

Group Difference in Differences can Identify Effect Heterogeneity in Non-Canonical Settings

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1 Introduction

Consider a very general setting in which data on an outcome of interest is collected in two ‘groups’ at two time periods, with certain group-periods deemed ‘treated’ and others ‘untreated’. A special case is the canonical Difference-in-Differences (DiD) setting in which one group is treated only in the second period while the other is treated in neither period. Then it is well known that under a parallel trends assumption across the two groups the classic DiD formula (subtracting the average change in the outcome across periods in the treated group by the average change in the outcome across periods in the untreated group) identifies the average treatment effect on the treated in the second period. But other relations between group, period, and treatment are possible (see Figure 1). For example, the groups might be demographic (or other baseline covariate) categories with all units in both groups treated in the second period and none treated in the first, i.e. a pre-post design. Or one group might be treated in both periods while the other is treated in neither. In these non-canonical settings (lacking a control group or a pre-period), some researchers still compute DiD estimators, while others avoid causal inference altogether. In this paper, we will elucidate the group-period-treatment scenarios and corresponding parallel trends assumptions under which a DiD formula identifies meaningful causal estimands and what those causal estimands are.

To make the issue more concrete, we introduce a running example, which we slightly simplify for clarity. Kim et al. [2024] look at the relationship between universal health coverage (UHC) and childhood immunization coverage at the country level in the years preceding and following the onset of the Covid-19 pandemic. The groups in this study are high and low UHC rating, and the periods are pre- and post-Covid. Kim et al. [2024] performed a DiD analysis (subtracting the average change in vaccination rates from pre- to post-Covid in high UHC countries by the same average change in low UHC countries) and summarized their results as: “Countries with high UHC scores prevented a 1.14% (95% CI: 0.39%, 1.90%) reduction in immunization coverage” in the post Covid period. One interpretation of the data is to take high UHC as the treatment, in which case there is no pre-treatment period as in panel (c) of Figure 1. Another interpretation is to define Covid as the treatment, in which case we would be in the pre-post design setting depicted in panel (b) of Figure 1.

In the pre-post setting, we will show that under the assumption that the expected counterfactual untreated outcomes in each group followed parallel trends pre- and post- treatment (‘group parallel trends’), the simple DiD expression across groups (‘group DiD’, or GDiD) identifies the difference between the treatment’s effects in the two groups (earlier noted in [Shahn, 2023]). In our running example, the causal estimand would be the difference between the impact of Covid-19 on national vaccination rates in high and low UHC countries. This identification result for effect modification by a baseline covariate holds even if we cannot identify the conditional treatment effect at any level of the baseline covariate. Effect modification estimands might be of particular interest when there are concerns surrounding equity and investigators wish to ensure that sensitive subgroups are not benefiting less from an intervention.

Of course, the group parallel trends assumption in the pre-post setting is strong and untestable. However, group parallel trends is not stronger (or weaker) than the standard DiD assumption of parallel trends across treatment groups when an untreated control group is available. Abadie [2005] describes how to characterize effect heterogeneity given a baseline covariate if a control group is available and the standard parallel trends assumption across treatment groups (conditional on the covariate) holds. Many analysts would happily conduct a traditional DiD study along the lines of [Abadie, 2005] but throw up their hands in the absence of a control group due to the typically less plausible assumptions required by pre-post designs (e.g. Chapter 6 of [Cook and Campbell, 2007]). Given a successful pre-trends test across groups in the pre-post setting, such analysts might wish to explore effect modification via GDiD instead of giving up altogether. This is particularly the case if effect modification is of primary interest, as in the equity scenario mentioned above.

In the setting where one group is always treated and the other group is never treated (panel (c) of Figure 1), we will show that under the assumption that the counterfactual untreated trends are parallel in the always and never treated groups, the simple GDiD formula identifies the difference between the effect of treatment in the treated in the second period and the effect of treatment in the treated in the first period. This change in effect in the treated across periods is identified despite the effect not being identified within either period. In our running example, the causal estimand would be the difference between the effect of high UHC on national vaccination rates in high UHC countries in the post-covid period and the effect of high UHC on national vaccination rates in high UHC countries in the pre-covid period. Unfortunately, pre-trends are not available to assess the plausibility of the parallel trends assumption in this context, as we never get to observe untreated outcomes in the treated group.

The organization of the paper is as follows. In Section 2, we formalize the problem and describe the data structures of interest. In Section 3, we briefly review the canonical setting. In Section 4, we discuss the interpretation of the GDiD expression under a group parallel trends assumption in the pre-post setting. In Section 5, we discuss the interpretation of the GDiD expression under a group parallel trends assumption in the no pre-period setting. In Section 6, we consider extensions such as group parallel trends conditional on other covariates and bounds on conditional ATTs. In Section 7, we discuss some connections to prior work, including DiD for a continuous or multi-valued treatment Callaway et al. [2024] and triple differences [Gruber, 1994, Olden and Møen, 2022]. In Section 8, we conclude.

2 Data Structures, Notation, and Assumptions

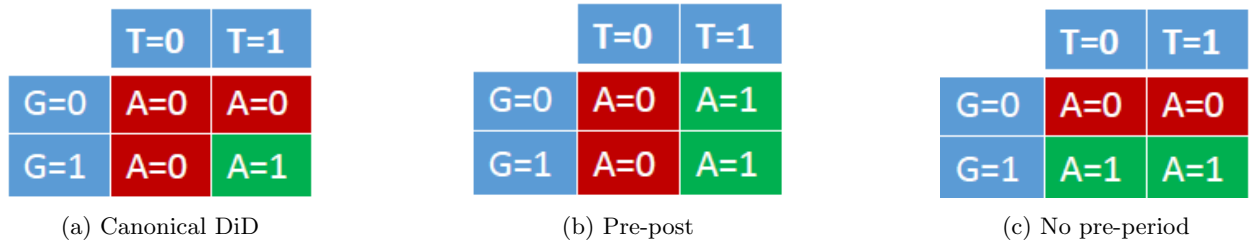


Figure 1: Three data structures in which we causally interpret the GDiD expression under Consistency (1) the group parallel trends assumption (3). Panel (a) depicts the canonical DiD setting in which all units are untreated in the first period, and group G indexes the second period treatment, i.e. $A_0 = 0$, $A_1 = G$. Panel (b) depicts a pre-post setting in which no units are treated in the first period, and all are treated in the second period, i.e. $A_0 = 0$ and $A_1 = 1$. Group G represents a category (e.g. a demographic group) defined by baseline covariates. Panel (c) depicts a setting without a pre-period in which group G indexes the treatment assignment received at both time periods, i.e. $A_0 = A_1 = G$.

To formalize the problem, assume the data comprise independent identically distributed realizations of the random variable $O = (G, A_0, Y_0, A_1, Y_1)$ in that temporal order, where G denotes a baseline group covariate, A_t denotes treatment at time t , and Y_t denotes outcome at time t for $t \in \{0, 1\}$. In the canonical

DiD setup depicted in panel (a) of Figure 1, $A_0 = 0$ and $A_1 = G$. In the pre-post setting depicted in panel (b) of Figure 1, $A_0 = 0$ and $A_1 = 1$. In the ‘group as treatment’ setting depicted in panel (c) of Figure 1, $A_t = G$ for all t . We let $Y_t(a)$ denote the potential outcome [Rubin, 1974] at time t that would be observed had, possibly contrary to fact, treatment at time t been set to value a . We make the standard consistency assumption that

$$\textbf{Consistency: } Y_t(a) = Y_t \text{ when } A_t = a. \quad (1)$$

We will proceed to consider the causal interpretation of the GDiD formula

$$E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] \quad (2)$$

under the group parallel trends assumption

$$E[Y_1(0) - Y_0(0)|G = 1] = E[Y_1(0) - Y_0(0)|G = 0] \quad (3)$$

in each data structure from Figure 1.

