Two Wheeled Mobile Manipulator Project Report

The main topic of my project is a manipulator mounted on a two-wheeled self-balancing mobile robot.

And this report mainly consists of following contents:

1. The mechanical structure of the robot
2. Control strategy and diagram
3. How to run this project?
4. Future work (how to improve?)

1. The mechanical structure of the robot

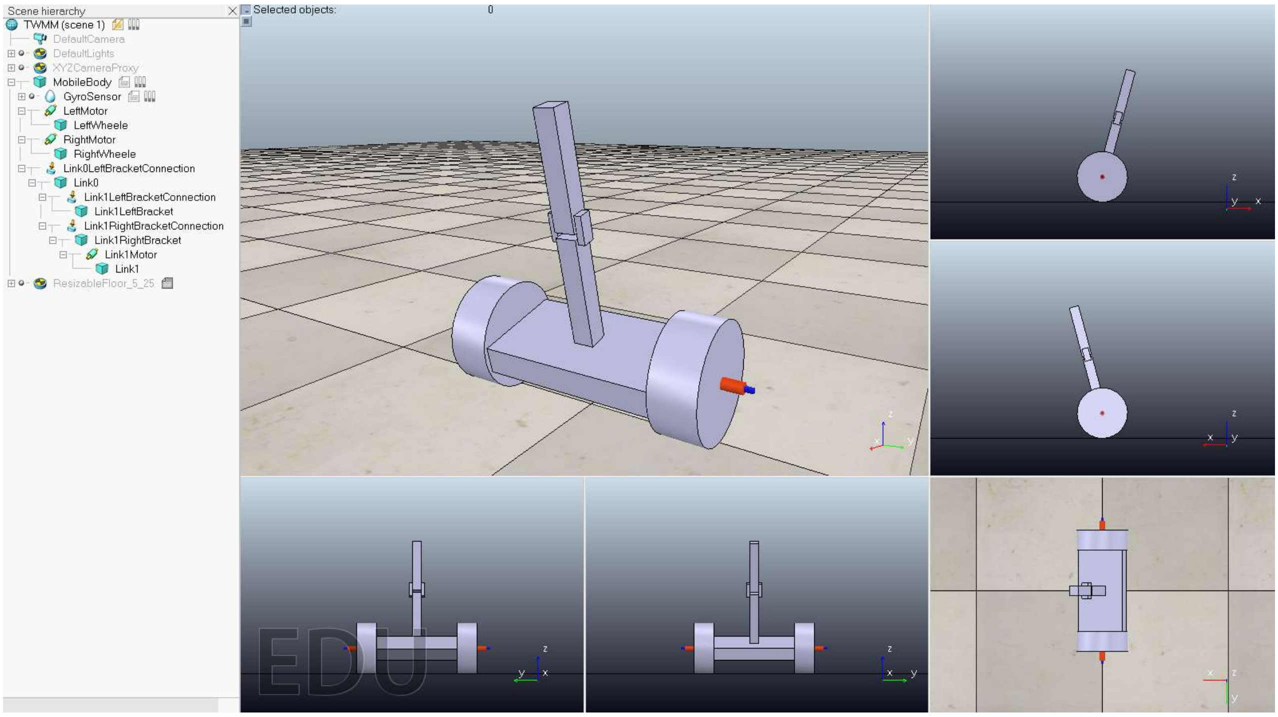


Fig 1: the appearance and the hierarchy of the robot

As shown in the figure 1, the robot is mainly consisting of a main body, 2 wheels, left and right motors, 2 link brackets, 1 link motor and 2 links. “Connections” are force sensors that were used to fix objects. And there is also a Gyro sensor at the center of the main body which can get the angular acceleration of the robot.

Detailed information is shown in the table 1. And each part of the robot is shown in figure 2.

