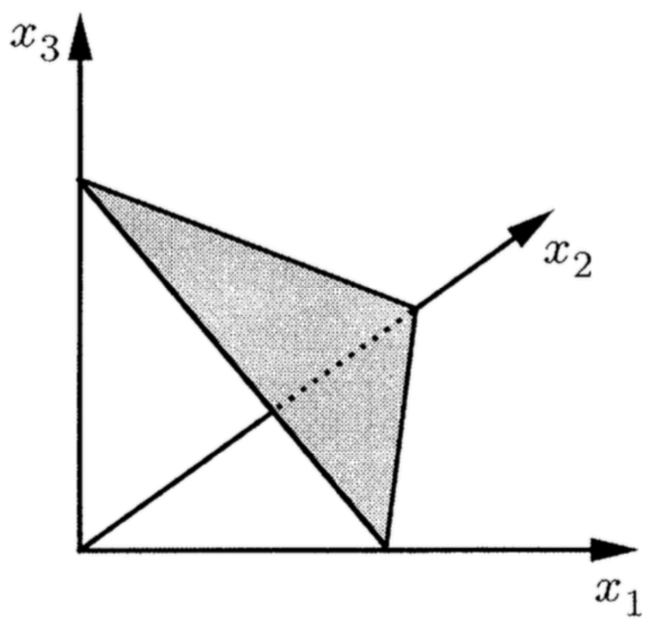
$$
\begin{array}{ccccccc}
c_1 & c_2 & c_3 & c_4 & \cdots & \cdots & c_n \\
x_1 & x_2 & x_3 & x_4 & \cdots & \cdots & x_n
\end{array}
$$

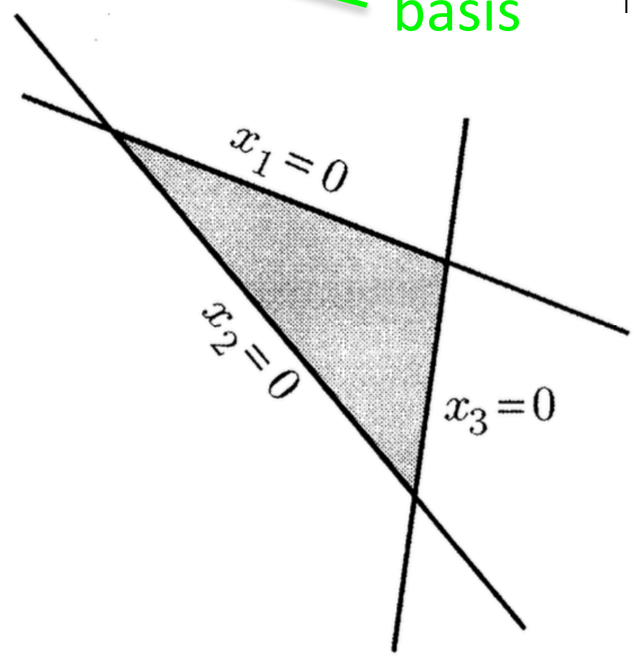
$$
\left[\begin{array}{ccccccc|c}
a_{11} & a_{12} & a_{13} & a_{14} & \cdots & \cdots & a_{1n} & b_1 \\
a_{21} & a_{22} & a_{23} & a_{24} & \cdots & \cdots & a_{2n} & b_2 \\
a_{31} & a_{32} & a_{33} & a_{34} & \cdots & \cdots & a_{3n} & b_3 \\
a_{41} & a_{42} & a_{43} & a_{44} & \cdots & \cdots & a_{4n} & b_4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \\
a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots & \cdots & a_{mn} & b_m
\end{array}\right]
$$

basis

例. 基本可行解及几何意义

$$
x_1 + x_2 + x_3 = 1,
$$
$$
x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.
$$



