CENG424

Logic for Computer Science

Assignment 2

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Question 1

1.

1.	$p \Rightarrow q$	Premise
2.	$q \Rightarrow r$	Premise
3.	$p \Rightarrow \neg r$	Premise (by Deduction Theorem)
4.	$(q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$	Implication Introduction (II)
5.	$p \Rightarrow (q \Rightarrow r)$	Modus Ponens (4, 2)
6.	$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	Implication Distribution (ID)
7.	$p \Rightarrow r$	Modus Ponens (6, 1)
8.	$(p \Rightarrow \neg r) \Rightarrow ((p \Rightarrow r) \Rightarrow \neg p)$	Contradiction Rule (CR)
9.	$(p \Rightarrow r) \Rightarrow \neg p$	Modus Ponens (8, 3)
10.	$\neg p$	Modus Ponens (9, 7)

It is shown that $(p \Rightarrow \neg r) \Rightarrow \neg p$.

2.

1.	$\neg s \vee \neg r \Rightarrow \neg t$	Premise
2.	$s \Rightarrow p \vee \neg r$	Premise
3.	$p \wedge s \Rightarrow q$	Premise
4.	t	Premise (by Deduction Theorem)
5.	$\neg(\neg t) \Rightarrow \neg(\neg s \vee \neg r)$	Contraposition
6.	$t \Rightarrow s \wedge r$	Double Negation Rule
7.	$s \wedge r$	Modus Ponens (6,4)
8.	s	Conjuction elimination (7)
9.	r	Conjuction elimination (7)
10.	$p \vee \neg r$	Modus Ponens (2,7)
11.	p	Disjunction elimination (10)
12.	$p \wedge s$	Conjuction introduction (8,12)
13.	q	Modus Ponens (3,12)

It is shown that $(\neg s \vee \neg r \Rightarrow \neg t) \wedge (s \Rightarrow p \vee \neg r) \wedge (s \Rightarrow p \vee \neg r) \Rightarrow (t \Rightarrow q)$.

Question 2

$$(r \Rightarrow s) \wedge ((t \wedge q) \Rightarrow \neg s) \Rightarrow ((t \wedge q) \Rightarrow \neg r)$$

First, we express the given logical formula in conjunctive normal form (CNF) and negate the goal.

Premises:

$r \Rightarrow s \equiv \neg r \vee s$

$(t \wedge q) \Rightarrow \neg s \equiv \neg t \vee \neg q \vee \neg s$

$(t \wedge q) \Rightarrow \neg r \equiv \neg t \vee \neg q \vee \neg r$

Thus, the premises are:

$$\begin{array}{rl} 1. & \neg r \vee s \\ 2. & \neg t \vee \neg q \vee \neg s \\ 3. & \neg t \vee \neg q \vee \neg r \end{array}$$

We negate the goal $(t \wedge q) \Rightarrow \neg r$:

$$\neg((t \wedge q) \Rightarrow \neg r) \equiv (t \wedge q) \wedge r$$

So, the negated goal gives us the following clauses:

$$\begin{array}{rl} 4. & t \\ 5. & q \\ 6. & r \end{array}$$

The premises and the negated goal:

$$\begin{array}{rl} 1. & \neg r \vee s \\ 2. & \neg t \vee \neg q \vee \neg s \\ 3. & \neg t \vee \neg q \vee \neg r \\ 4. & t \\ 5. & q \\ 6. & r \end{array}$$

We resolve the clauses step by step:

Resolving $\neg r \vee s$ (1) and r (6) gives us:

$$7. \quad s$$

Resolving $\neg t \vee \neg q \vee \neg s$ (2) and s (7) gives us:

$$8. \quad \neg t \vee \neg q$$

Resolving $\neg t \vee \neg q \vee \neg r$ (3) and $\neg t \vee \neg q$ (8) gives us:

$$9. \quad \neg r$$

Resolving r (6) and $\neg r$ (9) gives us:

$$10. \quad \square \quad \text{(Contradiction)}$$

Since we reached a contradiction (an empty clause, $\square$), the negation of the goal leads to a contradiction. Therefore, the original implication is valid.

Question 3

1.

If Paul has holidays and it is snowing, he will go skiing.	$(\textbf{HOL} \wedge \textbf{SNO}) \Rightarrow \textbf{GSK}$
He will go to France or Florida.	$\textbf{GFR} \vee \textbf{GFL}$
There is no skiing in Florida.	$\neg\textbf{SFL}$
There is skiing in France.	$\textbf{SFR}$
Paul has holidays and he will go to Florida.	$\textbf{HOL} \wedge \textbf{GFL}$

2.

Conjunctive Normal Form (CNF) of statements above:

$$(\mathbf{HOL} \wedge \mathbf{SNO}) \Rightarrow \mathbf{GSK} = \neg\ (\mathbf{HOL} \wedge \mathbf{SNO}) \Rightarrow \mathbf{GSK} = \neg\ \mathbf{HOL} \vee \neg\ \mathbf{SNO} \vee \mathbf{GSK}$$

$$(\neg\ \mathbf{HOL} \vee \neg\ \mathbf{SNO} \vee \mathbf{GSK}) \wedge (\mathbf{GFR} \vee \mathbf{GFL}) \wedge (\neg\mathbf{SFL}) \wedge (\mathbf{SFR}) \wedge (\mathbf{HOL}) \wedge (\mathbf{GFL})$$

Statement we need to prove: It is not snowing. ($\neg$ **SNO**)

We assume SNO (it is snowing) and will derive a contradiction.

The resolution process:

Resolving **SNO** and ($\neg$ **HOL** $\vee$ $\neg$ **SNO** $\vee$ **GSK**), end up with: ($\neg$ **HOL** $\vee$ **GSK**)

Resolve ($\neg$ **HOL** $\vee$ **GSK**) with **HOL**, end up with: **GSK**

Resolve **GFL** with **GSK**, end up with: **GFL** $\wedge$ **GSK**

Resolve **GFL** $\wedge$ **GSK** with $\neg$**SFL**, end up with a contradiction and an empty clause because Paul will go to Florida and he will go skiing, but there is no skiing in Florida.

Since we have reached a contradiction, we conclude that the assumption SNO is false, and therefore $\neg$SNO.