# Homework 3

Introduction to Machine Learning
Fall 2018
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Homework is due 10/19/2018.

## Problem 1 (14 points): Kernels

Recall the ways of constructing new kernels from previously defined kernels that we discussed on the lecture:

- $k(u, v) = \alpha k_1(u, v) + \beta k_2(u, v)$ for $\alpha, \beta \geq 0$
- $k(u, v) = k_1(u, v)k_2(u, v)$
- $k(u, v) = k_1(f(u), f(v))$, where $f : \mathcal{X} \to \mathcal{X}$
- $k(u, v) = g(u)g(v)$, where $g : \mathcal{X} \to \mathbb{R}$
- $k(u, v) = f(k_1(u, v))$, where $f$ is a polynomial with posivie coefficients
- $k(u, v) = \exp(k_1(u, v))$
- $k(u, v) = \exp\left(\frac{-\|u-v\|^2}{\sigma^2}\right)$,

where $k_1$ and $k_2$ are valid (symmetric, positive definite) kernels on $\mathcal{X}$. Prove that in all these 7 cases we obtain valid kernels.

## Problem 2 (26 points): SVMs

Build an SVM from Steve Gunn's code availabe at `http://www.isis.ecs.soton.ac.uk/resources/svminfo/`to classify the data in "shoesducks.mat" (images of shoes and ducks). In "svc.m" replace

[alpha lambda how] = qp();

with

[alpha lambda how] = quadprog(H,c,[],[],A,b,vlb,vub,x0);

You are required to submit the code. Describe well submitted code, shorter code is preferred. Clearly denote the various components and the function calls or scripts that execute your Matlab functions. To save the current figure in Matlab as a postscript file you can type:

print -depsc filename.eps

Extract the support vector machine code from Steve Gunn's GUI. This should let you learn a classifier with kernels with the decision boundary being represented as follows:

$$f(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t y_t K(x_t, x) + b\right)$$

Refer to the Gunn's code and Burges' tutorial for more details.

To test your SVM, build a simple object recognition system:

- download the images from "shoesducks.mat". You obtain
  - matrix $X$ of size $144 \times 768$: 144 images, 72 of ducks and 72 of shoes (e.g. to see the $4^{\text{th}}$ image type plot(X(4,:)) in Matlab), where each image is a contour profile of the top part of the object
  - vector $Y$ of size $144 \times 1$: label vector (label 1 denotes a duck)

Train your SVM on the half of the examples and test on the other half (or other subsets of the examples as you see fit). Show performance of the SVM for linear, polynomial and RBF kernels with different settings of the polynomial order, $\sigma$ and $C$ value. Pick-up a good setting of these parameters to obtain high recognition accuracy.

# Problem 3 (10 points): PCA

Download the "teapots.mat" data set containing 100 images of teapots of size $38 \times 50$. To view an image, say the second one in the data set type:
imagesc(reshape(teapotImages(2,:),38,50));
colormap gray;
Compute the data mean and top 3 eigenvectors of the data covariance matrix and show them as images. Reconstruct the data using PCA with least squares error using only the mean and a linear combination of the top 3 eigenvectors. Show 10 different images before and after reconstruction. Discuss results.