

Course Project, Spring 2016

## Cluster-State Quantum Computing

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The most widely used model of quantum computation at the present time is the quantum circuit model.

A set containing all single-qubit gates and at least one entangling two-qubit gate (such as the controlled-NOT or controlled-PHASE) is universal[Brylinski and Brylinski, 2002]

The quantum circuit model provides a convenient way to think about quantum computers as a system of circuits, analogous to our classical computers



A vastly different model, proposed by Briegel and Raussendorf [Raussendorf and Briegel, 2000], demonstrated that universal quantum computation could be achieved by *measurements alone!*

This so-called cluster model or *one-way quantum computer* (1WQC) relies on an entangled state of a large number of qubits or *cluster state* as the resource.

### Pure Quantum Model

These models have no evident classical analogues and they offer a new perspective on the role of entanglement in quantum computation



If we have a pool of maximally entangled states we can apply any unitary gate  $U$  to any (multi-qubit) input state  $|\psi\rangle$  by measurements alone.

A significant annoyance is that we do not get the exact desired result  $U|\psi\rangle$  but instead get  $PU|\psi\rangle$  where  $P$  is some Pauli operation (on each qubit) depending on the measurement outcome[Jozsa, 2006].



*Cluster states* form a class of multiparty entangled quantum states which belong to the larger set of so-called graph states.

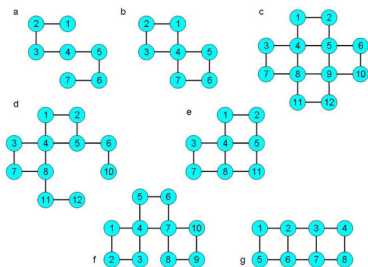
*Examples of graph states:*

- *Bell states*
- *Greenberger-Horne-Zeilinger (GHZ) states*
- *states that appear in quantum error correction*

Intuitively, graph states can be thought of as multi-qubit states that can be represented by a graph.

- Each qubit is represented by a vertex of the graph
- An edge between vertices represents an interacting pair of qubits

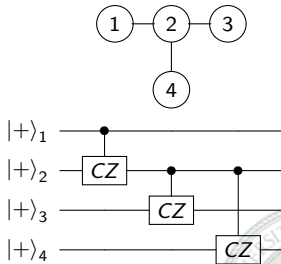




**Figure:** Figure showing representative 2-D cluster state shapes. The vertices are qubits with integer indices, and the edges indicate entanglement connectivity between select neighbors.

Any cluster state can be built with the following algorithm:

- 1 Prepare each of the  $n$  qubits in the state  $|+\rangle$
- 2 Apply controlled-PHASE gates between qubits whose corresponding graph vertices are connected.



A 4-node non-linear cluster state and its associated circuit

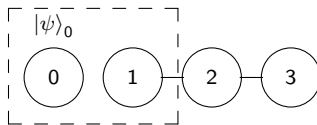
The spacial layout of the graph representation of the cluster state plays a role in the computational power of that state.

Operations on a linearly prepared cluster state can be efficiently simulated on a classical computer in  $O(n \log^c(1/n))$ , where  $n$  is the initial number of qubits, and  $c$  is the cost of floating point multiplication [Nielsen, 2006].

In general, measurement based models can be polynomial time reduced to the gate array model, and thus have the same power, but they are more easily parallelizable [Jozsa, 2006].



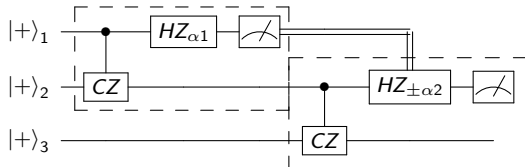


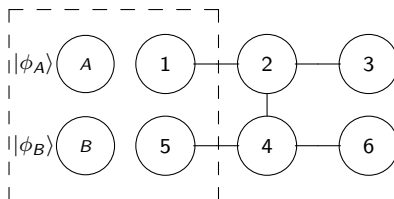


Gate  $C_z^{(0,1)}$ , followed by measurements  $M_X^{(0)}$ ,  $M_X^{(1)}$ , &  $M_X^{(2)}$ .



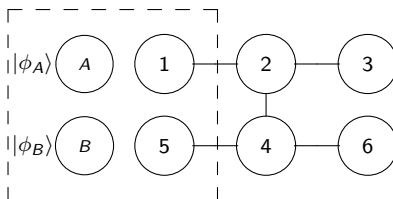
# Callback to teleportation discussion



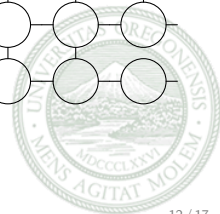
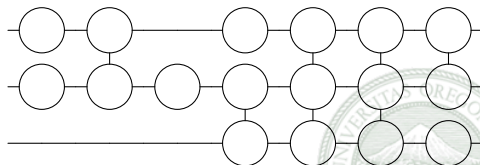
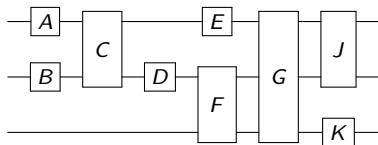


Apply  $C_z^{(A,1)}$  and  $C_z^{(B,5)}$  to input quantum information into cluster state.



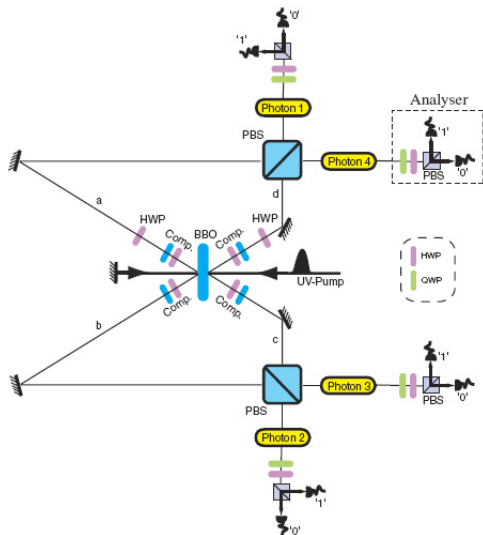


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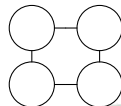




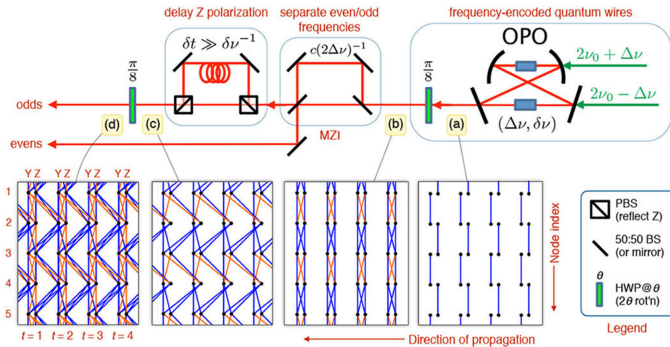
## Four Optical Qubit Cluster State



This generates a square cluster state. Two pairs of entangle photons are created and then separate pairs are entangled.



## Cluster States in Optical Frequency Combs







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