Course Project, Spring 2016

Cluster-State Quantum Computing

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CIS410/510 Introduction to Quantum Information Theory
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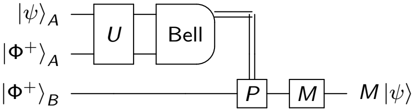
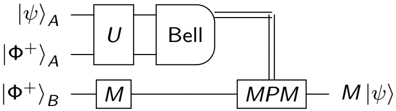
Teleportation

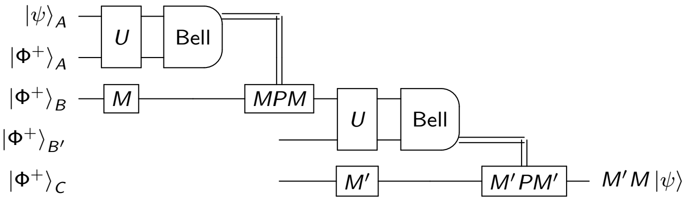


Single Quantum Gate Teleportation

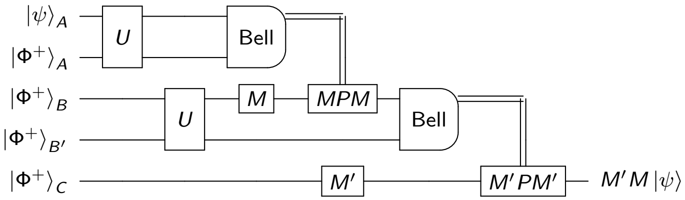


P depends on the two classical bits that Alice sends Bob.

Multiple Quantum Gate Teleportation



Rearrange the circuit to apply all of the entanglement gates first.



Cluster states form a class of multiparty entangled quantum states which belong to the larger set of so-called graph states.

Examples of graph states:

- *Bell states*
- *Greenberger-Horne-Zeilinger (GHZ) states*
- *states that appear in quantum error correction*

Intuitively, graph states can be thought of as multi-qubit states that can be represented by a graph.

- Each qubit is represented by a vertex of the graph
- An edge between vertices represents an interacting pair of qubits

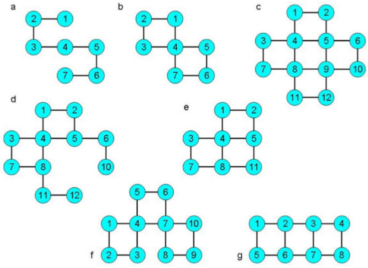
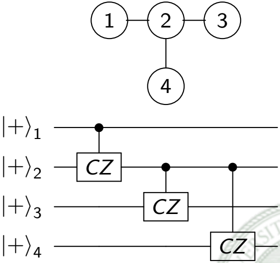


Figure: Figure showing representative 2-D cluster shapes. The vertices are qubits with integer indices, and the edges indicate entanglement connectivity between select neighbors.

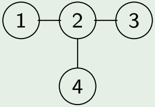
Any cluster state can be built with the following algorithm:

1. Prepare each of the n qubits in the state $|+\rangle$
2. Apply controlled-PHASE gates between qubits whose corresponding graph vertices are connected.



A 4-node non-linear cluster state and its associated circuit

4-node Cluster State



The state representing the above graph is given by:

$$\frac{|+\rangle_1 |0\rangle_2 |+\rangle_3 |+\rangle_4 + |-\rangle_1 |1\rangle_2 |-\rangle_3 |-\rangle_4}{\sqrt{2}}$$

If we were to measure qubit 4, this would 'sever' the vertex from the graph, and it would impart a Hadamard on every vertex that was adjacent, resulting in the following state:

$$\frac{|+\rangle_1 |+\rangle_2 |+\rangle_3 + |-\rangle_1 |-\rangle_2 |-\rangle_3}{\sqrt{2}}$$

This was an example of a dynamical operation, i.e. this is the 1WQC analogue of a gate!

The spacial layout of the graph representation of the cluster state plays a role in the computational power of that state.

Operations on a linearly prepared cluster state can be efficiently simulated on a classical computer in $O(n \log^c(1/n))$, where n is the initial number of qubits, and c is the cost of floating point multiplication [Nielsen, 2006]

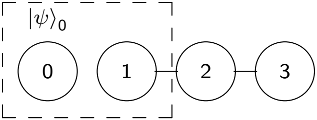
This fact means that (barring strange outcomes) one dimensional cluster states are not sufficient for universal quantum computation.

