# Assignment #4
# CIS 427/527

Group 2

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## 1

Complete the proof of soundness of propositional logic (given in val Dalen, Lemma 1.5.1) with the case of $\rightarrow_E$.

## Solution

## 2

Prove the soundness of the $\vee$ rules ($\vee I$ and $\vee E$).

## Solution

## 3

Do we have $\models (p \rightarrow q) \vee (q \rightarrow r)$?

## Solution

| p | q | r | $(p \rightarrow q)$ | $(q \rightarrow r)$ | $(p \rightarrow q) \vee (q \rightarrow r)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

## 4

Do we have $(q \rightarrow (p \vee (q \rightarrow p))) \vee \neg(p \rightarrow q) \models p$?

## Solution

## 5

Assuming the soundness and completeness of natural deduction for propositional logic, suppose that you need to show that $\phi$ is not a semantic consequence of $\phi_1, \phi_2, ..., \phi_n$, but that you are only allowed to base your argument on the use of natural deduction rules. Which judgement would you need to prove in order to guarantee that $\phi_1, \phi_2, ..., \phi_n \not\models \phi$? Do you need completeness and soundness for this to work out?

## Solution

## 6

Consider the following axiom based system, called Hilbert system:

$$(\phi \to (\psi \to \phi))$$
$$((\phi \to (\psi \to \sigma)) \to ((\phi \to \psi) \to (\phi \to \sigma)))$$
$$((\neg\phi \to \neg\psi) \to ((\neg\phi \to \psi) \to \phi))$$

Combined with the Modus Ponens inference rule, which corresponds to the elimination rule of the implication connective.

Prove according to this system the judgement $\vdash \phi \to \phi$.

## Solution

## 7

Consider classical logic given in the handout "Natural deduction in sequent form" in Figure 5. Prove the following judgements:
$\vdash \phi \vee \neg\phi$ (This is called Law of Excluded Middle).
$((\phi \to \psi) \to \phi) \to \phi$ (This is called Peirce's Law).

## Solution

## 8

Prove the following judgements:
$A \to B \to A$
$(A \to B \to C) \to (A \to B) \to (A \to C)$
$(A \wedge B \to C) \to (A \to B \to C)$
$(A \to B \to C) \to (A \wedge B \to C)$
Annotate each proof with lambda-terms.

## Solution