Assignment #3
CIS 427/527

Group 2

January 26, 2016

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Show that the following propositions are derivable:

(a) $\varphi \to \varphi$

$$\begin{prooftree}
\AxiomC{$[\varphi]^1$}
\RightLabel{$\to I^1$}
\UnaryInfC{$\varphi \to \varphi$}
\end{prooftree}$$

(b) $\bot \to \varphi$

$$\begin{prooftree}
\AxiomC{$[\bot]^1$}
\RightLabel{$\bot E$}
\UnaryInfC{φ}
\RightLabel{$\to I^1$}
\UnaryInfC{$\bot \to \varphi$}
\end{prooftree}$$

(c) $\neg(\varphi \wedge \neg\varphi)$

$$\begin{prooftree}
\AxiomC{$[\varphi \wedge \neg\varphi]_1$}
\RightLabel{$\wedge E$}
\UnaryInfC{φ}
\AxiomC{$[\varphi \wedge \neg\varphi]_1$}
\RightLabel{$\wedge E$}
\UnaryInfC{$\neg\varphi$}
\RightLabel{E}
\BinaryInfC{$\bot$}
\RightLabel{$\to RAA^1$}
\UnaryInfC{$\neg(\varphi \wedge \neg\varphi)$}
\end{prooftree}$$

(d) $(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$

Incomplete

$$\begin{prooftree}
\AxiomC{}
\UnaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{$\neg(\varphi \wedge \neg\psi)$}
\RightLabel{$\to I$}
\UnaryInfC{$(\varphi \to \psi) \to \neg(\varphi \wedge \neg\psi)$}
\AxiomC{$\varphi \to \psi$}
\RightLabel{$\to I$}
\UnaryInfC{$\neg(\varphi \wedge \neg\psi) \to (\varphi \to \psi)$}
\RightLabel{$\to I$}
\BinaryInfC{$(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$}
\end{prooftree}$$

(e) $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$

$$\begin{prooftree}
\AxiomC{$[\varphi \wedge \psi]_1$}
\RightLabel{$\wedge$E}
\UnaryInfC{ψ}
\AxiomC{$[\varphi \wedge \psi]_1$}
\RightLabel{$\wedge$E}
\UnaryInfC{φ}
\AxiomC{$[\varphi \to \neg\psi]_2$}
\RightLabel{$\to$I}
\BinaryInfC{$\neg\psi$}
\BinaryInfC{$\bot$}
\RightLabel{$\to$I^2}
\UnaryInfC{$\neg(\varphi \to \neg\psi)$}
\RightLabel{$\to$I^1}
\UnaryInfC{$(\varphi \wedge \psi) \to \neg(\varphi \to \neg\psi)$}
\AxiomC{$[\neg(\varphi \to \neg\psi)]_1$}
\AxiomC{$[\neg\psi]^2$}
\UnaryInfC{$\varphi \to \neg\psi$}
\RightLabel{RAA^2}
\BinaryInfC{$\bot$}
\RightLabel{$\wedge I^1$}
\UnaryInfC{$\varphi \wedge \psi$}
\RightLabel{$\to I$}
\UnaryInfC{$\neg(\varphi \to \neg\psi) \to (\varphi \wedge \psi)$}
\BinaryInfC{$(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$}
\end{prooftree}$$

(f) $\varphi \to (\psi \to (\varphi \wedge \psi))$

$$\begin{prooftree}
\AxiomC{$[\varphi]^1$}
\AxiomC{$[\psi]^2$}
\RightLabel{$\wedge I$}
\BinaryInfC{$\varphi \wedge \psi$}
\RightLabel{$\to I^2$}
\UnaryInfC{$\psi \to (\varphi \wedge \psi)$}
\RightLabel{$\to I^1$}
\UnaryInfC{$\varphi \to (\psi \to (\varphi \wedge \psi))$}
\end{prooftree}$$

2

Show that the following propositions are derivable:

(a) $(\varphi \to \neg\varphi) \to \neg\varphi$

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \neg\varphi]_1 \qquad [\varphi]_2}{\neg\varphi \wedge \varphi} \to E}{\bot}\, E}{\neg\varphi}\, RAA_2}{(\varphi \to \neg\varphi) \to \neg\varphi} \to I_1$$

(b) $[\varphi \to (\psi \to \sigma)] \leftrightarrow [\psi \to (\varphi \to \sigma)]$
(c) $(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi$
(d) $(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \psi]_1 \qquad [\varphi]_2}{\psi} \to E \qquad \dfrac{[\varphi]_2 \qquad [(\varphi \to (\psi \to \sigma))]_3}{\psi \to \sigma} \to E}{\sigma} \to E}{\varphi \to \sigma} \to I_2}{(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)} \to I_3}{(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]} \to I_1$$

3

Show:

(a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$
(b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
(c) $\neg\varphi \vdash (\varphi \to \psi) \leftrightarrow \neg\varphi$
(d) $\vdash \varphi \Rightarrow \vdash \psi \to \varphi$
(e) $\neg\varphi \vdash \varphi \to \psi$