# Cluster State Quantum Computing

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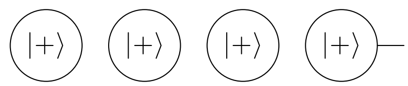
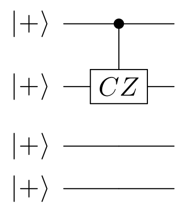
## I. 4-node Cluster State

### A. Linear Cluster State

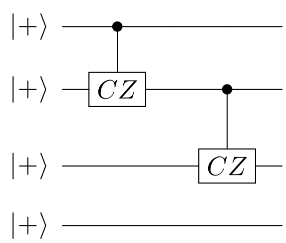



We start with four qubits in the $|+\rangle$ state and apply a CZ gate on the first two qubits to entangle them.



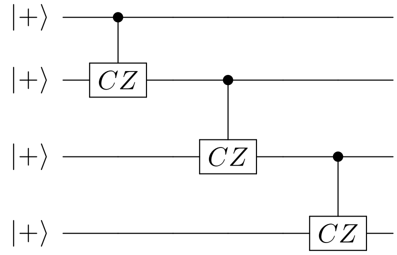

$$CZ_{12}\,|+\rangle_1\,|+\rangle_2\,|+\rangle_3\,|+\rangle_4 = \left(\frac{|0\rangle_1\,|+\rangle_2 + |1\rangle_1\,|-\rangle_2}{\sqrt{2}}\right)|+\rangle_3\,|+\rangle_4 = \left(\frac{|+\rangle_1\,|0\rangle_2 + |-\rangle_1\,|1\rangle_2}{\sqrt{2}}\right)|+\rangle_3\,|+\rangle_4$$

Now we apply a CZ gate to qubits 2 and 3.



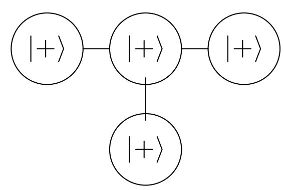

$$CZ_{23}\left(\frac{|+\rangle_1\,|0\rangle_2\,|+\rangle_3 + |-\rangle_1\,|1\rangle_2\,|+\rangle_3}{\sqrt{2}}\right)|+\rangle_4 = \left(\frac{|+\rangle_1\,|0\rangle_2\,|+\rangle_3 + |-\rangle_1\,|1\rangle_2\,|-\rangle_3}{\sqrt{2}}\right)|+\rangle_4$$
$$= \frac{1}{\sqrt{2}}\left[(|+\rangle_1\,|0\rangle_2 + |-\rangle\,|1\rangle)\,|0\rangle_3 + (|+\rangle_1\,|0\rangle_2 - |-\rangle\,|1\rangle)\,|1\rangle_3\right]|+\rangle_4$$

Finally we apply one last CZ gate on the last two qubits.



$$
\begin{aligned}
& CZ_{34}\frac{1}{\sqrt{2}}\left[\left(\left|+\right\rangle_1 \left|0\right\rangle_2 + \left|-\right\rangle \left|1\right\rangle\right) \left|0\right\rangle_3 \left|+\right\rangle_4 + \left(\left|+\right\rangle_1 \left|0\right\rangle_2 - \left|-\right\rangle \left|1\right\rangle\right) \left|1\right\rangle_3 \left|+\right\rangle_4\right] \\
&= \frac{1}{\sqrt{2}}\left[\left(\left|+\right\rangle_1 \left|0\right\rangle_2 + \left|-\right\rangle \left|1\right\rangle\right) \left|0\right\rangle_3 \left|+\right\rangle_4 + \left(\left|+\right\rangle_1 \left|0\right\rangle_2 - \left|-\right\rangle \left|1\right\rangle\right) \left|1\right\rangle_3 \left|-\right\rangle_4\right] \\
&= \frac{1}{\sqrt{2}}\left(\left|+\right\rangle_1 \left|0\right\rangle_2 \left|+\right\rangle_3 + \left|-\right\rangle_1 \left|1\right\rangle_2 \left|-\right\rangle_3\right) \left|0\right\rangle_4 + \frac{1}{\sqrt{2}}\left(\left|+\right\rangle_1 \left|0\right\rangle_2 \left|-\right\rangle_3 + \left|-\right\rangle_1 \left|1\right\rangle_2 \left|+\right\rangle_3\right) \left|1\right\rangle_4
\end{aligned}
$$

### *B. T-shaped Cluster State*



We can start with a three qubit entangled state and apply a CZ gate between the second qubit and a fourth qubit to create a t-shaped cluster state.