Zeynep Sıla EGÜL
2380335

CENG424

Assigment 1

Question 1

1. $\neg(pVr) \wedge (\neg p \Rightarrow (qVr)) \wedge \neg q$

Semantic tableaux:

$\neg(pVr) \wedge (\neg p \Rightarrow (qVr)) \wedge \neg q = 1$ (a)

| a

$\neg(pVr) = 1$ (b)

$\neg p \Rightarrow (qVr) = 1$ (c)

$\neg q = 1$ (d)

| b

$pVr = 0$ (e)

| e

$p = 0$
$r = 0$

c / c

$\neg p = 0$ (f) | $qVr = 1$ (g)

| f | d

$p = 1$ | $q = 0$

closed | g / g

$q = 1$ | $r = 1$

closed | closed

All branches are closed, thus formula is not satisfiable.

2. $\neg((p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q)))$

Semantic tableau:

$\neg((p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q))) = 1$ (a)

| a

$(p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q)) = 0$ (b)

| b

$p \Rightarrow q = 1$ (c)

$\neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q)) = 0$ (d)

| d

$(p \Rightarrow t) \wedge (p \wedge t \Rightarrow q) = 1$ (e)

| e

$p \Rightarrow t = 1$ (f)

$p \wedge t \Rightarrow q = 1$ (g)

g / \ g

$p \wedge t = 1$ (h) | $q = 1$

h | c / \ c

$p = 1$ | $p = 0$ | $q = 1$

$t = 1$ | f / \ f | f / \ f

c / \ c | $p = 0$ | $t = 1$ | $p = 0$ | $t = 1$

$p = 0$ | $q = 1$

closed | f / \ f

$p = 0$ | $t = 1$

closed

There are unclosed branches, thus formula is satisfiable.

Question 2:

1. $\neg(p \vee r) \wedge (\neg p \Rightarrow (q \vee r)) \wedge \neg q$

CNF form: $\neg p \wedge \neg r \wedge (\neg\neg p \vee q \vee r) \wedge \neg q$

$\equiv \neg p \wedge \neg q \wedge \neg r \wedge (p \vee q \vee r)$

DLL method:

$\neg p$
$\neg q$
$\neg r$
$p \vee q \vee r$

$\downarrow \neg p$

$\neg q$
$\neg r$
$q \vee r$

$\downarrow \neg q$

$\neg r$
r

$\downarrow$

 Contradiction

The formula is not satisfiable.

2. $\neg\Big(\big((p\wedge q\wedge\neg w)\Rightarrow r\big)\Rightarrow\neg\big(\big((p\wedge r)\Rightarrow(q\vee w)\big)\wedge p\wedge(q\Rightarrow\neg(r\wedge w))\big)\Big)\wedge(q\vee\neg r\vee\neg w)$

CNF form: $\neg\big(\neg(\neg p\vee\neg q\vee w\vee r)\vee\neg((\neg p\vee\neg r\vee q\vee w)\wedge p\wedge(\neg q\vee\neg r\vee\neg w))\big)\wedge(q\vee\neg r\vee\neg w)$

$\equiv(\neg p\vee\neg q\vee w\vee r)\wedge(\neg p\vee\neg r\vee q\vee w)\wedge p\wedge(\neg q\vee\neg r\vee\neg w)\wedge(q\vee\neg r\vee\neg w)$

DLL method:

$\neg p\vee\neg q\vee w\vee r$

$\neg p\vee\neg r\vee q\vee w$

p

$\neg q\vee\neg r\vee\neg w$

$q\vee\neg r\vee\neg w$

$\downarrow p$

$\neg q\vee w\vee r$

$\neg r\vee q\vee w$

$\neg q\vee\neg r\vee\neg w$

$q\vee\neg r\vee\neg w$

q → added

$\downarrow q$

$w\vee r$

$\neg r\vee\neg w$

w → added

$\downarrow w$

$\neg r$

$\downarrow$

$\square$

The formula is satisfiable with model $I=\{p\leftarrow 1, q\leftarrow 1, r\leftarrow 0, w\leftarrow 1\}$