Cluster State Quantum Computing

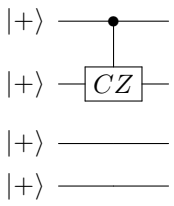
CIS:410/510 Midterm Report, Spring 2016

Dileep Reddy, Mayra Amezcua, Zach Schmidt
dileep@uoregon.edu, mamezcua@cas.uoregon.edu, zschmidt@cs.uoregon.edu

I. 4-node Cluster State

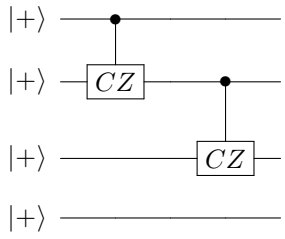
A. Linear Cluster State

We start with four qubits in the $|+\rangle$ state and apply a CZ gate on the first two qubits to entangle them.



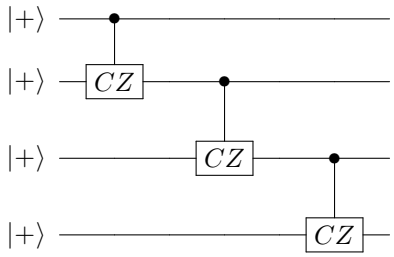
$$CZ_{12}\,|+\rangle_1\,|+\rangle_2\,|+\rangle_3\,|+\rangle_4 = \left(\frac{|0\rangle_1\,|+\rangle_2 + |1\rangle_1\,|-\rangle_2}{\sqrt{2}}\right)|+\rangle_3\,|+\rangle_4 = \left(\frac{|+\rangle_1\,|0\rangle_2 + |-\rangle_1\,|1\rangle_2}{\sqrt{2}}\right)|+\rangle_3\,|+\rangle_4$$

Now we apply a CZ gate to qubits 2 and 3.



$$CZ_{23}\left(\frac{|+\rangle_1\,|0\rangle_2\,|+\rangle_3 + |-\rangle_1\,|1\rangle_2\,|+\rangle_3}{\sqrt{2}}\right)|+\rangle_4 = \left(\frac{|+\rangle_1\,|0\rangle_2\,|+\rangle_3 + |-\rangle_1\,|1\rangle_2\,|-\rangle_3}{\sqrt{2}}\right)|+\rangle_4$$
$$= \frac{1}{\sqrt{2}}\left[\left(|+\rangle_1\,|0\rangle_2 + |-\rangle\,|1\rangle\right)|0\rangle_3 + \left(|+\rangle_1\,|0\rangle_2 - |-\rangle\,|1\rangle\right)|1\rangle_3\right]|+\rangle_4$$

Finally we apply one last CZ gate on the last two qubits.



$$CZ_{34}\frac{1}{\sqrt{2}}\left[\left(|+\rangle_1|0\rangle_2+|-\rangle|1\rangle\right)|0\rangle_3|+\rangle_4+\left(|+\rangle_1|0\rangle_2-|-\rangle|1\rangle\right)|1\rangle_3|+\rangle_4\right]$$

$$=\frac{1}{\sqrt{2}}\left[\left(|+\rangle_1|0\rangle_2+|-\rangle|1\rangle\right)|0\rangle_3|+\rangle_4+\left(|+\rangle_1|0\rangle_2-|-\rangle|1\rangle\right)|1\rangle_3|-\rangle_4\right]$$

$$=\frac{1}{\sqrt{2}}(|+\rangle_1|0\rangle_2|+\rangle_3+|-\rangle_1|1\rangle_2|-\rangle_3)|0\rangle_4+\frac{1}{\sqrt{2}}(|+\rangle_1|0\rangle_2|-\rangle_3+|-\rangle_1|1\rangle_2|+\rangle_3)|1\rangle_4$$

B. T-shaped Cluster State