MATH 584: Homework 2

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1 Question 1A

The goal is to construct and back-test the dynamic strategy that maximizes the expectation of power-utility.
Therefore, the following expression has been maximized over alpha:

$$\mathbb{E}(sgn(\zeta)(1+\alpha \cdot R_{t+1})^{\zeta} = 0.5((1+R+\alpha(\mu+\sigma-R))^{\zeta} + (1+R+\alpha(\mu-\sigma-R))^{\zeta}) \quad (1)$$

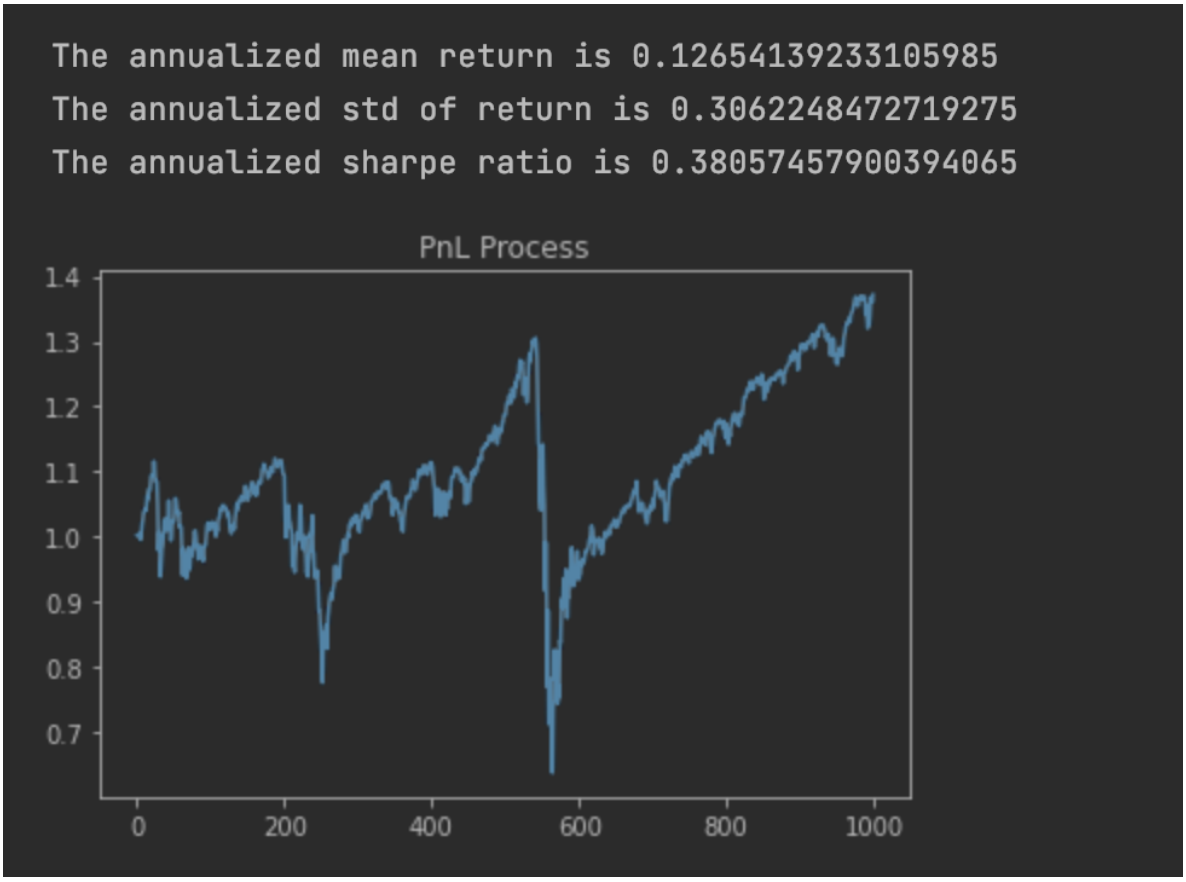

Figure 1: PnL process obtained without transaction costs

The Sharpe ratio obtained is equal to 0.38. Admittedly, the context is different as the number of assets studied is not the same, but we can still note a major difference between this Sharpe ratio and what we had in HW1. Clearly, we can see the Covid crisis in 2020 that impacted heavily the PnL process.

2 Question 1B

For this question, the goal was to apply transaction costs to the optimal strategy obtained in question A. In the lecture, it is said that we can consider the T-cost of the risk-less asset to be equal to zero. It was not specified in the homework, so two cases have been tested. First, I have considered that the risk-less asset was penalized by a T-cost of 0.02 and then, a second case where the risk-less is not penalized at all.

To obtain the PnL process with additional costs, we need to derive the following expression:

$$W_t(\alpha) = W_{t-1}(\alpha)(1 + \alpha_{t-1} \cdot (\mu + \epsilon_t)\lambda|\alpha_{t-1}A_{t-1}|), \tag{2}$$

based on:

$$A_t^i(\alpha) = \frac{\alpha_{t-1}^i(1 + \mu^i + \epsilon_t^i}{1 + \alpha_{t-1} \cdot (\mu + \epsilon_t)}$$

Here are the results obtained:

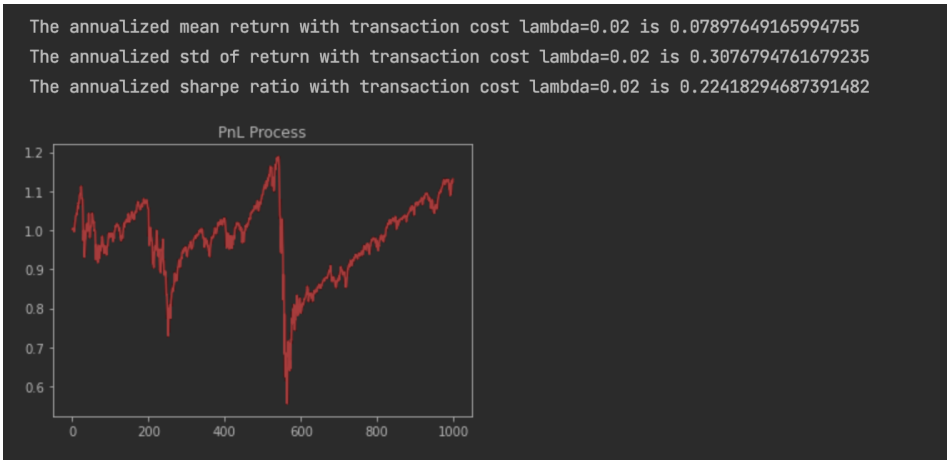

Figure 2: PnL process obtained using the strategy in 1A with transaction costs: T-cost for all the assets

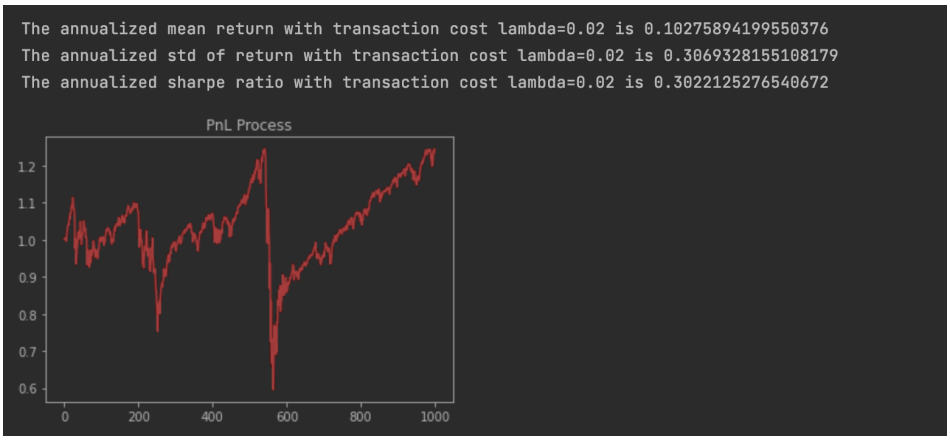

Figure 3: PnL process obtained using the strategy in 1A with transaction costs: T-cost just for risky asset

We can notice that transaction costs may have a significant impact on the Sharpe ratio. Therefore, the goal is to act in a dynamic way by rebalancing the weights over time via the Dynamic Programming Principle.

3 Question 1C **Time of execution: 30/35 minutes**

To improve the optimal strategy when transaction costs are involved, it is necessary to use the DPP. The different steps described in the lecture have been followed in order to get the PnL process as well as the Sharpe Ratio. Here are the results obtained: Thus, we can see a significant improvement as the

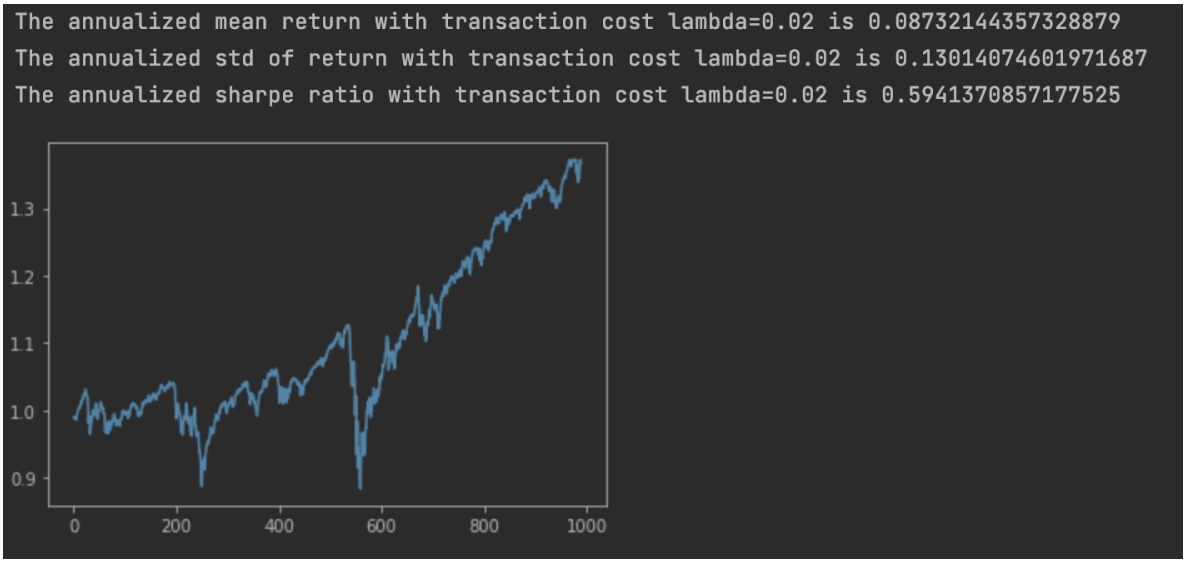

Figure 4: PnL process with transaction costs added: optimal strategy obtained by DPP

Sharpe Ratio is now equal to 0.60, which shows how relevant it is to be strategic when transaction costs are involved.
From what you showed in class, it seems that the Sharpe Ratio should have been equal to 0.7. I have spent hours/days trying to figure out where this small difference could come from but I have not found it.

Here is a recap of the different PnL processes seen in the homework:

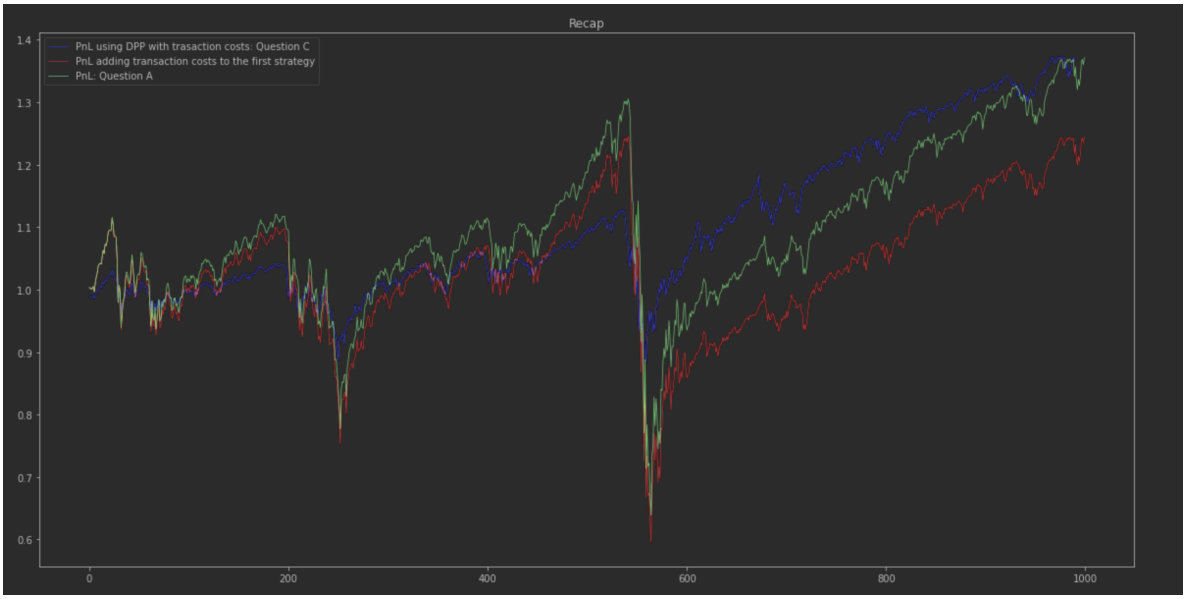

Figure 5: Recap