Course Project, Spring 2016

Cluster-State Quantum Computing

Mayra Amezcua, Dileep V. Reddy, Zach Schmidt

May 25, 2016

CIS410/510 Introduction to Quantum Information Theory
Lecturer: Prof. Xiaodi Wu
Computer and Information Science, University of Oregon

Table of Contents (optional frame. Can delete.)

The most widely used model of quantum computation at the present time is the quantum circuit model.

A set containing all single-qubit gates and at least one entangling two-qubit gate (such as the controlled-NOT or controlled-PHASE) is universal[Brylinski and Brylinski, 2002]

The quantum circuit model provides a convenient way to think about quantum computers as a system of circuits, analogous to our classical computers

A vastly different model, proposed by Briegel and Raussendorf [Raussendorf and Briegel, 2000], demonstrated that universal quantum computation could be achieved by *measurements alone!*

This so-called cluster model or *one-way quantum computer* (1WQC) relies on an entangled state of a large number of qubits or *cluster state* as the resource.

Pure Quantum Model

These models have no evident classical analogues and they offer a new perspective on the role of entanglement in quantum computation

If we have a pool of maximally entangled states we can apply any unitary gate U to any (multi-qubit) input state $|\psi\rangle$ by measurements alone.

A significant annoyance is that we do not get the exact desired result $U|\psi\rangle$ but instead get $PU|\psi\rangle$ where P is some Pauli operation (on each qubit) depending on the measurement outcome[Jozsa, 2006].

Cluster states form a class of multiparty entangled quantum states which belong to the larger set of so-called graph states.

Examples of graph states:

- *Bell states*
- *Greenberger-Horne-Zeilinger (GHZ) states*
- *states that appear in quantum error correction*

Intuitively, graph states can be thought of as multi-qubit states that can be represented by a graph.

- Each qubit is represented by a vertex of the graph
- An edge between vertices represents an interacting pair of qubits

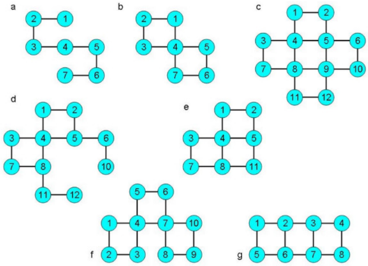
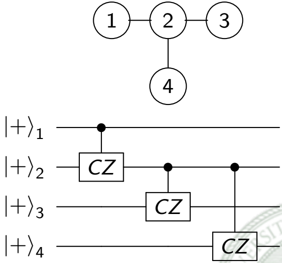


Figure: Figure showing representative 2-D cluster shapes. The vertices are qubits with integer indices, and the edges indicate entanglement connectivity between select neighbors.

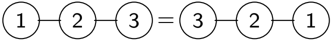
Any cluster state can be built with the following algorithm:

1. Prepare each of the n qubits in the state $|+\rangle$
2. Apply controlled-PHASE gates between qubits whose corresponding graph vertices are connected.



A 4-node non-linear cluster state and its associated circuit

Note that the numbering on the nodes is arbitrary:



This is immediately apparent when looking at the actual state:

$$\frac{|+\rangle\,|+\rangle\,|+\rangle + |-\rangle\,|-\rangle\,|-\rangle}{\sqrt{2}}$$

Notice this state is symmetric – 'numbering' the qubits is unnecessary in this case

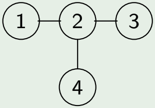
This symmetry can make it easy to see how these states will be built.
Consider an input of $|+++\rangle$. When this is run through a C_Z gate on qubits 1 and 2 produces the state:

$$\frac{(|0\rangle\,|+\rangle + |1\rangle\,|-\rangle)\,|+\rangle}{\sqrt{2}}$$

Which is now obviously equal to the state:

$$\frac{(|+\rangle\,|0\rangle + |-\rangle\,|1\rangle)\,|+\rangle}{\sqrt{2}}$$

4-node Cluster State



The state representing the above graph is given by:

$$\frac{|+\rangle_1 |0\rangle_2 |+\rangle_3 |+\rangle_4 + |-\rangle_1 |1\rangle_2 |-\rangle_3 |-\rangle_4}{\sqrt{2}}$$

If we were to measure qubit 4, this would 'sever' the vertex from the graph, and it would impart a Hadamard on every vertex that was adjacent, resulting in the following state:

$$\frac{|+\rangle_1 |+\rangle_2 |+\rangle_3 + |-\rangle_1 |-\rangle_2 |-\rangle_3}{\sqrt{2}}$$

This was an example of a dynamical operation, i.e. this is the 1WQC analogue of a gate!

