linear algebra in CS

> 你们觉得你们现在学的东西没有用，并不是因为它真的没有用，只是你们还没有遇到要用到那门课的时候

- 一个CS的学生, 在大学期间其实会多次重新学习线性代数
- 学习过的内容不需要你牢牢记住, 但是需要你知道它的存在, 以及它的用途, 以及清楚的知道你需要的时候去哪里找

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貪心不足
@Tanxinbuzu

「大学招生简直诡计多端...某个uc数学系的博士生说，现在愿意修线性代数的美国本科生越来越少了，后来他们改了个名字大概叫Math Foundationsof MachineLearning，爆满，学生给教授写邮件要去上这门课，其实还是线性代数。」

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- for algorithm and data structure / information theory

e.g. the Laplacian matrix of a graph

$$L_{ij} = \begin{cases} D_{ii} & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

Geometric Perspective

① Inner product $\langle X, Y\rangle = E[XY]$

$\langle X, Y\rangle = \langle Y, X\rangle$

$\langle \alpha X+\beta Y, Z\rangle = \alpha\langle X, Z\rangle + \beta\langle Y, Z\rangle$

$\langle X, X\rangle \geq 0$ and $\langle X, X\rangle = 0 \Rightarrow X = 0$

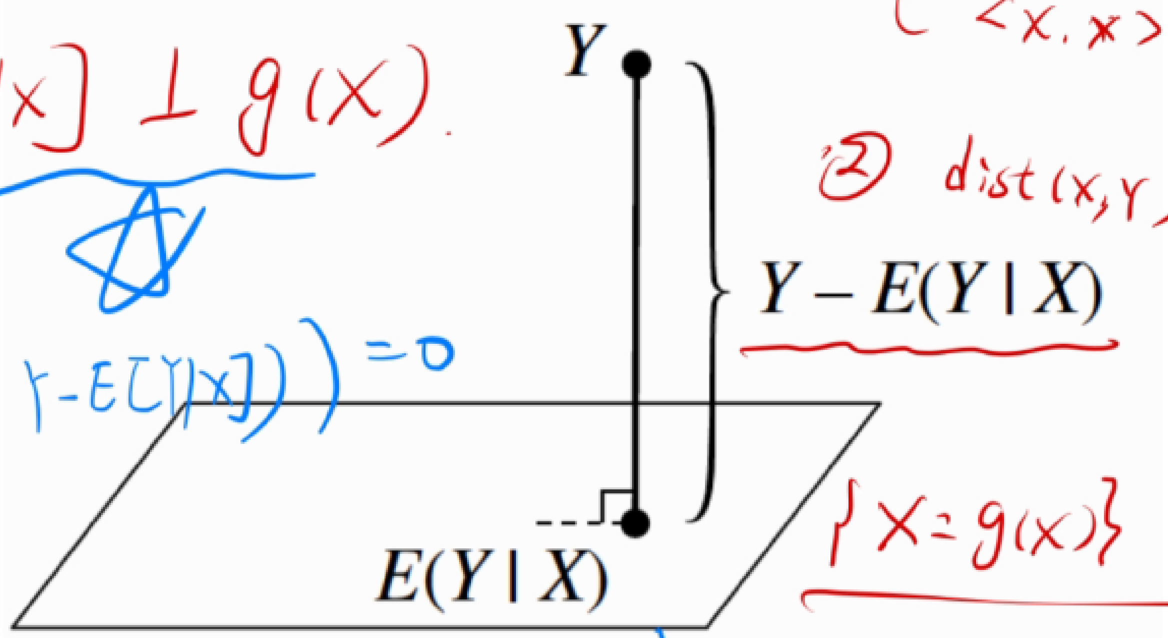
② $\text{dist}(X,Y) = \langle X-Y, X-Y\rangle^{\frac{1}{2}} = \sqrt{E((X-Y)^2)}$

③ X and Y are "orthogonal" 正交

If $E[XY] = 0$, $X \perp Y$

④ $Y - E[Y|X] \perp g(X)$

$E(g(X)(Y-E[Y|X])) = 0$



$\{X = g(x)\}$

$\{E(X^2) < \infty\}$

X的函数组成的空间

⑤ $E[Y|X]$ = a projection of Y onto the space of arbitrary functions of X.

