

Assignment #3

CIS 427/527

Group 2

January 24, 2016

1

Show that the following propositions are derivable:

(a) $\varphi \rightarrow \varphi$

$$\frac{\frac{[\varphi]^1}{\varphi}}{\varphi \rightarrow \varphi} \rightarrow I^1$$

(b) $\perp \rightarrow \varphi$

$$\frac{\frac{[\perp]^1}{\varphi} \perp E}{\perp \rightarrow \varphi} \rightarrow I^1$$

(c) $\neg(\varphi \wedge \neg\varphi)$

$$\frac{\frac{\varphi \quad \neg\varphi}{\varphi \wedge \neg\varphi} \wedge I}{\neg(\varphi \wedge \neg\varphi)} \rightarrow I^1$$

or

$$\frac{\frac{\frac{[\varphi \wedge \neg\varphi]^1}{\varphi} \wedge E \quad \frac{[\varphi \wedge \neg\varphi]^1}{\neg\varphi} \wedge E}{\perp} E}{\neg(\varphi \wedge \neg\varphi)} \rightarrow RAA^1$$

(d) $(\varphi \rightarrow \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$

(e) $(\varphi \wedge \psi) \leftrightarrow \neg(\varphi \rightarrow \neg\psi)$

(f) $\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$

$$\frac{\frac{\frac{[\varphi]^1 \quad [\psi]^2}{\varphi \wedge \psi} \wedge I}{(\psi \rightarrow (\varphi \wedge \psi))} \rightarrow I_2}{\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))} \rightarrow I_1$$

2

Show that the following propositions are derivable:

(a) $(\varphi \rightarrow \neg\varphi) \rightarrow \neg\varphi$

$$\frac{\frac{\frac{[\varphi \rightarrow \neg\varphi]^1 \quad [\varphi]^2}{\neg\varphi \wedge \varphi} \rightarrow E}{\perp} E}{\neg\varphi} RAA_2}{(\varphi \rightarrow \neg\varphi) \rightarrow \neg\varphi} \rightarrow I_1$$

(b) $[\varphi \rightarrow (\psi \rightarrow \sigma)] \leftrightarrow [\psi \rightarrow (\varphi \rightarrow \sigma)]$

(c) $(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi$

TYPO – NEED CLARIFICATION

(d) $(\varphi \rightarrow \psi) \rightarrow [(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)]$

$$\begin{array}{c}
 \frac{[\varphi \rightarrow \psi]_1 \quad [\varphi]_2}{\psi} \rightarrow E \quad \frac{[\varphi]_2 \quad [(\varphi \rightarrow (\psi \rightarrow \sigma))]_3}{\psi \rightarrow \sigma} \rightarrow E \\
 \frac{\sigma}{\varphi \rightarrow \sigma} \rightarrow I_2 \\
 \frac{(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)}{(\varphi \rightarrow \psi) \rightarrow [(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)]} \rightarrow I_3 \\
 \frac{}{(\varphi \rightarrow \psi) \rightarrow [(\varphi \rightarrow (\psi \rightarrow \sigma)) \rightarrow (\varphi \rightarrow \sigma)]} \rightarrow I_1
 \end{array}$$

3

Show:

- (a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$
- (b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
- (c) $\neg\varphi \vdash (\varphi \rightarrow \psi) \leftrightarrow \neg\varphi$
- (d) $\vdash \varphi \Rightarrow \vdash \psi \rightarrow \varphi$
- (e) $\neg\varphi \vdash \varphi \rightarrow \psi$