The spacial layout of the graph representation of the cluster state plays a role in the computational power of that state.

Operations on a linearly prepared cluster state can be efficiently simulated on a classical computer in $O(n \log^c(1/n))$, where n is the initial number of qubits, and c is the cost of floating point multiplication [Nielsen, 2006]

This fact means that (barring strange outcomes) one dimensional cluster states are not sufficient for universal quantum computation.

This begs the question: *How many dimensions are necessary?*

It turns out that the two dimensional cluster model is polynomial time equivalent to the gate array model[Jozsa, 2006]

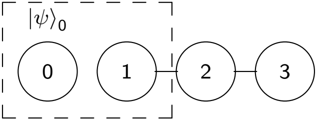
We can see that the gate array model allows for arbitrary measurements at any point, and we can further see that subsequent gates and measurements can be made to depend on earlier measurement outcomes. The idea is as follows:

- Attach an ancilla qubit A in state $|0\rangle$ to a state before applying some gate U to another qubit B
- Apply $U^{\dagger}$ to B and then apply C_X to BA
- Subsequent gates that depend on the measurement outcome are replaced by a corresponding controlled operation, controlled by the state of A

This (general) idea allows us to convert between the cluster and gate array models.

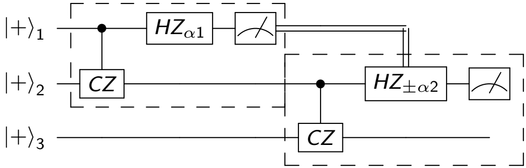
Why Bother?

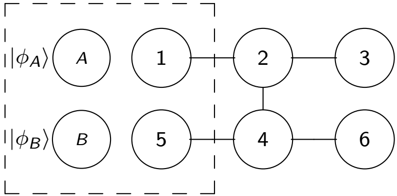
Since there is a polynomial time reduced to the gate array model (implying they have the same power) they may not seem to be worthy of further study, but they are more easily parallelizable [Jozsa, 2006]



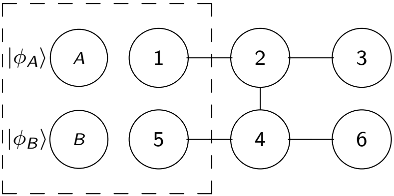
Gate $C_z^{(0,1)}$, followed by measurements $M_X^{(0)}$, $M_X^{(1)}$, & $M_X^{(2)}$.

Callback to teleportation discussion

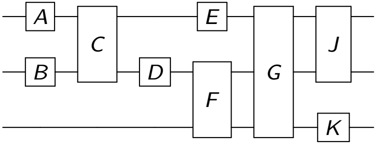


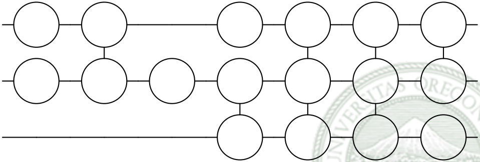


Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.



Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.





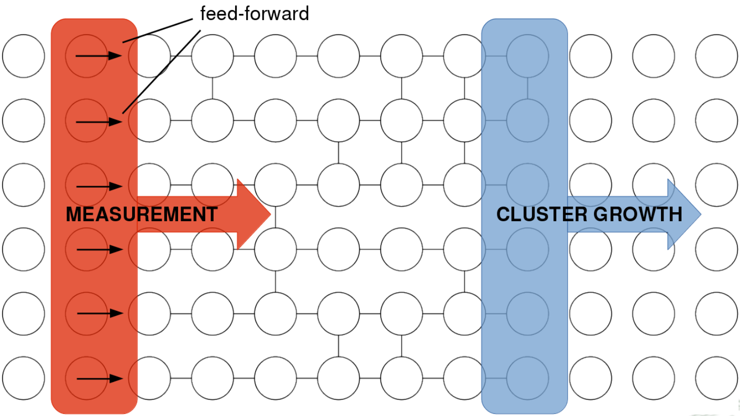


Figure: The controlled-phase operations commute with unitaries and measurements on other parts of the cluster state. This allows one to conserve and reuse physical resourses, as well as maintain coherence on smaller cluster sizes at any given time.

