Math 584 (Math for Algo Trading), Spring 2022.

Homework 2

Due: Tue, Feb 22, 2022, NO LATER than 5pm.

For the ticker ^GSPC, download its closing price for each business day between Jan 1, 2014, and Jan 1, 2022. This is your sample. All questions listed below must be answered using this sample. Assume the existence of a riskless return 0.01 (annualized). All exercises must be solved using only the modules listed in the file 'Template.ipynb'.

1. In this question, you back-test dynamic trading strategies that maximize the expected power utility. (Do not forget that the riskless return is a part of the universe of available assets.) Use the power utility with $\zeta = -3$, the size $T = 100$ of each trading window (measured in days), the size $N = 1000$ of each estimation window, and the number $d = 2$ of basic assets (including the riskless asset).

   (a) (8 pts) Construct and back-test the dynamic strategy that maximizes $\mathbb{E}\left((W_T(\alpha))^\zeta/\zeta\right)$ over all strategies $\alpha = (\alpha_0, \ldots, \alpha_{T-1})$. The model for the risky return is

   $$R_t = \mu + \varepsilon_t,$$

   with i.i.d. $\{\varepsilon_t\}_{t=1}^T$, s.t.

   $$\varepsilon_t = \begin{cases} \sigma, & \text{prob. } 1/2, \\ -\sigma, & \text{prob. } 1/2, \end{cases}$$

   where μ and σ should be approximated via the sample mean and the sample standard deviation of the returns of the risky asset (over each estimation window).

   Plot the PnL process of this strategy. Print the annualized mean, standard deviation and the Sharpe ratio of the returns of the generated wealth process.

   In parts b and c, it makes a difference what initial values of the "weights before rebalancing" you use at the beginning of each trading window. While other choices are possible, herein, you need to assume that the initial "weights before rebalancing" at the beginning of a trading window are given by the weights obtained at the end of the previous window.

   (b) (4 pts) Using the strategy computed in part (a), construct its PnL process in the presence of proportional transaction costs of size $\lambda = 0.02$. Plot the resulting PnL process. Print the annualized mean, standard deviation and the Sharpe ratio of the returns of the generated wealth process.

   (c) (16 pts) Repeat part (a) with proportional transaction costs of size $\lambda = 0.02$. Note that, unlike part (b), here you need to find the optimal strategy in the presence of transaction costs, as opposed to re-using the strategy computed in part (a).

   To compute the value function and the feedback optimal strategy via DPP, use an equidistant grid on $[-1, 2.5]$, consisting of 100 points, for the possible values of the "weights before rebalancing". To compute the value function and the optimal strategy outside of the grid points, use linear interpolation between the grid points and constant extrapolation outside of $[-1, 2.5]$.

   On the very first trading day in your sample, i.e. on day N, your capital is fully invested in the riskless asset before rebalancing (i.e., right before you decide on the optimal portfolio weights to be used at that time).

Plot the PnL process of this strategy. Print the annualized mean, standard deviation and the Sharpe ratio of the returns of the generated wealth process.