# SI251 - Convex Optimization homework 2

## Deadline: 2024-4-10 23:59:59

1. You can use Word, Latex or handwriting to complete this assignment. If you want to submit a handwritten version, scan it clearly.
2. The **report** has to be submitted as a PDF file to Gradescope, other formats are not accepted.
3. The submitted file name is **student_id+your_student_name.pdf**.
4. Late policy: You have 4 free late days for the quarter and may use up to 2 late days per assignment with no penalty. Once you have exhausted your free late days, we will deduct a late penalty of 25% per additional late day. Note: The timeout period is recorded in days, even if you delay for 1 minute, it will still be counted as a 1 late day.
5. You are required to follow ShanghaiTech's academic honesty policies. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious sanctions.

**Any plagiarism will get Zero point.**

1. (50 pts) **Robust quadratic programming**. In the lecture, we have learned about robust linear programming as an application of second-order cone programming. Now we will consider a similar robust variation of the convex quadratic program

$$\begin{array}{ll} \text{minimize} & (1/2)x^T Px + q^T x + r \\ \text{subject to} & Ax \preceq b. \end{array}$$

For simplicity, we assume that only the matrix $P$ is subject to errors, and the other parameters $(q, r, A, b)$ are exactly known. The robust quadratic program is defined as

$$\begin{array}{ll} \text{minimize} & \sup_{P\in\mathcal{E}} \left((1/2)x^T Px + q^T x + r\right) \\ \text{subject to} & Ax \prec b \end{array}$$

where $\mathcal{E}$ is the set of possible matrices $P$.

For each of the following sets $\mathcal{E}$, express the robust QP as a convex problem in a standard form (e.g., QP, QCQP, SOCP, SDP).

(a) A finite set of matrices: $\mathcal{E} = \{P_1, ..., P_K\}$, where $P_i \in S^n_+, i = 1, ..., K$.

(b) A set specified by a nominal value $P_0 \in S^n_+$ plus a bound on the eigenvalues of the deviation $P - P_0$:

$$\mathcal{E} = \{P \in \mathbf{S}^n \mid -\gamma I \preceq P - P_0 \preceq \gamma I\}$$

where $\gamma \in \mathbf{R}$ and $P_0 \in \mathbf{S}^n_+$.

(c) An ellipsoid of matrices:

$$\mathcal{E} = \left\{ P_0 + \sum_{i=1}^{K} P_i u_i \mid \|u\|_2 \leq 1 \right\}.$$

You can assume $P_i \in \mathbf{S}^n_+, i = 0, \ldots, K$.

2. (50 pts) **Water-filling.** Please consider the convex optimization problem and calculate its solution

$$\begin{array}{ll} \text{minimize} & -\sum_{i=1}^n \log\left(\alpha_i + x_i\right) \\ \text{subject to} & x \succeq 0, \quad \mathbf{1}^T x = 1, \end{array}$$