It turns out that all we need is two dimensions to allow universal quantum computation.

Teleportation

A linear cluster state acts as a wire to teleport information from one end to the other.

A single bit of quantum information is stored in $|\psi\rangle_0$. The state is teleported to qubit 3 by entangling $|\psi\rangle_0$ and $|1\rangle$ and then making a series of measurements.



Gate $C_z^{(0,1)}$, followed by measurements $M_X^{(0)}$, $M_X^{(1)}$, & $M_X^{(2)}$, with outcomes m_0, m_1, m_2, $\in \{0, 1\}$.

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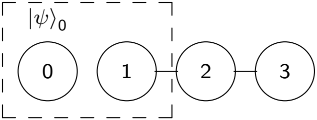

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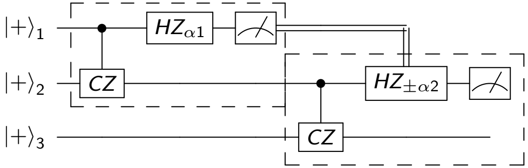
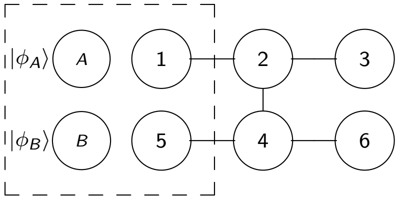
$$C_z^{(0,1)} |\psi\rangle_0 \otimes |\mathcal{LC}\rangle_{123} \xrightarrow{M_X^{(0,1,2)}} X^{m_2} Z^{m_1} X^{m_0} H |\psi\rangle_3 , \qquad m_j \in \{0, 1\}.$$

Gates can be applied as a series of one-bit teleportation. In this case these two one-bit teleportations can be implemented in any order because they commute! This also means that you can grow the cluster state to be more complex.



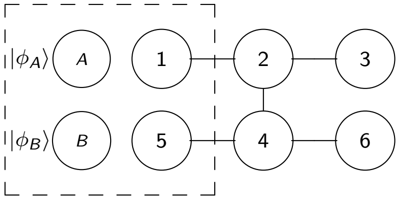
$$X^{m_2} HZ_{\pm\alpha 2} X^{m_1} HZ_{\alpha 1} \left|+\right\rangle_3 \equiv X^{m_2} Z^{m_1} HZ_{\alpha_2} HZ_{\alpha_1} \left|+\right\rangle_3$$



Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.

$$\frac{1}{\sqrt{2}}\left[|\mathcal{LC}\rangle_{123}|0\rangle_4|+\rangle_5|+\rangle_6 + Z_2|\mathcal{LC}\rangle_{123}|0\rangle_4|-\rangle_5|-\rangle_6\right]$$

$$\otimes|\phi_A\rangle_A|\phi_B\rangle_B\,,$$



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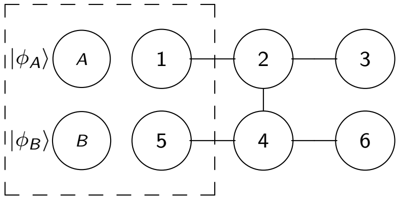
$$\to X_6^{m_4}Z_6^{m_5}X_6^{m_B}C_Z^{(2,6)}|\mathcal{LC}\rangle_{123}|\phi_B\rangle_6\otimes|\phi_A\rangle_A\,.$$



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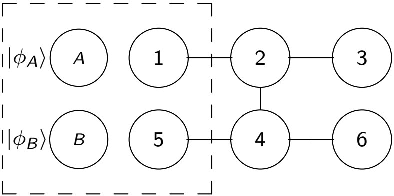
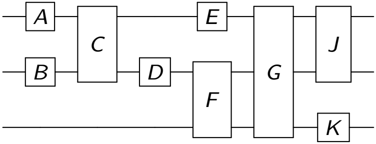
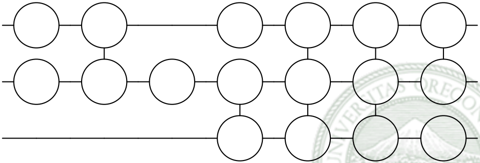
Measurement sequences: $(M_X^{(A)}, M_X^{(B)}) \to (M_X^{(1)}, M_X^{(5)}) \to (M_X^{(2)}, M_X^{(4)})$, or $(M_X^{(B)} \to M_X^{(5)} \to M_X^{(4)}) \to (M_X^{(A)} \to M_X^{(1)} \to M_X^{(2)})$



Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.