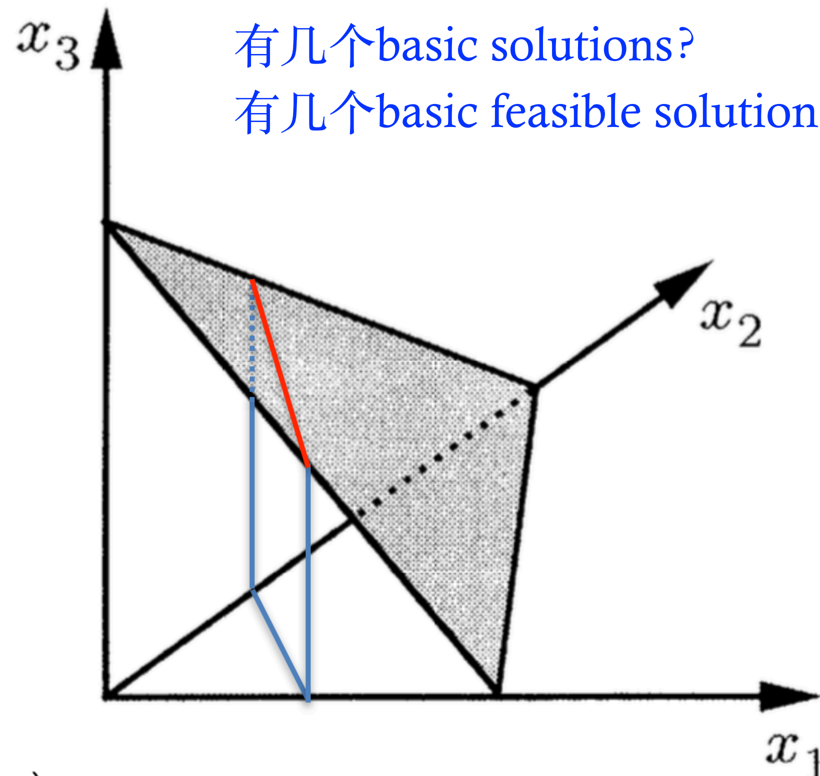


例. 基本可行解及几何意义

$$x_1 + x_2 + x_3 = 1$$
$$2x_1 + 3x_2 = 1$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

$B = [a_1, a_3]$ and $B = [a_3, a_1]$

是同一组基吗?



有几个basic solutions?
有几个basic feasible solutions?

- 基本可行解的个数不超过 $\binom{n}{m} = \frac{n!}{m!(n-m)!}$
- 退化基本可行解：某个或某些基变量取零的基本可行解！

问题：基本可行解与基的对应关系是一一对应的吗?

