SI251 - Convex Optimization Quiz

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1. You can use Word, Latex or handwriting to complete this assignment. If you want to submit a handwritten version, scan it clearly.
2. The **report** has to be submitted as a PDF file to Blackboard, other formats are not accepted.
3. The submitted file name is **student_id+your_student_name.pdf**.
4. Late policy: You have 4 free late days for the quarter and may use up to 2 late days per assignment with no penalty. Once you have exhausted your free late days, we will deduct a late penalty of 25% per additional late day. Note: The timeout period is recorded in days, even if you delay for 1 minute, it will still be counted as a 1 late day.
5. You are required to follow ShanghaiTech's academic honesty policies. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious sanctions.

Any plagiarism will get Zero point.

1. (30 pt) **Dual cone of psd.** What is the dual cone of $\mathbb{S}_+^n$? Please prove that. Hint: The matrix inner product is $< A, B >= \mathbf{tr}(\mathrm{AB}^{\mathrm{T}})$.

2. (30 pt) **Convexity of hyperbolic sets.** Please show that $\left\{x \in \mathbb{R}_+^n \mid \prod_{i=1}^n x_i \geq 1\right\}$ is a convex set. Hint: If $a, b \geq 0$ and $0 \leq \theta \leq 1$, then $a^\theta b^{1-\theta} \leq \theta a + (1-\theta)b$.

3. (40 pt) **Eigenvalue functions.** Please prove:

   (a) $f(\boldsymbol{X}) = \lambda_{\max}(\boldsymbol{X}) = \sup_{\boldsymbol{y}\neq\boldsymbol{0}} \frac{\boldsymbol{y}^T\boldsymbol{X}\boldsymbol{y}}{\boldsymbol{y}^T\boldsymbol{y}}$ is convex on $\mathbb{S}^n$.

   (b) $f(\boldsymbol{X}) = \mathbf{tr}(\boldsymbol{X}) = \lambda_1(\boldsymbol{X}) + \cdots + \lambda_n(\boldsymbol{X})$ is linear on $\mathbb{S}^n$.

   (c) $f(\boldsymbol{X}) = \mathbf{tr}(\boldsymbol{X}^{-1}) = \frac{1}{\lambda_1(\boldsymbol{X})} + \cdots + \frac{1}{\lambda_n(\boldsymbol{X})}$ is convex on $\mathbb{S}^n_{++}$.