条件期望就是投影

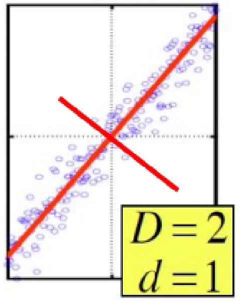
- for machine learning(ML)

Principal Component Analysis (PCA)

$(\mathrm{X}\,\mathrm{X}^{\mathrm{T}})\mathrm{v} = \lambda\mathrm{v}$, so v (the first PC) is the eigenvector of sample correlation/covariance matrix $X\,X^T$

Sample variance of projection $\mathrm{v}^T X\,X^T \mathrm{v} = \lambda \mathrm{v}^T \mathrm{v} = \lambda$

Thus, the eigenvalue λ denotes the amount of variability captured along that dimension (aka amount of energy along that dimension).

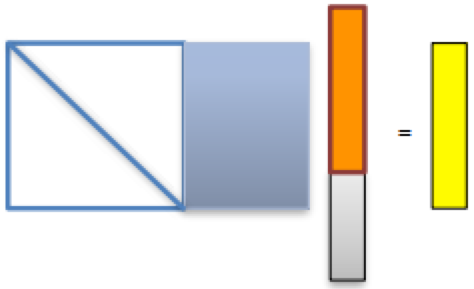


Eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$

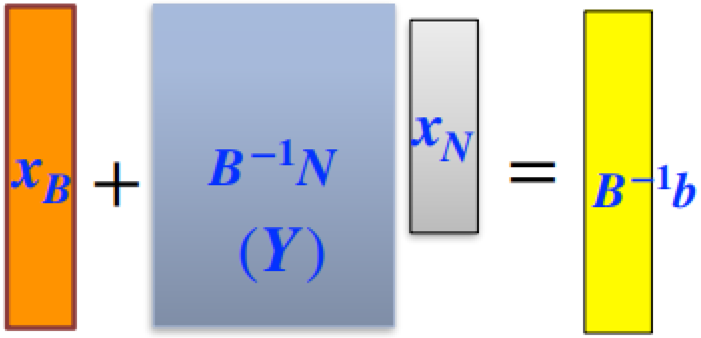
- The 1st PC v_1 is the the eigenvector of the sample covariance matrix $X\,X^T$ associated with the largest eigenvalue
- The 2nd PC v_2 is the the eigenvector of the sample covariance matrix $X\,X^T$ associated with the second largest eigenvalue
- And so on …

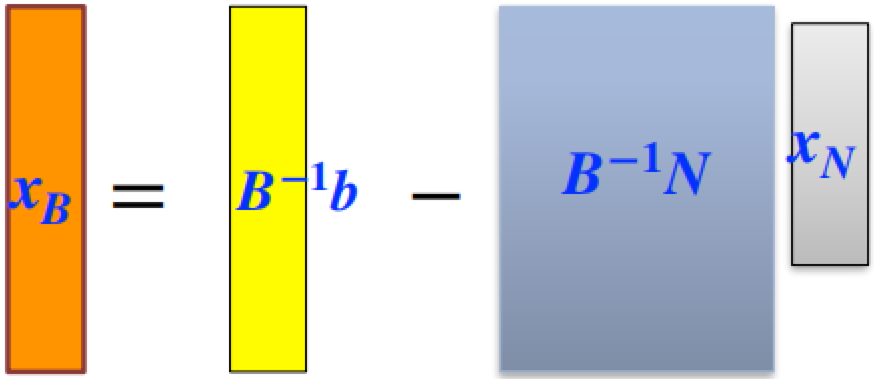
- for optimization

将 $\boldsymbol{Ax}=\boldsymbol{b}$ 的任一解 $\boldsymbol{x}$ 用非基变量表示为



$$
\begin{aligned}
x_1 &= \bar{b}_1 - \sum_{j=m+1}^{n} y_{1j}x_j \\
x_2 &= \bar{b}_2 - \sum_{j=m+1}^{n} y_{2j}x_j \\
&\vdots \\
x_m &= \bar{b}_m - \sum_{j=m+1}^{n} y_{mj}x_j
\end{aligned}
$$

