

SI251 - Convex Optimization, 2024 Spring
Homework 2

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1. **(50 pts) Robust quadratic programming.** In the lecture, we have learned about robust linear programming as an application of second-order cone programming. Now we will consider a similar robust variation of the convex quadratic program

$$\begin{aligned} & \text{minimize} && (1/2)x^T Px + q^T x + r \\ & \text{subject to} && Ax \preceq b. \end{aligned}$$

For simplicity, we assume that only the matrix P is subject to errors, and the other parameters (q, r, A, b) are exactly known. The robust quadratic program is defined as

$$\begin{aligned} & \text{minimize} && \sup_{P \in \mathcal{E}} ((1/2)x^T Px + q^T x + r) \\ & \text{subject to} && Ax \prec b \end{aligned}$$

where \mathcal{E} is the set of possible matrices P .

For each of the following sets \mathcal{E} , express the robust QP as a convex problem in a standard form (e.g., QP, QCQP, SOCP, SDP).

- (a) A finite set of matrices: $\mathcal{E} = \{P_1, \dots, P_K\}$, where $P_i \in S_+^n, i = 1, \dots, K$.
- (b) A set specified by a nominal value $P_0 \in S_+^n$ plus a bound on the eigenvalues of the deviation $P - P_0$:

$$\mathcal{E} = \{P \in \mathbf{S}^n \mid -\gamma I \preceq P - P_0 \preceq \gamma I\}$$

where $\gamma \in \mathbf{R}$ and $P_0 \in \mathbf{S}_+^n$.

- (c) An ellipsoid of matrices:

$$\mathcal{E} = \left\{ P_0 + \sum_{i=1}^K P_i u_i \mid \|u\|_2 \leq 1 \right\}.$$

You can assume $P_i \in \mathbf{S}_+^n, i = 0, \dots, K$.

Solution:

2. **(50 pts) Water-filling.** Please consider the convex optimization problem and calculate its solution

$$\begin{aligned} & \text{minimize} && - \sum_{i=1}^n \log(\alpha_i + x_i) \\ & \text{subject to} && x \succeq 0, \quad \mathbf{1}^T x = 1 \end{aligned}$$

Solution: