Assignment #6
CIS 427/527

Group 2

March 2, 2016

1

Prove the following using resolution:

(a) $(A \to B) \wedge (B \to C) \wedge \neg(A \to C) \models \bot$

(1) Convert to CNF: $\{\{\neg A, B\}, \{\neg B, C\}, \{A\}, \{\neg C\}\}$
(2) Perform resolution:

$$\dfrac{\dfrac{\dfrac{\{\neg A, B\} \qquad \{A\}}{\{B\}} \qquad \{\neg B, C\}}{\{C\}} \qquad \{\neg C\}}{\{\}}$$

(b) $(A \vee B \vee C) \wedge (\neg B \vee D) \wedge (\neg A \vee D) \wedge (\neg C \vee D) \models D$

(1) Convert to CNF: $\{\{A, B, C\}, \{\neg B, D\}, \{\neg A, D\}, \{\neg C, D\}\{\neg D\}\}$
(2) Perform resolution:

$$\dfrac{\dfrac{\dfrac{\dfrac{\{A, B, C\} \qquad \{\neg B, D\}}{\{A, C, D\}} \qquad \{\neg A, D\}}{\{C, D\}} \qquad \{\neg C, D\}}{\{D\}} \qquad \{\neg D\}}{\{\}}$$

(c) $A \to B, B \to C, D \to C, C \vee D \models \neg A \vee C$

(1) Convert to CNF: $\{\{\neg A, B\}, \{\neg B, C\}, \{\neg D, C\}, \{C, D\}, \{A\}, \{\neg C\}\}$
(2) Perform resolution:

$$\dfrac{\dfrac{\{\neg D, C\} \qquad \{C, D\}}{\{C\}} \qquad \{\neg C\}}{\{\}}$$

2

Which of the following sets are inconsistent?

(a) $\{\{p1, p2, p3\}, \{p1, \neg p3\}, \{\neg p1, \neg p2\}\}$
(b) $\{\{p1, \neg p2, p3\}, \{p1, \neg p3\}, \{p1, p2\}\}$

Solution

Neither set is inconsistent – the first can be satisfied by having both $p1$ and $\neg p2$ be true (since a logic is consistent iff it is satisfiable), and the second can be satisfied by simply having $p1$ be true.

5

For each of the following set of clauses S, write down the Herbrand domain H, the Herbrand base HB, and two interpretations, one that satisfies S and one that does not.

(a) $S = \{\{A(x), \neg B(y, x)\}, \{\neg A(y), C(c)\}\}$

$$\begin{aligned} H &= \{a\} \\ HB &= \{A(a), B(a,a), C(a)\} \\ T &: \{A(a), C(a)\} \\ F &: \{A(a)\} \end{aligned}$$

(b) $S = \{\{A(f(x)), \neg B(y, x)\}, \{\neg A(c), C(x)\}\}$

$$\begin{aligned} H &= \{a, f(a), f(f(a)), ...\} \\ HB &= \{A(a), B(a,a), C(a), A(f(a)), B(f(a), f(a)), C(f(a)), ...\} \\ T &: \{A(a), C(a)\} \\ F &: \{A(a)\} \end{aligned}$$

6

Show, using Herbrand interpretations, that the following set of clauses is unsatisfiable.

(a) $S = \{\{A(x), \neg B(x, a)\}, \{A(a), B(y, a)\}, \{\neg A(y)\}\}$

$$\begin{aligned} H &= \{a\} \\ HB &= \{A(a), B(a,a)\} \end{aligned}$$

Since all the interpretations are $\{A(a), B(a,a)\}, \{A(a)\}, \{B(a,a)\}$, or $\emptyset$, S is not satisfiable.

(b) $S = \{\{A(x)\}, \{\neg A(x), B(f(x))\}, \{\neg B(f(a))\}\}$

$$\begin{aligned} H &= \{a, f(a), f(f(a)), ...\} \\ HB &= \{A(a), B(a), A(f(a)), B(f(a), f(a)), ...\} \end{aligned}$$

Since all the interpretations are $\{A(a), B(a)\}, \{A(a)\}, \{B(a)\}$, or $\emptyset$, S is not satisfiable.

(c) $S = \{\{A(x)\}, \{\neg A(y)\}\}$

$$\begin{aligned} H &= \{a\} \\ HB &= \{A(a)\} \end{aligned}$$

Since all the interpretations are $\{A(a)\}$ or $\emptyset$, S is not satisfiable.