	Size(length*width*height, m)	Initial position(m)	Mass(kg)
Wheels	diameter 0.2, width 0.08	0, ±0.2, 0.1	0.3142
Main body	0.18*0.32*0.05	0, 0, 0.1	0.72
Link 0	0.0375*0.0375*0.2	0, 0, 0.225	0.2813
Link bracket	0.025*0.125*0.05	0, ±0.025, 0.4	0.01563
Link 1 motor	/	0, 0, 0.35	/
Link1	0.0375*0.0375*0.2	0, 0, 0.435	0.2813

Table 1: detailed information of each part

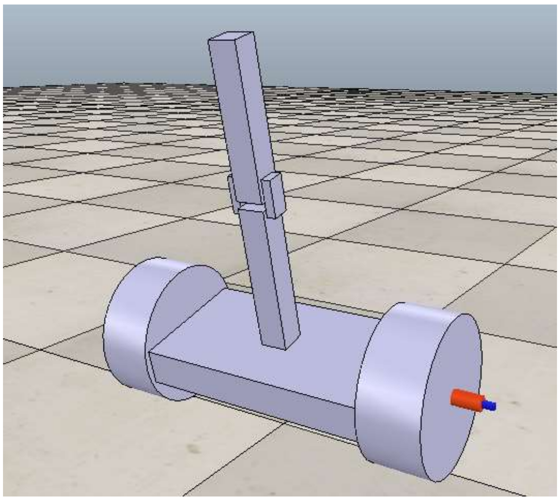


Fig 2: each part of the robot

2. Control strategy and diagram

Two-wheeled self-balancing robot can be modeled as an inverted pendulum or a double inverted pendulum. There are 3 parts of the control strategy: attitude control, speed control and direction control. Attitude control is the main part of the whole system. In a way, the speed is also controlled by the angle control loop.

In my project, I use PD control for the attitude control and PI control for the speed control. The control diagram is shown in figure 3.

