Homework 5: MATH 548

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Contents

1 Introduction

The goal of this project is to obtain the price of call and put options thanks to a binomial model that must be realized optimally. Previously, the pricing was made for "European style" options. Here, it will be done for "American style" options. Then, the results will be compared.

2 Presentation of the zip file

2.1 Data used

For the data, I only used two sets.
The first set is called $data4.csv$. $data4.csv$ contains the real call options prices of Apple taken on the 15th of November for an expiration on the 18th of February 2022. On the 15th of November, $S_0 = 149.87$. Based on the volume observed, I decided to study strike prices from 140 to 190 (11 values).

The second set is called $data5.csv$. $data5.csv$ contains the real put option prices of Apple with the same dates as above. Based on the volume observed, I decided to study strike prices from 120 to 170 (11 values).

For both sets, there are 70 trading days until maturity.

Then, another parameter becomes important which is dividend. Indeed, a cash dividend payment of \$ 0.22 per share was scheduled to be paid on November 11, 2021. Therefore, we do not take that one in account in our computations (our set starts on the 15th of November). However, another cash dividend will happen on the 17th of February 2022, which concerns us. The value of the dividend is not known yet. Therefore, I decided, first, to proceed as if there were not any dividends paid, meaning that I considered Apple as a non-dividend paying stock. At last, I took this parameter into account by considering the next dividend equal to \$ 1.

2.2 Functions

From the previous homework, I have come to the conclusion that the methods of calibrating or estimating the parameters tend, in my case, to give similar results even though calibrating seemed to be more efficient. This is why I only focused on the calibrating method in this project.
However, what differs from the previous homework is the way that American options are priced. Thus, the pricing method must be adapted. Once again, in the previous work, I realized that the pricing using risk-neutral probability was less time-consuming during the execution than the method that consists in finding H_0 and H_1 at each time step and each state. This is why, I only focused on adapting the "risk-neutral" pricing method.

The file contains two main script files which are:

- long-expiration-calls.m
- long-expiration-puts.m

It is from those two files that I get my results.
Then, the other functions enable me to obtain American style pricing:

- binomial-tree.m
- risk-neutral-pricing-usa.m

At last, to compare my results with European style pricing, I kept from the previous homework the file $risk - neutral - pricing.m$ which does the pricing for European options.

Thus, aside from the script files that I modified, the main change compared to HW4 comes from the pricing. Indeed, for American options, when we go through the tree backwards, the maximum between the intrinsic value and what would be the common European style value must be taken.

```
% Every price obtained is place in the pricing list
sum1 = 0;
sum2 = 0;
for k=T:-1:1
        length_vector = length(Binomialtree(:,k)) - sum(isnan(Binomialtree(:,k)));
        a=0;
        for i=length_vector-1:-1:1
            C = (1/1+r)*(q1*Pricing_list(a+i)+(1-q1)*Pricing_list(a+i+1));
            % American style: Must choose the max between intrinsic value
            % and european style at each time and each state w
            if (C <= Payoff_tree(i,k-1))
                C = Payoff_tree(i,k-1);
                sum1 = sum1+1;
            else
                C=C;
                sum2 = sum2+1;
            end
            Pricing_list = [C Pricing_list];
            a=a+1;
        end
```

Figure 1: Main change to price American options

The variables $sum1$ and $sum2$ were a way for me to check the good behavior of the function.

3 Presentation of the results

3.1 Expectations

Considering American options implies that normally some key properties must be verified.

So, first of all, the price of an American option is higher than the price of a European option. Indeed, getting the right but not the obligation to exercise before maturity is a clear advantage for the buyer of the option and this why the buyer pays more for an American option.

Secondly, for non-paying dividend stocks, American options and European options must have the same value. In other words, it is always optimal to wait maturity to exercise in that case. That is an interesting point, as first, I consider Apple as a non-paying dividend stock (which can be justified by the dates I have chosen).

At last, it can be optimal to exercise put options before maturity.

3.2 Put options

First of all, I did not consider any dividends. First, I needed to obtain the optimal u, and I set $d = \frac{1}{u}$. Let's recall what I obtained for European options:

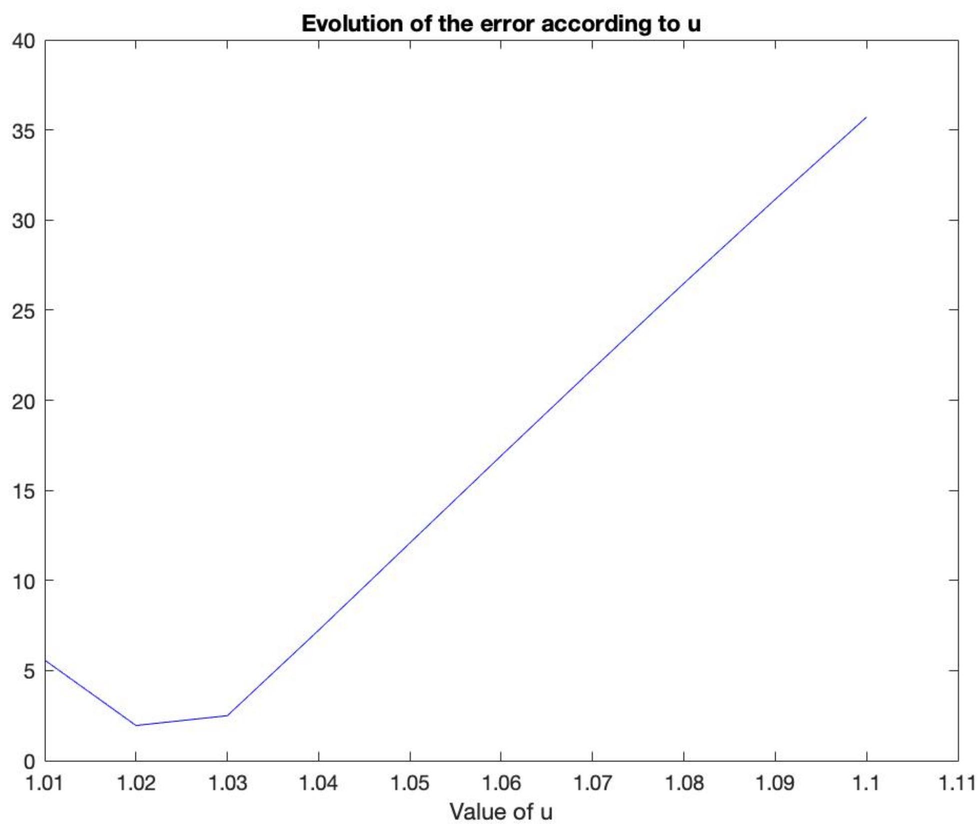

Figure 2: Optimal u for European put options

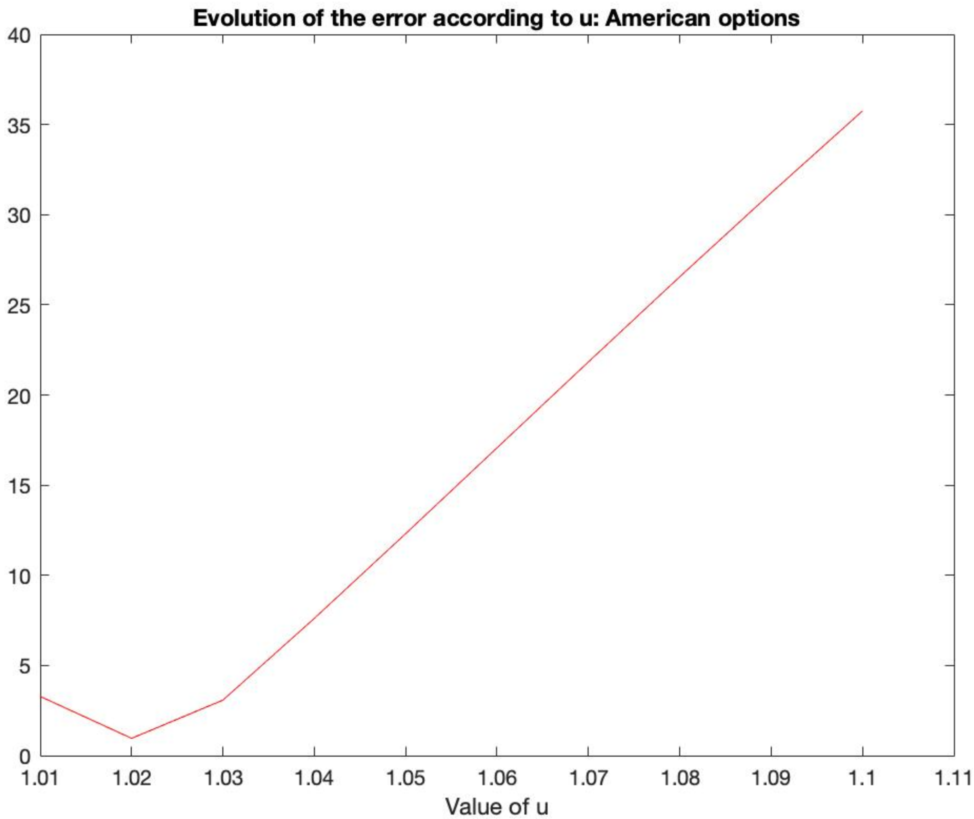

Figure 3: Optimal u for American put options

Therefore, for both styles, the optimal u that minimizes the $RMSE$ in the calibrating method is equal to 1.02.

Here is what is obtained for the pricing:

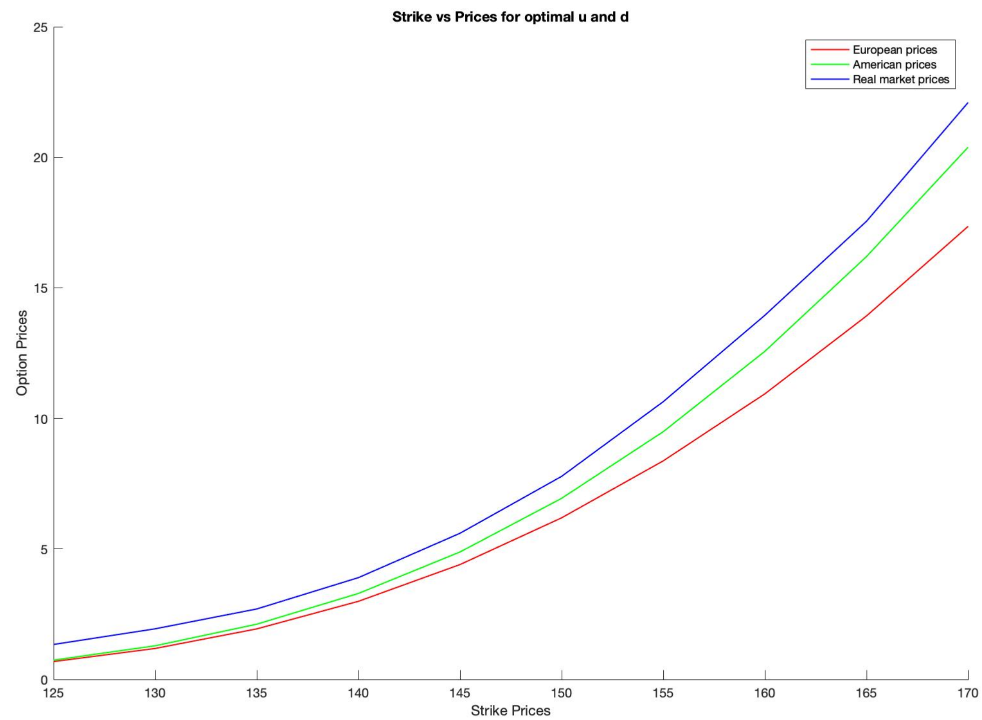

Figure 4: Put options price: real market, American style, European Style

What we can see it that, first, the price of the American option is always greater than the price of the corresponding European option, regardless of the strike price. This is a good point that needed to be verified.
Then, we can notice what is obtained with American options is more accurate than what is obtained with European options. I have not been able to figure out what was the style of the options on Yahoo Finance. They indicate to look at the the nomenclature of the option but it is only written P not PE or PA as they suggest. Nonetheless, American options are widely used compared to European options so their price might be the one on Yahoo Finance.

3.3 Call options

The same work has been realized for call options. Once again, first, I did not consider any dividends. The same optimal value for u has been obtained for American and European call options. Therefore, $u = 1.007$.

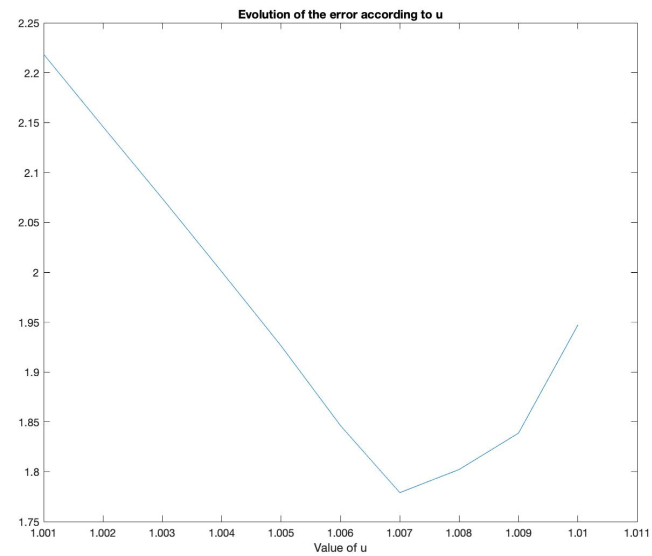

Figure 5: Optimal u for European call options

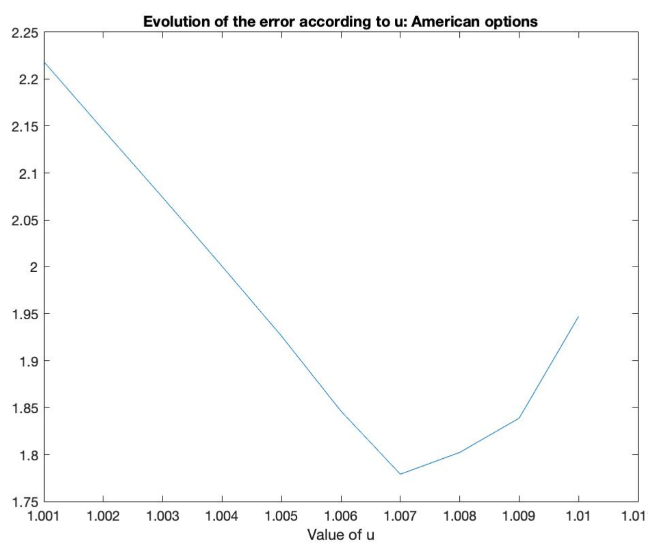

Figure 6: Optimal u for American call options

For this optimal u, call options prices have been compared. What can be noticed is that the curve for European options is exactly the same as the one for American options (this is why it does not appear). This is a good point as in absence of dividends, "American=European".

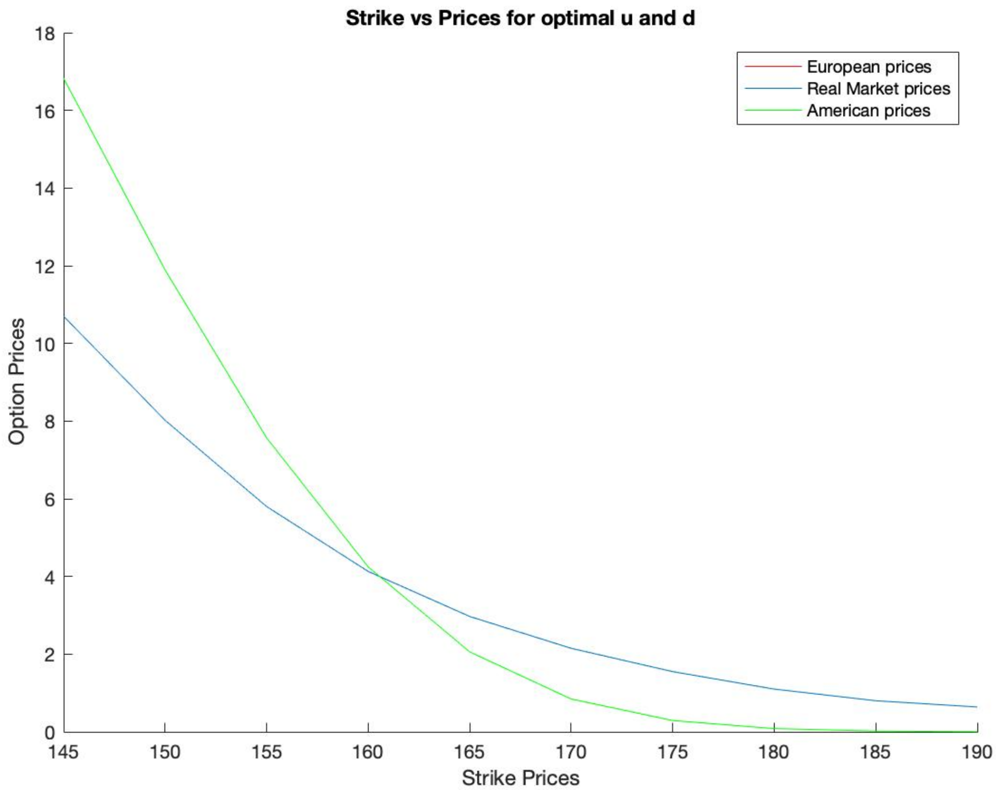

Figure 7: Call options price: real market, American style, European Style

4 Possible improvement

At last, one possible improvement would be to consider dividends. To do so, I created a file called $binomial - dividend.m$. The purpose of this file is to obtain the binomial tree with the right dividend by precising as inputs the time-step corresponding to the the day where the dividend is paid and the dividend.
Here, the trading day where the dividend happens is the 69th and I took it equal to one dollar.

```
function [ Binomialtree ] = binomial_dividends(T,u,d,S_0,div,T_div)
% Build the binomial tree for u and d

Binomialtree = nan(T,T);
Binomialtree(1,1) = S_0;
for i=2:T_div
    %if it goes up, the price at t=t-1 is multiplied by u
    Binomialtree(1:i-1,i) = Binomialtree(1:i-1,i-1)*u;
    %if it goes down, the price at t=t-1 is multiplied by d
    Binomialtree(i,i) = Binomialtree(i-1,i-1)*d;
end
Binomialtree = Binomialtree - div;
for i=T_div+1:T
    %if it goes up, the price at t=t-1 is multiplied by u
    Binomialtree(1:i-1,i) = Binomialtree(1:i-1,i-1)*u;
    %if it goes down, the price at t=t-1 is multiplied by d
    Binomialtree(i,i) = Binomialtree(i-1,i-1)*d;
end

end
```

Figure 8: Code with dividends

The optimal u is again 1.007. For that value, here is what is obtained:

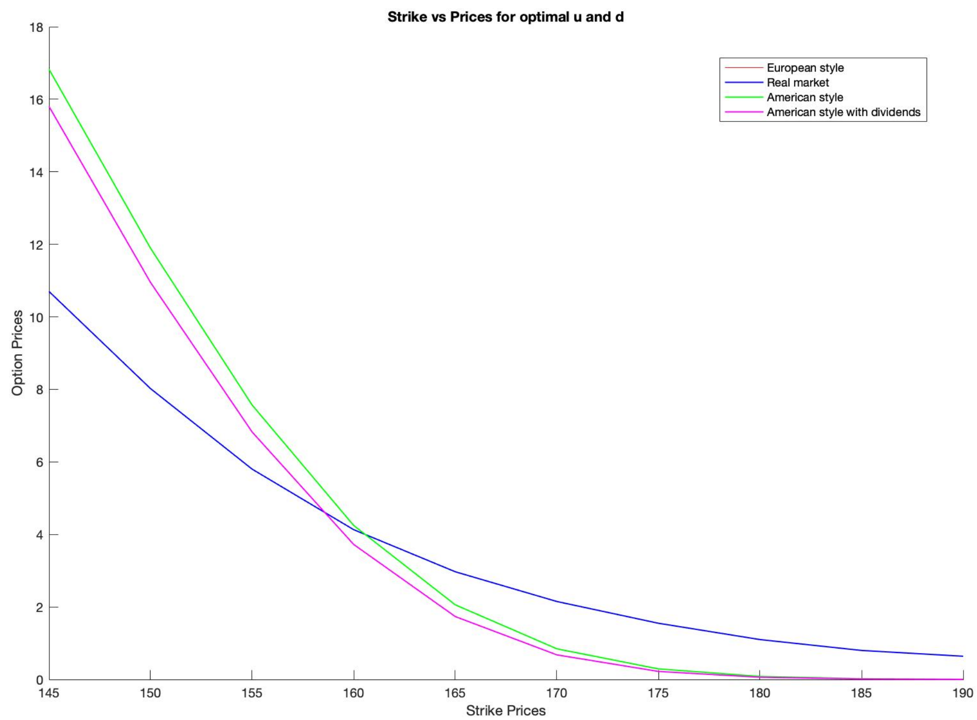

Figure 9: Call options price: real market, American style, European Style, American style with dividends

What can be seen is that adding dividends improves the estimation of the option price. However, I took a dividend equal to 1 dollar which is an unlikely value. It will probably be close to 22 cents. So, this is not a significant improvement in my case.