

Assignment #3

CIS 427/527

Group 2

January 26, 2016

1

Show that the following propositions are derivable:

(a) $\varphi \rightarrow \varphi$

$$\frac{[\varphi]^1}{\varphi \rightarrow \varphi} \rightarrow I^1$$

(b) $\perp \rightarrow \varphi$

$$\frac{\frac{[\perp]^1}{\varphi} \perp E}{\perp \rightarrow \varphi} \rightarrow I^1$$

(c) $\neg(\varphi \wedge \neg\varphi)$

$$\frac{\frac{[\varphi \wedge \neg\varphi]_1}{\varphi} \wedge E \quad \frac{[\varphi \wedge \neg\varphi]_1}{\neg\varphi} \wedge E}{\frac{\perp}{\neg(\varphi \wedge \neg\varphi)} \rightarrow I^1} \text{EFQ}$$

(d) $(\varphi \rightarrow \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$

$$\frac{\frac{\frac{[\varphi \wedge \neg\psi]_1}{\varphi} \wedge E \quad \frac{[\varphi \rightarrow \psi]_2}{\psi}}{\frac{\perp}{\neg(\varphi \wedge \neg\psi)} \rightarrow I^1} \rightarrow E \quad \frac{\frac{[\neg(\varphi \wedge \neg\psi)]_2}{\frac{\perp}{\psi} \text{RAA}^1} \rightarrow E \quad \frac{[\varphi]_3 \quad [\neg\psi]_1}{\varphi \wedge \neg\psi} \wedge I}{\frac{\frac{\perp}{\neg(\varphi \wedge \neg\psi)} \rightarrow I^1 \quad \frac{\frac{\perp}{\psi} \text{RAA}^1}{\varphi \rightarrow \psi} \rightarrow I^3}{\frac{(\varphi \rightarrow \psi) \rightarrow \neg(\varphi \wedge \neg\psi)}{\neg(\varphi \wedge \neg\psi) \rightarrow (\varphi \rightarrow \psi)} \rightarrow I^2} \rightarrow I^2$$

$$\frac{(\varphi \rightarrow \psi) \rightarrow \neg(\varphi \wedge \neg\psi) \quad \neg(\varphi \wedge \neg\psi) \rightarrow (\varphi \rightarrow \psi)}{(\varphi \rightarrow \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)}$$

(e) $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \rightarrow \neg\psi)$

$$\frac{\frac{[\varphi \wedge \psi]_1}{\varphi} \wedge E \quad \frac{[\varphi \wedge \psi]_1}{\psi} \wedge E}{\frac{\perp}{\neg(\varphi \rightarrow \neg\psi)} \rightarrow I^2} \rightarrow I^1 \quad \frac{\frac{[\neg(\varphi \rightarrow \neg\psi)]_1}{\frac{\perp}{\varphi \wedge \psi} \wedge I^1} \text{RAA}^2 \quad \frac{[\neg\psi]^2}{\varphi \rightarrow \neg\psi}}{\frac{\perp}{\neg(\varphi \rightarrow \neg\psi) \rightarrow (\varphi \wedge \psi)} \rightarrow I} \rightarrow I$$

$$\frac{(\varphi \wedge \psi) \rightarrow \neg(\varphi \rightarrow \neg\psi) \quad \neg(\varphi \rightarrow \neg\psi) \rightarrow (\varphi \wedge \psi)}{(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \rightarrow \neg\psi)}$$

(f) $\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$

$$\frac{\frac{[\varphi]^1 \quad [\psi]^2}{\varphi \wedge \psi} \wedge I \quad \frac{\varphi \wedge \psi}{\psi \rightarrow (\varphi \wedge \psi)} \rightarrow I^2}{\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))} \rightarrow I^1$$

2

Show that the following propositions are derivable:

(a) $(\varphi \rightarrow \neg\varphi) \rightarrow \neg\varphi$

$$\frac{\frac{\frac{[\varphi \rightarrow \neg\varphi]_1}{\neg\varphi} \rightarrow E \quad [\varphi]_2 \rightarrow E}{\frac{\perp}{\neg\varphi} \rightarrow I^2} \rightarrow E}{(\varphi \rightarrow \neg\varphi) \rightarrow \neg\varphi} \rightarrow I_1$$

