SI251 - Convex Optimization, 2024 Spring

Homework 2

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Due 23:59 (CST), Apr. 10, 2024

1. (50 pts) **Robust quadratic programming**. In the lecture, we have learned about robust linear programming as an application of second-order cone programming. Now we will consider a similar robust variation of the convex quadratic program

$$\begin{array}{ll} \text{minimize} & (1/2)x^T Px + q^T x + r \\ \text{subject to} & Ax \preceq b. \end{array}$$

For simplicity, we assume that only the matrix P is subject to errors, and the other parameters (q, r, A, b) are exactly known. The robust quadratic program is defined as

$$\begin{array}{ll} \text{minimize} & \sup_{P \in \mathcal{E}} \left((1/2)x^T Px + q^T x + r\right) \\ \text{subject to} & Ax \preceq b \end{array}$$

where $\mathcal{E}$ is the set of possible matrices P.

For each of the following sets $\mathcal{E}$, express the robust QP as a convex problem in a standard form (e.g., QP, QCQP, SOCP, SDP).

(a) A finite set of matrices: $\mathcal{E} = \{P_1, \ldots, P_K\}$, where $P_i \in S^n_+, i = 1, \ldots, K$.

(b) A set specified by a nominal value $P_0 \in S^n_+$ plus a bound on the eigenvalues of the deviation $P - P_0$:

$$\mathcal{E} = \{P \in \mathbf{S}^n \mid -\gamma I \preceq P - P_0 \preceq \gamma I\}$$

where $\gamma \in \mathbf{R}$ and $P_0 \in \mathbf{S}^n_+$.

(c) An ellipsoid of matrices:

$$\mathcal{E} = \left\{ P_0 + \sum_{i=1}^{K} P_i u_i \mid \|u\|_2 \leq 1 \right\}.$$

You can assume $P_i \in \mathbf{S}^n_+, i = 0, \ldots, K$.

Solution:

2. (50 pts) **Water-filling**. Please consider the convex optimization problem and calculate its solution

$$
\begin{array}{ll}
\text{minimize} & -\sum_{i=1}^{n} \log\left(\alpha_i + x_i\right) \\
\text{subject to} & x \succeq 0, \quad \mathbf{1}^T x = 1
\end{array}
$$

Solution: