Cluster State Computing

Dileep Reddy, Maira Amezcua, Zach Schmidt
dileep@uoregon.edu, mamezcua@cas.uoregon.edu, zschmidt@cs.uoregon.edu

Abstract—**Computation can be done via measurement only, the outcome of which is dependant on the initial entangled state – the *cluster state*. This paper will attempt to provide background regarding computation using only measurements, a brief forray into the preparation of cluster states, a discussion of one way quantum computers (1WQC), and the computational power of various configurations of a 1WQC.**

I. Background

This paper will attempt to discuss some background on cluster states, a correspondence between 1WQC and the more traditional gate array model, and the computational power of this new model.

A. Cluster States

A cluster state is characterized by a set of eigenvalue equations, which are are determined by the distribution of the qubits on some lattice[1]. Briegel and Raussendorf show that any quantum logic circuit can be implemented on a cluster state, which demonstrates universality of the proposed scheme[2]. Nielsen[3] extended this result to no longer require coherent dynamics, instead relying on a method to teleport quantum gates, and he provided a concise algorithm to do it. A method to prepare a one-dimensional cluster state is given in [4], consisting of "cascading" Controlled-Z (C_z) gates on n qubits, where:

$$C_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The action of the C_z gate in the computational basis can be seen to be $|x, y\rangle \to (-1)^{xy} |x, y\rangle$. The set of all single qubit unitaries, coupled with either C_x or C_z has been shown to be universal[5].

B. One-Way Quantum Computation

The unidirectionality of cluster state computation is inherent, due to the fact that the entanglement is progressively consumed at every step. An execution on a one-way quantum computer is a sequence of one-qubit measurements on a cluster state[4].

C. Gate Array Correspondence

In his analysis of the reducability of 1WQC to the gate array model, Richard Jozsa gives a polynomial time algorithm to perform the conversion between the two computational models[6].

D. Computational Power and Complexity

The gate teleportion algorithm[3] has a time complexity of $O(\log(1/\epsilon)$, where ϵ is the failure probability. In general, measurement based models can be polynomial time reduced to the gate array model, and thus have the same power, but they are more easily parallelizable[6].

References

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