Cluster State Quantum Computing

CIS:410/510 Midterm Report, Spring 2016

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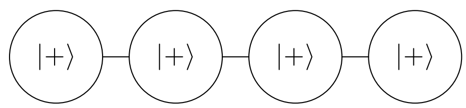
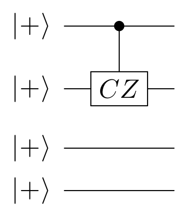
I. 4-node Cluster State

A. Linear Cluster State

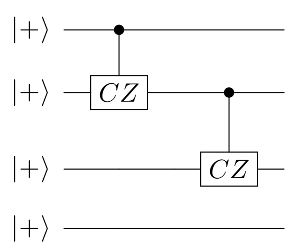


We start with four qubits in the $|+\rangle$ state and apply a CZ gate on the first two qubits to entangle them.



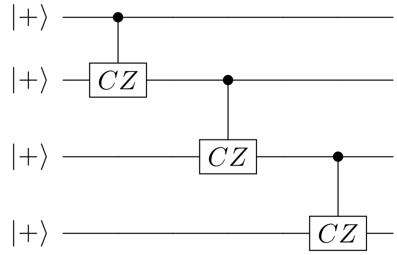
$$CZ_{12}\,|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4 = \left(\frac{|0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2}{\sqrt{2}}\right)|+\rangle_3|+\rangle_4 = \left(\frac{|+\rangle_1|0\rangle_2 + |-\rangle_1|1\rangle_2}{\sqrt{2}}\right)|+\rangle_3|+\rangle_4$$

Now we apply a CZ gate to qubits 2 and 3.



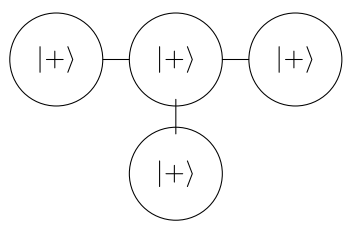
$$CZ_{23}\left(\frac{|+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|+\rangle_3}{\sqrt{2}}\right)|+\rangle_4 = \left(\frac{|+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3}{\sqrt{2}}\right)|+\rangle_4$$
$$= \frac{1}{\sqrt{2}}\left[\left(|+\rangle_1|0\rangle_2 + |-\rangle|1\rangle\right)|0\rangle_3 + \left(|+\rangle_1|0\rangle_2 - |-\rangle|1\rangle\right)|1\rangle_3\right]|+\rangle_4$$

Finally we apply one last CZ gate on the last two qubits.

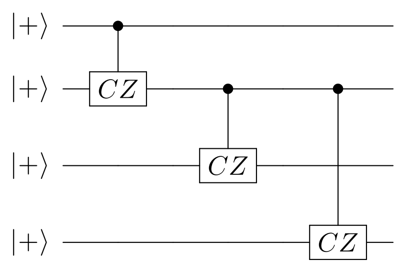


$$
\begin{aligned}
&CZ_{34}\frac{1}{\sqrt{2}}\left[\left(|+\rangle_1|0\rangle_2+|-\rangle|1\rangle\right)|0\rangle_3|+\rangle_4+\left(|+\rangle_1|0\rangle_2-|-\rangle|1\rangle\right)|1\rangle_3|+\rangle_4\right]\\
&=\frac{1}{\sqrt{2}}\left[\left(|+\rangle_1|0\rangle_2+|-\rangle|1\rangle\right)|0\rangle_3|+\rangle_4+\left(|+\rangle_1|0\rangle_2-|-\rangle|1\rangle\right)|1\rangle_3|-\rangle_4\right]\\
&=\frac{1}{\sqrt{2}}\left(|+\rangle_1|0\rangle_2|+\rangle_3+|-\rangle_1|1\rangle_2|-\rangle_3\right)|0\rangle_4+\frac{1}{\sqrt{2}}\left(|+\rangle_1|0\rangle_2|-\rangle_3+|-\rangle_1|1\rangle_2|+\rangle_3\right)|1\rangle_4
\end{aligned}
$$

B. T-shaped Cluster State



We can start with a three qubit entangled state and apply a CZ gate between the second qubit and a fourth qubit to create a t-shaped cluster state.



$$
CZ_{24}\left(\frac{|+\rangle_1|0\rangle_2|+\rangle_3|+\rangle_4+|-\rangle_1|1\rangle_2|+\rangle_3|+\rangle_4}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}\left(|+\rangle_1|0\rangle_2|+\rangle_3|+\rangle_4+|-\rangle_1|1\rangle_2|+\rangle_3|-\rangle_4\right)
$$