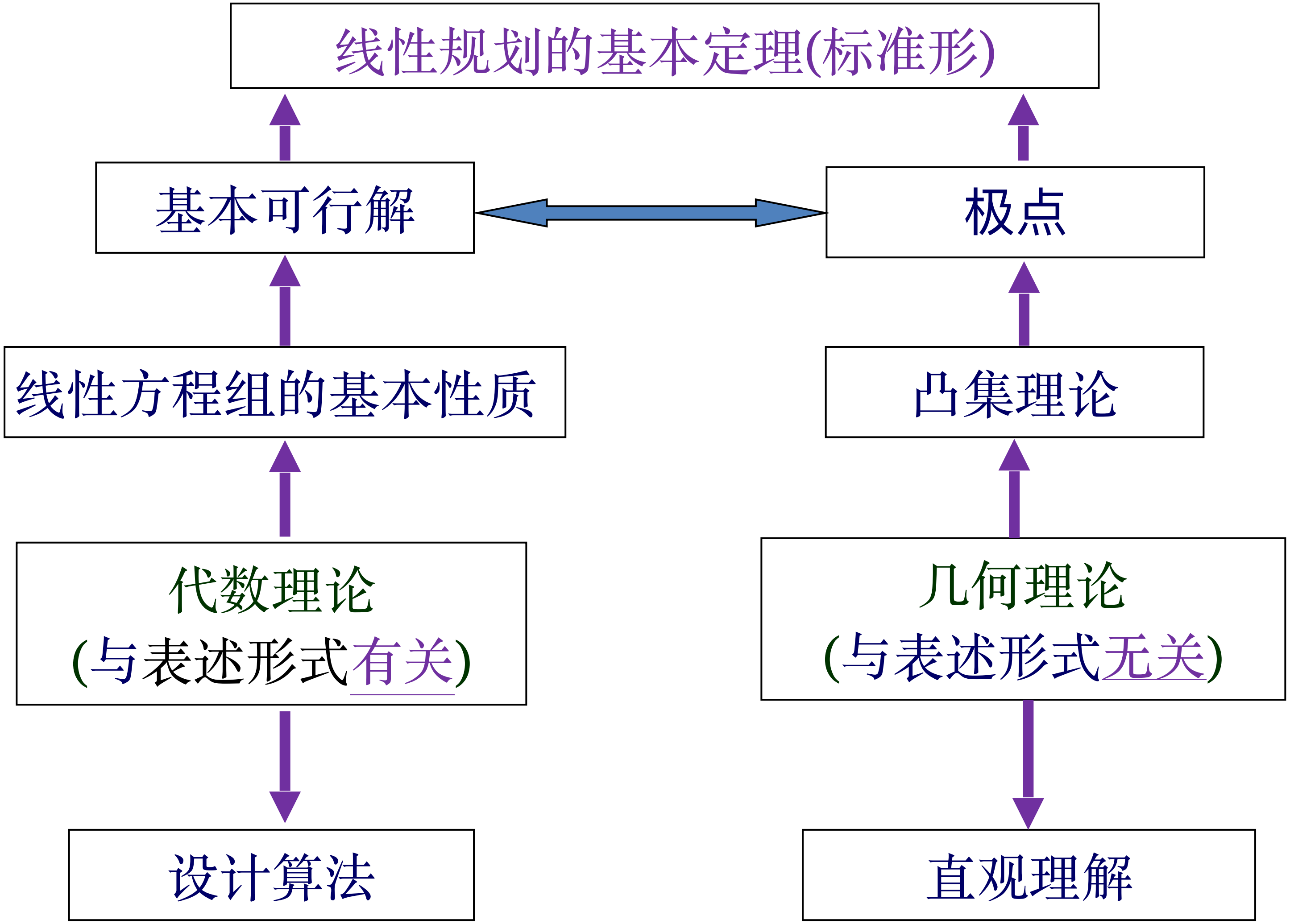
基本可行解的存在性与基本定理（标准型）

定理 (**BFS**的存在性) 考虑具有标准形的线性规划问题，其中$\boldsymbol{A}$ 是秩为 $\boldsymbol{m}$ 的$\boldsymbol{m\times n}$ 矩阵，如果问题有可行解，则必存在基本可行解.

事实：设 $\boldsymbol{x}$ 是线性规划的可行解，则 $\boldsymbol{x}$ 是线性规划的基本可行解当且仅当$\boldsymbol{x}$的正分量对应的列线性无关.

定理(线性规划基本定理) 考虑具有标准形的线性规划问题，其中$\boldsymbol{A}$ 是秩为 $\boldsymbol{m}$ 的$\boldsymbol{m\times n}$ 矩阵，如果问题有解，则必有某个基本可行解是最优解. (Theorem 2.7 [2])

几何直观



— *"You don't understand anything until you learn it more than one way." Marvin Minsky*

极点与基本可行解的等价性(标准型)

考虑具有标准形的线性规划问题，其中A是秩为m的 $m \times n$矩阵，可行集$C = \{Ax = b, x \geq 0\}$则x是C的极点, 当且仅当x是C的基本可行解(非负基本解).

推论:

(i) 若标准型非空，则至少有一个极点(corollary 2.2 [2]).

(ii) 若标准型有解，则必有一个极点是最优解(section 2.6 [2]).

(iii) $\boldsymbol{C}$ 的极点集是有限集.