(b) $[\varphi \rightarrow (\psi \rightarrow \sigma)] \leftrightarrow [\psi \rightarrow (\varphi \rightarrow \sigma)]$

$$\frac{\frac{\frac{[\psi \rightarrow (\varphi \rightarrow \sigma)]_3}{\varphi \rightarrow \sigma} \rightarrow E \quad [\psi]_1 \rightarrow E}{\frac{\sigma}{\psi \rightarrow \sigma} \rightarrow I^1} \rightarrow E \quad \frac{[\varphi]_2 \rightarrow E}{\frac{\varphi \rightarrow (\psi \rightarrow \sigma)}{\psi \rightarrow (\varphi \rightarrow \sigma)} \rightarrow I^2} \rightarrow I^3}{[\psi \rightarrow (\varphi \rightarrow \sigma)] \rightarrow [\varphi \rightarrow (\psi \rightarrow \sigma)]} \rightarrow I^3$$

$$\frac{\frac{\frac{[\varphi \rightarrow (\psi \rightarrow \sigma)]_3}{\psi \rightarrow \sigma} \rightarrow E \quad [\varphi]_1 \rightarrow E}{\frac{\sigma}{\varphi \rightarrow \sigma} \rightarrow I^1} \rightarrow E \quad \frac{[\psi]_2 \rightarrow E}{\frac{\psi \rightarrow (\varphi \rightarrow \sigma)}{\varphi \rightarrow (\psi \rightarrow \sigma)} \rightarrow I^2} \rightarrow I^3}{[\varphi \rightarrow (\psi \rightarrow \sigma)] \rightarrow [\psi \rightarrow (\varphi \rightarrow \sigma)]} \rightarrow I^3$$

$$[\varphi \rightarrow (\psi \rightarrow \sigma)] \leftrightarrow [\psi \rightarrow (\varphi \rightarrow \sigma)]$$

(c) $(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi$

$$\frac{\frac{[(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \neg\psi)]_2}{\varphi \rightarrow \psi} \wedge E \quad [\varphi]_1 \rightarrow E}{\psi} \rightarrow E$$

$$\frac{\frac{[(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \neg\psi)]_2}{\varphi \rightarrow \neg\psi} \wedge E \quad [\varphi]_1 \rightarrow E}{\neg\psi} \rightarrow E$$

$$\frac{\frac{\perp}{\neg\varphi} \rightarrow I^1}{(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi} \rightarrow I^2$$

(d) $(\varphi \rightarrow \psi) \rightarrow [(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)]$

$$\frac{\frac{[\varphi \rightarrow \psi]_1}{\psi} \rightarrow E \quad [\varphi]_2 \rightarrow E}{\psi} \rightarrow E$$

$$\frac{\frac{[\varphi]_2}{\psi \rightarrow \sigma} \rightarrow E \quad [(\varphi \rightarrow (\psi \rightarrow \sigma))]_3 \rightarrow E}{\frac{\sigma}{\varphi \rightarrow \sigma} \rightarrow I_2} \rightarrow E$$

$$\frac{(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)}{(\varphi \rightarrow \psi) \rightarrow [(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)]} \rightarrow I_1$$

3

Show:

(a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$

$$\frac{\frac{[\neg\varphi \wedge \psi]_1}{\neg\varphi} \wedge E \quad \varphi}{\frac{\perp}{\neg(\neg\varphi \wedge \psi)} \rightarrow I_1} \rightarrow E$$

(b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$

$$\frac{\frac{[\neg\psi]_1}{\varphi \wedge \neg\psi} \wedge I \quad \neg(\varphi \wedge \neg\psi)}{\frac{\perp}{\psi} \text{RAA}_1} \rightarrow E$$

(c) $\neg\varphi \vdash (\varphi \rightarrow \psi) \leftrightarrow \neg\varphi$

$$\frac{\frac{\frac{[\neg\varphi]_2 \quad [\varphi]_1}{\perp} \text{EFQ}}{\psi} \rightarrow\text{I}^1}{\neg\varphi \rightarrow (\varphi \rightarrow \psi)} \rightarrow\text{I}^2 \quad \frac{\frac{[\varphi \rightarrow \psi]_1 \quad \neg\varphi}{(\varphi \rightarrow \psi) \wedge (\neg\varphi)} \wedge\text{I}}{\frac{\neg\varphi}{(\varphi \rightarrow \psi) \rightarrow \neg\varphi} \rightarrow\text{I}^1} \wedge\text{E}$$

$$\frac{}{(\varphi \rightarrow \psi) \leftrightarrow \neg\varphi}$$

(d) $\vdash \varphi \Rightarrow \vdash \psi \rightarrow \varphi$

(e) $\neg\varphi \vdash \varphi \rightarrow \psi$

$$\frac{[\varphi]_1 \quad \neg\varphi}{\perp} \rightarrow\text{E}$$

$$\frac{\perp}{\psi} \text{EFQ}$$

$$\frac{}{\varphi \rightarrow \psi} \rightarrow I_1$$