3 The Canonical DiD setting: Panel (a)

In the canonical DiD setup, G indexes the treatment group in the second period. Thus, the group parallel trends assumption (3) reduces to the familiar parallel trends assumption that untreated expected outcome trends are equal across treatment groups, and the GDiD formula (2) reduces to the familiar DiD formula. It is well known that in this canonical setup, under parallel trends assumption (3) the GDiD formula (2) identifies the average treatment effect on the treated (ATT) in the second time period, i.e.

$$\begin{aligned} & E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] \\ &= (E[Y_1 - Y_0|G = 1] - E[Y_1(0) - Y_0(0)|G = 1]) - (E[Y_1 - Y_0|G = 0] - E[Y_1(0) - Y_0(0)|G = 0]) \\ &= (E[Y_1(1) - Y_0|G = 1] - E[Y_1(0) - Y_0|G = 1]) - (E[Y_1 - Y_0|G = 0] - E[Y_1 - Y_0|G = 0]) \\ &= E[Y_1(1) - Y_1(0)|G = 1] \\ &= E[Y_1(1) - Y_1(0)|A_1 = 1]. \end{aligned}$$

Parallel trends (3) implies that the second line is obtained from the first by adding and subtracting the same quantity. The third line follows from applications of Consistency (1) given that $A_0 = 0$ for all units and $A_1 = G$ in this data structure. The fourth line follows from simple cancellations. The final equality follows because $A_1 = G$. The plausibility of the parallel trends assumption can be partially assessed by looking at pre-trends in the outcome prior to time 0 in the treated and untreated groups. We do not discuss the canonical case further here but hope it serves as a familiar basis for comparison.

4 The Pre-Post Setting: Panel (b)

In the pre-post setting of panel (b) in Figure 1, under Consistency (1) and the group parallel trends assumption (3), the GDiD expression (2) identifies the difference between the second time period ATT across groups, i.e.

$$E[Y_1(1) - Y_1(0)|G = 1] - E[Y_1(1) - Y_1(0)|G = 0]. \quad (4)$$

The derivation of this result is similar to the canonical case and proceeds as follows:

$$\begin{aligned} & E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] \\ &= (E[Y_1 - Y_0|G = 1] - E[Y_1(0) - Y_0(0)|G = 1]) - (E[Y_1 - Y_0|G = 0] - E[Y_1(0) - Y_0(0)|G = 0]) \\ &= (E[Y_1(1) - Y_0(0)|G = 1] - E[Y_1(0) - Y_0(0)|G = 1]) - (E[Y_1(1) - Y_0(0)|G = 0] - E[Y_1(0) - Y_0(0)|G = 0]) \\ &= E[Y_1(1) - Y_1(0)|G = 1] - E[Y_1(1) - Y_1(0)|G = 0] \end{aligned} \quad (5)$$

As in the canonical setting, (3) implies that the second line of the derivation is obtained from the first line by adding and subtracting the same quantity. The third line again follows from applications of Consistency (1), this time given that $A_0 = 0$ and $A_1 = 1$ for all units. The fourth line again follows by simple cancellations. Fewer terms cancel than in the canonical case because we do not observe $Y_1(0)$.

The plausibility of the group parallel trends assumption (3) in the pre-post setting can be partially assessed by comparing pre-baseline outcome trends in the two groups. Indeed, Kim et al. [2024] found that the assumption was plausible in their setting, i.e. they found that average national vaccination rates in high and low UHC countries fluctuated similarly prior to Covid-19. Under (3), and interpreting ‘treatment’ as Covid-19 and ‘group’ as UHC status, one might summarize the results of Kim et al. [2024] as: Covid-19 was estimated to reduce immunization coverage by 1.14 fewer percentage points (95% CI: 0.39%, 1.90%) in high UHC countries than low UHC countries. (This phrasing does not ascribe a causal role to UHC in modifying the effect of Covid-19. Even under our identifying assumptions, UHC may simply be associated with varying effects of Covid-19 without being responsible for that variation [VanderWeele and Robins, 2007].)