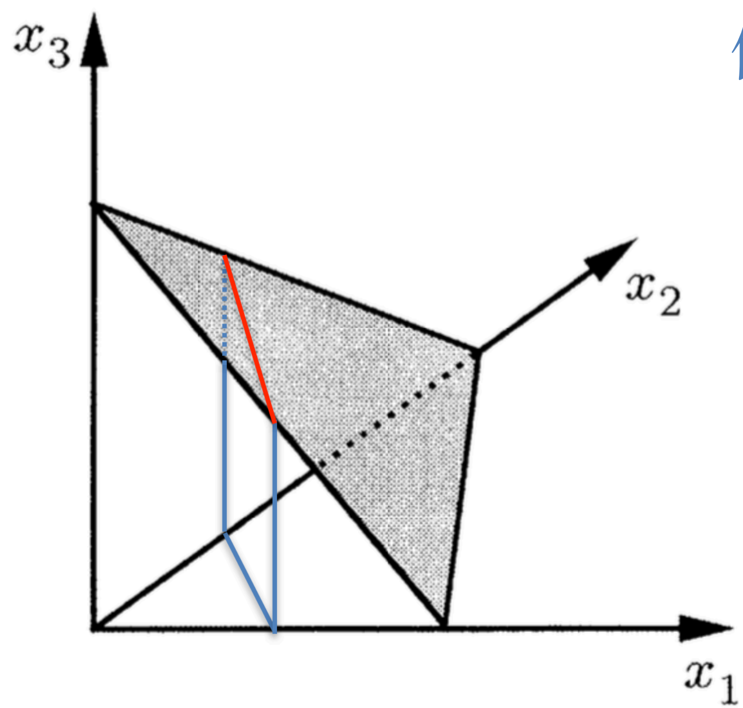
(iv) $\boldsymbol{C}$ 中的点 $\boldsymbol{x}$ 是极点当且仅当它的正分量对应的列线性无关.

例1. $C=\{x\in\mathbb{R}^n \mid x_1+x_2+x_3=1, x_1\geq 0, x_2\geq 0, x_3\geq 0\}$

C 有3个极点

$\Updownarrow$

$x_1+x_2+x_3=1$有3个基本解，均可行



例2. $$\begin{aligned} &x_1+x_2+x_3=1\\ &2x_1+3x_2=1\\ &x_1\geq 0, x_2\geq 0, x_3\geq 0 \end{aligned}$$