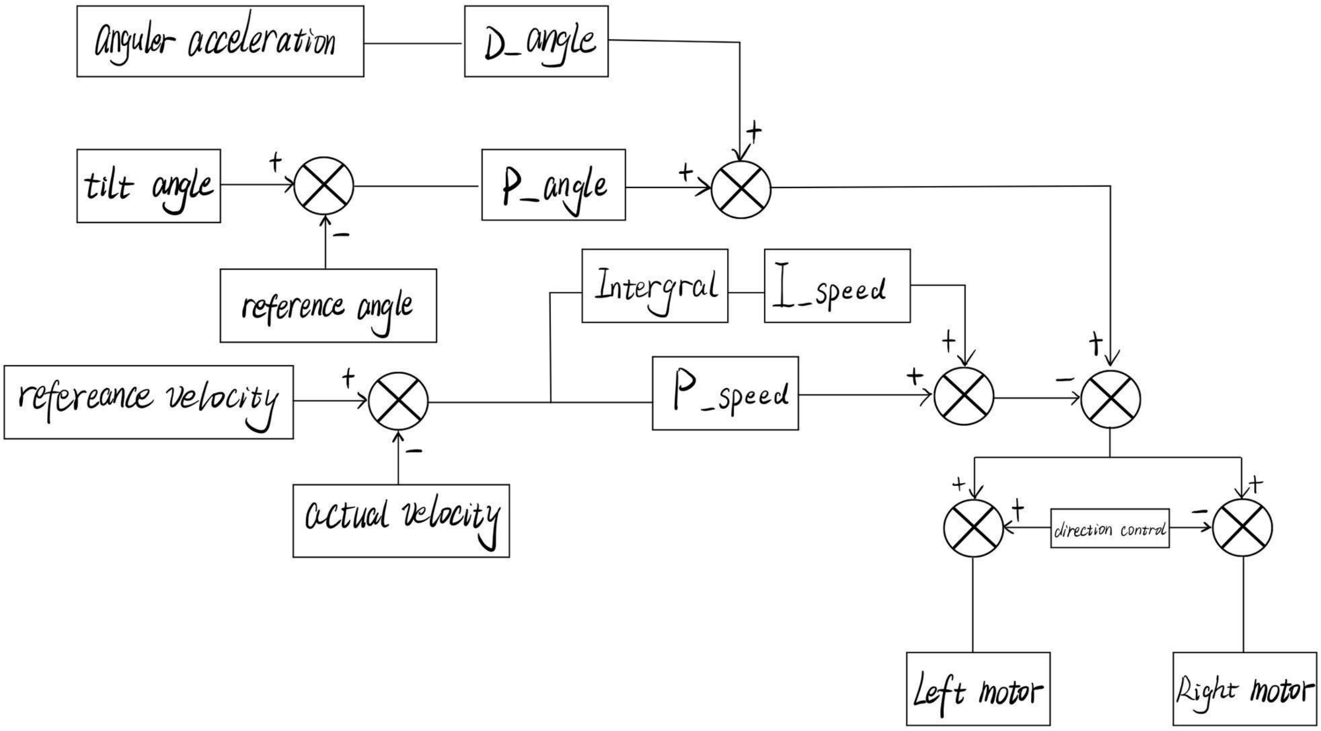


Fig 3: control diagram

How to compute the tilt angle of the robot? We can only get the Euler angle from V-rep. The rotation matrix is R = $R_x(\alpha)*R_y(\beta)*R_z(\gamma)$. Then we can get the X-axis unit vector after rotation $[X_x, Y_y, X_z]=[\cos\beta * \cos\gamma, \cos\alpha * \sin\gamma + \cos\gamma * \sin\alpha * \sin\beta, \sin\alpha * \sin\gamma - \cos\alpha * \cos\gamma * \sin\beta]$. Then tilt angle equals to $\tan^{-1}(\frac{X_z}{\sqrt{X_x^2+X_y^2}})$.

$$R = R_x(\alpha)*R_y(\beta)*R_z(\gamma)=\begin{pmatrix} \cos\beta * \cos\gamma & -\cos\beta * \sin\gamma & \sin\beta \\ \cos\alpha * \sin\gamma + \cos\gamma * \sin\alpha * \sin\beta & \cos\alpha * \cos\gamma - \sin\alpha * \sin\beta * \sin\gamma & -\cos\beta * \sin\alpha \\ \sin\alpha * \sin\gamma - \cos\alpha * \cos\gamma * \sin\beta & \cos\gamma * \sin\alpha + \cos\alpha * \sin\beta * \sin\gamma & \cos\alpha * \cos\beta \end{pmatrix}$$

Then how to compute the velocity of the robot? Similar to the tilt angle computation, we can only get the velocity relative to the world frame. The value of the velocity is then $\sqrt{v_x^2 + v_y^2 + v_z^2}$ where v_z is nearly 0. But we also need to get the orientation of the velocity relative to the robot frame to know the robot is moving forward or backward. My judgement standard here is the sign of $[X_x\ Y_y]\cdot[v_x\ v_y]^T$. If it's positive, then the robot is moving forward, otherwise, backward.

3. How to run this project?

The code for this project is divided into two parts. Main part is the Matlab code connecting to the V-rep as remote API. And there is also some Lua code in V-rep file.

To run this project, open the folder named "TwoWheeledMobileManipulator" and open the file "TWMM.ttt" in V-rep and open the file "TWMM.m" in Matlab. Make sure there is only one V-rep progress running on the computer, otherwise Matlab might connects to other progresses. Also, very importantly, use the physics engine "Newton" and set the simulatiom time step dt equals to 5ms. Then simply click "Run" to run "TWMM.m" (or to run "Main.m" in the same folder). The simulation will then start with a UI window popped up in V-rep(as shown in figure 4). There are 3 sliders: "Robot Speed", "Link1 rotate" and "Turn Left or Right". You can adjust robot speed, angle of the 2^{th} link and the forward direction of the robot using these sliders.