5 The No Pre-Period Setting: Panel (c)

In the no pre-period setting of panel (c) in Figure 1, under Consistency (1) the group parallel trends assumption (3), the GDiD expression (2) identifies the difference between the ATT in the second time period and the ATT in the first time period, i.e.

$$E[Y_1(1) - Y_1(0)|G = 1] - E[Y_0(1) - Y_0(0)|G = 1]. \quad (6)$$

The derivation of this result is similar to the canonical and pre-post cases and proceeds as follows:

$$\begin{aligned} & E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] \\ &= (E[Y_1 - Y_0|G = 1] - E[Y_1(0) - Y_0(0)|G = 1]) - (E[Y_1 - Y_0|G = 0] - E[Y_1(0) - Y_0(0)|G = 0]) \\ &= (E[Y_1(1) - Y_0(1)|G = 1] - E[Y_1(0) - Y_0(0)|G = 1]) - (E[Y_1(0) - Y_0(0)|G = 0] - E[Y_1(0) - Y_0(0)|G = 0]) \\ &= E[Y_1(1) - Y_1(0)|G = 1] - E[Y_0(1) - Y_0(0)|G = 1] \\ &= E[Y_1(1) - Y_1(0)|A = 1] - E[Y_0(1) - Y_0(0)|A = 1] \end{aligned} \quad (7)$$

As in the previous derivations, (3) implies that the second line of the derivation is obtained from the first line by adding and subtracting the same quantity. The third line again follows from applications of Consistency (1), this time given that $A = G$ for all units. The fourth line again follows by simple cancellation, and the final line just substitutes A for G to emphasize that this is a difference between ATTs.

Unlike the canonical and pre-post settings, the plausibility of the group parallel trends assumption (3) cannot be assessed via pre-trends in the no pre-period setting because we do not get to observe untreated outcomes in the treated group (unless the treated group was untreated prior to baseline, in which case a standard DiD analysis focusing on the period when treatment began might be more desirable). The pre-trends tests performed by Kim et al. [2024] would therefore not be relevant if one wanted to interpret their GDiD analysis as an estimate of (6) with UHC as the treatment. If one did wish to interpret the analysis in this way, their results might be summarized as: high UHC was estimated to increase immunization coverage by 1.14 additional percentage points (95% CI: 0.39%, 1.90%) in the post-Covid period compared to the pre-Covid period in high UHC countries. (This phrasing does not ascribe a causal role to Covid in modifying the effect of high UHC over time. Even under our identifying assumptions, Covid may simply be associated with time-varying effects of UHC without being responsible for that variation [VanderWeele and Robins, 2007].)

6 Extensions

6.1 Conditional group parallel trends

Sometimes the subgroup parallel trends assumption may be more plausible within levels of other covariates Z . Suppose we make the conditional subgroup parallel trends assumption

$$E[Y_1(0) - Y_0|G = 1, Z] = E[Y_1(0) - Y_0|G = 0, Z]. \quad (8)$$

Then we can adapt Abadie's [Abadie, 2005] inverse probability weighted DiD identification formula for the ATT to an inverse probability weighted GDiD identification formula for effect modification of the ATT:

$$E[Y_1(1) - Y_1(0)|G = 1] - E[Y_1(1) - Y_1(0)|G = 0] = E\left[\frac{Y_1 - Y_0}{P(G = 1)} \frac{G - P(G = 1|Z)}{1 - P(G = 1|Z)}\right]. \quad (9)$$

The proof exactly follows the proof of Lemma 3.1 of [Abadie, 2005]. A corresponding plug-in estimator is given by

$$\mathbb{P}_n\left\{\frac{Y_1 - Y_0}{\hat{P}(G = 1)} \frac{G - \hat{P}(G = 1|Z)}{1 - \hat{P}(G = 1|Z)}\right\}$$

where \mathbb{P}_n denotes sample average, $\hat{P}(G = 1)$ denotes the sample proportion of $G = 1$, and $\hat{P}(G = 1|Z)$ denotes an estimate (e.g. via logistic regression) of the conditional probability that $X = 1$ given covariate(s) Z .

