# Assignment #3
# CIS 427/527

Group 2

January 26, 2016

## 1

Show that the following propositions are derivable:

**(a)** $\varphi \to \varphi$

$$\frac{[\varphi]^1}{\varphi \to \varphi} \to I^1$$

**(b)** $\bot \to \varphi$

$$\frac{\dfrac{[\bot]^1}{\varphi} \bot E}{\bot \to \varphi} \to I^1$$

**(c)** $\neg(\varphi \wedge \neg\varphi)$

$$\frac{\dfrac{\dfrac{[\varphi \wedge \neg\varphi]_1}{\varphi} \wedge E \quad \dfrac{[\varphi \wedge \neg\varphi]_1}{\neg\varphi} \wedge E}{\bot} E}{\neg(\varphi \wedge \neg\varphi)} \to RAA^1$$

**(d)** $(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$

Incomplete

$$\frac{\dfrac{\dfrac{\dfrac{\bot}{\neg(\varphi \wedge \neg\psi)}\,\text{RAA}}{(\varphi \to \psi) \to \neg(\varphi \wedge \neg\psi)} \to I \quad \dfrac{\varphi \to \psi}{\neg(\varphi \wedge \neg\psi) \to (\varphi \to \psi)} \to I}{(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)} \to I$$

**(e)** $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$

$$\frac{\dfrac{\dfrac{\dfrac{[\varphi \wedge \psi]_1}{\psi} \wedge\text{E} \quad \dfrac{[\varphi \wedge \psi]_1}{\varphi} \wedge\text{E} \quad [\varphi \to \neg\psi]_2}{\neg\psi} \to\text{I}}{\dfrac{\bot}{\neg(\varphi \to \neg\psi)} \to\text{I}^2}{(\varphi \wedge \psi) \to \neg(\varphi \to \neg\psi)} \to\text{I}^1 \quad \dfrac{\dfrac{\dfrac{[\neg(\varphi \to \neg\psi)]_1 \quad \dfrac{[\neg\psi]^2}{\varphi \to \neg\psi}}{\bot}\,\text{RAA}^2}{\varphi \wedge \psi} \wedge I^1}{\neg(\varphi \to \neg\psi) \to (\varphi \wedge \psi)} \to I}{(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)}$$

**(f)** $\varphi \to (\psi \to (\varphi \wedge \psi))$

$$\frac{\dfrac{\dfrac{[\varphi]^1 \quad [\psi]^2}{\varphi \wedge \psi} \wedge I}{\psi \to (\varphi \wedge \psi)} \to I^2}{\varphi \to (\psi \to (\varphi \wedge \psi))} \to I^1$$

## 2

Show that the following propositions are derivable:

**(a)** $(\varphi \to \neg\varphi) \to \neg\varphi$

$$\dfrac{\dfrac{\dfrac{[\varphi \to \neg\varphi]_1 \qquad [\varphi]_2}{\neg\varphi \wedge \varphi}\to E}{\dfrac{\bot}{\neg\varphi}\,E}\,RAA_2}{(\varphi \to \neg\varphi) \to \neg\varphi}\to I_1$$

**(b)** $[\varphi \to (\psi \to \sigma)] \leftrightarrow [\psi \to (\varphi \to \sigma)]$
**(c)** $(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi$
**(d)** $(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \psi]_1 \qquad [\varphi]_2}{\psi}\to E \qquad \dfrac{[\varphi]_2 \qquad [(\varphi \to (\psi \to \sigma))]_3}{\psi \to \sigma}\to E}{\sigma}\to E}{\varphi \to \sigma}\to I_2}{(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)}\to I_3}{(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]}\to I_1$$

## 3

Show:

**(a)** $\varphi \vdash \neg(\neg\varphi \wedge \psi)$

$$\dfrac{\dfrac{\dfrac{[\neg\varphi \wedge \psi]_1}{\neg\varphi}\wedge\mathrm{E} \qquad \varphi}{\bot}\to\mathrm{E}}{\neg(\neg\varphi \wedge \psi)}\to I_1$$

**(b)** $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
**(c)** $\neg\varphi \vdash (\varphi \to \psi) \leftrightarrow \neg\varphi$
**(d)** $\vdash \varphi \Rightarrow \vdash \psi \to \varphi$
**(e)** $\neg\varphi \vdash \varphi \to \psi$