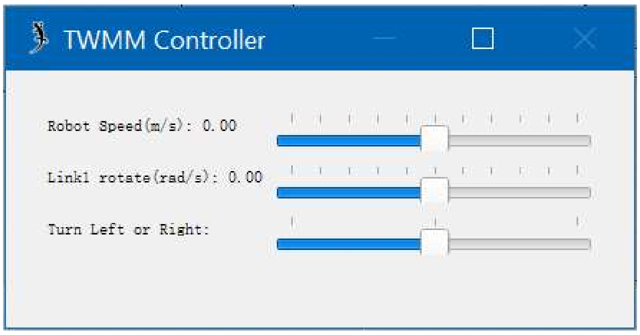


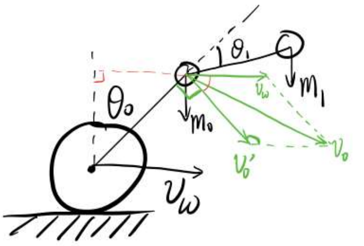
Fig 4: UI window

4. Future work

Apparently, the control is not good enough(response not fast enough, not robust enough). Not to mention most of the project is merely attitude and speed control of the robot.

Secondly, I intended to build a more complex manipulator mounted on the mobile robot using dynamic characteristics to control the whole system. For example, PID control with feedforward or PID control with feedback linearization. And the end-effector can reach certain points using forward and inverse kinematics. Since the robot can be regarded as a double inverted pendulum, I even computed the dynamic function using Euler-LaGrange function, as shown in the page 4 and 5. But the workload turns out to be too big. So, in the end, I only used a PID with gravity compensation(τ=PID+G(q)). I computed the position of center of mass relative to the initial balancing posture and get a reference angle as balanced position. Then I use PID to control the robot to stay at the balanced position.

Finally, I think a counterweight or a "counter torque" can be used on the robot. On the one hand, it can adjust the center of gravity due to manipulators' postures change or objects carrying. On the other hand, it can balance the torques produced by the other actions.

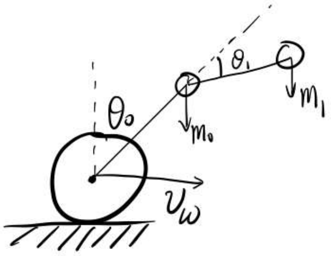


The law of Cosines

$2v_0' v_\omega \cos(\pi-\theta_0) = v_0'^2 + v_\omega^2 - v_0^2$

$\Rightarrow v_0^2 = v_0'^2 + v_\omega^2 + 2v_0' v_\omega \cos\theta_0$

Similarly, $v_1^2 = v_1'^2 + v_\omega^2 + 2v_1' v_\omega \cos$



$$\dot{x}_1 = l_0\dot{\theta}_0 C_0 + l_1(\dot{\theta}_0+\dot{\theta}_1) C_{01}$$

$$\dot{y}_1 = -l_0\dot{\theta}_0 S_0 - l_1(\dot{\theta}_0+\dot{\theta}_1) S_{01}$$

$$\vec{v}_0 = \vec{v}_0' + \vec{v}_\omega \Rightarrow v_{0x} = v_{0x}' + v_\omega = l_0\dot{\theta}_0 C_0 + v_\omega$$

$$v_{0y} = v_{0y}' = -l_0\dot{\theta}_0 S_0$$

$$\vec{v}_0^{\,2} = v_{0x}^2 + v_{0y}^2 = l_0^2\dot{\theta}_0^2 + v_\omega^2 + 2v_\omega l_0\dot{\theta}_0 C_0 = l_0^2\dot{\theta}_0^2 + \dot{x}_\omega^2 + 2\dot{x}_\omega l_0\dot{\theta}_0 C_0$$

$$\vec{v}_1 = \vec{v}_1' + \vec{v}_\omega \Rightarrow v_{1x} = v_{1x}' + v_\omega = l_0\dot{\theta}_0 C_0 + l_1(\dot{\theta}_0+\dot{\theta}_1)C_{01} + \dot{x}_\omega$$