C 有2个极点

$\Updownarrow$

有3个基本解，2个可行

例3. minimize $-2x_1 - x_2$

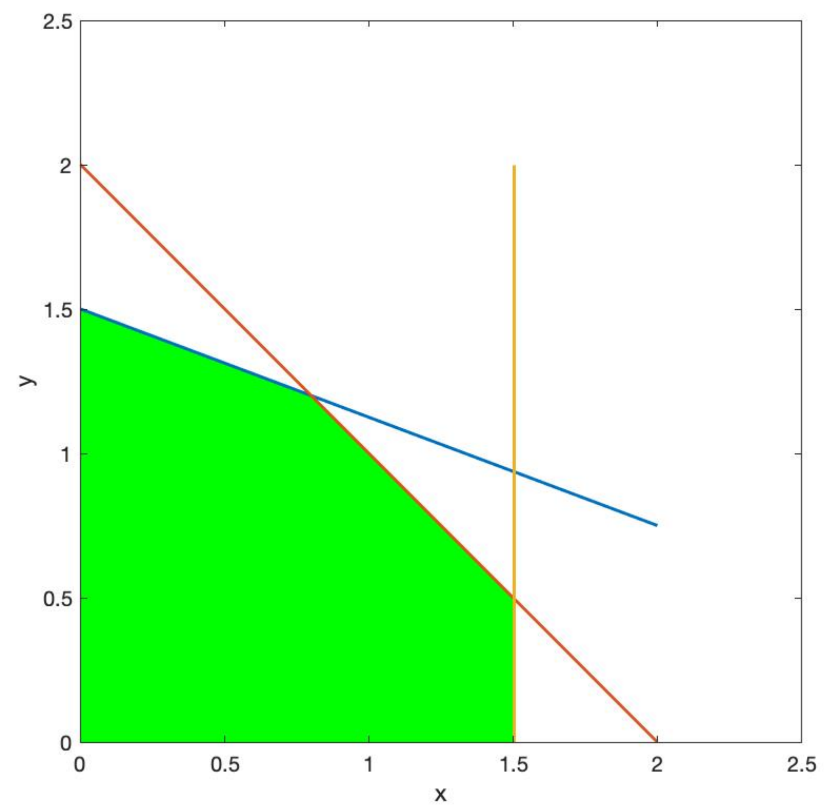
subject to $x_1+\frac{8}{3}x_2 \leq 4,$

$x_1 + x_2 \leq 2,$

$2x_1 \leq 3,$

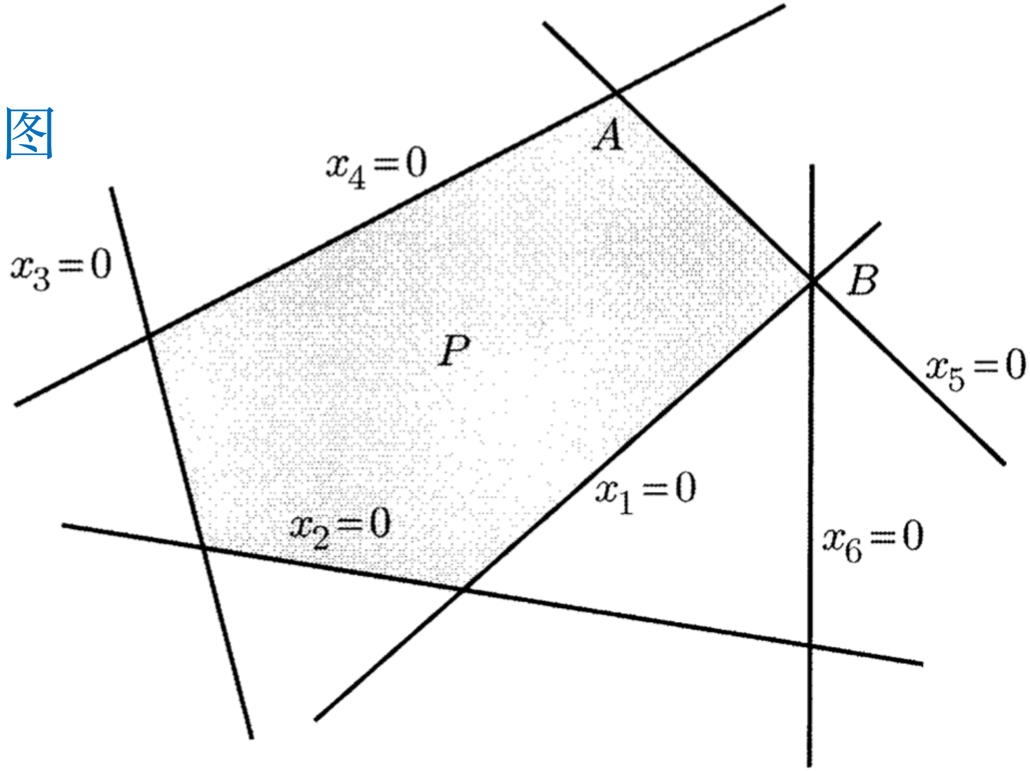
$x_1 \geq 0,\ x_2 \geq 0$

5个极点

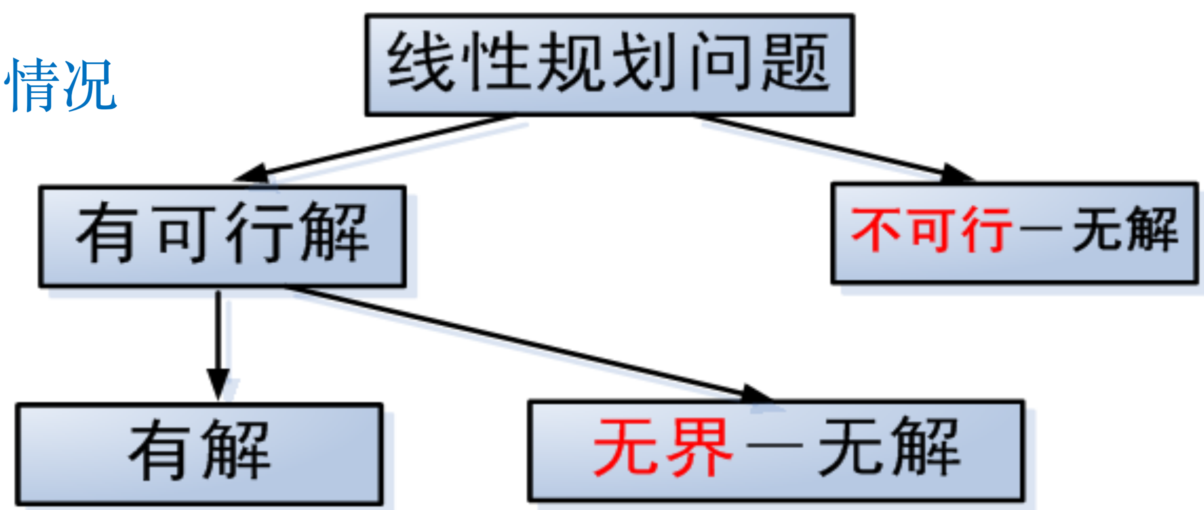


最优解: 极点 $x^* = (\frac{3}{2}, \frac{1}{2})^T$

高维情况基本可行解示意图



线性规划问题解的几种情况



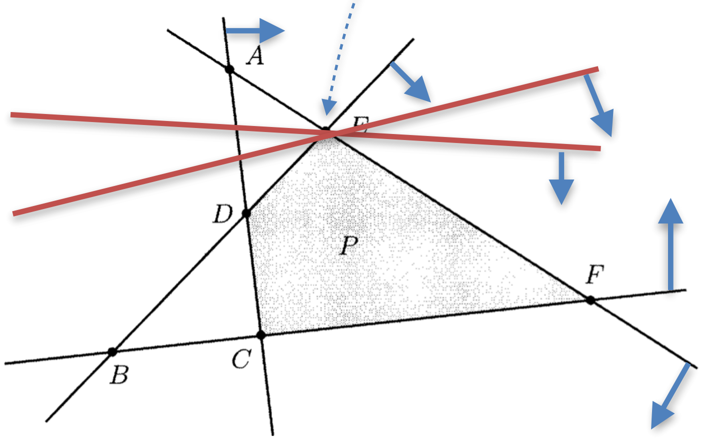
唯一解、无穷多解、退化解

Degeneracy（退化）

Definition 2.10 *A basic solution* $\mathbf{x} \in \Re^n$ *is said to be* **degenerate** *if more than* n *of the constraints are active at* $\mathbf{x}$.

Definition 2.11 *Consider the standard form polyhedron* $P = \{\mathbf{x} \in \Re^n \mid \mathbf{A}\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0}\}$ *and let* $\mathbf{x}$ *be a basic solution. Let* m *be the number of rows of* $\mathbf{A}$. *The vector* $\mathbf{x}$ *is a* **degenerate** *basic solution if more than* $n - m$ *of the components of* $\mathbf{x}$ *are zero.*

退化的基本解



$$Ax = b,\ x \geq 0$$

$$x_1 > 0,\ x_2 > 0,\ \ldots,\ x_{m-1} = 0,\ x_m = 0$$

$$x_{m+1} = 0,\ x_{m+2} = 0,\ \ldots\ x_n = 0$$

Which constraints are "unnecessarily" active?

问题/作业

1、证明以下两个集合的极点是一一对应的！

$$S_1 = \{x \in \mathbb{R}^n : Ax \le b, x \ge 0\}$$

$$S_2 = \{(x, y) \in \mathbb{R}^n \times \mathbb{R}^m : Ax + y = b, x \ge 0, y \ge 0\}$$

2、集合$P = \{x \in \mathbb{R}^2 \mid 0 \le x_1 \le 1\}$有极点吗？它的标准型是什么？它的标准型有极点吗？若有，则给出一个极点，并解释为什么为极点。