Assignment #6
CIS 427/527

Group 2

March 2, 2016

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Prove the following using resolution:

(a) $(A \to B) \land (B \to C) \land \neg(A \to C) \models \bot$
(b) $(A \lor B \lor C) \land (\neg B \lor D) \land (\neg A \lor D) \land (\neg C \lor D) \models D$
(c) $A \to B, B \to C, D \to C, C \lor D \models \neg A \lor C$

Solution

(a) $(A \to B) \land (B \to C) \land \neg(A \to C) \models \bot$

- **(1)** Convert to CNF: $\{\{\neg A, B\}, \{\neg B, C\}, \{A\}, \{\neg C\}\}$
- **(2)** Perform resolution:

$$\dfrac{\dfrac{\dfrac{\{\neg A, B\} \qquad \{A\}}{\{B\}} \qquad \{\neg B, C\}}{\{C\}} \qquad \{\neg C\}}{\{\}}$$

(b) $(A \lor B \lor C) \land (\neg B \lor D) \land (\neg A \lor D) \land (\neg C \lor D) \models D$

- **(1)** Convert to CNF: $\{\{A, B, C\}, \{\neg B, D\}, \{\neg A, D\}, \{\neg C, D\}\{\neg D\}\}$
- **(2)** Perform resolution:

$$\dfrac{\dfrac{\dfrac{\dfrac{\{A, B, C\} \qquad \{\neg B, D\}}{\{A, C, D\}} \qquad \{\neg A, D\}}{\{C, D\}} \qquad \{\neg C, D\}}{\{D\}} \qquad \{\neg D\}}{\{\}}$$

(c) $A \to B, B \to C, D \to C, C \lor D \models \neg A \lor C$

- **(1)** Convert to CNF: $\{\{\neg A, B\}, \{\neg B, C\}, \{\neg D, C\}, \{C, D\}, \{A\}, \{\neg C\}\}$
- **(2)** Perform resolution:

$$\dfrac{\dfrac{\{\neg D, C\} \qquad \{C, D\}}{\{C\}} \qquad \{\neg C\}}{\{\}}$$

2

Which of the following sets are inconsistent?

(a) $\{\{p1, p2, p3\}, \{p1, \neg p3\}, \{\neg p1, \neg p2\}\}$
(b) $\{\{p1, \neg p2, p3\}, \{p1, \neg p3\}, \{p1, p2\}\}$

Solution

Neither set is inconsistent – the first can be satisfied by having both $p1$ and $\neg p2$ be true (since a logic is consistent iff it is satisfiable), and the second can be satisfied by simply having $p1$ be true.

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For each of the following set of clauses S, write down the Herbrand domain H, the Herbrand base HB, and two interpretations, one that satisfies S and one that does not.

(a) $S = \{\{A(x), \neg B(y,x)\}, \{\neg A(y), C(c)\}\}$
(b) $S = \{\{A(f(x)), \neg B(y,x)\}, \{\neg A(c), C(x)\}\}$

Solution

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Show, using Herbrand interpretations, that the following set of clauses is unsatisfiable.

Solution

(a) $S = \{\{A(x), \neg B(x,a)\}, \{A(a), B(y,a)\}, \{\neg A(y)\}\}$
(b) $S = \{\{A(x)\}, \{\neg A(x), B(f(x))\}, \{\neg B(f(a))\}\}$
(c) $S = \{\{A(x)\}, \{\neg A(y)\}\}$