6.2 Continuous or multi-valued G

Suppose G is a continuous variable. In the pre-post setting, G might be a continuous baseline covariate such as income. In the no pre-period setting, G can represent a continuous dose of treatment. For two values g and g' of G , we can make the group parallel trends assumption

$$E[Y_1(0) - Y_0(0)|G = g] = E[Y_1(0) - Y_0(0)|G = g']. \quad (10)$$

Substituting g for 1 and g' for 0 in the derivations from the previous sections shows that under (10) the GDiD expression $E[Y_1 - Y_0|G = g] - E[Y_1 - Y_0|G = g']$ identifies $E[Y_1(1) - Y_1(0)|G = g] - E[Y_1(1) - Y_1(0)|G = g']$ in the pre-post setting and $E[Y_1(1) - Y_1(0)|G = g] - E[Y_0(1) - Y_0(0)|G = g']$ in the no pre-period setting. Note that separate subgroup parallel trends assumptions are required for any two levels of G to be compared. Furthermore, a plugin GDiD estimator

$$\hat{E}[Y_1 - Y_0|G = g] - \hat{E}[Y_1 - Y_0|G = g'],$$

where $\hat{E}[\cdot|\cdot]$ denotes estimated conditional expectations, would require a regression model for $E[Y_1 - Y_0|G]$ in the continuous case where sample averages would suffice in the binary setting.

6.3 Bounds (or point identification with a zero effect subgroup)

Note that if, as in triple differences, there exists a subgroup in which treatment is known to have no effect, GDiD might identify conditional effects in other subgroups. That is, if group parallel trends (3) holds with subgroups $G = 1$ and $G = 0$ and the intervention is known not to have any effect in subgroup $G = 0$, then the second term in (4) is equal to 0 and the GDiD expression identifies the ATT in subgroup $G = 1$ (i.e. $E[Y_1(1) - Y_1(0)|G = 1]$) in the pre-post setting. In this setting, the zero effect subgroup is essentially an untreated control group and GDiD reduces to canonical DiD with a control group.

We can generalize this reasoning to get bounds on subgroup effects on the treated. If we assume that $\tau_l < E[Y_1(1) - Y_1(0)|G = 0] < \tau_u$, then we have that

$$E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] + \tau_l < E[Y_1(1) - Y_1(0)|G = 1] < E[Y_1 - Y_0|G = 1] - E[Y_1 - Y_0|G = 0] + \tau_u.$$

7 Connections to Other Work

7.1 Different versions of a pre-post treatment

A special case of pre-post GDiD arises when everybody in the cohort receives ‘treatment’ starting in the second time period, though different groups receive different versions of treatment. For example, different subgroups might receive different doses of the same treatment [Callaway et al., 2024] or different treatments altogether. Then the group parallel trends assumption (3) states that trends in the absence of any treatment would have been parallel in the groups receiving different versions of treatment indexed by G , and the GDiD expression identifies the difference between the effects of each treatment version in those who received it.

Consider a cohort of patients who all receive an antihypertensive medication, but some receive a beta blocker and some receive an ACE inhibitor. (Patients may be prescribed treatment on different dates, with data aligned to time of treatment.) In this setting, the group parallel trends assumption would state that the expected counterfactual blood pressure absent any treatment at all would have progressed similarly in patients who were prescribed a beta blocker and patients who were prescribed an ACE inhibitor. Formally, $E[Y_1(0) - Y_0|A = \text{‘beta blocker’}] = E[Y_1(0) - Y_0|A = \text{‘ACE inhibitor’}]$. If average blood pressure follows parallel trajectories in the two treatment groups leading up to the prescription date in a pre-trends test, that would lend some credibility to the assumption. Then $E[Y_1 - Y_0|A = \text{‘beta blocker’}] - E[Y_1 - Y_0|A = \text{‘ACE inhibitor’}]$ identifies $E[Y_1(\text{beta blocker}) - Y_1(0)|A = \text{‘beta blocker’}] - E[Y_1(\text{ACE inhibitor}) - Y_1(0)|A = \text{‘ACE inhibitor’}]$. That is, GDiD in this context identifies whether beta blocker users or ACE inhibitor users benefited more from the treatment they received. However, note that this difference between treatment effects on the treated does not directly tell us anything about comparative effectiveness because it is not itself a causal effect, i.e. it is not a contrast between counterfactuals in the same population. Even if beta blocker users benefited more than ACE inhibitor users compared to no treatment, it could still be the case that every individual would benefit more from ACE inhibitors than beta blockers.

