Course Project, Spring 2016

Cluster-State Quantum Computing

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CIS410/510 Introduction to Quantum Information Theory
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The most widely used model of quantum computation at the present time is the quantum circuit model.

A set containing all single-qubit gates and at least one entangling two-qubit gate (such as the controlled-NOT or controlled-PHASE) is universal[Brylinski and Brylinski, 2002]

A vastly different model, proposed by Briegel and Raussendorf [Raussendorf and Briegel, 2000], demonstrated that universal quantum computation could be achieved by *measurements alone!*

This so-called cluster model or *one-way quantum computer* (1WQC) relies on an entangled state of a large number of qubits or *cluster state* as the resource.

> *These models have no evident classical analogues and they offer a new perspective on the role of entanglement in quantum computation*

Gates through teleportation

If we have a pool of maximally entangled states we can apply any unitary gate U to any (multi-qubit) input state $|\psi\rangle$ by measurements alone.

A significant annoyance is that we do not get the exact desired result $U|\psi\rangle$ but instead get $PU|\psi\rangle$ where P is some Pauli operation (on each qubit) depending on the measurement outcome[Jozsa, 2006].

Cluster states form a class of multiparty entangled quantum states which belong to the larger set of so-called graph states.

Examples of graph states:

- *Bell states*
- *Greenberger-Horne-Zeilinger (GHZ) states*
- *states that appear in quantum error correction*

Intuitively, graph states can be thought of as multi-qubit states that can be represented by a graph.

- Each qubit is represented by a vertex of the graph
- An edge between vertices represents an interacting pair of qubits

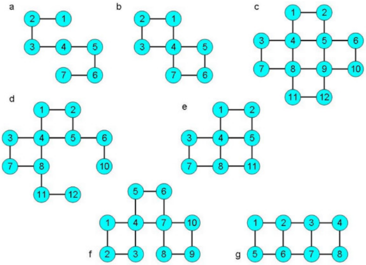
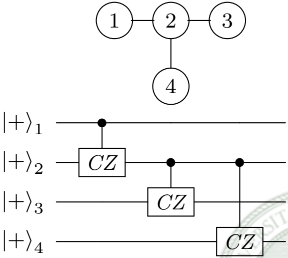


Figure: Figure showing representative 2-D cluster shapes. The vertices are qubits with integer indices, and the edges indicate entanglement connectivity between select neighbors.

Any cluster state can be built with the following algorithm:

1. Prepare each of the n qubits in the state $|+\rangle$
2. Apply controlled-PHASE gates between qubits whose corresponding graph vertices are connected.

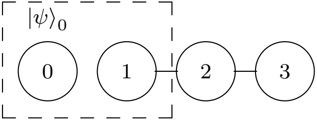


A 4-node non-linear cluster state and its associated circuit

The spacial layout of the graph representation of the cluster state plays a role in the computational power of that state.

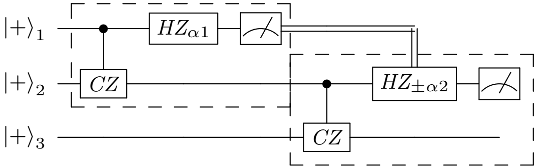
Operations on a linearly prepared cluster state can be efficiently simulated on a classical computer in $O(n \log^c(1/n))$, where n is the initial number of qubits, and c is the cost of floating point multiplication [Nielsen, 2006].

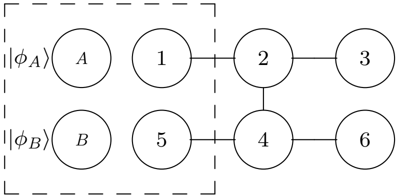
In general, measurement based models can be polynomial time reduced to the gate array model, and thus have the same power, but they are more easily parallelizable [Jozsa, 2006].



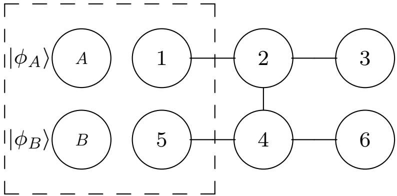
Gate $C_z^{(0,1)}$, followed by measurements $M_X^{(0)}$, $M_X^{(1)}$, & $M_X^{(2)}$.

Callback to teleportation discussion





Apply $C_z^{(A,1)}$ and $C_z^{(B,5)}$ to input quantum information into cluster state.



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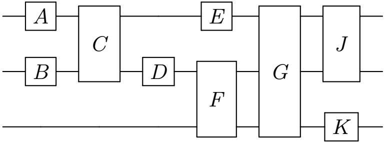

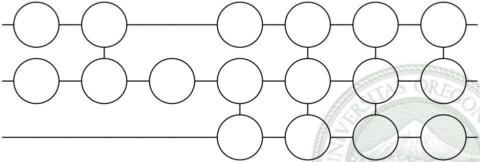

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