线性代数(2023-2024)第一次作业

1 复习知识点

- 记住三种初等行变换。
- 阶梯型与简化阶梯型的定义。
- 如何将一个矩阵通过初等行变换转化为阶梯型或简化阶梯型。
- 主元与自由元的定义，如何通过阶梯型找出主元与自由元。
- 如何通过高斯消元法或高斯-若尔当消元法求解线性方程组。
- 线性方程组通解的的表示。

建议在做习题之前首先阅读以下内容：

- 英文教材第16页 Example 5，第18页 Example 6，第20页 Example 7。

2 习题部分

Problem A(6 Points). Consider the following linear system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10. \end{aligned}$$

1. (1 point) Write down the augmented matrix of this linear system.
2. (3 points) Transform the augmented matrix to a row echelon form or to a reduced row echelon form, and indicate which elementary row operation is used in every step.
3. (1 point) Determine the leading variables and free variables of this linear system.
4. (1 point) Solve this linear system by using the row echelon form or reduced row echelon form obtained in 2., and give the general solution.

翻译：考虑以下线性方程组

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10. \end{aligned}$$

1. (1 分) 写出该方程组对应的增广矩阵。
2. (3 分) 将该增广矩阵转化为阶梯型或者简化阶梯型 (任选一种类型即可)，在转化过程中的每一步都需要标明使用了哪个初等行变换。
3. (1 分) 确定该方程组里的主元与自由元。
4. (1 分) 通过在第2问中得到的阶梯型或者简化阶梯型确定方程组的解，并写出通解。

Problem B(6 Points). Consider the following linear system

$$\begin{aligned} 3x_1 + x_2 + x_3 + x_4 &= 0 \\ 5x_1 - x_2 + x_3 - x_4 &= 10. \end{aligned}$$

1. (1 point) Write down the augmented matrix of this linear system.
2. (3 points) Transform the augmented matrix to a row echelon form or to a reduced row echelon form, and indicate which elementary row operation is used in every step.
3. (1 point) Determine the leading variables and free variables of this linear system.
4. (1 point) Solve this linear system by using the row echelon form or reduced row echelon form obtained in 2., and give the general solution.

Problem C(6 Points). Consider the following linear system

$$\begin{aligned} v + 3w - 2x &= 0 \\ 2u + v - 4w + 3x &= 0 \\ 2u + 3v + 2w - x &= 0 \\ -4u - 3v + 5w - 4x &= 0. \end{aligned}$$

1. (1 point) Write down the augmented matrix of this linear system.
2. (3 points) Transform the augmented matrix to a row echelon form or to a reduced row echelon form, and indicate which elementary row operation is used in every step.
3. (1 point) Determine the leading variables and free variables of this linear system.
4. (1 point) Solve this linear system by using the row echelon form or reduced row echelon form obtained in 2., and give the general solution.

Problem D(6 Points). Determine the values of a for which the linear system

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

has

1. (2 points) no solution,
2. (2 points) exactly one solution,
3. (2 points) infinitely many solution.

翻译：考虑以下线性方程组

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

求a取何值时

1. (2 分) 该方程组无解，
2. (2 分) 该方程组有且仅有一个解，
3. (2 分) 该方程组有无穷多个解。

Problem E(6 Points). Solve the following system of nonlinear equations for x, y and z.

$$\begin{aligned} x^2 + y^2 + z^2 &= 6 \\ x^2 - y^2 + 2z^2 &= 2 \\ 2x^2 + y^2 - z^2 &= 3. \end{aligned}$$

翻译：求解以下非线性方程组

$$\begin{aligned} x^2 + y^2 + z^2 &= 6 \\ x^2 - y^2 + 2z^2 &= 2 \\ 2x^2 + y^2 - z^2 &= 3. \end{aligned}$$

Deadline: 22:00, October 15.

作业提交截止时间：十月十五日晚上22：00。