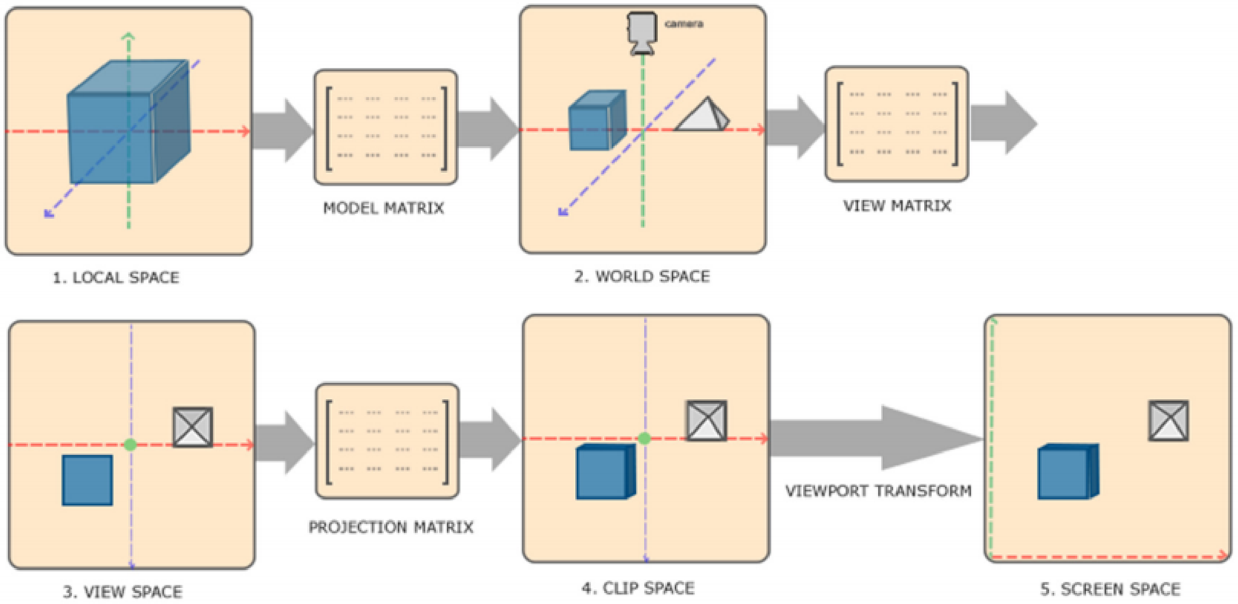




- for computer graphics(CG)

Coordinate spaces

- **The global picture**
  - Space transformations using matrices



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- for computer vision(CV)

homography projective transformation(单应性变换)

- 想要任意变换(正方形变为任意四边形)

$$H = K_2 R_2 \left(I - \frac{1}{d}(-R_1^{-1} t_1 + R_2^{-1} t_2) n^T R_1\right) R_1^T K_1^{-1}$$

