

# Assignment #1

## CIS 427/527

Group 2

January 18, 2016

### 1

Since two of the men are telling the truth, we have a proposition of the form  $P = (A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$ .

- If Adams is telling the truth, then Brown is lying ( $A \implies \neg B$ ).
- If Brown is telling the truth, then both Adams and Clark are lying ( $B \implies (\neg A \wedge \neg C)$ ).
- If Clark is telling the truth, then Brown is lying ( $C \implies \neg B$ ).

If Brown were telling the truth, then  $P$  could never be satisfied, therefore Brown is lying, which makes him the killer.

### 2

2. Show that  $((\rightarrow) \notin PROP$

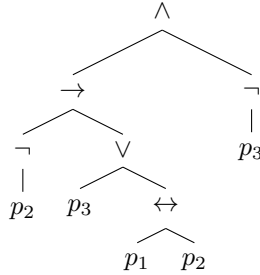
Suppose  $((\rightarrow) \in X$  and  $X$  satisfies (i), (ii), (iii) of Definition 2.1.2. We claim that  $Y = X \setminus \{((\rightarrow)\}$  also satisfies (i), (ii), and (iii).

- (i)  $\perp, p_i \in Y$ ,
- (ii)  $\varphi, \psi \in Y$  and  $(\varphi \Box \psi) \neq ((\rightarrow)$ , it is clear that  $(\varphi \Box \psi) \in Y$
- (iii) Similarly,  $\varphi \in Y$  and  $(\neg \varphi) \neq ((\rightarrow)$ , so  $(\neg \varphi) \in Y$

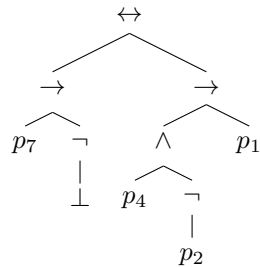
Therefore  $X$  is not the smallest set satisfying (i), (ii), and (iii), so  $((\rightarrow)$  cannot belong to  $PROP$ .

7. (a) Determine the trees of the proposition in Exercise 1

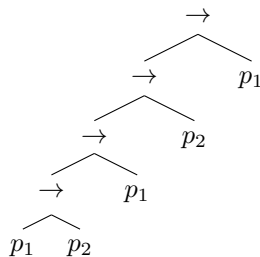
$$(\neg p_2 \rightarrow (p_3 \vee (p_1 \leftrightarrow p_2))) \wedge \neg p_3$$



$$(p_7 \rightarrow \neg \perp) \leftrightarrow ((p_4 \wedge \neg p_2) \rightarrow p_1$$



$$(((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow p_2 \rightarrow p_1$$



(b) Determine the propositions with the trees given

$$\neg\neg\neg\perp$$

$$(p_0 \rightarrow \perp) \rightarrow ((p_0 \leftrightarrow p_1) \wedge p_5)$$

$$\neg((\neg p_1) \rightarrow (\neg p_1))$$

9. Show that a proposition with  $n$  connectives has at most  $2n + 1$  subformulas

**Base Case** A proposition  $P$  with zero connectives has  $\leq 2(0) + 1 = 1$  subformula(s).

**Inductive Case** Inductive Hypothesis: A proposition  $\varphi$  with  $n$  connectives has at most  $2n + 1$  subformulas, where  $s(\varphi)$  gives the number of subformulas.

Assume propositions  $p, q$  (where  $s(p) = n$  and  $s(q) = m \leq n$  connectives) have the property above. If we create a new proposition  $r$  from  $p$  and  $q$  we have:

$$s(r) = s(p) + s(q) + 1$$

$$s(r) = n + m + 1$$

$$s(r) \leq 2n + 1$$

Since the desired property holds for  $r$  the inductive hypothesis holds  $\forall n \in \mathbb{N}$ .

### 3

(a)

$(\neg\varphi \vee \psi) \iff (\psi \rightarrow \varphi)$	$\varphi$	$\psi$	
1	0	0	
0	0	1	Not a tautology.
1	1	0	
1	1	1	

(f)

$(\varphi \vee \neg\varphi)$	$\varphi$	
1	0	Is a tautology.
1	1	

(h)

$(\perp \rightarrow \varphi)$	$\varphi$	
1	0	Is a tautology.
1	1	

### 4

(a)  $\varphi \models \varphi$

(b)  $\varphi \models \psi$  and  $\psi \models \sigma \implies \varphi \models \sigma$

(c)  $\models \varphi \rightarrow \psi \iff \varphi \models \psi$

### 5

### 6

$$\# = \neg(\neg(\neg p \vee \neg q) \vee \neg(p \vee q))$$

### 7

A conjunctive normal form is a tautology iff every clause is also a tautology.

### 8

### 9

$P \rightarrow Q$ : True when  $P$  is false; false when  $P$  is true and  $Q$  is false.

$P \vee Q \rightarrow P \wedge Q$ : True when  $P \iff Q$ ; false otherwise.

$\neg(P \vee Q \vee R)$ : True when  $P, Q, R$  are all false; false when any  $P, Q, R$  are true.

$\neg(P \wedge Q) \wedge \neg(Q \vee R) \wedge (P \vee R)$ : True when  $P$  is true,  $Q$  is false,  $R$  is false; false when  $P$  is false and  $R$  is false.