Assignment #5
CIS 427/527

Group 2

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2.2.1

Which of the following strings are formulas in predicate logic?

Solution

(a),(b),(f),(g) are formulas.

(c) isn't, as $f(m)$ is a term.
(d) isn't, as B is expecting two terms, yet $B(m, x)$ is a formula.
(e) isn't, as $B(m)$ doesn't have enough arguments.
(h) isn't, as $B(x)$ doesn't have enough arguments.

2.5.3

Solution

2.5.11

Solution

2.6.1

Consider the formula

$$\phi = \forall x \; \exists y \; Q(g(x, y), g(y, y), z)$$

Find two models M and M' with respective environments l and l' such that $M \models_l \phi$ but $M' \not\models_l \phi$

Solution

M
$A = \{a, b\}$
$Q^M = \{(a, a, b)\}$
$g^M(x, y) = a$
$z^M = c$

Then, $\forall x \; \forall y \; Q(g(x, y), g(y, y), z) = Q(a, a, b)$, which holds, so $M \models \phi$.

M'
$A = \{a, b\}$
$Q^{M'} = \{(a, a, c)\}$
$g^{M'} = c$
$z^{M'} = a$

Then, $\forall x \; \forall y \; Q(g(x, y), g(y, y), z) = Q(c, c, a)$, but $(c, c, a) \notin Q^{M'}$, so $M' \not\models \phi$.

2.6.2

Consider the sentence

$$\phi = \forall x\ \exists y\ \exists z\ (P(x,y) \land P(z,y) \land (P(x,z) \rightarrow P(z,x)))$$

Which of the following models satisfies ϕ?

(a) $P^M = \{(m,n)|m < n\}$
(b) $P^{M'} = \{(m, 2*m)|m \text{ natural number}\}$
(c) $P^{M''} = \{(m,n)|m < n+1\}$

Solution

(a) This model does not satisfy ϕ, because we either need to force $P(x,z)$ to be false by requiring z to be smaller than x (in which case we can escape the natural numbers), or by having $x < z \land z < x$, which cannot happen.
(b) Yes, because the first two properties say $y = 2*x$ and $y = 2*z$, which means $x = z$ making $P(x,z)$ always false.
(c) Yes, let $y = z = x$, then all the properties hold.

2.6.3

Let P be a predicate with two arguments. Find a model M which satisfies $\forall x\ \neg P(x,x)$. Find also a model M' such that $M' \not\models \forall x\ \neg P(x,x)$.

Solution

- Let $P^M = \{(x,y)|x < y\}$, then we have $\forall x\ \neg P(x,x)$.
- Let $P^{M'} = \{(x,y)|x = y\}$, then we have $M' \not\models \forall x\ \neg P(x,x)$ as desired.

2.7.5

Show $\forall x(P(x) \bigvee Q(x)) \not\models \forall x P(x) \bigvee \forall x Q(x)$. Thus, find a model which satisfies $\forall x(P(x) \bigvee Q(x))$, but not $\forall x P(x) \bigvee \forall x Q(x)$

Solution

- Let A = natural numbers, and let $P^M = \{x|x \leq 5\}$, $Q^M = \{x|x > 5\}$.

Then $\forall x.x \leq 5 \bigvee x > 5$ holds, but it is not true that $\forall x.x \leq 5 \bigvee \forall x.x > 5$.