

Assignment #5

CIS 427/527

Group 2

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2.2.1

Which of the following strings are formulas in predicate logic?

Solution

(a),(b),(f),(g) are formulas.

(c) isn't, as $f(m)$ is a term.

(d) isn't, as B is expecting two terms, yet $B(m, x)$ is a formula.

(e) isn't, as $B(m)$ doesn't have enough arguments.

(h) isn't, as $B(x)$ doesn't have enough arguments.

2.5.3

Solution

2.5.11

Solution

2.6.1

Consider the formula

$$\phi = \forall x \exists y Q(g(x, y), g(y, y), z)$$

Find two models M and M' with respective environments l and l' such that $M \models_l \phi$ but $M' \not\models_{l'} \phi$

Solution

M

$$A = \{a, b\}$$

$$Q^M = \{(a, a, b)\}$$

$$g^M(x, y) = a \text{ for all input}$$

$$z^M = b$$

Then, $\forall x \forall y Q(g(x, y), g(y, y), z) = Q(a, a, b)$, which holds, so $M \models \phi$.

M'

$$A = \{a, b\}$$

$$Q^{M'} = \{(a, a, c)\}$$

$$g^{M'}(x, y) = c \text{ for all input}$$

$$z^{M'} = a$$

Then, $\forall x \forall y Q(g(x, y), g(y, y), z) = Q(c, c, a)$, but $(c, c, a) \notin Q^{M'}$, so $M' \not\models \phi$.

2.6.2

Consider the sentence

$$\phi = \forall x \exists y \exists z (P(x, y) \wedge P(z, y) \wedge (P(x, z) \rightarrow P(z, x)))$$

Which of the following models satisfies ϕ ?

- (a) $P^M = \{(m, n) | m < n\}$
- (b) $P^{M'} = \{(m, 2 * m) | m \text{ natural number}\}$
- (c) $P^{M''} = \{(m, n) | m < n + 1\}$

Solution

- (a) This model does not satisfy ϕ , because we either need to force $P(x, z)$ to be false by requiring z to be smaller than x (in which case we can escape the natural numbers), or by having $x < z \wedge z < x$, which cannot happen.
- (b) Yes, because the first two properties say $y = 2 * x$ and $y = 2 * z$, which means $x = z$ making $P(x, z)$ always false.
- (c) Yes, let $y = z = x$, then all the properties hold.

2.6.3

Let P be a predicate with two arguments. Find a model M which satisfies $\forall x \neg P(x, x)$. Find also a model M' such that $M' \not\models \forall x \neg P(x, x)$.

Solution

- Let $P^M = \{(x, y) | x < y\}$, then we have $\forall x \neg P(x, x)$.
- Let $P^{M'} = \{(x, y) | x = y\}$, then we have $M' \not\models \forall x \neg P(x, x)$ as desired.

2.7.5

Show $\forall x (P(x) \vee Q(x)) \not\models \forall x P(x) \vee \forall x Q(x)$. Thus, find a model which satisfies $\forall x (P(x) \vee Q(x))$, but not $\forall x P(x) \vee \forall x Q(x)$.

Solution

- Let $A = \text{natural numbers}$, and let $P^M = \{x | x \leq 5\}$, $Q^M = \{x | x > 5\}$.

Then $\forall x. x \leq 5 \vee x > 5$ holds, but it is not true that $\forall x. x \leq 5 \vee \forall x. x > 5$.