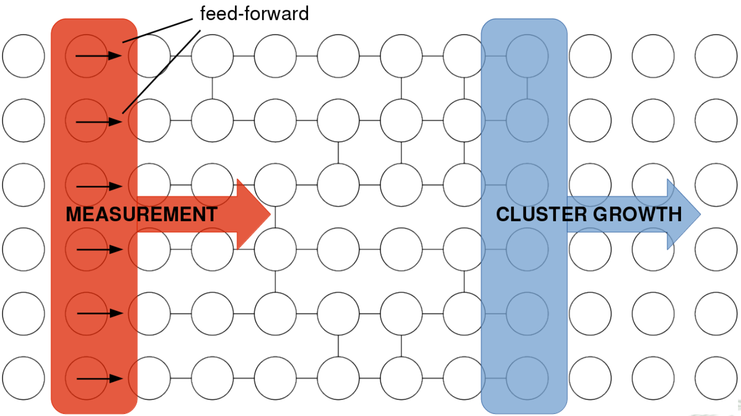


Figure: The controlled-phase operations commute with unitaries and measurements on other parts of the cluster state. This allows one to conserve and reuse physical resourses, as well as maintain coherence on smaller cluster sizes at any given time.

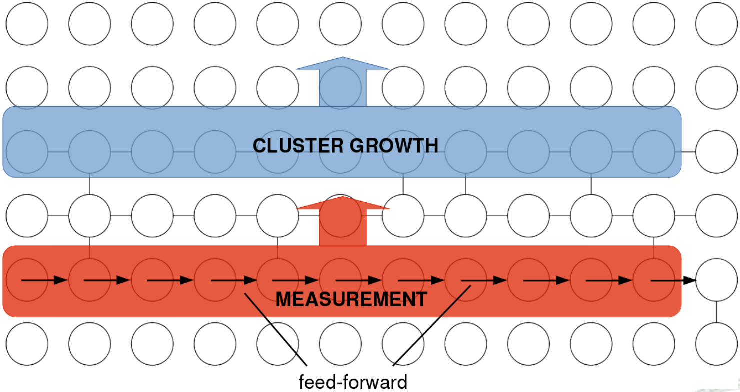
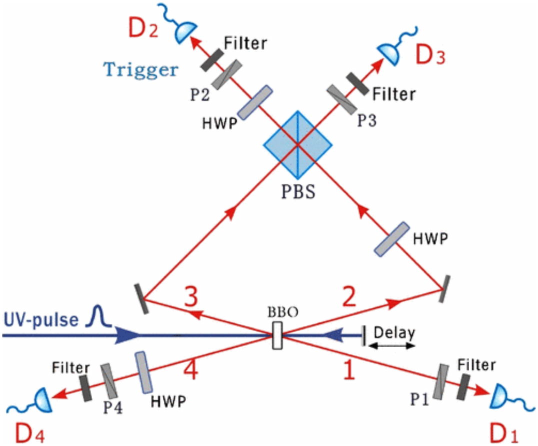


Figure: X-measurements do not sever "vertical" links despite destruction of qubits. This allows different linear layers to be processed in any order.

Three Optical Qubit (Photon) Cluster State



[Zhang et al., 2006]

- This generates a linear cluster state by fusing together two pairs of entangled photons.
- Uses type-I fusion, which annihilates a photon but entangles it's partner with the second pair.
- Optical components act as gates. HWP = Hadamard gate

It turns out that the two dimensional cluster model is polynomial time equivalent to the gate array model[Jozsa, 2006]

We can see that the gate array model allows for arbitrary measurements at any point, and we can further see that subsequent gates and measurements can be made to depend on earlier measurement outcomes. The idea is as follows:

- Attach an ancilla qubit A in state $|0\rangle$ to a state before applying some gate U to another qubit B
- Apply $U^\dagger$ to B and then apply C_X to BA
- Subsequent gates that depend on the measurement outcome are replaced by a corresponding controlled operation, controlled by the state of A

This (general) idea allows us to convert between the cluster and gate array models.

Why Bother?

Since there is a polynomial time reduced to the gate array model (implying they have the same power) they may not seem to be worthy of further study, but they are more easily parallelizable [Jozsa, 2006]

[Jozsa, 2006] Jozsa, R. (2006).

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