Assignment #3
CIS 427/527

Group 2

January 26, 2016

1

Show that the following propositions are derivable:

(a) $\varphi \to \varphi$

$$\dfrac{[\varphi]^1}{\varphi \to \varphi} \to I^1$$

(b) $\bot \to \varphi$

$$\dfrac{\dfrac{[\bot]^1}{\varphi}\ \bot E}{\bot \to \varphi} \to I^1$$

(c) $\neg(\varphi \wedge \neg\varphi)$

$$\dfrac{\dfrac{\dfrac{[\varphi \wedge \neg\varphi]_1}{\varphi}\ \wedge E \qquad \dfrac{[\varphi \wedge \neg\varphi]_1}{\neg\varphi}\ \wedge E}{\bot}\ \text{EFQ}}{\neg(\varphi \wedge \neg\varphi)} \to \text{I}^1$$

(d) $(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$

Incomplete

$$\dfrac{\dfrac{\dfrac{\bot}{\neg(\varphi \wedge \neg\psi)}\ \text{RAA}}{(\varphi \to \psi) \to \neg(\varphi \wedge \neg\psi)} \to I \qquad \dfrac{\varphi \to \psi}{\neg(\varphi \wedge \neg\psi) \to (\varphi \to \psi)} \to I}{(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)} \to I$$

(e) $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \wedge \psi]_1}{\psi}\ \wedge\text{E} \qquad \dfrac{\dfrac{[\varphi \wedge \psi]_1}{\varphi}\ \wedge\text{E} \qquad [\varphi \to \neg\psi]_2}{\neg\psi} \to\text{I}}{\bot}}{\neg(\varphi \to \neg\psi)} \to\text{I}^2}{(\varphi \wedge \psi) \to \neg(\varphi \to \neg\psi)} \to\text{I}^1 \qquad \dfrac{\dfrac{\dfrac{[\neg(\varphi \to \neg\psi)]_1 \qquad \dfrac{[\neg\psi]^2}{\varphi \to \neg\psi}}{\bot}\ \text{RAA}^2}{\varphi \wedge \psi}\ \wedge I^1}{\neg(\varphi \to \neg\psi) \to (\varphi \wedge \psi)} \to I}{(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)}$$

(f) $\varphi \to (\psi \to (\varphi \wedge \psi))$

$$\dfrac{\dfrac{\dfrac{[\varphi]^1 \qquad [\psi]^2}{\varphi \wedge \psi}\ \wedge I}{\psi \to (\varphi \wedge \psi)} \to I^2}{\varphi \to (\psi \to (\varphi \wedge \psi))} \to I^1$$

2

Show that the following propositions are derivable:

(a) $(\varphi \to \neg\varphi) \to \neg\varphi$

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \neg\varphi]_1 \qquad [\varphi]_2}{\neg\varphi \wedge \varphi} \to E}{\bot}\, E}{\neg\varphi}\, RAA_2}{(\varphi \to \neg\varphi) \to \neg\varphi} \to I_1$$

(b) $[\varphi \to (\psi \to \sigma)] \leftrightarrow [\psi \to (\varphi \to \sigma)]$
(c) $(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi$
(d) $(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \psi]_1 \qquad [\varphi]_2}{\psi} \to E \qquad \dfrac{[\varphi]_2 \qquad [(\varphi \to (\psi \to \sigma))]_3}{\psi \to \sigma} \to E}{\sigma} \to E}{\varphi \to \sigma} \to I_2}{(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)} \to I_3}{(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]} \to I_1$$

3

Show:

(a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$

$$\dfrac{\dfrac{\dfrac{[\neg\varphi \wedge \psi]_1}{\neg\varphi} \wedge\mathrm{E} \qquad \varphi}{\bot} \to\mathrm{E}}{\neg(\neg\varphi \wedge \psi)} \to I_1$$

(b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
(c) $\neg\varphi \vdash (\varphi \to \psi) \leftrightarrow \neg\varphi$
(d) $\vdash \varphi \Rightarrow \vdash \psi \to \varphi$
(e) $\neg\varphi \vdash \varphi \to \psi$

$$\dfrac{\dfrac{\dfrac{[\varphi]_1 \qquad \neg\varphi}{\bot} \to\mathrm{E}}{\psi}\, \mathrm{EFQ}}{\varphi \to \psi} \to I_1$$