$$v_{1y} = v_{1y}' = -l_0\dot{\theta}_0 S_0 - l_1(\dot{\theta}_0+\dot{\theta}_1)S_{01}$$

$$\vec{v}_1^{\,2} = v_{1x}^2 + v_{1y}^2 = l_0^2\dot{\theta}_0^2 + l_1^2(\dot{\theta}_0+\dot{\theta}_1)^2 + 2l_0 l_1\dot{\theta}_0(\dot{\theta}_0+\dot{\theta}_1)C_1 + \dot{x}_\omega^2 + 2\dot{x}_\omega\left[l_0\dot{\theta}_0 C_0 + l_1(\dot{\theta}_0+\dot{\theta}_1)C_{01}\right]$$

For the whole system:

$$KE = \frac{1}{2}(m_\omega+m_0+m_1)v_\omega^2 + \frac{1}{2}I_\omega\dot{\theta}_\omega^2 + \frac{1}{2}m_0 v_0^2 + \frac{1}{2}m_1 v_1^2$$

$$= \frac{1}{2}(m_\omega+m_0+m_1)\dot{x}_\omega^2 + \frac{1}{2}\frac{I_\omega}{R^2}\dot{x}_\omega^2 + \frac{1}{2}m_0 l_0^2\dot{\theta}_0^2 + \frac{1}{2}m_1\left[l_0^2\dot{\theta}_0^2 + l_1^2(\dot{\theta}_0+\dot{\theta}_1)^2 + 2l_0 l_1\dot{\theta}_0(\dot{\theta}_0+\dot{\theta}_1)C_1\right] + m_0\dot{x}_\omega l_0\dot{\theta}_0 C_0 + m_1\dot{x}_\omega\left[l_0\dot{\theta}_0 C_0 + l_1(\dot{\theta}_0+\dot{\theta}_1)C_{01}\right]$$

$$KP = m_0 g l_0 S_0 + m_1 g(l_0 S_0 + l_1 S_{01})$$

$$L = KE - KP$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i \quad \text{where } q = [x_\omega, \theta_0, \theta_1]^T$$

$$\frac{\partial L}{\partial \dot{x}_\omega} = \left(m_\omega + m_0 + m_1 + \frac{I_\omega}{R^2}\right)\dot{x}_\omega + m_0 l_0\dot{\theta}_0 C_0 + m_1\left[l_0\dot{\theta}_0 C_0 + l_1(\dot{\theta}_0+\dot{\theta}_1)C_{01}\right]$$

$$\frac{\partial L}{\partial \dot\theta_0} = m_0 l_0^2 \dot\theta_0 + m_1 l_0^2 \dot\theta_0 + m_1 l_1^2 (\dot\theta_0 + \dot\theta_1) + m_1 l_0 l_1 (2\dot\theta_0 + \dot\theta_1) C_1 + m_0 \dot x_w l_0 C_0 + m_1 \dot x_w [l_0 C_0 + l_1 C_{01}]$$

$$\frac{\partial L}{\partial \dot\theta_1} = m_1 l_1^2 (\dot\theta_0 + \dot\theta_1) + m_1 l_0 l_1 \dot\theta_0 C_1 + m_1 \dot x_w l_1 C_{01}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot x_w} = (m_w + m_0 + m_1)\ddot x_w + (m_0+m_1) l_0 \ddot\theta_0 C_0 - (m_0+m_1) l_0 \dot\theta_0^2 S_0 + m_1 l_1 (\ddot\theta_0 + \ddot\theta_1) C_{01} - m_1 l_1 (\dot\theta_0 + \dot\theta_1)^2 S_{01}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_0} = m_0 l_0^2 \ddot\theta_0 + m_1 l_0^2 \ddot\theta_0 + m_1 l_1^2 (\ddot\theta_0 + \ddot\theta_1) + m_1 l_0 l_1 (2\ddot\theta_0 + \ddot\theta_1) C_1 - m_1 l_0 l_1 (2\dot\theta_0 + \dot\theta_1) S_1 \dot\theta_1$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = m_1 l_1^2 (\ddot\theta_0 + \ddot\theta_1) + m_1 l_0 l_1 \ddot\theta_0 C_1 - m_1 l_0 l_1 \dot\theta_0 S_1 \dot\theta_1$$