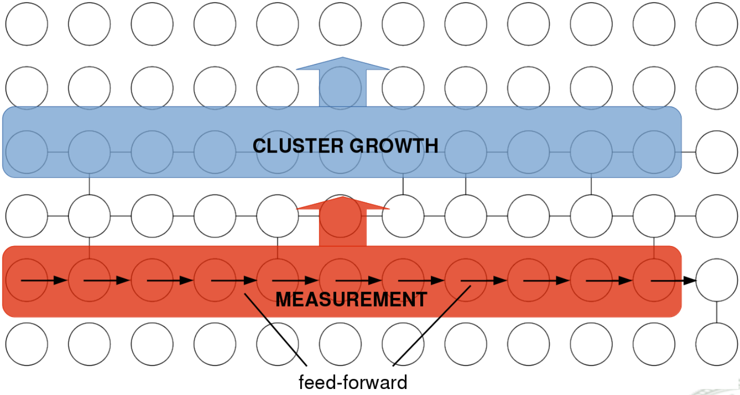
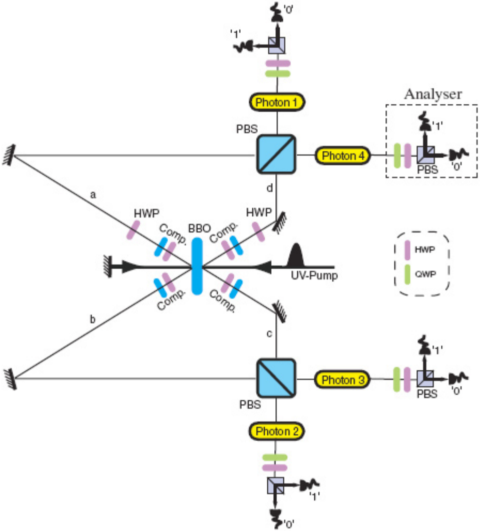
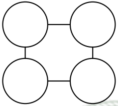


Figure: X-measurements do not sever "vertical" links despite destruction of qubits. This allows different linear layers to be processed in any order.

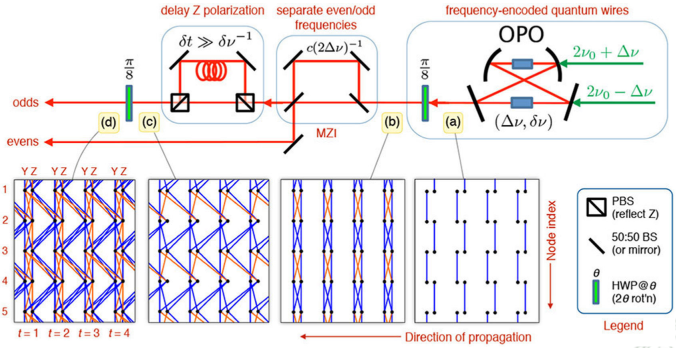
Four Optical Qubit Cluster State



This generates a square cluster state. Two pairs of entangle photons are created and then separate pairs are entangled.



Cluster States in Optical Frequency Combs



Coming soon

Text about polynomial time classical simulation of linear cluster chain operations, and necessity of at least 2-D cluster states for true quantum speed up of certain algorithms. This separates product-state preparation from entangling operations.

[Brylinski and Brylinski, 2002] Brylinski, J.-L. and Brylinski, R. (2002).
Universal quantum gates.
Mathematics of Quantum Computation, 20022356.

[Jozsa, 2006] Jozsa, R. (2006).
An introduction to measurement based quantum computation.
NATO Science Series, III: Computer and Systems Sciences. Quantum Information Processing-From Theory to Experiment, 199:137–158.

[Nielsen, 2006] Nielsen, M. A. (2006).
Cluster-state quantum computation.
Reports on Mathematical Physics, 57(1):147–161.

[Raussendorf and Briegel, 2000] Raussendorf, R. and Briegel, H. J. (2000).
Quantum computing via measurements only.
eprint arXiv:quant-ph/0010033.