7.2 Triple differences

GDiD might also be compared to triple differences, which also rests on assumptions related to trends within subgroups. Triple differences as originally formulated assumes that there is some subgroup $G = 0$ in which the intervention has no effect at all and another group $G = 1$ which might be impacted by the intervention. All units are untreated in the first period, and some but not all units from each group are treated in the second period. Furthermore, it is assumed that the difference in trends in these groups is equal in the treated and the untreated, i.e.

$$\begin{aligned} &E[Y_1(0) - Y_0|G = 0, A_1 = 1] - E[Y_1(0) - Y_0|G = 1, A_1 = 1] = \\ &E[Y_1(0) - Y_0|G = 0, A_1 = 0] - E[Y_1(0) - Y_0|G = 1, A_1 = 0]. \end{aligned} \quad (11)$$

The formula

$$(E[Y_1 - Y_0|G = 1, A = 1] - E[Y_1 - Y_0|G = 0, A = 1]) - (E[Y_1 - Y_0|G = 1, A = 0] - E[Y_1 - Y_0|G = 0, A = 0]). \quad (12)$$

then identifies the ATT in $G = 1$ in the second period. Unlike pre-post GDiD, triple differences still requires a control group and, as described above, does not estimate effect heterogeneity other than between the impacted and unimpacted groups.

Some authors have recently used triple differences type estimators to estimate the same effect heterogeneity contrast (4) as the pre-post GDiD estimator. For example, Moriya and Chakravarty [2023] consider the disparate effects of Medicaid expansion on health outcomes in Blacks and Whites. They estimate this difference in effects as

$$\begin{aligned} &(\hat{E}[Y_1 - Y_0|Race = \text{‘Black’}, A = 1] - \hat{E}[Y_1 - Y_0|Race = \text{‘White’}, A = 1]) - \\ &(\hat{E}[Y_1 - Y_0|Race = \text{‘Black’}, A = 0] - \hat{E}[Y_1 - Y_0|Race = \text{‘White’}, A = 0]). \end{aligned} \quad (13)$$

The GDiD estimator is actually the first line of (13). Their estimator makes use of a control group, however, and rests on the parallel trends assumption (11) that the expected counterfactual untreated trends in the outcome disparity across subgroups are the same in the treated and untreated groups. This assumption is not necessarily stronger or weaker than the group parallel trends assumption (3) required by the GDiD estimator in the pre-post setting. Each assumption could be partially assessed using distinct naturally corresponding pre-trends.

8 Discussion

It is a widely held belief that a basic requirement for DiD to work is a data set comprising treated and control groups and pre- and post- periods. In this paper, we have presented some very simple yet surprising facts about DiD formulas. First, in pre-post designs without a control group, the GDiD expression identifies effect modification of the ATT by group under a group parallel trends assumption that is not stronger than that required by canonical DiD and also lends itself to pre-trends assessments. While the conditional ATTs are not identified in each group, effect heterogeneity can be of interest in its own right. In our running example application of Kim et al. [2024], effect modification by high UHC is suggestive that UHC might have been protective against harms from Covid. Another prime example would be if equity is at issue. Consider the application of Moriya and Chakravarty [2023] discussed in Section 7.2. Even if a control group were unavailable, one would still be able to target their estimand of interest (the disparity in the effects of Medicaid expansion on health outcomes in Blacks and Whites) using GDiD under a comparable assumption. If a control group were available, the group parallel trends assumption might still be more plausible and, again, could be partially assessed via pre-trends.

Secondly, even in the absence of a pre-period, the G-DiD formula identifies variation in the ATT across time periods under the group parallel trends assumption. Thus, a program evaluator might be able to ascertain whether the impact of a program is diminishing over time using only data gathered from program participants and controls after the program was implemented, even if entry into the program was driven by unobserved variables that are prognostic for outcomes of interest and no instrumental variable is available. However, we emphasize that the group parallel trends assumption in this setting cannot be assessed via pre-trends.

We are sure that GDiD estimates have been computed countless times by practitioners, but we hope we have helped to clarify their interpretation and justification. We also hope that these simple yet surprising results might inspire additional reassessments of well worn topics.

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