$$\frac{\partial L}{\partial x_w} = 0$$

$$\frac{\partial L}{\partial \theta_0} = 0 - m_0 g l_0 C_0 - m_1 g (l_0 C_0 + l_1 C_{01})$$

$$\frac{\partial L}{\partial \theta_1} = -m_1 l_0 l_1 \dot\theta_0 (\dot\theta_0 + \dot\theta_1) S_1 - m_1 g l_1 C_{01}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot x_w} - \frac{\partial L}{\partial x_w} = \tau_w \Rightarrow (m_w + m_0 + m_1)\ddot x_w = \tau_w$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_0} - \frac{\partial L}{\partial \theta_0} = \tau_0 \Rightarrow m_0 l_0^2 \ddot\theta_0 + m_1 l_0^2 \ddot\theta_0 + m_1 l_1^2 (\ddot\theta_0 + \ddot\theta_1) + m_1 l_0 l_1 (2\ddot\theta_0 + \ddot\theta_1) C_1 - m_1 l_0 l_1 (2\dot\theta_0 + \dot\theta_1) S_1 \dot\theta_1 + m_0 g l_0 C_0 + m_1 g (l_0 C_0 + l_1 C_{01}) = \tau_0$$

$$[(m_0+m_1) l_0^2 + m_1 l_1^2 + 2 m_1 l_0 l_1 C_1]\ddot\theta_0 + (m_1 l_1^2 + m_1 l_0 l_1 C_1)\ddot\theta_1 - m_1 l_0 l_1 (2\dot\theta_0 + \dot\theta_1) S_1 \dot\theta_1 + [m_0 l_0 C_0 + m_1 (l_0 C_0 + l_1 C_{01})] g = \tau_0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} - \frac{\partial L}{\partial \theta_1} = \tau_1 \Rightarrow m_1 l_1^2 (\ddot\theta_0 + \ddot\theta_1) + m_1 l_0 l_1 \ddot\theta_0 C_1 - m_1 l_0 l_1 \dot\theta_0 S_1 \dot\theta_1 + m_1 l_0 l_1 \dot\theta_0 (\dot\theta_0 + \dot\theta_1) S_1 + m_1 g l_1 C_{01} = \tau_1$$

$$(m_1 l_1^2 + m_1 l_0 l_1 C_1)\ddot\theta_0 + m_1 l_1^2 \ddot\theta_1 + m_1 l_0 l_1 \dot\theta_0^2 S_1 + m_1 l_1 C_{01} g = \tau_1$$

In form of $M(q)\ddot q + h(q,\dot q) + g(q) = \tau$

$M_{11} = m_w + m_0 + m_1$

$M_{12} = M_{13} = 0$

$M_{21} = 0$

$M_{22} = (m_0+m_1) l_0^2 + m_1 l_1^2 + 2 m_1 l_0 l_1 C_1$

$M_{23} = m_1 l_1^2 + m_1 l_0 l_1 C_1$

$M_{31} = 0$

$M_{32} = m_1 l_1^2 + m_1 l_0 l_1 C_1$

$M_{33} = m_1 l_1^2$

$h_1 = 0$

$h_2 = -m_1 l_0 l_1 (2\dot\theta_0 + \dot\theta_1) S_1 \dot\theta_1$

$h_3 = m_1 l_0 l_1 \dot\theta_0^2 S_1$

$g_1 = 0$

$g_2 = [m_0 l_0 C_0 + m_1 (l_0 C_0 + l_1 C_{01})] g$

$g_3 = m_1 l_1 C_{01} g$