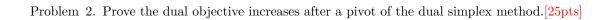
Numerical Optimization, 2023 Fall Homework 3

Name: Zhou Shouchen Student ID: 2021533042

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Problem 1. Prove the dual of the dual of a linear programming (standard form) is itself. [25pts]

So above all, the dual of the dual of a linear programming (standard form) is itself.



Problem 3. Let $L(x, \lambda)$ be the Lagrangian of a linear programming problem, and (x^*, λ^*) be the optimal primal-dual solution. Prove that

$$L(\boldsymbol{x}, \boldsymbol{\lambda}^*) \ge L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) \ge L(\boldsymbol{x}^*, \boldsymbol{\lambda}),$$

for any primal feasible \boldsymbol{x} and dual feasible $\boldsymbol{\lambda}.[25 \mathrm{pts}]$

Problem 4. Construct a linear programming problem for which both the primal and the dual problem has no feasible solution. [25pts]

Construct a linear programming problem that is:

$$\min_{x_1, x_2} \quad x_1 - 2x_2
\text{s.t.} \quad x_1 - x_2 \le 1
\quad x_1 - x_2 \ge 2
\quad x_1, x_2 \le 0$$
(1)

Since it is impossible to satisfy $x_1 - x_2 \le 1$ and $x_1 - x_2 \ge 2$ at the same time, so the primal problem has no feasible solution.

And the dual problem is that:

$$\begin{aligned} \max_{\lambda_1, \lambda_2} & \lambda_1 + 2\lambda_2 \\ \text{s.t.} & \lambda_1 + \lambda_2 \le 1 \\ & -\lambda_1 - \lambda_2 \le -2 \\ & \lambda_1 \le 0, \lambda_2 \ge 0 \end{aligned} \tag{2}$$

The second constrain $-\lambda_1 - \lambda_2 \le -2$ can be written as $\lambda_1 + \lambda_2 \ge 2$.

Since it is impossible to satisfy $\lambda_1 + \lambda_2 \leq 1$ and $\lambda_1 + \lambda_2 \geq 2$ at the same time, so the dual problem has no feasible solution.

So above all, the above construction's primal and dual problem has no feasible solution.