Course Project, Spring 2016

Cluster-State Quantum Computing

Mayra Amezcua, Dileep V. Reddy, Zach Schmidt

May 25, 2016

CIS410/510 Introduction to Quantum Information Theory
Lecturer: Prof. Xiaodi Wu
Computer and Information Science, University of Oregon

Table of Contents (optional frame. Can delete.)

template frame (delete me)

test block

some text

test varblock

Variable block (here 4cm)

test alert

some alert

test example

some example citation [1]

[1] Auth, DV, 123, 2001.

The most widely used model of quantum computation at the present time is the quantum circuit model.

A set containing all single-qubit gates and at least one entangling two-qubit gate (such as the controlled-NOT or controlled-PHASE) is universal[Brylinski and Brylinski, 2002]

The quantum circuit model provides a convenient way to think about quantum computers as a system of circuits, analogous to our classical computers

A vastly different model, proposed by Briegel and Raussendorf [Raussendorf and Briegel, 2000], demonstrated that universal quantum computation could be achieved by *measurements alone!*

This so-called cluster model or *one-way quantum computer* (1WQC) relies on an entangled state of a large number of qubits or *cluster state* as the resource.

Pure Quantum Model

These models have no evident classical analogues and they offer a new perspective on the role of entanglement in quantum computation

If we have a pool of maximally entangled states we can apply any unitary gate U to any (multi-qubit) input state $|\psi\rangle$ by measurements alone.

A significant annoyance is that we do not get the exact desired result $U|\psi\rangle$ but instead get $PU|\psi\rangle$ where P is some Pauli operation (on each qubit) depending on the measurement outcome[Jozsa, 2006].

Cluster states form a class of multiparty entangled quantum states which belong to the larger set of so-called graph states.

Examples of graph states:

- *Bell states*
- *Greenberger-Horne-Zeilinger (GHZ) states*
- *states that appear in quantum error correction*

Intuitively, graph states can be thought of as multi-qubit states that can be represented by a graph.

- Each qubit is represented by a vertex of the graph
- An edge between vertices represents an interacting pair of qubits

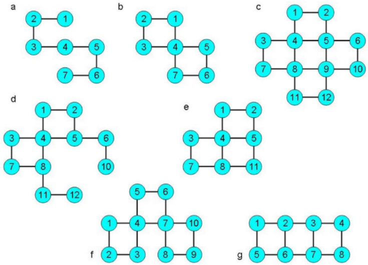
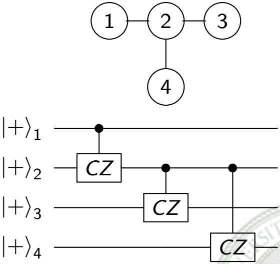


Figure: Figure showing representative 2-D cluster shapes. The vertices are qubits with integer indices, and the edges indicate entanglement connectivity between select neighbors.

Any cluster state can be built with the following algorithm:

1. Prepare each of the *n* qubits in the state $|+\rangle$
2. Apply controlled-PHASE gates between qubits whose corresponding graph vertices are connected.

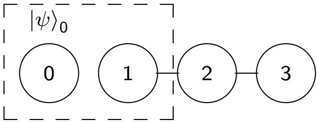


A 4-node non-linear cluster state and its associated circuit

The spacial layout of the graph representation of the cluster state plays a role in the computational power of that state.

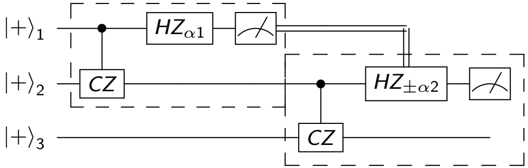
Operations on a linearly prepared cluster state can be efficiently simulated on a classical computer in $O(n \log^c(1/n))$, where n is the initial number of qubits, and c is the cost of floating point multiplication [Nielsen, 2006].

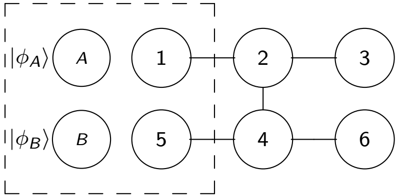
In general, measurement based models can be polynomial time reduced to the gate array model, and thus have the same power, but they are more easily parallelizable [Jozsa, 2006].



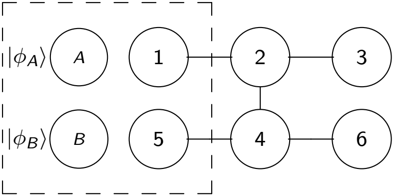
Gate $C_z^{(0,1)}$, followed by measurements $M_X^{(0)}$, $M_X^{(1)}$, & $M_X^{(2)}$.

Callback to teleportation discussion

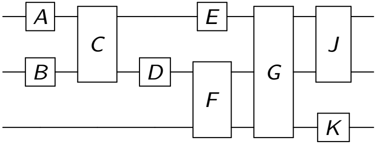


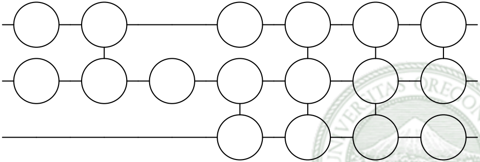


Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.

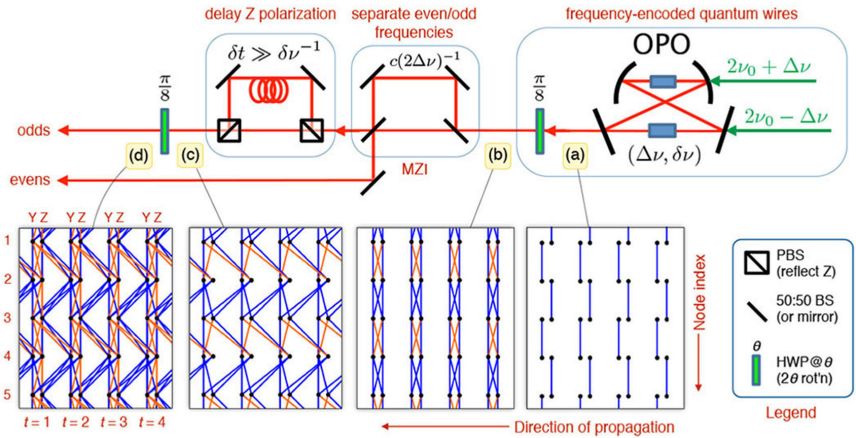


Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.





Cluster States in Frequency Combs



[Brylinski and Brylinski, 2002] Brylinski, J.-L. and Brylinski, R. (2002).
Universal quantum gates.
Mathematics of Quantum Computation, 20022356.

[Jozsa, 2006] Jozsa, R. (2006).
An introduction to measurement based quantum computation.
NATO Science Series, III: Computer and Systems Sciences. Quantum Information Processing-From Theory to Experiment, 199:137–158.

[Nielsen, 2006] Nielsen, M. A. (2006).
Cluster-state quantum computation.
Reports on Mathematical Physics, 57(1):147–161.

[Raussendorf and Briegel, 2000] Raussendorf, R. and Briegel, H. J. (2000).
Quantum computing via measurements only.
eprint arXiv:quant-ph/0010033.