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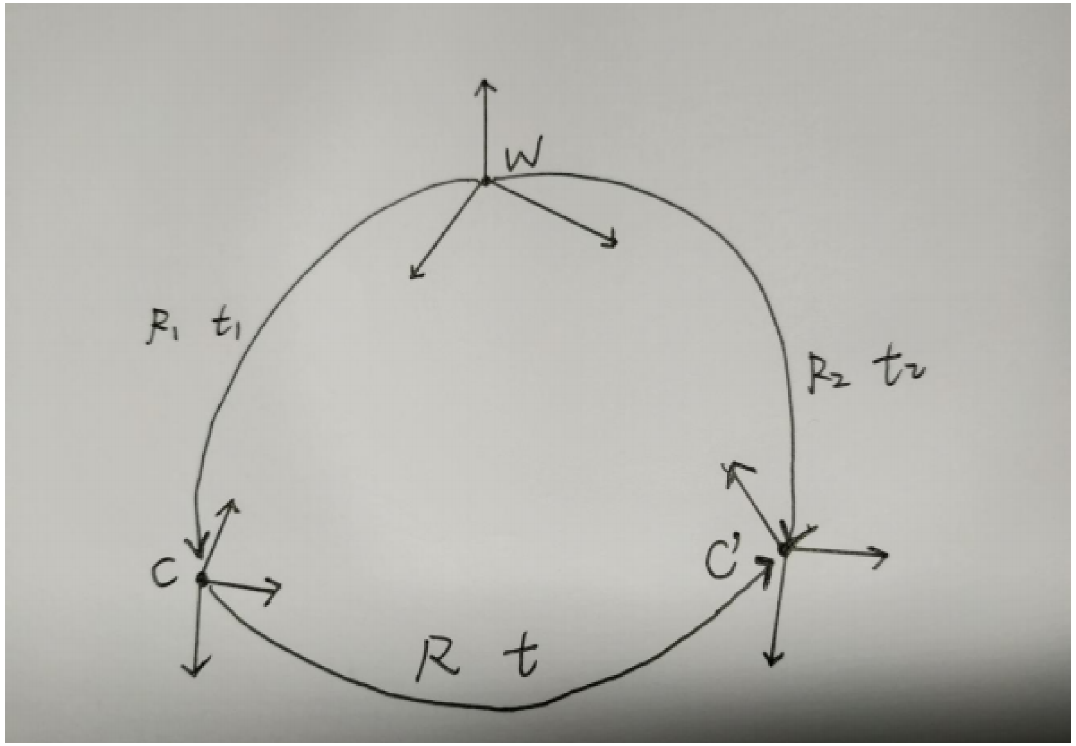
- for computer vision(CV)



$$\begin{bmatrix} R_2 & t_2 \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix}^{-1} \begin{bmatrix} R & t \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} P = P$$

$$\Downarrow$$

$$\begin{bmatrix} R & t \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} = \begin{bmatrix} R_2 R_1^{-1} & -R_2 R_1^{-1} t_1 + t_2 \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix}$$

$$\Downarrow$$

$$R = R_2 R_1^{-1}$$

$$t = -R_2 R_1^{-1} t_1 + t_2$$

$$\boxed{H = K_2 (R - t\frac{n^T}{d}) K_1^{-1}}$$

$$H = K_2 (R_2 R_1^{-1} - (-R_2 R_1^{-1} t_1 + t_2)\frac{n^T}{d}) K_1^{-1}$$

$$\boxed{H = K_2 R_2 (I - \frac{1}{d}(-R_1^{-1} t_1 + R_2^{-1} t_2) n^T R_1) R_1^T K_1^{-1}}$$

- for computer vision(CV)

Camera calibration: Linear method

$$\begin{bmatrix} 0^T & \mathbf{X}_1^T & -y_1\mathbf{X}_1^T \\ \mathbf{X}_1^T & 0^T & -x_1\mathbf{X}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{X}_n^T & -y_n\mathbf{X}_n^T \\ \mathbf{X}_n^T & 0^T & -x_n\mathbf{X}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0 \qquad \mathbf{Ap} = \mathbf{0}$$

- P has 11 degrees of freedom
- One 2D/3D correspondence gives us two linearly independent equations
  - 6 correspondences needed for a minimal solution
- Homogeneous least squares: find $\mathbf{p}$ minimizing $\|\mathbf{Ap}\|^2$
  - Solution given by eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue

- for natural language processing(NLP)
- for robotics
- for data mining
- for data science

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