Assignment #3
CIS 427/527

Group 2

January 24, 2016

1

Show that the following propositions are derivable:

(a) $\varphi \to \varphi$

$$\frac{[\varphi]^1}{\varphi \to \varphi} \to I^1$$

(b) $\bot \to \varphi$

$$\frac{\dfrac{[\bot]^1}{\varphi}\bot E}{\bot \to \varphi} \to I^1$$

(c) $\neg(\varphi \wedge \neg\varphi)$

$$\frac{\dfrac{\neg\varphi \qquad \dfrac{[\varphi]}{\neg\varphi \to \varphi}}{\varphi} \qquad \dfrac{\varphi \qquad \dfrac{[\neg\varphi]}{\varphi \to \neg\varphi}}{\neg\varphi}}{\dfrac{\bot}{\neg(\varphi \wedge \neg\varphi)} \to I^1} \wedge I$$

or

$$\frac{\dfrac{\dfrac{[\varphi \wedge \neg\varphi]_1}{\varphi}\wedge E \qquad \dfrac{[\varphi \wedge \neg\varphi]_1}{\neg\varphi}\wedge E}{\bot} E}{\neg(\varphi \wedge \neg\varphi)} \to RAA^1$$

(d) $(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$
(e) $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$
(f) $\varphi \to (\psi \to (\varphi \wedge \psi))$

$$\frac{\dfrac{\dfrac{\dfrac{[\varphi]^1 \qquad [\psi]^2}{\varphi \wedge \psi}\wedge I}{\psi \to (\varphi \wedge \psi)} \to I^2}{\varphi \to (\psi \to (\varphi \wedge \psi))} \to I^1$$

2

Show that the following propositions are derivable:

(a) $(\varphi \to \neg\varphi) \to \neg\varphi$

$$\frac{\dfrac{\dfrac{[\varphi \to \neg\varphi]_1 \qquad [\varphi]_2}{\neg\varphi \wedge \varphi} \to E}{\dfrac{\bot}{\neg\varphi} RAA_2} E}{(\varphi \to \neg\varphi) \to \neg\varphi} \to I_1$$

(b) $[\varphi \to (\psi \to \sigma] \leftrightarrow [\psi \to (\varphi \to \sigma)]$ TYPO – NEED CLARIFICATION
(c) $(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi$
(d) $(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]$

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \psi]_1 \qquad [\varphi]_2}{\psi} \to E \qquad \dfrac{[\varphi]_2 \qquad [(\varphi \to (\psi \to \sigma))]_3}{\psi \to \sigma} \to E}{\sigma} \to E}{\varphi \to \sigma} \to I_2}{(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)} \to I_3}{(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]} \to I_1$$

3

Show:

(a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$
(b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
(c) $\neg\varphi \vdash (\varphi \to \psi) \leftrightarrow \neg\varphi$
(d) $\vdash \varphi \Rightarrow \vdash \psi \to \varphi$
(e) $\neg\varphi \vdash \varphi \to \psi$