Project Proposal: Simulating Simple Pendulum Damped Harmonic Oscillation

Introduction:
In this project proposal, I outline the approach I will take to analyze the equation for damped oscillation for a pendulum. Additionally, I will hopefully explore how to expand this understanding by modeling a coupled pendulum system. This project involves data collection, data analysis, applying quality filters, fitting with error, and providing explanations for the model fit.

Chosen Phenomenon and Data Source:
The chosen phenomenon for this project is the damping effect of simple pendulums due to drag. To study this, I will create a swinging pendulum with objects I have and use Logger Pro to collect data about this set up. To better understand this data and qualify it, I will derive an equation for this mechanical motion. I will compared the idealized, theoretical result with the actual results I gather. I will later try to apply this situation to a coupled oscillator so I may achieve a model and collect data. I will compare the data I collect both experimentally and the data I generate in my simulation.

Equation to Fit Data:
The equation to fit the data will depend on characteristics of the environment observed tests have been conducted in. I will study how this system behaves with forces and equations of motion at the beginning to guide my simulation. I will derive a general equation for this phenomena and compare my results with real-world data obtained from experimentation. To fit the data I collect in my experiment, I will be guided by trigonometric models that model the periodic nature of pendulums and exponential curves to bound my graph to detail how the amplitude will decrease over time.

Exponential decay:
$A(t) = A_0e^{\lambda t}$
$A(t)$ = amplitude at some time
A_0 = initial displacement from equilibrium (initial amplitude)
λ = growth constant ($\lambda<0$ will result in a decay function)

General sinusoidal equation for pendulum:
$x(t) = A_0\cos(\omega_0 t + C)$
$x(t)$ = position (displacement from equilibrium) of pendulum at some time
ω_0 = idealized frequency of pendulum (simple harmonic motion) = sqrt(g/L)
C = phase shift (if the pendulum was already moving or something and I started recording data form after I let go of the pendulum)
t = time

Force equation and solution to damped oscillation:
Force on pendulum:
Y-direction: all cancel
X-direction: $F = -mg\sin\theta$ (restoring force)

θ = angle between equilibrium and displaced position
Small angle approximation:
$\sin\theta = \theta$ as $\theta \rightarrow 0$
$\sin\theta = x/L$
$x = L\theta$
L = length of string
Therefore, $F = -mg\theta = -mgx/L$

Derivation of equation of motion for pendulum:
Torque = $I\alpha = R \times F$
I = moment of inertia = $\Sigma r_i^2 m_i$
α = angular acceleration = a/L
a = linear acceleration
$(mL^2)\alpha = -mgL\sin\theta$
$L(d^2\theta/dt^2) = -g\theta$
$(d^2x/dt^2) + (g/L)x = 0$
ω_0 = simple harmonic angular frequency of pendulum = sqrt(g/L)

Drag:
-bv = force of drag
b = damping constant
v = velocity
ω = angular velocity = v/L
$F_{drag} = -bL(\omega)$

$m(d\theta^2/d^2t) = -mgL\theta - bL(d\theta/dt)$
$(d\theta^2/d^2t) + (g/L)\theta + (b/m)(d\theta/dt) = 0$
m = mass

The characteristic equation that corresponds to this is $r^2 + (b/m)r + (g/L) = 0$. If we substitute $2\gamma = b/m$ and $g/L = \omega_0^2$, we get $r^2 + 2\gamma r + \omega_0^2 = 0$

*I have gotten to this point, but need to explore why the 2γ substitution is important in this derivation.

Solution to second order differential equation:
$x(t) = A_0 e^{-\gamma t}\cos(\omega' t + C)$
ω' = new frequency, which is not necessarily ω_0 = sqrt(ω_0^2 - γ^2)
γ = b/(2m)

*more derivation will be done fully during report

This model makes sense because we know that oscillations happen periodically, and damped oscillations are bounded by decaying curves because their amplitudes are decreasing over time.

I expect to model my simulation based on this equation and expect my experimental results to follow a similar curve. I will compare both data sets with each other.

Data Generation for Testing:
In addition to using data I will collect from my own setup, I will generate random test data to evaluate the performance of the model. This synthetic data will help ensure that the model can handle different scenarios. The test data will be generated using Python's NumPy library to mimic the characteristics of different systems, such as different gravitational fields, mass, string lengths, displacements, and drag coefficients.

Data Filtering:
To ensure the quality and reliability of the dataset, I will apply several quality filters and data preprocessing steps:
a. Outlier Removal: Outliers, which are data points that deviate significantly from expected ranges, will be identified and excluded from the dataset. This will be achieved through statistical methods and comparisons with other collected data.
b. Duplicate Data: Any duplicate entries will be detected and removed to maintain data consistency.
c. Quality Flags: I will filter out data with low-quality data (like when I mess up the apparatus, bump into the pendulum, release at awkward trajectories, or any other outside influence I do not want) to prioritize high-quality observations.

Data Fitting With Error:
To analyze the distribution of exoplanets, I will fit the data with appropriate mathematical models while considering the associated errors. The choice of the model will be guided by the theoretical considerations, and the error estimation will be done using statistical methods.
- Model Selection: The selection of models will depend on the observed position versus time graphs. Common models include bounded trigonometric functions. The choice will be based on the data patterns.
- Error Estimation: Error estimation is essential to understand the uncertainties in the model parameters. I will use statistical techniques, like taking the difference between the two graphs and using an Nth degree remainder Taylor Series calculation, to quantify the errors associated with the model fit.

Explanation of Model Fit:
After fitting the data with a chosen model, I will provide a detailed explanation of the model's relevance to the observed data. This explanation will include:
- How the selected model represents the damping of a simple harmonic pendulum.
- The interpretation of model parameters in the context of pendulum systems.
- Evaluation of the goodness of fit and the significance of the results.

The explanation will provide insights into damping motion and its implications for our understanding of physical systems.