Assignment #3
CIS 427/527

Group 2

January 26, 2016

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Show that the following propositions are derivable:

(a) $\varphi \to \varphi$

$$\frac{[\varphi]^1}{\varphi \to \varphi} \to I^1$$

(b) $\bot \to \varphi$

$$\frac{\dfrac{[\bot]^1}{\varphi} \bot E}{\bot \to \varphi} \to I^1$$

(c) $\neg(\varphi \land \neg\varphi)$

$$\frac{\dfrac{\dfrac{[\varphi \land \neg\varphi]_1}{\varphi} \land E \quad \dfrac{[\varphi \land \neg\varphi]_1}{\neg\varphi} \land E}{\bot} \text{EFQ}}{\neg(\varphi \land \neg\varphi)} \to I^1$$

(d) $(\varphi \to \psi) \leftrightarrow \neg(\varphi \land \neg\psi)$

$$\frac{\dfrac{\dfrac{\dfrac{[\varphi \land \neg\psi]_1}{\varphi} \land E \quad [\varphi \to \psi]_2}{\psi} \quad \dfrac{[\varphi \land \neg\psi]_1}{\neg\psi} \land E}{\dfrac{\bot}{\neg(\varphi \land \neg\psi)} \to I^1}}{(\varphi \to \psi) \to \neg(\varphi \land \neg\psi)} \to I^2 \qquad \frac{\dfrac{[\neg(\varphi \land \neg\psi)]_2 \quad \dfrac{[\varphi]_3 \quad [\neg\psi]_1}{\varphi \land \neg\psi} \land I}{\dfrac{\bot}{\psi} \text{RAA}^1} \to E}{\dfrac{\varphi \to \psi}{\neg(\varphi \land \neg\psi) \to (\varphi \to \psi)} \to I^2} \to I^3$$

$$(\varphi \to \psi) \leftrightarrow \neg(\varphi \land \neg\psi)$$

(e) $(\varphi \land \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$

$$\frac{\dfrac{\dfrac{[\varphi \land \psi]_1}{\psi} \land E \quad \dfrac{\dfrac{[\varphi \land \psi]_1}{\varphi} \land E \quad [\varphi \to \neg\psi]_2}{\neg\psi} \to I}{\dfrac{\bot}{\neg(\varphi \to \neg\psi)} \to I^2}}{(\varphi \land \psi) \to \neg(\varphi \to \neg\psi)} \to I^1 \qquad \frac{\dfrac{[\neg(\varphi \to \neg\psi)]_1 \quad \dfrac{[\neg\psi]^2}{\varphi \to \neg\psi} \text{RAA}^2}{\dfrac{\bot}{\varphi \land \psi} \land I^1}}{\neg(\varphi \to \neg\psi) \to (\varphi \land \psi)} \to I$$

$$(\varphi \land \psi) \leftrightarrow \neg(\varphi \to \neg\psi)$$

(f) $\varphi \to (\psi \to (\varphi \land \psi))$

$$\frac{\dfrac{\dfrac{[\varphi]^1 \quad [\psi]^2}{\varphi \land \psi} \land I}{\psi \to (\varphi \land \psi)} \to I^2}{\varphi \to (\psi \to (\varphi \land \psi))} \to I^1$$

2

Show that the following propositions are derivable:

(a) $(\varphi \to \neg\varphi) \to \neg\varphi$

$$\cfrac{\cfrac{\cfrac{[\varphi \to \neg\varphi]_1 \quad [\varphi]_2}{\neg\varphi} \to E \quad [\varphi]_2}{\cfrac{\cfrac{\bot}{\neg\varphi} \to I^2}{(\varphi \to \neg\varphi) \to \neg\varphi} \to I_1} \to E}{}$$

(b) $[\varphi \to (\psi \to \sigma)] \leftrightarrow [\psi \to (\varphi \to \sigma)]$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[\psi \to (\varphi \to \sigma)]_3 \quad [\psi]_1}{\varphi \to \sigma} \to E \quad [\varphi]_2}{\sigma} \to E}{\psi \to \sigma} \to I^1}{\varphi \to (\psi \to \sigma)} \to I^2}{[\psi \to (\varphi \to \sigma)] \to [\varphi \to (\psi \to \sigma)]} \to I^3 \qquad \cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[\varphi \to (\psi \to \sigma)]_3 \quad [\varphi]_1}{\psi \to \sigma} \to E \quad [\psi]_2}{\sigma} \to E}{\varphi \to \sigma} \to I^1}{\psi \to (\varphi \to \sigma)} \to I^2}{[\varphi \to (\psi \to \sigma)] \to [\psi \to (\varphi \to \sigma)]} \to I^3$$
$$\overline{[\varphi \to (\psi \to \sigma)] \leftrightarrow [\psi \to (\varphi \to \sigma)]}$$

(c) $(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi$

$$\cfrac{\cfrac{\cfrac{\cfrac{[(\varphi \to \psi) \wedge (\varphi \to \neg\psi)]_2}{\varphi \to \psi} \wedge E \quad [\varphi]_1}{\psi} \to E}{\cfrac{\cfrac{\bot}{\neg\varphi} \to I^1}{(\varphi \to \psi) \wedge (\varphi \to \neg\psi) \to \neg\varphi} \to I^2}}{} \qquad \cfrac{\cfrac{[(\varphi \to \psi) \wedge (\varphi \to \neg\psi)]_2}{\varphi \to \neg\psi} \wedge E \quad [\varphi]_1}{\neg\psi} \to E$$

(d) $(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[\varphi \to \psi]_1 \quad [\varphi]_2}{\psi} \to E \quad \cfrac{[\varphi]_2 \quad [(\varphi \to (\psi \to \sigma))]_3}{\psi \to \sigma} \to E}{\sigma} \to E}{\varphi \to \sigma} \to I_2}{(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)} \to I_3}{(\varphi \to \psi) \to [(\varphi \to (\psi \to \sigma)) \to (\varphi \to \sigma)]} \to I_1$$

3

Show:

(a) $\varphi \vdash \neg(\neg\varphi \wedge \psi)$

$$\cfrac{\cfrac{\cfrac{[\neg\varphi \wedge \psi]_1}{\neg\varphi} \wedge E \quad \varphi}{\bot} \to E}{\neg(\neg\varphi \wedge \psi)} \to I_1$$

(b) $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$

(c) $\neg\varphi \vdash (\varphi \to \psi) \leftrightarrow \neg\varphi$

$$\cfrac{\cfrac{\cfrac{\cfrac{[\neg\varphi]_2 \quad [\varphi]_1}{\bot} \to E}{\cfrac{\psi}{\varphi \to \psi} \to I^1} \text{EFQ}}{\neg\varphi \to (\varphi \to \psi)} \to I^2 \qquad \cfrac{\cfrac{\cfrac{[\varphi \to \psi]_1 \quad \neg\varphi}{(\varphi \to \psi) \wedge (\neg\varphi)} \wedge I}{\neg\varphi} \wedge E}{(\varphi \to \psi) \to \neg\varphi} \to I^1}{(\varphi \to \psi) \leftrightarrow \neg\varphi}$$

(d) $\vdash \varphi \Rightarrow \vdash \psi \to \varphi$

(e) $\neg\varphi \vdash \varphi \rightarrow \psi$

$$\cfrac{\cfrac{\cfrac{[\varphi]_1 \qquad \neg\varphi}{\bot}\ {\rightarrow}\mathrm{E}}{\psi}\ \mathrm{EFQ}}{\varphi \rightarrow \psi}\ \rightarrow I_1$$